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# ON THE STRUCTURAL CHARACTERIZATION THROUGH K-SPACE METHODS: ASSESSMENTS AND VALIDATIONS

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**Key words:** Periodic structure, wavenumber, damping loss factor, dispersion curves

**Abstract.** The physical characteristics of periodic structures are employed to analyze the vibro-acoustic response of several complex structures in the wavenumber domain. This work is focused on the analysis of elastic periodic structures, designed in order to obtain a wave attenuation in certain frequency bands, generating the so-called band-gaps (mainly related to the Bragg's effect), and on the identification of the material properties of complex structures. In this context, an inverse wavenumber correlation method is developed to obtain the dispersion characteristics and the damping information of complex periodic structures. The wavenumbers and the damping loss factor are identified exciting the structure by a unit harmonic force and using the complete vibrational field as primary input.

## 1 INTRODUCTION

In literature, the complex wavenumber recovery of a vibrating structure is an open issue. Several methods are available to correctly extract the real part of the wavenumber, but the estimation of the related damping information is still an open challenge. In the domain of punctual harmonic excited structures, McDaniel et al.<sup>1,2</sup> developed a semi-analytical approach based on damped plane wave propagation (1D guided waves), using an expression of the plane wave

of type  $e^{\pm ikx}$ , where  $i$  is the imaginary unit,  $k$  is the complex wavenumber and  $x$  is the spatial coordinate. Different methods based on Prony series have been developed over the past years; Grosh et al.<sup>3</sup> applied their method on cylindrical shells subjected to point harmonic excitation, obtaining a good estimation of the dispersion relation. Using a similar approach presented by Grosh et al.,<sup>3</sup> another method, called *Inhomogeneous Wave Correlation (IWC)* method has been developed by Berthaut et al.;<sup>4</sup> the proposed approach estimates the flexural wavenumber and the dispersion relation of a vibrating plate under punctual harmonic excitation, correlating the complete displacement field and a damped traveling plane wave, of the type  $e^{\pm ik(x\cos\theta+y\sin\theta)}$ , where  $\theta$  is the propagation angle of the inhomogeneous wave. An interesting application of the IWC method is presented in Ichchou et al.,<sup>5</sup> obtaining the complete  $\theta$ -dependent dispersion relation of a ribbed plate. The presented approaches based on plane waves propagation suffered of a disadvantage related to plane wave hypothesis: the vibrational field should be acquired in a steady state condition, trying to avoid the excitation region where the plane wave assumption is not valid. One of the main advantage of the IWC method is the estimation of the complex wavenumber, with the related damping information; Cherif et al.<sup>6</sup> applied the IWC method to an aluminum panel and to a two different laminated composite panels with honeycomb core, correlating the measured vibrational field and the damped inhomogeneous wave; the flexural wavenumber and the damping loss factor are both successful estimated. Van Damme et al.<sup>7,8</sup> obtained the dispersion relation and the complex wavenumber for complex 1D structures, showing a good description of the Bragg's band gaps, adopting a 1D formulation of the IWC method. An enriched formulation of the IWC method is shown in Van Belle et al.,<sup>9</sup> where they obtained the dispersion relation of a metamaterial plate with periodic resonators; this formulation takes in account the location of the force to have a better estimation of the wave attenuation. An IWC formulation for curved and axial-symmetric structures is shown in Tufano et al.,<sup>10</sup> obtaining a good description of the k-space domain.

To describe the vibrational field of a structure subjected to punctual harmonic excitation, the Green's functions and the related image source method are widely used in literature. An application of the method of images is shown in Gunda et al.<sup>11</sup>; the authors used this approach to describe the harmonic response of beams and rectangular plates. Based on the method of the images, Cuenca et al.<sup>12</sup> described the vibrational field of a finite plate with simply supported boundary conditions by the reconstruction of the Green's function of a point excited plate. The same method has been applied by Cuenca et al.<sup>13</sup> to estimate the equivalent material properties of a plate covered by a viscoelastic layer. Recently, Roozen et al.<sup>14</sup> used the Green's function of a point excited plate, by using a set of Hankel's functions, into a complex wavenumber fit procedure, making a comparison between the acquired vibrational field and the Green's function; the procedure here described allows to retrieve the complex wavenumber and the equivalent material properties.

