



**HAL**  
open science

## Ultrafast laser micro and nano processing with nondiffracting and curved beams

François Courvoisier, Razvan Stoian, Arnaud Couairon

► **To cite this version:**

François Courvoisier, Razvan Stoian, Arnaud Couairon. Ultrafast laser micro and nano processing with nondiffracting and curved beams. *Optics and Laser Technology*, 2016, 80, pp.125 - 137. hal-02392845

**HAL Id: hal-02392845**

**<https://hal.science/hal-02392845>**

Submitted on 4 Dec 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Ultrafast laser micro and nano processing with nondiffracting and curved beams

F. Courvoisier<sup>1</sup>

*Institut FEMTO-ST, UMR 6174 CNRS Université de Bourgogne Franche-Comté,  
Besançon, France*

R. Stoian

*Laboratoire Hubert Curien, UMR CNRS 5516, Université de Lyon, Université Jean  
Monnet, F-42000 Saint-Etienne, France*

A. Couairon

*Centre de Physique Théorique, CNRS, Ecole Polytechnique, F-91128 Palaiseau, France*

---

## Abstract

Ultrafast laser materials processing has undergone an important change with the development of non-diffracting beams. These beams enable overcoming many of the difficulties usually encountered with standard Gaussian-beam focusing in materials. We review the techniques of non-diffracting and accelerating beam shaping that generates lines, tubes or curved segments of focused light on distances that exceed the Gaussian Rayleigh range by several orders of magnitude. We review the benefits and applications of nondiffracting beams for laser micro- and nano-processing in the general context of materials processing with ultra-short pulses in the filamentation regime. We highlight applications on ultra-high aspect ratio nano-drilling and direct laser processing along curves.

*Keywords:* nondiffracting beams, Bessel beams, Accelerating beams, ultrafast laser materials processing

*2010 MSC:* 00-01, 99-00

---

<sup>1</sup>electronic mail: francois.courvoisier@femto-st.fr

## 1. Introduction

Ultrashort pulse lasers allow for materials processing at micro and sub-micrometric scales. Cutting, drilling, bulk or surface processing operations are possible with side damage in the sub-micron range. Ultrashort pulses deposit  
5 energy with ultra-high contrast between affected and non-affected zones. Ultrafast laser processing has entered in a novel development stage since high average power sources are available. While initially dedicated to niche markets and fast prototyping, ultrafast lasers now entered the markets of mass fabrication (solar technologies, consumer electronics, automotive) [1].

10 In this technological context, high-speed processing of large surfaces and deep drilling of materials are needed. A number of recent progress in terms of laser scanner technology and parallel processing have already greatly increased the throughput of laser processing. In this review, we show that using *non-diffracting beams* is an extremely powerful leverage to change the efficiency of  
15 the laser-matter interaction for high speed material processing and ablation.

Beam shaping has been developed and used for all types of laser processing techniques, mainly in 2D surface processing and shallow drilling of materials: the intensity distribution of the beam was optimized for one specific focusing plane to produce the desired mark (e.g. flat-top profile on a square or on a  
20 disk[2], arrays of spots[3] etc). In contrast, we will focus in this review on beam shapes optimized over long beam propagation distances.

*Non-diffracting* beams or diffraction-free beams are defined as propagation invariant solutions of the Helmholtz equation. Finite-energy realizations of these beams define extended focal lines, on distances much longer than the Rayleigh  
25 range. Several publications review works on laser processing with Bessel beams but separately from nonlinear optics of nondiffracting beams [4, 5, 6]. Here, we review the aspects of beam shaping and nonlinear optics that are directly relevant to ultrashort laser processing. In addition, we will review the extension to recently developed shaping techniques for the generation of “accelerating  
30 beams” that share many of the properties and benefits with nondiffracting Bessel

beams.

We will first review the basics of the generation of nondiffracting beams, then the main important results of their nonlinear propagation in transparent media and their applications to laser processing, both in terms of surface processing and high aspect ratio deep drilling in transparent media. This will be performed in the general context of materials processing with ultrashort filaments, or more precisely, within elongated focal regions. The last section is dedicated to accelerating beams, which possess an extended focus that is curved in space. The typical example for accelerating beams, the *Airy beam*, is propagation-invariant *i.e.* diffraction-free, with a parabolic trajectory[7]. Other beams that exhibit an intensity peak propagating along arbitrary trajectories have been developed and have found applications in terms of laser processing of curved profiles. We will detail the generation of accelerating beams and its application to laser ablation of curved profiles.

## 2. Diffraction Free Bessel beams

*Bessel beams.* Bessel beams are produced by the interference between plane waves propagating along the surface of a cylindrically symmetric cone, *i.e.*, having their  $k$  vector along a generatrix of the cone, crossing the optical axis at the same angle  $\theta$  [8].

The interference produces a central lobe with high intensity, surrounded by circular lobes. McLeod showed in 1954 that Bessel beams can be generated from conical lenses, *i.e.* axicons (cf figure 1)[9]. It is only in 1987 that Durnin *et al* generated experimental beams exhibiting the diffraction-free property of Bessel beams, *i.e.* quasi propagation-invariant beams [10]. Indeed, the cylindrically symmetric field amplitude  $E(r, z) = E_0 J_0(k \sin \theta r) e^{ik \cos \theta z}$  is a solution to the scalar Helmholtz equation.  $k$  is the modulus of the wavevector and  $\theta$  is the angle made by the geometrical rays with the optical axis, which we will refer to as “cone angle”.

This formal solution is of infinite energy. Experimentally, only apodized

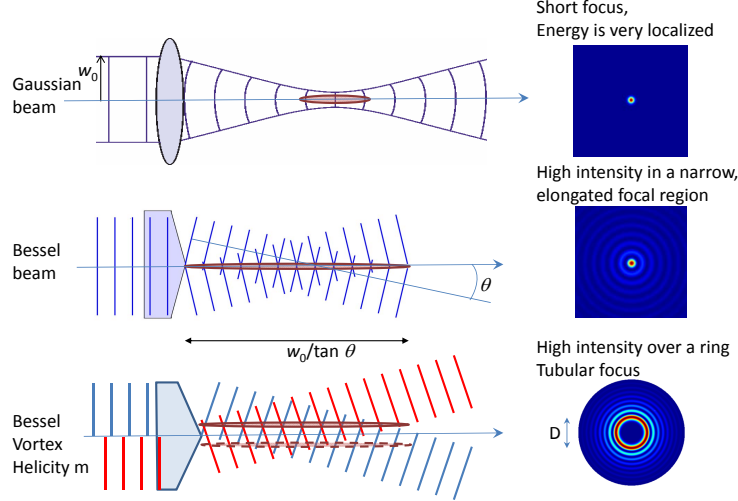


Figure 1: Schematic wavefronts and transverse intensity distribution for (top) Gaussian beam, (center) Bessel beam, (bottom) Bessel vortex beams

60 versions are physically realizable due to the necessary limitation in numerical aperture.

As an example, Bessel Gauss beams are obtained by sending a Bessel beam through an axicon and can be viewed as an ideal Bessel beam apodized by a large Gaussian beam. As the large Gaussian beam follows the laws for Gaussian optics, the Bessel-Gauss beam has invariant beam lobe positions the peak intensity and the number of visible lobes decrease far from the focus [11]. In the propagation direction, the full width at half maximum of the focal region for an apodized Bessel beam defines the Bessel zone. A Bessel-Gauss beam is produced by a Gaussian beam carrying a linear spatial phase  $\Phi(r) = k \sin(\theta)r$  which reshapes the Gaussian beam into a Bessel profile. Within the stationary phase approximation of the Fresnel integral, the on-axis intensity profile is, in air :

65  $I(r = 0, z) = 4P_0 k z \sin^2 \theta e^{-2(z \sin \theta/w)^2} / w^2$ , where  $P_0$  and  $w$  are respectively the power and the waist of the input Gaussian beam [12, 13]. The spatial spectrum of a Bessel beam is a thin annulus of intensity centered at the mean transverse

75 wavevector  $k_{r0} = k \sin \theta$  with a FWHM width  $\Delta k_r$  inversely proportional to the

Gaussian beam waist  $w$ [13].

Therefore, close to the optical axis (*i.e.* over very few lobes) and in the scalar approximation, a typical Bessel beam radial amplitude distribution can be approximated by  $E(r) = E_0 J_0(k \sin \theta r)$ , where the peak amplitude  $E_0$  can be  
80 determined by the stationary phase approximation. The polarization of Bessel beams is usually very close to the one of the input beam provided the cone angle is within the paraxial approximation. Detailed calculations and measurements have been reported since longitudinal polarization can be created with radially polarized beams[14, 15].

85 Bessel vortices are very similar to Bessel beams and are also diffraction-free solutions : they only differ by the fact that they carry an optical angular momentum, *i.e.* a vortex charge , and the main lobe in their intensity distribution is ring shaped as for high-order Bessel functions [16]. Details of the structure can be found in references [17, 18].

90 Figure 1 compares the structure of the geometrical rays and wavefronts propagating in a Gaussian, Bessel and Bessel vortex beams. In Bessel beams, all rays cross the optical axis with the same angle. In Bessel vortex beams, the rays have all the same angle with respect to the optical axis, but are tangent to a tube, forming a caustic [17, 18].

95 *Experimental realization.* A number of experimental techniques have been developed to produce Bessel beams, which can be split in two main families. “Direct” space shaping transforms a plane wave or its approximation into a Bessel beam by applying a linear ramp of phase with cylindrical symmetry. This can be performed with an axicon [9], sometimes processed at the exit side of an  
100 optical fiber [19, 20, 21]. More details on fiber-based Bessel beam shaping are reviewed in reference [4]. Other ways of shaping use holograms [22], spatial light modulators (SLM) [23, 24, 25] or diffractive optical elements [26, 27]. This approach allows for high-energy throughput, which is particularly important for applications in nonlinear optics or laser processing. When using axicon lenses,  
105 one experimental difficulty is the roundness of the tip, at the beam center. The

roundness generates imperfections in the beam that can be corrected by further Fourier filtering [28, 29]. Approximation of Bessel beams can be generated with tunable acoustic gradient index of refraction lenses [30]. For applications, 4-f telescopes are used to generate Bessel beams with long working distance and in  
110 combination with Fourier filtering [31, 24, 25].