All the methods based on plane wave assumption are affected by some limitations due to the nature of the plane wave itself. The vibrational field should be acquired in a steady-state condition, sufficiently far from the excitation location (this is a singularity point where the plane wave hypothesis is not valid) and trying to distinguish between the direct, reflected and evanescent fields. In this paper, a method is proposed to combine advantages of the IWC method and of the image source method, which uses a set of Hankel's functions to obtain the vibrational

field of a point excited structure. The main advantage of this approach is the possibility to be close the excitation region, avoiding the problems related to the plane wave assumption. The proposed approach estimates the complex wavenumber and the dispersion relation of several complex structures, showing a good description of the periodicity effect (Bragg's band gaps); the damping loss factor is also well estimated.

## 2 METHODOLOGY

The complex dispersion relation of different structures is here estimated by a wave correlation method. The presented approach estimates, at each frequency  $f_0$ , the complex wavenumber as the point of best agreement between the acquired vibrational field and the Green's function of the point excited structure. The Green's function is derived by the classical Kirchhoff's thin plate theory<sup>15</sup>; according to this theory, for an isotropic, homogeneous plate and of a constant thickness  $h$ , the flexural vibration, in terms of transverse displacement  $w$ , is expressed as follows

$$D\nabla^4 w(\vec{x}, t) + \rho h \frac{\partial^2 w(\vec{x}, t)}{\partial t^2} = P(\vec{x}_0, t) \quad (1)$$

where  $D = \frac{Eh^3}{12(1-\nu^2)}$  is the bending stiffness,  $E$  is the Young's modulus,  $\nu$  is the Poisson's coefficient,  $\rho$  is the mass density,  $\nabla^4$  is the biharmonic operator,  $t$  is the time variable,  $P$  is the force per unit area and  $\vec{x}$ ,  $\vec{x}_0$  are the position vectors of the acquisition and excitation points, respectively. Assuming an harmonic point excitation, the expression of the force becomes  $P(\vec{x}_0, t) = F(\vec{x}_0)e^{i\omega t}$ , being  $\omega$  the angular frequency; consequently, the steady state response of the transversal vibrational field is  $w(\vec{x}, t) = u(\vec{x})e^{i\omega t}$ . Substituting these two expressions in Eq. 1, the governing equation assumes the following form

$$D\nabla^4 u(\vec{x}) - k^4 u(\vec{x}) = F(\vec{x}_0) \quad (2)$$

where  $k^4 = \rho h \omega^2 / D$  is the flexural wavenumber.

For a plate of infinite lateral dimensions, the solution of Eq. 2 is given by

$$G_\infty(x, y) = \frac{1}{8k^2 D} \left[ H_0^1(kr) - H_0^1(ikr) \right] \quad (3)$$

with  $G_\infty$  the Green's function of the infinite plate,  $H_0^1$  the Hankel's function of first kind and order 0 and  $r = \|x - x_0, y - y_0\|$  is the distance between the acquisition  $(x, y)$  and excitation  $(x_0, y_0)$  points.

At each frequency  $f_0$ , the complex flexural wavenumber is found as the point of best agreement between the measured vibrational field and the Green's function expressed in the Eq. 3. Considering an acquisition region of area  $S$  in the plane  $(x, y)$ , indicating with  $\tilde{w}$  and  $w$  the measured vibrational field and the one described by Eq. 3, respectively, the normalized correlation function has the following expression

$$\mathcal{F}(k_{\mathbb{R}}, k_{\mathbb{I}}) = \frac{\left| \iint_S \tilde{w} \cdot w(k_{\mathbb{R}}, k_{\mathbb{I}})^* dx dy \right|}{\sqrt{\iint_S |\tilde{w}|^2 dx dy \cdot \iint_S |w(k_{\mathbb{R}}, k_{\mathbb{I}})|^2 dx dy}} \quad (4)$$

where  $*$  denotes the complex conjugate. For discrete points measurement, the integrals in Eq. 4 are replaced by summations over the entire domain. The complex wavenumber is identified as the location of the maximum of the correlation function. From the estimated wavenumber amplitude, an estimation of the damping loss factor can be obtained, at each frequency, by the following relationship<sup>2</sup>

$$\eta = \left| \frac{\Im(k^4)}{\Re(k^4)} \right| \quad (5)$$

By choosing a proper direction of propagation, Eq. 3 can be applied to a narrow beam subjected to punctual harmonic excitation. Keeping the same notation and indicating with  $L$  the length of the acquisition region, the correlation function showed in Eq. 4 becomes

$$\mathcal{F}(k_{\mathbb{R}}, k_{\mathbb{I}}) = \frac{\left| \int_L \tilde{w} \cdot w(k_{\mathbb{R}}, k_{\mathbb{I}})^* dx \right|}{\sqrt{\int_L |\tilde{w}|^2 dx \cdot \int_L |w(k_{\mathbb{R}}, k_{\mathbb{I}})|^2 dx}} \quad (6)$$

The dispersion curves of the different structures have been validated using the Wave Finite Element Method (WFEM)<sup>16-18</sup>; this method is not here described.

### 3 NUMERICAL RESULTS AND VALIDATIONS

The proposed approach is applied on different structures. Firstly, two different beams are taken in account: isotropic and periodic (with varying material and cross-section along the length). Then, application cases on two different plates are shown, taking in account an isotropic and homogeneous plate and a composite one, made of three different layers in the thickness direction.

#### 3.1 Isotropic and periodic narrow plates

The first application of the proposed approach has been conducted on a simple isotropic beam. The total length of the beam is 1.0 m and the cross-section has dimensions 10 x 1 mm<sup>2</sup>; the finite element model of the beam is shown in Fig. 1a. The employed material is a general thermoplastic polymer (ABS) with Young's modulus  $E = 1.0 \text{ GPa}$ , density  $\rho = 980.0 \text{ kg m}^{-3}$  and Poisson's coefficient  $\nu = 0.35$ ; two constant values of structural damping  $\eta$  are assumed: 2% and 4%. An analytical expression of the dispersion relation can be derived using Timoshenko's model for vibrating beams, resulting in

$$EIk^4 - \rho A \omega^2 - \left( \rho I + \frac{EI\rho}{KG} \right) k^2 \omega^2 + \frac{\rho^2 I}{KG} \omega^4 = 0 \quad (7)$$

where  $G$  is the shear modulus,  $I = bh^3/12$  is the second moment of area,  $A$  is the beam's cross-section area and  $K = 5/6$  is a constant to take in account the shear force variation.

A periodic beam is also investigated, to prove the feasibility of the proposed approach in describing the band-gaps due to the geometric periodicity of the structure. The finite element model of the unit cell of the considered structure is shown in Fig. 1b. The unit cell has total length of 0.04 m and it's characterized by a double periodicity: material and cross-section. The