The second family of nondiffracting Bessel beam shaping is Fourier space shaping, where an *amplitude* modulation is performed. In the spatial frequencies, i.e. Fourier space, an ideal Bessel beam is a circle of amplitude  $A(k_r) = \delta(k_r - k_{r0})$  [10]. Experimental realizations form an annulus of light  
115 in the back focal plane of a lens, and the Bessel beam is formed around the front focal plane, as the Fourier transformation. Beam shaping is based either on a hard aperture annular slit, or with Spatial Light Modulators or holograms, where only a thin annulus of light is formed in the first order of diffraction [10, 32, 22, 33, 34, 35, 8]. The longitudinal on-axis distribution of intensity  
120 in the Bessel beam  $I(r = 0, z)$  depends on the exact distribution of the annular distribution of amplitude in the Fourier plane. For instance, a perfect flat-top ring distribution generates a Bessel beam with a longitudinal profile  $I(r = 0, z) \sim (\sin(az)/z)^2$  [33].

Other means include i/conical diffraction. The coupling between propaga-  
125 tion direction and polarization in a biaxial crystal, has been recently used to produce Bessel beams [36]; and ii/ lasers designed to produce Bessel modes. The principle of operation is the use of a diffractive optical element as the end-cavity mirror[37].

Bessel vortex beams are produced either by applying a vortex to a Bessel  
130 beam, for instance by adding the vortex phase in the phase mask of the SLM, or by generating a Bessel beam from a Gaussian beam already carrying a vortex charge [22, 13, 17]. The vortex charge can be added with a vortex plate either in the Fourier or in the direct space [38, 39].

*Spatio-temporal aspects.* In the case of ultrashort pulses, the way of Bessel  
135 beams are generated is important because it determines the phase relation be-

tween the different temporal frequencies of the pulse. With refractive axicons, if dispersion is neglected over the pulse bandwidth, the spatial phase applied by refractive axicon is  $\phi(r, \omega) = \frac{\omega}{c} r \sin \theta$ . Using holograms the spatial phase applied is constant with frequency  $\Phi(r) = k_0 r \sin \theta$  [40, 41]. The dependance of  
140 spatial phase on the temporal frequency impacts on the spatio-temporal shape of the Bessel pulse. In the first case, when spatial phase varies linearly with frequency, the generated Bessel pulse is called an *X-wave* because the amplitude distribution in the spatio-temporal spectrum  $(k_r, \omega)$  has the shape of a X. The pulse is also X-shaped in  $(r, t)$  space. Importantly, the apparent speed of  
145 propagation of the on-axis lobe is  $v = c / \cos \theta$  where  $c$  is the speed of light. The superluminal ( $v > c$ ) propagation speed has generated a great interest but is still physical because the photons all propagate at speed  $c$ . Superluminality can be interpreted as a “scissor effect” [40, 5, 42]. This behavior has been experimentally characterized in [43] by measuring the speed of the ionization front of a  
150 femtosecond pulse by shadowgraphy and in [44] by direct FROG measurement. It does have important applications in nonlinear optics since nonlinear index deformations can be generated with superluminal speeds [45].

In contrast, when the spatial phase is constant with temporal frequency, the Bessel beams generated are *Pulsed Bessel Beams*. The shape of the pulse is  
155 spatially a Bessel beam with a temporal Gaussian envelope. In this case, the virtual on-axis speed of the pulse is  $v = c \cos \theta$  [46, 42], and is sub-luminal.

Generating Bessel pulses with Spatial Light Modulators is a specific case because the phase applied to the Gaussian laser beam by the liquid crystal array is not constant with the optical frequency. This phase is equal to the one  
160 applied by an axicon, but wrapped (most often over  $2\pi$ ). The spatio-temporal behavior is locally similar to Bessel-X pulses but can be approximated to Pulsed Bessel Beams over long propagation distance. It has been detailed in reference [47]. It is important to note that in all cases, the pulse duration of the input laser beam is preserved on the optical axis.



165 *Modifications of the intensity patterns in Nondiffracting beams.* For applica-  
tions in microscopy, nonlinear optics or laser materials processing, it is some-  
times important to modify the evolution of the on-axis intensity. Cizmar *et al*  
have developed a technique based on shaping the Fourier spectrum to engineer  
intensity profiles such as flat-top or linearly increasing ramps of intensity [33].  
170 Ouadghiri-Idrissi *et al* have developed a similar approach that preseves the en-  
ergy throughput [48]. In these approaches, the radius of the main intensity lobe  
of the Bessel beam is preserved but the relative intensities between the lateral  
lobes of the beam are modified.

A number of transverse profile variations of Bessel beams have been de-  
175 veloped such as spiraling Bessel beams and helicon waves [49, 50] that can be  
viewed as accelerating beams [51]. Slight distortions of the input hologram allow  
for tilting and curving the beams along arbitrary trajectories [52]. Suppressing  
the side lobes of Bessel beams generates “needle beams”. These beams have  
been generated by adapting the diameter of the axicon to generate only a single  
180 central lobe, thus performing an automatic apodization [53].

*Self-reconstruction and resistance to aberrations.* Bessel beams are ”self-healing”:  
the main central lobe can reconstruct after encountering an obstacle. This prop-  
erty arises from the conical flux of energy with the angle  $\theta$  with respect to the op-  
tical axis. This has first been used for optical trapping with many particles [54].  
185 The reconstruction distance after an obstacle of diameter  $a$  is  $\Delta z = a/(2 \cos \theta)$   
[55, 53].

The spatial spectrum of a nondiffracting Bessel beam is highly peaked around  
the radial wavevector  $k_{r0}$ . This strongly reduces the impact of optical aberrations  
such as spherical aberration of a glass plate. Broky *et al* have investigated  
190 the resistance of Bessel beams to turbulent conditions such as those occurring  
in the atmosphere [56, 57].

The properties of spot-size invariance, resistance to obstacles and aberrations  
accelerated the interest for the use of nondiffracting beams in the fields of  
particle trapping [8, 58, 59] and light sheet microscopy [60, 61].

### 195 **3. Laser surface processing with nondiffracting beams**

The nondiffracting property of Bessel beams has been very early identified for laser processing. In contrast with Gaussian beams where the transverse and longitudinal dimensions of the focal zone are linked by a single parameter, the length of the Bessel zone and the main lobe transverse size can be inde-  
200 pendently adjusted by two free parameters: the initial Gaussian beam waist and the crossing angle  $\theta$  determined by the axicon apex. Therefore, small spot size (down to  $\lambda/2$ , with  $\lambda$  the laser central wavelength) can be achieved over arbitrary propagation distances.

This offers the possibility of maintaining identical illumination conditions on  
205 a workpiece whatever its position within the Bessel zone. This requires however that only the central lobe is involved in the process.

In terms of surface processing, this has opened three main applications, not necessarily implying ultrashort pulses: i/ laser processing of curved surfaces [62]; ii/ laser processing of biological material where the position of the sample is  
210 not well known [63, 20] and iii/ ultrafast laser processing with non-critical positioning [25]. Ultrashort pulses can induce optical breakdown and ablation down to nanometric transverse dimensions, but in return, this requires nanometric absolute positioning of the workpiece versus the Gaussian focal spot [64, 65]. Nondiffracting beams solve this issue and were used to drill sub-micron holes  
215 in glass, graphene and metallic films [25, 66, 67]. We note that surface processing with nondiffracting beams is more beneficial for films or materials that are opaque at the laser wavelength. In transparent materials, the long depth of focus of Bessel beams can induce a damage or index modification well below the surface of the material, as it will be described in section 4.

220 Reference [68] reported Bessel beam formation with annular slits and their application to surface ablation over subwavelength diameter. Nondiffracting beams with modified shapes or vortices allowed the generation of micro and nano-disks on graphene films [69] and the investigation of ablation threshold and the ablation probability [70, 71]. Cheng and Polynkin reported annular

225 surface crater formation with ultrashort pulse ablation of fused silica with high  
order Bessel beams [72].

Combination of lens and axicon focusing allows for reducing the Bessel zone  
over which laser pulse energy is spread. It allows for preserving the Bessel  
profile and enhances the local intensity on the central lobe: this approach has  
230 been used for silicon micro-drilling [73].

#### 4. High aspect ratio ultrafast laser processing with nondiffracting beams

*High aspect ratio laser processing.* Deep drilling in materials is an issue for  
many technological fields. The aspect ratio is a representative figure of merit  
235 of the technical difficulty for drilling: it is the ratio of the channel depth over  
the channel diameter. While aspect ratios up to 10:1 are common whatever the  
drilling or cutting technique (laser, mechanical drill, water jet, DRIE, Focused  
Ion beam.. etc), it is much more difficult to extract matter out of channels with  
high aspect ratio exceeding  $\sim 10 : 1$ . This is even more difficult for channels in  
240 sub-micron diameter range.

Nondiffracting Bessel beams and needle beams are *a priori* potential can-  
didates for high aspect ratio drilling because of the ultra-long focal distance.  
However, it is important to note that drilling with Bessel beams is only possible  
in transparent materials. Indeed, in opaque materials, the conical flux of energy  
245 generating the central lobe cannot propagate. Bessel beam drilling of metals  
generates V-shaped craters where the opening angle is close to the Bessel beam  
angle [74]. Therefore, in the following, our interest will be only for transparent  
materials, unless explicitly mentioned.