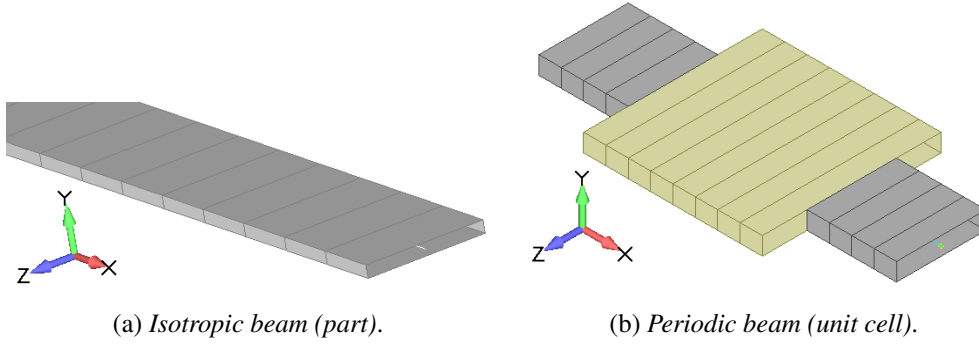


Figure 1: Finite element models of the isotropic and periodic beams

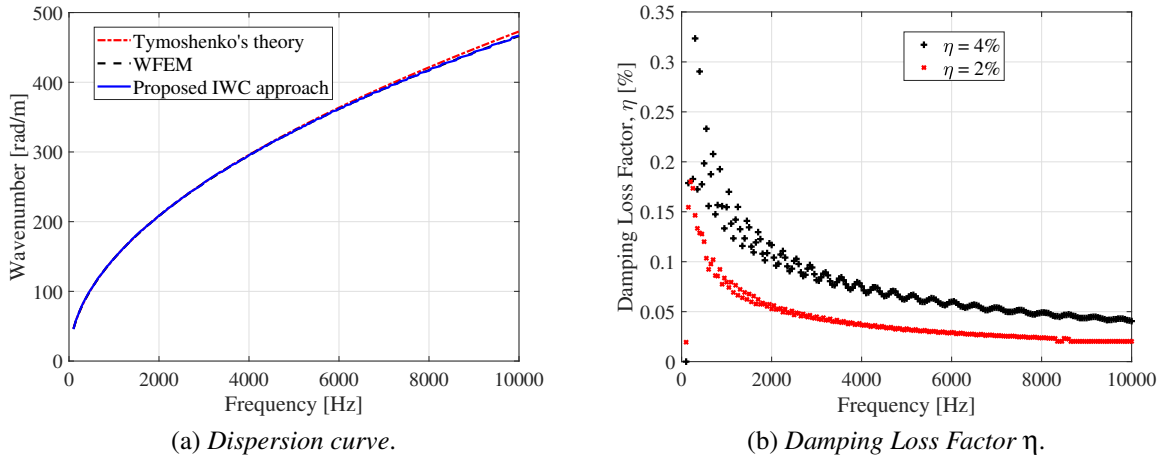


Figure 2: Dispersion curve and Damping Loss Factor estimation for the isotropic beam

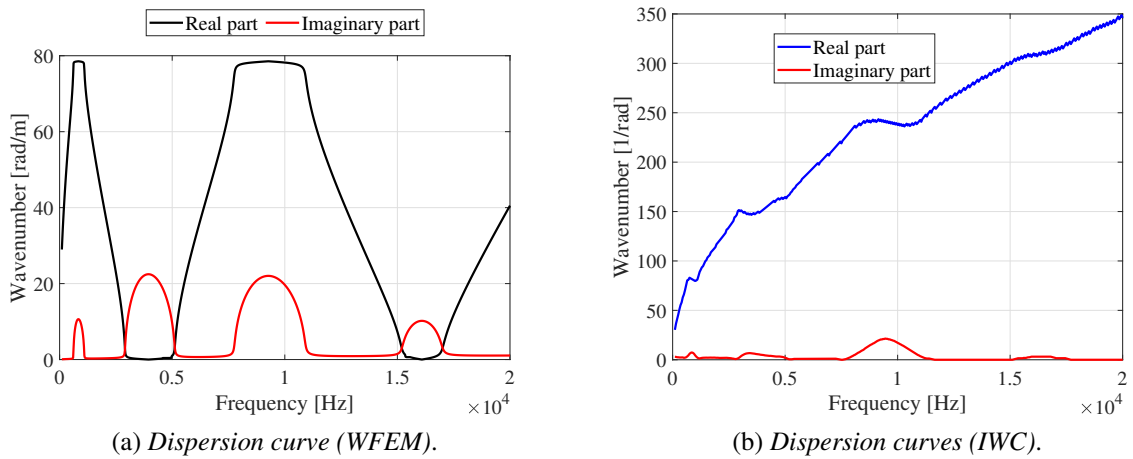


Figure 3: Real and imaginary dispersion curves of the periodic beam

employed materials are a standard aluminum alloy, with Young's modulus  $E = 70.0 \text{ GPa}$ , density  $\rho = 2700.0 \text{ kg m}^{-3}$  and Poisson's coefficient  $\nu = 0.33$ , and the ABS previously described. The aluminum part has a cross-section of dimensions  $10 \times 2 \text{ mm}^2$ ; the ABS part has dimensions  $20 \times 2 \text{ mm}^2$ . The propagative part of the dispersion curve for the isotropic beam is shown in Fig. 2a; the proposed IWC approach exhibits a very good agreement with the analytical model (cfr. Eq. 7) and the WFE method. In Fig. 2b is shown the damping loss factor estimation; in both cases, the identified complex wavenumbers give a reasonable estimation of the structural damping previously introduced.

The dispersion curve of the periodic beam are shown in Fig. 3; the dispersion curve calculated with the WFE method are limited to the first Brillouin zone, where the wavelengths are longer than one period of the structure (Fig. 3a). The real part of the wavenumber has periodicity  $\pi/\Delta$ , with  $\Delta$  the length of the unit cell, showing multiples band-gaps, but always with  $k_{\Re} \in [0, \pi/\Delta]$ . The dispersion curve estimation with the proposed approach is shown in Fig. 3b, being in good agreement with the one calculated with the WFE method, identifying the same wide band-gaps zone. The proposed IWC approach takes in account the whole structure, avoiding the periodicity issue previously described in the wavenumbers estimation.

### 3.2 Isotropic and laminated plates

The proposed approach has been extended to 2D structures, an isotropic plate, made of ABS, and a composite one, constituted of three layers: aluminum-ABS-aluminum, of thickness 1 - 8 - 1 mm, respectively; both plates have geometrical dimensions  $1.0 \times 0.6 \times 0.01 \text{ m}^3$ . The finite element models of the two unit cells are shown in Fig. 4; both unit cells are used in the WFE method to validate the results obtained with the proposed IWC approach. For both test cases, two different constant values of structural damping  $\eta$  are assumed: 2% and 5%.

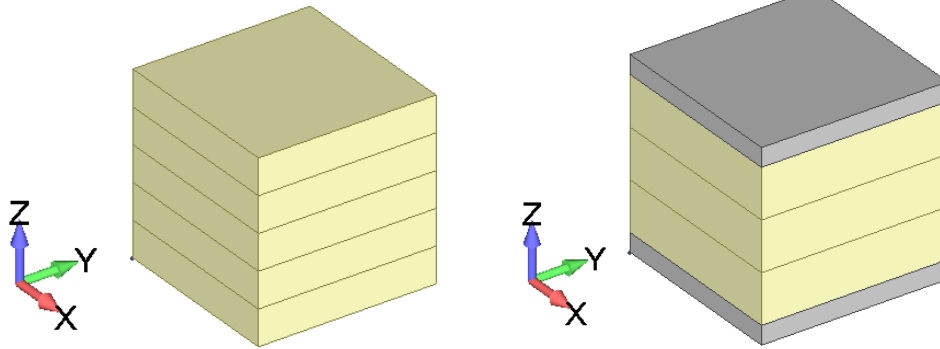
For what concern the isotropic plate, an analytical expression of the dispersion relation can be derived by Kirchhoff's thin plate theory, obtaining the following expression for the flexural wavenumber

$$k = \sqrt{\omega} \left( \frac{\rho h}{D} \right)^{1/4} \quad (8)$$

The propagative part of the dispersion curve is shown in Fig. 5a; the proposed approach is in good agreement with the analytical formulation of Eq. 8 and the WFE method, showing a low overestimation in the high-frequency region. In Fig. 5b, the damping loss factor estimations are shown; the relation of Eq. 5 is used, based on the estimated complex wavenumbers.