*Transparent materials processing by ultrashort pulses.* Ultrafast lasers are well  
250 suited to processing transparent materials because they can deposit energy at  
the surface or in the material bulk with an extreme degree of confinement.  
In brief, ultrashort laser material modification process is the following. High  
intensities (typ.  $10^{13}$  W.cm<sup>-2</sup>) reached in ultrashort pulses allow multiphoton

ionization to generate free-electrons in the conduction band. The transition  
 255 involves typically between 2 and 15 photons depending on the bandgap and  
 illumination wavelength [75, 76], where the number of photons is determined  
 by the ratio between the energy to promote an electron from the valence band  
 to the conduction band and the photon energy. The plasma of free-electrons  
 and holes can further absorb light through collisions. The hot electron gas  
 260 then relaxes by carrier-phonon collisions and transfers its energy to the initially  
 cool lattice. Phase transformations, diffusion, hydrodynamic effects occur on  
 timescales from several picoseconds to microseconds. The full description of  
 laser-matter interaction and relaxation processes is out of the scope of this  
 review and more details can be found in references [77, 78, 79, 80].

265 *Nonlinear propagation in transparent materials.* It is important to realize that  
 the propagation of light into the medium is nonlinear and highly impacts on ma-  
 terial ablation or modification process. The nonlinear propagation of Gaussian  
 beam pulse is difficult to predict, even in the simplest case of the propagation  
 in defect-free media (*i.e.* single shot damage). Three main sources of nonlin-  
 270 earities are : i/ the nonlinear Kerr effect that modifies the index of refraction  
 of the medium  $n_0$  as  $n = n_0 + n_2 I$ , with  $I$  the local intensity of the field ,  
 ii/nonlinear ionization, which rate can be described by Keldysh formulation as  
 a first approximation [81] and iii/ the free-electron plasma. The latter induces  
 defocusing since, in first approximation, it induces a refractive index change  
 275 of  $\Delta n = -\frac{\rho(\mathbf{r},t)}{2\rho_c}$  where  $\rho(\mathbf{r},t)$  is the local density of free electrons and  $\rho_c$  the  
 critical plasma density at the laser central wavelength ( $\rho_c \sim 1.7 \times 10^{21} \text{ cm}^{-3}$  at  
 800 nm) [82]. The free-electron plasma also absorbs the laser pulse. Losses are  
 typically modelled through the plasma Drude model [81].

These physical effects also greatly affect the pulse in time and spectrum  
 280 through spectral broadening, self-steepening etc [82]. The previous list is not  
 exhaustive : several other effects might need to be taken into account for rel-  
 atively long propagation distances ( $> \text{cm}$ ) or extremely short pulses (typically  
 $< 20 \text{ fs}$ ) : group velocity dispersion, Raman contributions to nonlinearities, etc.

Depending on actual intensities, saturations of the nonlinear effects do occur.

285 At moderate intensities and focusing (focal spot diameter  $\sim 10\mu\text{m}$ ), it is relevant to describe pulse propagation and laser-matter interaction by two coupled equations. The pulse amplitude is described by the nonlinear Schrödinger equation [81, 83] describing the evolution of the envelope of the electric field, that is coupled to a rate equation describing ionization processes (multiphoton  
290 and tunnel ionization described by Keldysh formulation and impact ionization). Additional saturation terms taking into account the finite number of electrons in the valence band were described in reference [84] and the influence of self-trapped excitons was also modelled by several groups [77, 85].

We note that modelling laser-matter interaction in dielectrics is still a hot  
295 topic because of the technical difficulty to take into account the numerous effects of out-of equilibrium physics, with large gradients [80, 86, 87, 88]. Tractable numerical models require numerous approximations on plasma dynamics, which strongly restrict the domain of validity of the models. Linking pulse propagation to the damage is an even higher challenge [80, 85].

300 In conclusion of this section, the nonlinear propagation of Gaussian pulses in transparent materials at high intensities yields highly distorted pulses, both in space and time.

*Filamentation.* Under the combined action of Kerr self-focusing and plasma defocusing, the nonlinear propagation of Gaussian pulses typically tends to a  
305 dynamical regime of light propagation where one or several hot-spots self-form in the beam over extended propagation distances, leaving long tracks of plasma in their wake. These structures are called filaments [89, 90, 82].

The extended propagation length exceeds by several orders of magnitude the Rayleigh range corresponding to the diameter of the hot spots. This effect occurs  
310 both in solids and gases and the self-healing properties of filaments rendered them particularly interesting for atmospheric applications [91].

In this research direction, the formation of elongated damages by filaments were attractive for laser materials processing at high aspect ratio.

*High aspect ratio processing by filaments.* High aspect ratio damages induced  
315 in by filaments in transparent solids were reported by several groups [92, 81,  
93, 94, 95, 96]. The process has been also investigated by means of pump-  
probe analysis of the transient dynamics of free-carrier generation and material  
damage [97]. It has been observed that the filamentation process is enhanced  
by spherical aberration, such as the aberration induced by propagation through  
320 thick samples (rear side focusing conditions). Nanochannels with aspect ratio  $\sim$   
20 : 1 were generated by single shot focusing on rear side of glass [98]. Elongated  
channels were obtained in single shot within the bulk of fused silica and PMMA  
[99, 100, 101, 102]. More recently, Herbstman and Hunt created high aspect ratio  
nanochannels in single shot focusing at the exit side of fused silica plates[103].  
325 Ahmed *et al* reported that the spherical aberration induced by thick glass plates  
inserted between the focusing lens and the sample allow drilling extremely long  
voids in glass in single shot [104].

A glass cutting technique has been developed, where filamentation tracks  
create elongated damages in glass: creating those tracks next to each other,  
330 at translation speed of the workpiece on the order of 1 m/s, guides mechan-  
ical cleaving or self-cleaving, depending on the type of glass and illumination  
conditions [102].

However, the structure of the filament in this case is bounded to the de-  
gree of control of the competition between Kerr effect and plasma defocusing.  
335 The complex ionization dynamics of glasses makes this technique highly dif-  
ficult and sensitive to input conditions (beam diameter, beam quality, pulse  
duration, energy, position of the sample, etc).

*Nonlinear propagation of nondiffracting beams.* Nondiffracting Bessel beams are  
advantageous over Gaussian beams for laser bulk processing because their non-  
340 linear propagation permits a stationary regime with negligible spatio-temporal  
dynamics and generates a uniform plasma track. This regime is stabilized by  
high nonlinear losses and allows for a high degree of control of light-plasma in-  
teraction in the bulk of the material with reduced spatio-temporal distortions.

Nonlinear propagation of nondiffracting Bessel beams has generated an intense interest when it was realized that filamentation of Gaussian beams spontaneously generates structures that are highly similar to Bessel beams: conical waves and self-healing properties [105, 106, 107, 108, 109, 110].

Polesana *et al* identified three propagation regimes for intense Bessel beams [111]: i/ “weakly nonlinear”, where losses are negligible and is featured by oscillating peak intensity and transverse lobe compression [112]; ii/ “unsteady Bessel filamentation”, characterized by high peak intensities yielding periodic optical breakdown and damage [113] and where the pulse propagation shows spatio-temporal dynamics; iii/ “steady Bessel filamentation” shows an absence of dynamics. In the latter regime, high losses occurring in the central lobe are compensated by the energy flow from the surrounding lobes (conical flux, playing the role of a reservoir).

Propagation invariant and monochromatic solutions to the nonlinear Schrödinger equation exist [114]. In comparison to the Bessel beams, they are featured by lobe compression reflecting the optical Kerr effect and to a power flux from the periphery toward the main lobe where nonlinear losses are localized [115]. The low intensity tail of the beam can be viewed as a superposition of two cylindrical Hankel waves with unequal amplitudes. Nonlinear losses in the intense part correspond to the difference between the power flux for the inward and the outward Hankel waves. This feature is at the origin of the name of Nonlinear Unbalanced Bessel Beams (NLUBBs) given to these solutions. Like Bessel beams, NLUBBs have apodized counterparts carrying finite energy, which play the role of attractors in the filamentation dynamics of Gaussian beams. NLUBBs with similar features can be found whatever the type of nonlinear losses or nonlinear refraction index.

The stationary, or steady, regime of Bessel filamentation was investigated in the context of applications that require the generation of a uniform plasma channel in air or in liquids [116, 117, 118]. Due to the conical structure of Bessel beams, the photons creating the plasma in the central lobe have travelled along trajectory crossing regions only with low intensity, with moderate nonlinearity

375 and losses. This allows minimizing the effect of plasma defocusing and opti-  
mizing localization of light as energy arrives from the side in the focal volume  
without crossing the plasma [119].

Experimentally, the choice of input conditions impacts on the dynamics of  
Bessel filaments. Polesana *et al* have shown that the progressive intensity rise  
380 and Bessel beam formation in the nonlinear medium yield stationary filamen-  
tation regime. In contrast, when the Bessel beam is formed before the lin-  
ear/nonlinear interface, it can lead to the dynamical unsteady filamentation  
regime [120].

*Laser processing of transparent materials with Bessel beams.* Marcinkevicius *et*  
385 *al* reported the first optical damage by Bessel beams in glass with ultrashort  
pulses [31]. After multiple shot illumination of glass, straight modifications of  
the index of refraction were observed in fused silica. J. Amako *et al* realized sim-  
ilar marking in glass with Bessel beams, followed by selective chemical etching  
to process high-aspect ratio channels [26]. Grooves in fused silica were also pro-  
390 cessed with a similar approach [121]. In the multishot regime, the progressive  
build-up of a damaged zone can be interpreted as a spatially nonlinear absorp-  
tion : the stationary regime is favored by the stronger absorption of defect  
centers within the central lobe. The latter regime has also been used to write  
volume Bragg gratings in fused silica [122].

395 Bhuyan *et al* produced the first application in the ablation regime in single  
shot [123]. A key element is the high cone angle used:  $\theta$  was varied between 16  
to 26 degrees in air, corresponding to 11 to 17 degrees in glass. Nanochannels  
could be drilled in glass with aspect ratio up to 100:1. At that time, no other  
technological mean could reach so high aspect ratio in short processing time.

400 Figure 3 summarizes the drilling mechanism at play. During the propagation  
of the laser pulse in the dielectric, a plasma of electron-hole pairs is created by  
the Bessel beam. High cone angles, exceeding  $15^\circ$  play a key role in maintaining  
a stationary regime even at high intensities and in building a near uniform  
plasma density. Lower cone angles do not always provide a stationary regime.



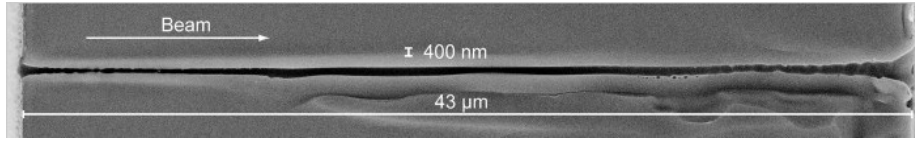


Figure 2: Proof of principle machining of a through channel using Bessel beam conical half angle  $\theta = 11^\circ$  in glass and a  $43\mu\text{m}$  thick glass membrane at single shot pulse energy of  $3.1\mu\text{J}$ . Reprinted with permission from reference [123]

405 At cone angles  $\theta \sim 7^\circ$  in glass, transitions to unsteady Bessel filamentation regime were observed when the Bessel zone crosses the sample entrance surface [124]. A novel technique of chirped pulse spectral transmission confirmed the stationary behavior of high-angle Bessel beams at ablation-level conditions [125].

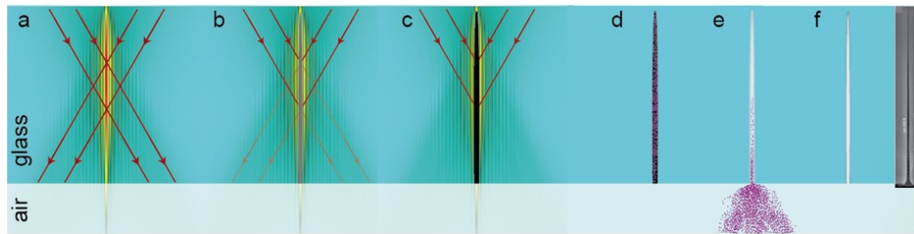


Figure 3: Physical sequence of drilling. (a) Bessel beam propagation; (b) creation of a plasma channel of electron-hole pairs; (c) plasma is highly absorbing and creates avalanche; (d) energy transfer from the electron-hole plasma to the lattice; (e) phase change and material expulsion and/or material compaction; (f) cooling and an elongated void is left.

After the build-up of the plasma of electron-hole pairs, the hot electron  
 410 gas transfers its energy to the cool lattice, which undergoes rapid phase change (melting, evaporation, atomization). Two mechanisms for channel opening are at play : i/ material compression such as in the case of formation of voids in silica and sapphire[126]; ii/material expulsion through the channel itself.

In the case of nanochannel drilling in Corning 0211 glass [123], material re-  
 415 moval is the main mechanism at play. It was observed that drilling occurred only when the Bessel zone was crossing the exit side. Focusing in the bulk of the material led to elongated index modifications with no void. In contrast, another study with Bessel beams with picosecond pulse duration, elongated voids could

be realized completely in the bulk of fused silica, suggesting the second mechanism [127]. The influence of cone angle, thermal diffusivity coefficient of the material (linked to rapid or fast cooling from liquid to solid phase), cooling and ejection dynamics of the plasma column are potential parameters influencing the difference in behaviour. Understanding the thermomechanical evolution of the material after laser energy deposition remains a challenging problem calling for further investigations.

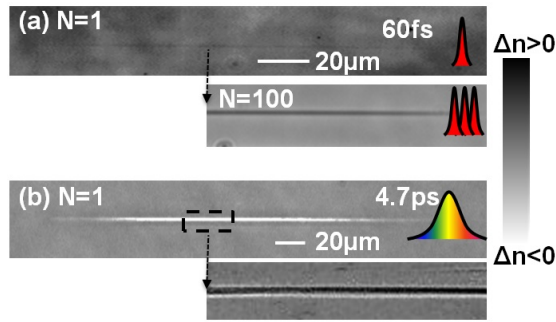


Figure 4: Phase contrast microscopy images of Bessel beam induced high-aspect-ratio structures in fused silica at moderate focusing conditions ( $\theta = 8.3$ ). (a) Single-shot positive refractive index structure at  $1.72 \mu\text{J}$  pulse energy and 60 fs. (inset) Multipulse ( $N=100$ ) smooth refractive index structure for  $1 \mu\text{J}$ , 60 fs pulse irradiation conditions. (b) Uniform void structure at  $7 \mu\text{J}/\text{pulse}$ , 4.7 ps. (inset) FIB/SEM Cross sectional profile of the fabricated structure at ps pulse durations

The influence of input pulse duration has been investigated [127, 128]. It was reported that picosecond pulse durations led to enhanced damage in fused silica. Figure 4 shows that moderate focusing of Bessel beams with variable pulse durations can tune positive and negative index changes. Indirect measurements of free-carriers density in the plasma indicate the higher absorptivity of the plasma created by picosecond pulse durations [129]. However, microchannel drilling in borosilicate glass was reported with aspect ratio 1200:1 for 50 fs pulse duration[130]. The interpretation of the differences between picosecond and femtosecond regimes is nontrivial because at high degrees of ionization, a number of different mechanisms change light absorption : bandgap shrinking,

ultrafast phase change during the picosecond pulse, dynamics of trapped states, etc. [86, 131, 85, 132].

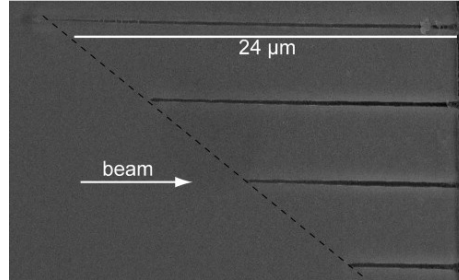


Figure 5: SEM image showing four machined channels with sample displacement differing by  $4\mu\text{m}$  in the horizontal direction. Reprinted with permission from reference [123].

In contrast with Gaussian beam filamentation, the longitudinal position of the channels is highly predictable, as shown in figure 5 [123]. These results  
440 highly impact on the field of glass cutting. Single shot writing of channels in lines generates a stress in glass that can be used to cleave the material at speeds on the order of 1 m/s. Stress generation by Bessel beams in glass and kerf-free cutting of glass have been investigated by Tsai *et al* [133] in the femtosecond regime. High speed cutting of glass was reported using picosecond regime with a slightly  
445 larger kerf [128]. Processing of silicon with Bessel beams was investigated in the transparency window at laser wavelength of  $1.3\mu\text{m}$  [134]. It was reported the absence of damage in the bulk, which was attributed to nonlinear losses before light reaches the focus.

## 5. Tubular Plasmas

450 It is attractive to drill large diameter holes (several tens of microns) with annular beams, such as those produced by optical vortices. However, during nonlinear propagation of annular beams in dielectrics, modulation instability tends to break-up the ring of intense light into several hot spots [135, 136, 137].

Recent work from several groups has shown that Bessel beams carrying vortex charge, *i.e.* “Bessel vortices” can also sustain stationary propagation.  
455

Monochromatic and propagation invariant nonlinear solutions of the nonlinear Schrödinger equation in form of a Bessel function carrying angular momentum has been found [138, 139]. Experimental and numerical investigations of femtosecond Bessel vortices in fused silica have shown the transition between three regimes when the pulse energy was increased. The stationary regime creates a tube of plasma in glass or in air following the main lobe of intensity with obvious applications to laser materials processing [140, 141]. This regime is sustained over a finite range of energies. This energy range is extended when the cone angle is increased, as for the case of zero-order Bessel beams. At higher intensities, nonlinear wave mixing generates supplementary spatial frequencies. First, the main intensity lobe breaks-up into several hot spots spiraling around the optical axis. This regime is close to what was observed in water by Shiffler *et al* [142]. At much higher intensities, in “speckle-like” propagation regime, multiple hot spots appear and disappear along the propagation distance. This regime is unexploitable for laser processing while is it still reproducible from shot to shot.

Proof of principle tubular index modification has been reported in glass from single shot Bessel vortices in the stationary regime [141, 143]. Picosecond laser processing with vortices has been investigated in single and multiple shot regime by Jedrkiewicz *et al* [144]. Although tubular modification could be reported (see figure 6), drilling of cylindrical holes remains difficult because of the distortions due to the roughness appearing after a few laser shots. Energy deposition is not efficient enough to realize cylindrical voids. Novel strategies are needed for this aim.

## 6. Accelerating beams and laser processing of curved profiles

The technological field of the fabrication of screens, smartphones, flat panels, microelectronics requires profiling of glass with rounded edges. Brittle materials with rounded edges are less prone to fracture than those cut with hard edges. In this section, we show that another family of beams, “accelerating beams”,

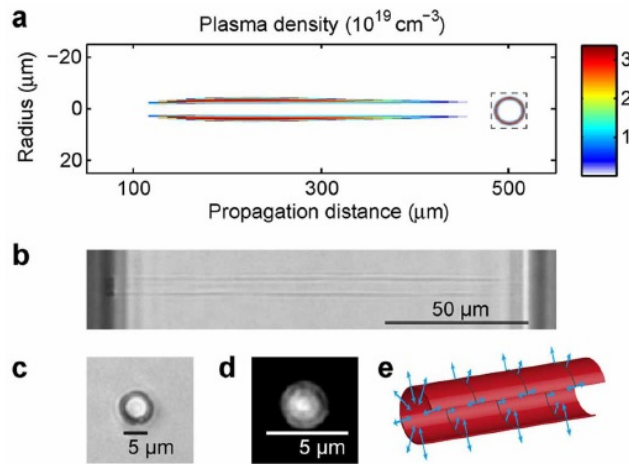


Figure 6: Tubular plasma generated in the stationary regime. (a) Tubular plasma density distribution at a pulse energy of  $5 \mu\text{J}$ . The inset shows the cross section of the plasma distribution at a distance of  $300 \mu\text{m}$  (b) longitudinal view under transmission optical microscopy of a tubular damage produced in glass by single shot, high energy, stationary Bessel vortex, pulse energy  $35 \mu\text{J}$  and pulse duration  $1 \text{ ps}$ . The damage extends from one sample side to the other. (c) Transverse section of the damage observed in identical conditions the same beam at pulse energy of  $20 \mu\text{J}$ . The bright central region shows high index modification of the core of the tubular region. (d) Image of near-field output guided light in the structure shown in (c), at an input wavelength of  $632 \text{ nm}$ . (e) Schematic view of the propagation direction of mechanical and thermal waves expanding outward and inward (arrows) from the excited tubular sheet volume (circle). Reprinted with permission from reference[141].