The propagative part of the dispersion curve for the composite plate is shown in Fig. 6a; a good agreement between the proposed approach and the WFE method is obtained in the whole frequency band of investigation. A very good identification of the damping loss factor is performed, and shown in Fig. 6b. Using an inverse approach based on the estimated wavenumber, the equivalent bending stiffness and the equivalent Young's modulus of the structure have been calculated by the following formulas, directly derived from Eq. 2

$$D_{eq} = \frac{\omega^2 \rho h}{k^4} \quad \text{and} \quad E_{eq} = \frac{D_{eq} 12(1 - \nu^2)}{h^3} \quad (9)$$



(a) *Isotropic plate (unit cell).*

(b) *Composite plate (unit cell).*

Figure 4: Finite element models of the isotropic and composite plates

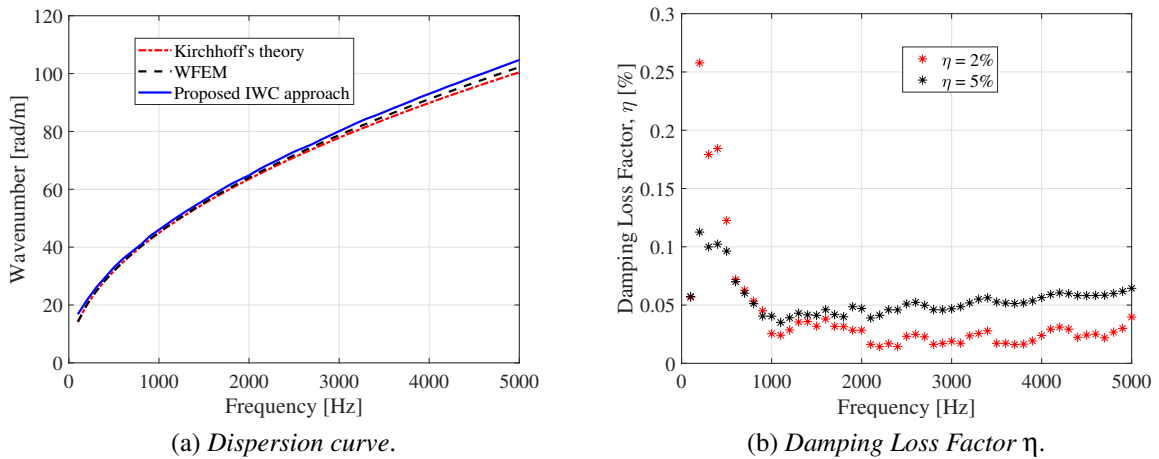


Figure 5: Dispersion curve and Damping Loss Factor estimation for the isotropic plate