485 provides interesting pathways to solve this issue.

Currently, after laser cutting, glass panels are mechanically polished to reach rounded edges. This is time consuming and not useable for thin glass panels. Glass with bull nose profile can be obtained by straight cutting of chamfers ( +45 and -45°). Mathis *et al* developed an approach for direct curved laser  
490 profiling with arbitrary profiles[145]. In this section, we will review recent results obtained with accelerating beams whose main intensity lobe propagates along an arbitrarily curved trajectory. The beam shaping technique for generating accelerating beams conserves the pulse duration on the trajectory, and therefore allows for ultrafast laser processing. We will review results in terms of laser  
495 curved profiling and curved cutting.

*Accelerating beams.* In 2007, Siviloglou and Christodoulides identified the analogy between a nonspreading quantum wavepacket [146] and an optical field propagating along a parabola [7, 147]. The Airy beam is a solution of the paraxial wave propagation equation. Its transverse intensity profile is invariant  
500 along the propagation, while the position of its maximum follows a parabola. As Bessel beams, Airy beams are of infinite energy but apodized versions have been well identified [7]. These results have triggered an intense interest. Applications have been developed in almost all fields of optics and photonics [148]: optical trapping [149], microscopy [150], nonlinear optics [151, 152, 153], plasmonics  
505 [154] and even electron waves [155].

Two important results for the field of laser processing are that, like Bessel beams, Airy beams are self-healing [56] and Airy beams sustain stationary nonlinear propagation [156]. As for nonlinear Bessel beams, stationarity is supported by an energy flux from the low intensity tail of the beam toward the  
510 intense main lobe where nonlinear absorption occurs. Curved plasma generation in air with paraxial Airy femtosecond pulses has been reported in 2009 [157].

Experimentally, Airy beams are generated by applying spatially cubic phase in the focal plane of a lens, realizing an optical Fourier transform. However,

515 for applications to laser processing of curved profiles, Airy beams do not show  
enough bending on short distances. Froehly *et al* [158] and Courvoisier *et al*  
[159] have therefore developed an approach to shape non-paraxial beams. This  
has been performed by identifying that Airy beams can be interpreted in terms  
of optical caustics, *i.e.* a family of optical rays tangent to a curved trajectory  
520 [160]. In the framework of optical catastrophes, the transverse Airy function  
arise from a fold catastrophe (see supplementary information in reference [161]).

From these observations, shaping along arbitrary convex trajectories could  
be realized experimentally by applying a spatial phase to a Gaussian beam  
[161, 158].

525 Accelerating beams cover the family of beams having a main intensity lobe  
following a curved trajectory. The transverse profiles are not necessarily diffraction-  
free. In 2D, the FWHM of the main intensity lobe of an accelerating beam is  
linked to the local radius of curvature of the trajectory:  $\Delta x = 1.630[R/2k^2]^{1/3}$ ,  
where  $k$  is the wavevector and  $R$  the local radius of curvature of the trajec-  
530 tory [159]. The transverse lobe dimension therefore evolves: higher focusing is  
reached for highly curved trajectories.

This work has been further extended in highly non-paraxial regime (numerical  
aperture exceeding 0.4) with Fourier-space shaping. In this case, experi-  
mental realization requires a correct description of the correspondance between  
535 the optical frequencies and the position in the back focal plane of high numeri-  
cal aperture microscope objectives. Arbitrary nonparaxial trajectories (circular,  
parabolic, quadratic) were experimentally generated with bending angles filling  
the full numerical aperture of a microscope objective of NA 0.8 [162]. Simultane-  
ously, analytical derivation from Maxwell equations predicted the same results  
540 [163, 148] and was extended since then to many other geometries and properties  
[52].

As for the case of Bessel beams, the spatio-temporal properties of Airy beams  
depend on the space and mean of shaping. Direct space shaping [159] and  
Fourier space shaping [164] were investigated and showed that in both case,  
545 pulse duration is preserved on the caustic trajectory. In the first case, the pulse

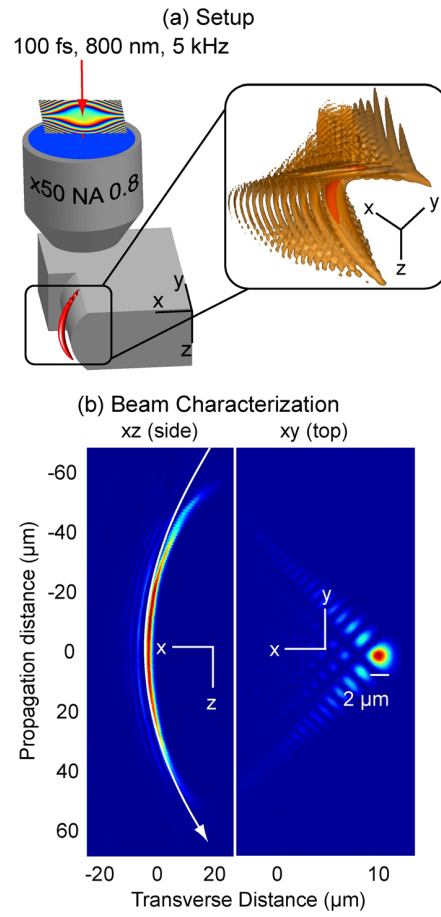


Figure 7: (a) Concept of laser processing of curved profiles with accelerating beams. (b) Intensity distribution of the accelerating beam in the longitudinal  $xz$  plane and the transverse  $xy$  plane. Reprinted with permission from reference [145].



travels along the curved trajectory while in the second, the whole trajectory is illuminated simultaneously.

Experimentally, accelerating beams are generated with spatial light modulators, diffractive optical elements or phase plates produced in fused silica [165].

550 *Laser processing of curved profiles.* Early demonstration of curved damage in glass induced by an accelerating femtosecond pulse was reported by Froehly *et al* [158]. Mathis *et al* investigated two approaches of materials processing with accelerating beams: side processing of edges and direct curved trench machining[145]. In both cases, the beam used was obtained by interference  
555 of two 1D accelerating beam profiles, locally generating an intensity pattern of  $I(x, y) \sim |Ai(x/w_0)Ai(y/w_0)|^2$ . A main intensity lobe propagates along predefined curved trajectory. This allows reaching higher peak intensities and avoiding break-up of 1D profiles into hot spots by modulation instability.

It is however important to note that only a part of the input pulse energy  
560 contributes to build the main intensity lobe along the propagation, in contrast with Bessel beams. As shown by Kaganovsky *et al*, only a part of the optical rays in a 2D Airy beam formed by such interference cross the central lobe [166].

Curved edge processing of silicon and diamond were reported. This is performed by back and forth translation of the beam along the edge with progressive  
565 penetration toward the central part of the workpiece, as shown in the concept figure 7 adapted from reference [145]. With this approach, arbitrary profiles could be imprinted on the material with excellent agreement with expected profiles. The limitation of this approach is the formation of diffusing roughnesses on the edge of the structure. Depending on illumination conditions, the roughnesses modify the local ablation rate and discrepancies with expected profile  
570 appear.

The second approach developed in this work was detailed in reference [167]: direct translation of the accelerating beam into an opaque material such as silicon for a femtosecond laser centered at 800 nm, opens a curved trench. In  
575 this case, the intensity distribution on the top surface of the workpiece is an

essential parameter. Indeed, the opening of the trench let the light penetrate into the medium and follow the curvature of the beam trajectory.

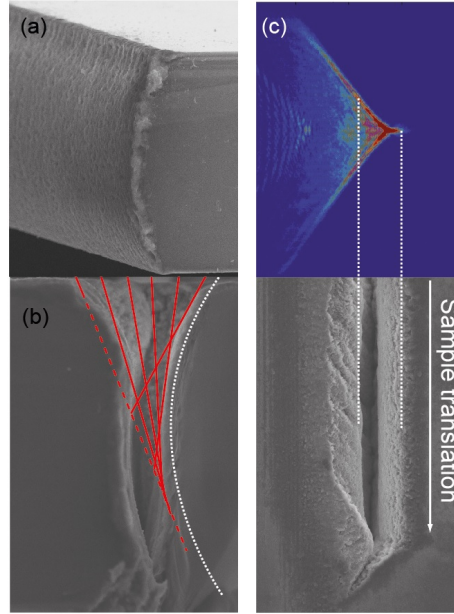


Figure 8: (a) Curved processing of a silicon wafer of thickness  $100 \mu\text{m}$ . (b) direct curved trench machining of silicon. The geometrical rays show where the drilling stops following the curved trajectory. (c) Intensity distribution on the top surface of the silicon sample : the trench opening is performed by the two side lobes of the beam.

In principle, this approach can be applied to transparent materials. However, refraction at the air/dielectric interface reduces the effective curvature of the accelerating beam inside the material. The multishot drilling process is then  
 500 expected to become more complex.

Accelerating beams have also found another application in the field of laser processing. *Abruptly autofocusing waves*, produced by the cylindrically-symmetric interference of Airy beams, produce an intense focus over a propagation distance  
 585 shorter than the equivalent Rayleigh range [168, 169]. This property has been used to create localized ablation at the exit surface of transparent material, without damage in the region preceding the ablation zone [170].

## 7. Unsolved problems and future directions

Nondiffracting beams have opened up a very novel way to create intense interaction over extended propagation distances. Particularly, the physics at play during channel formation is only partially uncovered. Pump-probe imaging of the plasma densities generated by Bessel beams have been realized [129], but a key difficulty of the analysis of the experiments lies in the extremely high gradients of plasma density generated over subwavelength transverse dimensions. Future work includes the investigation of the exact influence of cone angle, thermal diffusivity and the hydrodynamics of the plasma column.

The investigation of the compression mechanisms and plasma/matter ejection during the formation of the nanochannels is an important task to explain channel formation and the different regimes of channel fragmentation observed by Bhuyan *et al* [127]. This understanding is also essential to extend single shot nanochannel drilling to other transverse geometries such as channel drilling with vortex beams to obtain tubular channels or with curved accelerating beams. A key aspect is to control the *energy density* deposited during the laser-matter interaction.

## 605 Conclusion

The recent research developments on nondiffracting Bessel and accelerating beams open up new possibilities for fast processing of glasses. Nonlinear beam propagation in transparent dielectrics supports stationary regimes of Bessel and Bessel vortex filamentation. These regimes have generated unique pathways to control high aspect ratio ultrafast laser structuring and cutting. In contrast with Gaussian beams, the elongated focal line of nondiffracting beams allows for fast surface structuring at nanometric diameters without critical positioning of the beam on the workpiece. Thanks to their higher stability at high intensity, Bessel beams provide means to circumvent beam distortions that originate from nonlinear Kerr effect, ionization and defocusing by free carriers. This is of prior importance to control bulk processing of transparent materials.

The properties of accelerating beams are close to those of Bessel beams and very novel approaches for material separation and edge processing have been developed. These results have already attracted a wide interest in optics and photonics as well as in technological fields such as glass processing market. We anticipate that demanding applications will foster this research into solving those novel challenges.

*Acknowledgements.* The authors acknowledge funding from Region Franche-Comte, French ANR, contract 2011-BS04-010-01 NANOFLAM. This work has been performed in cooperation with the Labex ACTION program, contract ANR-11-LABX-01-01. This work was partly supported by the French RENATECH network.

## References

- [1] K. Sugioka, Y. Cheng, Ultrafast lasers—reliable tools for advanced materials processing, *Light: Science & Applications* 3 (4) (2014) e149. doi:10.1038/lisa.2014.30.  
URL <http://dx.doi.org/10.1038/lisa.2014.30>
- [2] N. Sanner, N. Huot, E. Audouard, C. Larat, J.-P. Huignard, B. Loiseaux, Programmable focal spot shaping of amplified femtosecond laser pulses, *Optics Letters* 30 (12) (2005) 1479. doi:10.1364/ol.30.001479.  
URL <http://dx.doi.org/10.1364/ol.30.001479>
- [3] S. Hasegawa, Y. Hayasaki, N. Nishida, Holographic femtosecond laser processing with multiplexed phase fresnel lenses, *Optics Letters* 31 (11) (2006) 1705. doi:10.1364/ol.31.001705.  
URL <http://dx.doi.org/10.1364/ol.31.001705>
- [4] M. Duocastella, C. Arnold, Bessel and annular beams for materials processing, *Laser & Photonics Reviews* 6 (5) (2012) 607–621. doi:10.1002/lpor.201100031.  
URL <http://dx.doi.org/10.1002/lpor.201100031>

- 645 [5] H. E. Hernández-Figueroa, M. Zamboni-Rached, E. Recami (Eds.), *Localized Waves*, Wiley-Blackwell, 2008. doi:10.1002/9780470168981.  
URL <http://dx.doi.org/10.1002/9780470168981>
- [6] H. E. Hernández-Figueroa, E. Recami, M. Zamboni-Rached (Eds.), *Non-Diffracting Waves*, Wiley-Blackwell, 2013. doi:10.1002/9783527671519.  
650 URL <http://dx.doi.org/10.1002/9783527671519>
- [7] G. A. Siviloglou, D. N. Christodoulides, Accelerating finite energy airy beams, *Optics Letters* 32 (8) (2007) 979. doi:10.1364/ol.32.000979.  
URL <http://dx.doi.org/10.1364/ol.32.000979>
- [8] D. McGloin, K. Dholakia, Bessel beams: Diffraction in a new  
655 light, *Contemporary Physics* 46 (1) (2005) 15–28. doi:10.1080/0010751042000275259.  
URL <http://dx.doi.org/10.1080/0010751042000275259>
- [9] J. H. McLeod, The axicon: A new type of optical element, *Journal of the Optical Society of America* 44 (8) (1954) 592. doi:10.1364/josa.44.  
660 000592.  
URL <http://dx.doi.org/10.1364/josa.44.000592>
- [10] J. Durnin, J. J. Miceli, J. H. Eberly, Diffraction-free beams, *Phys. Rev. Lett.* 58 (15) (1987) 1499–1501. doi:10.1103/physrevlett.58.1499.  
URL <http://dx.doi.org/10.1103/physrevlett.58.1499>
- 665 [11] F. Gori, G. Guattari, C. Padovani, Bessel-gauss beams, *Optics Communications* 64 (6) (1987) 491–495. doi:10.1016/0030-4018(87)90276-8.  
URL [http://dx.doi.org/10.1016/0030-4018\(87\)90276-8](http://dx.doi.org/10.1016/0030-4018(87)90276-8)
- [12] G. Roy, R. Tremblay, Influence of the divergence of a laser beam on the axial intensity distribution of an axicon, *Optics Communications* 34 (1)  
670 (1980) 1–3. doi:10.1016/0030-4018(80)90145-5.  
URL [http://dx.doi.org/10.1016/0030-4018\(80\)90145-5](http://dx.doi.org/10.1016/0030-4018(80)90145-5)

- [13] V. Jarutis, R. Paškauskas, A. Stabinis, Focusing of laguerre–gaussian beams by axicon, *Optics Communications* 184 (1-4) (2000) 105–112. doi:10.1016/S0030-4018(00)00961-5.  
675 URL [http://dx.doi.org/10.1016/S0030-4018\(00\)00961-5](http://dx.doi.org/10.1016/S0030-4018(00)00961-5)
- [14] Y. Zhang, Analytical expression for the diffraction field of an axicon using the ray-tracing and interference method, *Applied Physics B* 90 (1) (2007) 93–96. doi:10.1007/s00340-007-2830-4.  
URL <http://dx.doi.org/10.1007/s00340-007-2830-4>
- 680 [15] A. Dudley, Y. Li, T. Mhlanga, M. Escuti, A. Forbes, Generating and measuring nondiffracting vector bessel beams, *Optics Letters* 38 (17) (2013) 3429. doi:10.1364/ol.38.003429.  
URL <http://dx.doi.org/10.1364/ol.38.003429>
- [16] S. M. Barnett, L. Allen, *Optical Angular Momentum*, Informa UK Limited, 2003.  
685
- [17] C. Xie, R. Giust, V. Jukna, L. Furfaro, M. Jacquot, P.-A. Lacourt, L. Froehly, J. Dudley, A. Couairon, F. Courvoisier, Light trajectory in bessel–gauss vortex beams, *Journal of the Optical Society of America A* 32 (7) (2015) 1313. doi:10.1364/josaa.32.001313.  
690 URL <http://dx.doi.org/10.1364/JOSAA.32.001313>
- [18] M. V. Berry, K. T. McDonald, Exact and geometrical optics energy trajectories in twisted beams, *Journal of Optics A: Pure and Applied Optics* 10 (3) (2008) 035005. doi:10.1088/1464-4258/10/3/035005.  
URL <http://dx.doi.org/10.1088/1464-4258/10/3/035005>
- 695 [19] T. Grosjean, S. S. Saleh, M. A. Suarez, I. A. Ibrahim, V. Piquerey, D. Charraut, P. Sandoz, Fiber microaxicons fabricated by a polishing technique for the generation of bessel-like beams, *Appl. Opt.* 46 (33) (2007) 8061. doi:10.1364/ao.46.008061.  
URL <http://dx.doi.org/10.1364/AO.46.008061>

- 700 [20] X. Tsampoula, K. Taguchi, T. Cizmár, V. Garces-Chavez, N. Ma, S. Mohanty, K. Mohanty, F. Gunn-Moore, K. Dholakia, Fibre based cellular transfection, *Opt. Express* 16 (21) (2008) 17007. doi:10.1364/oe.16.017007.  
URL <http://dx.doi.org/10.1364/OE.16.017007>
- 705 [21] C. Xie, M. Hu, Z. Xu, W. Wu, H. Gao, P. Qin, D. Zhang, B. Liu, C.-Y. Wang, High power femtosecond bessel-x pulses directly from a compact fiber laser system, *Appl. Phys. Lett.* 101 (15) (2012) 151111. doi:10.1063/1.4758687.  
URL <http://dx.doi.org/10.1063/1.4758687>
- 710 [22] A. Vasara, J. Turunen, A. T. Friberg, Realization of general nondiffracting beams with computer-generated holograms, *Journal of the Optical Society of America A* 6 (11) (1989) 1748. doi:10.1364/josaa.6.001748.  
URL <http://dx.doi.org/10.1364/JOSAA.6.001748>
- [23] N. Chattrapiban, E. A. Rogers, D. Cofield, I. Wendell T. Hill, R. Roy,  
715 Generation of nondiffracting bessel beams by use of a spatial light modulator, *Optics Letters* 28 (22) (2003) 2183. doi:10.1364/ol.28.002183.  
URL <http://dx.doi.org/10.1364/OL.28.002183>
- [24] T. Cizmár, V. Kollárová, X. Tsampoula, F. Gunn-Moore, W. Sibbett,  
720 Z. Bouchal, K. Dholakia, Generation of multiple bessel beams for a biophotonics workstation, *Opt. Express* 16 (18) (2008) 14024. doi:10.1364/oe.16.014024.  
URL <http://dx.doi.org/10.1364/OE.16.014024>
- [25] F. Courvoisier, P.-A. Lacourt, M. Jacquot, M. K. Bhuyan, L. Furfaro,  
725 J. M. Dudley, Surface nanoprocessing with nondiffracting femtosecond bessel beams, *Optics Letters* 34 (20) (2009) 3163. doi:10.1364/ol.34.003163.  
URL <http://dx.doi.org/10.1364/OL.34.003163>