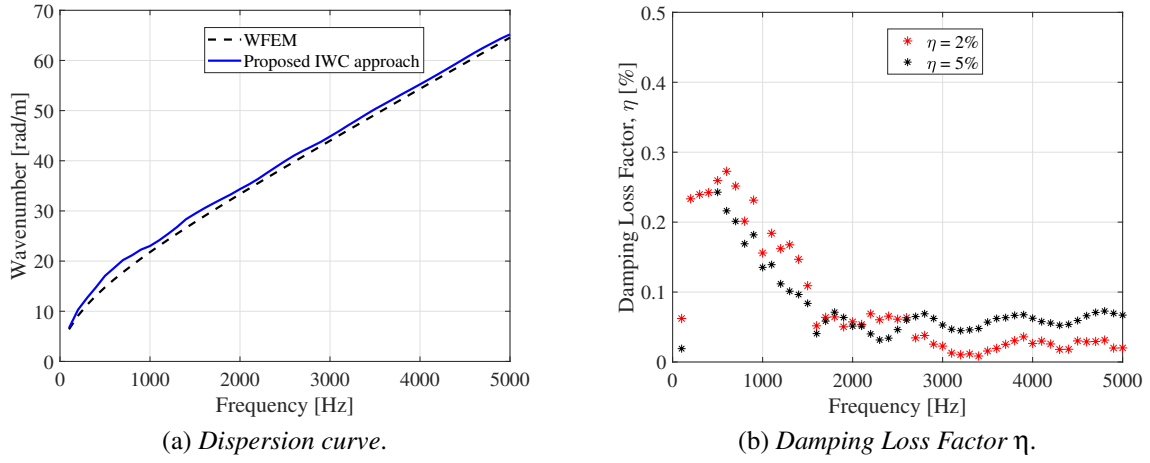


Figure 6: Dispersion curve and Damping Loss Factor estimation for the composite plate

The equivalent plate properties are shown in Fig. 7.

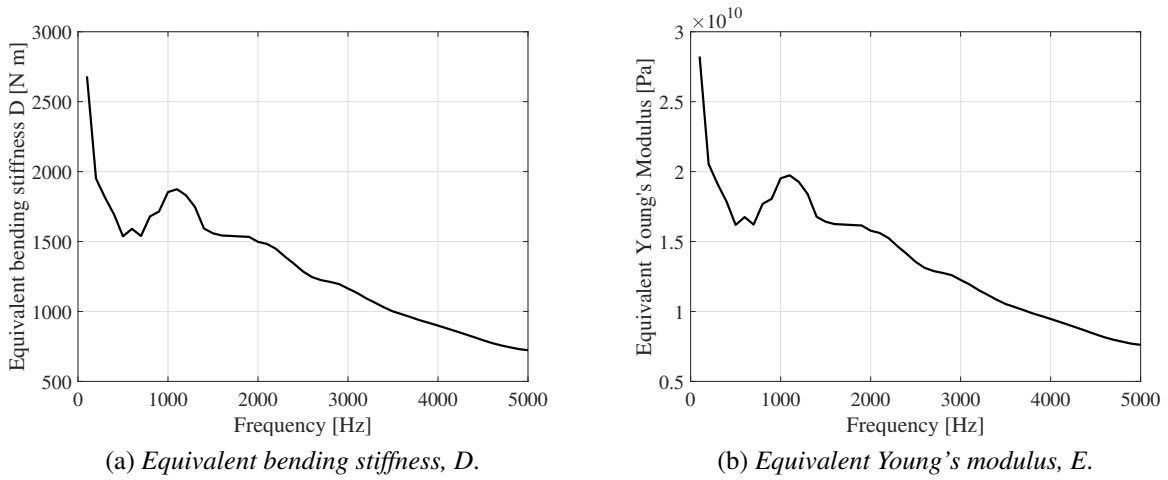


Figure 7: Dispersion curve and Damping Loss Factor estimation for the composite plate

#### 4 CONCLUDING REMARKS

In this work, different ideas are successfully implemented in this extended IWC approach, showing a very good prediction of the complex dispersion relation of periodic narrow plates and laminated structures. An estimation of the damping loss factor is obtained; the calculated values of  $\eta$  fast converge to the structural damping introduced. The feasibility of this method in estimating the wave attenuation in a structure is successfully demonstrated.

The proposed approach allows to identify the resonance zones (stop-bands) due to the geometrical periodicity of the structure, showing a good agreement with the WFE method, based on

the unit cell theory, avoiding some limitations related to the latter method.

The proposed approach can be used in an inverse way to estimate the frequency dependent equivalent material properties of complex structures, when analytical models are not available or difficult to obtain.

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