- [26] J. Amako, D. Sawaki, E. Fujii, Microstructuring transparent materials by use of nondiffracting ultrashort pulse beams generated by diffractive optics, *Journal of the Optical Society of America B* 20 (12) (2003) 2562. doi:10.1364/josab.20.002562. URL <http://dx.doi.org/10.1364/JOSAB.20.002562>
- [27] S. N. Khonina, V. V. Kotlyar, R. V. Skidanov, V. A. Soifer, K. Jefimovs, J. Simonen, J. Turunen, Rotation of microparticles with bessel beams generated by diffractive elements, *Journal of Modern Optics* 51 (14) (2004) 2167–2184. doi:10.1080/09500340408232521. URL <http://dx.doi.org/10.1080/09500340408232521>
- [28] O. Brzobohatý, T. Čižmár, P. Zemánek, High quality quasi-bessel beam generated by round-tip axicon, *Opt. Express* 16 (17) (2008) 12688. doi:10.1364/oe.16.012688. URL <http://dx.doi.org/10.1364/OE.16.012688>
- [29] S. Akturk, C. L. Arnold, B. Prade, A. Mysyrowicz, Generation of high quality tunable bessel beams using a liquid-immersion axicon, *Optics Communications* 282 (16) (2009) 3206–3209. doi:10.1016/j.optcom.2009.05.026. URL <http://dx.doi.org/10.1016/j.optcom.2009.05.026>
- [30] E. McLeod, A. B. Hopkins, C. B. Arnold, Multiscale bessel beams generated by a tunable acoustic gradient index of refraction lens, *Optics Letters* 31 (21) (2006) 3155. doi:10.1364/ol.31.003155. URL <http://dx.doi.org/10.1364/OL.31.003155>
- [31] A. Marcinkevicius, S. Juodkazis, S. Matsuo, V. Mizeikis, H. Misawa, Application of bessel beams for microfabrication of dielectrics by femtosecond laser, *Jpn. J. Appl. Phys.* 40 (Part 2, No. 11A) (2001) L1197–L1199. doi:10.1143/jjap.40.L1197. URL <http://dx.doi.org/10.1143/JJAP.40.L1197>



- [32] J. Turunen, A. Vasara, A. T. Friberg, Holographic generation of diffraction-free beams, *Appl. Opt.* 27 (19) (1988) 3959. doi:10.1364/ao.27.003959.  
URL <http://dx.doi.org/10.1364/AO.27.003959>
- 760 [33] T. Čižmár, K. Dholakia, Tunable bessel light modes: engineering the axial propagation, *Opt. Express* 17 (18) (2009) 15558. doi:10.1364/oe.17.015558.  
URL <http://dx.doi.org/10.1364/OE.17.015558>
- [34] Y.-Y. Yu, D.-Z. Lin, L.-S. Huang, C.-K. Lee, Effect of subwavelength  
765 annular aperture diameter on the nondiffracting region of generated bessel beams, *Opt. Express* 17 (4) (2009) 2707. doi:10.1364/oe.17.002707.  
URL <http://dx.doi.org/10.1364/OE.17.002707>
- [35] C.-Y. Chen, Y.-H. Lee, C.-J. Chien, M.-H. Chung, Y.-Y. Yu, T.-H. Chen, C.-K. Lee, Generating a sub-wavelength bessel-like light beam using a  
770 tapered hollow tube, *Optics Letters* 37 (21) (2012) 4537. doi:10.1364/ol.37.004537.  
URL <http://dx.doi.org/10.1364/OL.37.004537>
- [36] C. F. Phelan, D. P. O'Dwyer, Y. P. Rakovich, J. F. Donegan, J. G. Lunney, Conical diffraction and bessel beam formation with a high optical quality  
775 biaxial crystal, *Opt. Express* 17 (15) (2009) 12891. doi:10.1364/oe.17.012891.  
URL <http://dx.doi.org/10.1364/OE.17.012891>
- [37] A. Hakola, S. Buchter, T. Kajava, H. Elfström, J. Simonen, P. Pekkari, J. Turunen, Bessel-gauss output beam from a diode-pumped nd:YAG  
780 laser, *Optics Communications* 238 (4-6) (2004) 335–340. doi:10.1016/j.optcom.2004.05.012.  
URL <http://dx.doi.org/10.1016/j.optcom.2004.05.012>
- [38] C. Paterson, R. Smith, Higher-order bessel waves produced by axicon-type computer-generated holograms, *Optics Communications* 124 (1-2) (1996)

- 785 121–130. doi:10.1016/0030-4018(95)00637-0.  
URL [http://dx.doi.org/10.1016/0030-4018\(95\)00637-0](http://dx.doi.org/10.1016/0030-4018(95)00637-0)
- [39] J. Arlt, K. Dholakia, Generation of high-order bessel beams by use of an axicon, *Optics Communications* 177 (1-6) (2000) 297–301. doi:10.1016/S0030-4018(00)00572-1.  
790 URL [http://dx.doi.org/10.1016/S0030-4018\(00\)00572-1](http://dx.doi.org/10.1016/S0030-4018(00)00572-1)
- [40] P. Saari, K. Reivelt, Evidence of *X*-shaped propagation-invariant localized light waves, *Phys. Rev. Lett.* 79 (1997) 4135–4138. doi:10.1103/PhysRevLett.79.4135.  
URL <http://link.aps.org/doi/10.1103/PhysRevLett.79.4135>
- 795 [41] P. Fischer, H. Little, R. L. Smith, C. Lopez-Mariscal, C. T. A. Brown, W. Sibbett, K. Dholakia, Wavelength dependent propagation and reconstruction of white light bessel beams, *Journal of Optics A: Pure and Applied Optics* 8 (5) (2006) 477–482. doi:10.1088/1464-4258/8/5/018.  
URL <http://dx.doi.org/10.1088/1464-4258/8/5/018>
- 800 [42] M. Clerici, D. Faccio, A. Lotti, E. Rubino, O. Jedrkiewicz, J. Biegert, P. D. Trapani, Finite-energy, accelerating bessel pulses, *Opt. Express* 16 (24) (2008) 19807. doi:10.1364/oe.16.019807.  
URL <http://dx.doi.org/10.1364/OE.16.019807>
- [43] I. Alexeev, K. Y. Kim, H. M. Milchberg, Measurement of the superluminal  
805 group velocity of an ultrashort bessel beam pulse, *Phys. Rev. Lett.* 88 (7) (2002) 073901. doi:10.1103/physrevlett.88.073901.  
URL <http://dx.doi.org/10.1103/PhysRevLett.88.073901>
- [44] H. Valtna-Lukner, P. Bownan, M. Lõhmus, P. Piksarv, R. Trebino, P. Saari, Direct spatiotemporal measurements of accelerating ultrashort  
810 bessel-type light bullets, *Opt. Express* 17 (17) (2009) 14948. doi:10.1364/oe.17.014948.  
URL <http://dx.doi.org/10.1364/OE.17.014948>

- [45] F. Belgiorno, S. L. Cacciatori, M. Clerici, V. Gorini, G. Ortenzi, L. Rizzi, E. Rubino, V. G. Sala, D. Faccio, Hawking radiation from ultrashort laser pulse filaments, *Phys. Rev. Lett.* 105 (20) (2010) 203901. doi:10.1103/physrevlett.105.203901.  
URL <http://dx.doi.org/10.1103/PhysRevLett.105.203901>
- [46] C. J. R. Sheppard, Bessel pulse beams and focus wave modes, *Journal of the Optical Society of America A* 18 (10) (2001) 2594. doi:10.1364/josaa.18.002594.  
URL <http://dx.doi.org/10.1364/JOSAA.18.002594>
- [47] L. Froehly, M. Jacquot, P. A. Lacourt, J. M. Dudley, F. Courvoisier, Spatiotemporal structure of femtosecond bessel beams from spatial light modulators, *Journal of the Optical Society of America A* 31 (4) (2014) 790. doi:10.1364/josaa.31.000790.  
URL <http://dx.doi.org/10.1364/JOSAA.31.000790>
- [48] I. Ouadghiri-Idrissi, A. Gil-Villalba, L. Froehly, R. Giust, L. Furfaro, M. Jacquot, P.-A. Lacourt, J. Dudley, F. Courvoisier, to be submitted (2015).
- [49] V. Jarutis, A. Matijošius, P. D. Trapani, A. Piskarskas, Spiraling zero-order bessel beam, *Optics Letters* 34 (14) (2009) 2129. doi:10.1364/ol.34.002129.  
URL <http://dx.doi.org/10.1364/OL.34.002129>
- [50] C. Paterson, R. Smith, Helicon waves: propagation-invariant waves in a rotating coordinate system, *Optics Communications* 124 (1-2) (1996) 131–140. doi:10.1016/0030-4018(95)00636-2.  
URL [http://dx.doi.org/10.1016/0030-4018\(95\)00636-2](http://dx.doi.org/10.1016/0030-4018(95)00636-2)
- [51] C. Vetter, T. Eichelkraut, M. Ornigotti, A. Szameit, Generalized radially self-accelerating helicon beams, *Phys. Rev. Lett.* 113 (18) (2014) 183901. doi:10.1103/physrevlett.113.183901.  
URL <http://dx.doi.org/10.1103/PhysRevLett.113.183901>

- [52] J. Zhao, I. D. Chremmos, D. Song, D. N. Christodoulides, N. K. Efremidis, Z. Chen, Curved singular beams for three-dimensional particle manipulation, *Sci. Rep.* 5 (2015) 12086. doi:10.1038/srep12086.  
845 URL <http://dx.doi.org/10.1038/srep12086>
- [53] R. Grunwald, U. Neumann, U. Griebner, G. Steinmeyer, G. Stibenz, M. Bock, V. Kebbel, Self-reconstruction of pulsed optical x-waves, in: *Localized Waves*, Wiley-Blackwell, 2008, pp. 299–313. doi:10.1002/9780470168981.ch11.  
850 URL <http://dx.doi.org/10.1002/9780470168981.ch11>
- [54] V. Garcés-Chávez, D. McGloin, H. Melville, W. Sibbett, K. Dholakia, Simultaneous micromanipulation in multiple planes using a self-reconstructing light beam, *Nature* 419 (6903) (2002) 145–147. doi:10.1038/nature01007.  
855 URL <http://dx.doi.org/10.1038/nature01007>
- [55] Z. Bouchal, J. Wagner, M. Chlup, Self-reconstruction of a distorted nondiffracting beam, *Optics Communications* 151 (4-6) (1998) 207–211. doi:10.1016/S0030-4018(98)00085-6.  
URL [http://dx.doi.org/10.1016/S0030-4018\(98\)00085-6](http://dx.doi.org/10.1016/S0030-4018(98)00085-6)
- 860 [56] J. Broky, G. A. Siviloglou, A. Dogariu, D. N. Christodoulides, Self-healing properties of optical airy beams, *Opt. Express* 16 (17) (2008) 12880. doi:10.1364/oe.16.012880.  
URL <http://dx.doi.org/10.1364/OE.16.012880>
- [57] W. Nelson, J. P. Palastro, C. C. Davis, P. Sprangle, Propagation of bessel and airy beams through atmospheric turbulence, *Journal of the Optical Society of America A* 31 (3) (2014) 603. doi:10.1364/josaa.31.000603.  
865 URL <http://dx.doi.org/10.1364/JOSAA.31.000603>
- [58] E. Mcleod, C. B. Arnold, Subwavelength direct-write nanopatterning using optically trapped microspheres, *Nature Nanotech* 3 (7) (2008) 413–417.

- 870       doi:10.1038/nnano.2008.150.  
URL <http://dx.doi.org/10.1038/nnano.2008.150>
- [59] K. Dholakia, T. Čižmár, Shaping the future of manipulation, *Nature Photonics* 5 (6) (2011) 335–342. doi:10.1038/nphoton.2011.80.  
URL <http://dx.doi.org/10.1038/nphoton.2011.80>
- 875 [60] T. A. Planchon, L. Gao, D. E. Milkie, M. W. Davidson, J. A. Galbraith, C. G. Galbraith, E. Betzig, Rapid three-dimensional isotropic imaging of living cells using bessel beam plane illumination, *Nature Methods* 8 (5) (2011) 417–423. doi:10.1038/nmeth.1586.  
URL <http://dx.doi.org/10.1038/nmeth.1586>
- 880 [61] F. O. Fahrbach, P. Simon, A. Rohrbach, Microscopy with self-reconstructing beams, *Nature Photonics* 4 (11) (2010) 780–785. doi:10.1038/nphoton.2010.204.  
URL <http://dx.doi.org/10.1038/NPHOTON.2010.204>
- [62] J. Amako, K. Yoshimura, D. Sawaki, T. Shimoda, Laser-based micro-  
885 processes using diffraction-free beams generated by diffractive axicons, in: J. Fieret, P. R. Herman, T. Okada, C. B. Arnold, F. G. Bachmann, W. Hoving, K. Washio, Y. Lu, D. B. Geohegan, F. Trager, J. J. Dubowski (Eds.), *Photon Processing in Microelectronics and Photonics IV*, SPIE-Intl Soc Optical Eng, 2005. doi:10.1117/12.585135.  
890 URL <http://dx.doi.org/10.1117/12.585135>
- [63] X. Tsampoula, V. Garceés-Chavez, M. Comrie, D. J. Stevenson, B. Agate, C. T. A. Brown, F. Gunn-Moore, K. Dholakia, Femtosecond cellular transfection using a nondiffracting light beam, *Appl. Phys. Lett.* 91 (5) (2007) 053902. doi:10.1063/1.2766835.  
895 URL <http://dx.doi.org/10.1063/1.2766835>
- [64] A. P. Joglekar, H. Liu, G. J. Spooner, E. Meyhofer, G. Mourou, A. J. Hunt, A study of the deterministic character of optical damage by femtosecond

laser pulses and applications to nanomachining, *Applied Physics B-Lasers And Optics* 77 (1) (2003) 25–30.

- 900 [65] L. Mercadier, J. Peng, Y. Sultan, T. A. Davis, D. M. Rayner, P. B. Corkum, Femtosecond laser desorption of ultrathin polymer films from a dielectric surface, *Appl. Phys. Lett.* 103 (6) (2013) 061107. doi:10.1063/1.4817816.  
URL <http://dx.doi.org/10.1063/1.4817816>
- 905 [66] B. Yalizay, T. Ersoy, B. Soylu, S. Akturk, Fabrication of nanometer-size structures in metal thin films using femtosecond laser bessel beams, *Appl. Phys. Lett.* 100 (3) (2012) 031104. doi:10.1063/1.3678030.  
URL <http://dx.doi.org/10.1063/1.3678030>
- [67] R. Sahin, Y. Morova, E. Simsek, S. Akturk, Bessel-beam-written nanoslit  
910 arrays and characterization of their optical response, *Appl. Phys. Lett.* 102 (19) (2013) 193106. doi:10.1063/1.4805358.  
URL <http://dx.doi.org/10.1063/1.4805358>
- [68] Y.-Y. Yu, C.-K. Chang, M.-W. Lai, L.-S. Huang, C.-K. Lee, Laser ablation  
915 of silicon using a bessel-like beam generated by a subwavelength annular aperture structure, *Appl. Opt.* 50 (34) (2011) 6384. doi:10.1364/ao.50.006384.  
URL <http://dx.doi.org/10.1364/A0.50.006384>
- [69] B. Wetzell, C. Xie, P.-A. Lacourt, J. M. Dudley, F. Courvoisier, Femtosecond laser fabrication of micro and nano-disks in single layer graphene  
920 using vortex bessel beams, *Appl. Phys. Lett.* 103 (24) (2013) 241111. doi:10.1063/1.4846415.  
URL <http://dx.doi.org/10.1063/1.4846415>
- [70] R. Sahin, E. Simsek, S. Akturk, Nanoscale patterning of graphene through  
925 femtosecond laser ablation, *Appl. Phys. Lett.* 104 (5) (2014) 053118. doi:10.1063/1.4864616.  
URL <http://dx.doi.org/10.1063/1.4864616>

- [71] A. Gil-Villalba, C. Xie, R. Salut, L. Furfaro, R. Giust, M. Jacquot, P. A. Lacourt, J. M. Dudley, F. Courvoisier, Deviation from threshold model in ultrafast laser ablation of graphene at sub-micron scale, *Appl. Phys. Lett.* 107 (6) (2015) 061103. doi:10.1063/1.4928391.  
930 URL <http://dx.doi.org/10.1063/1.4928391>
- [72] W. Cheng, P. Polynkin, Micromachining of borosilicate glass surfaces using femtosecond higher-order bessel beams, *Journal of the Optical Society of America B* 31 (11) (2014) C48. doi:10.1364/josab.31.000c48.  
935 URL <http://dx.doi.org/10.1364/JOSAB.31.000C48>
- [73] R. Inoue, K. Takakusaki, Y. Takagi, T. Yagi, Micro-ablation on silicon by femtosecond laser pulses focused with an axicon assisted with a lens, *Applied Surface Science* 257 (2) (2010) 476–480. doi:10.1016/j.apsusc.2010.07.016.  
940 URL <http://dx.doi.org/10.1016/j.apsusc.2010.07.016>
- [74] Y. Matsuoka, Y. Kizuka, T. Inoue, The characteristics of laser micro drilling using a bessel beam, *Appl. Phys. A* 84 (4) (2006) 423–430. doi:10.1007/s00339-006-3629-6.  
URL <http://dx.doi.org/10.1007/s00339-006-3629-6>
- 945 [75] A. Salimnia, J.-P. Bérubé, R. Vallée, Refractive index-modified structures in glass written by 266nm fs laser pulses, *Opt. Express* 20 (25) (2012) 27410. doi:10.1364/oe.20.027410.  
URL <http://dx.doi.org/10.1364/OE.20.027410>
- [76] D. Grojo, S. Leyder, P. Delaporte, W. Marine, M. Sentis, O. Utéza, Long-wavelength multiphoton ionization inside band-gap solids, *Phys. Rev. B* 88 (19) (2013) 195135. doi:10.1103/physrevb.88.195135.  
950 URL <http://dx.doi.org/10.1103/PhysRevB.88.195135>
- [77] S. Mao, F. Quéré, S. Guizard, X. Mao, R. Russo, G. Petite, P. Martin, Dynamics of femtosecond laser interactions with dielectrics, *Appl. Phys.*

- 955 A 79 (7) (2004) 1695–1709. doi:10.1007/s00339-004-2684-0.  
URL <http://dx.doi.org/10.1007/s00339-004-2684-0>
- [78] R. R. Gattass, E. Mazur, Femtosecond laser micromachining in transparent materials, *Nature Photonics* 2 (4) (2008) 219–225.
- [79] G. D. Valle, R. Osellame, P. Laporta, Micromachining of photonic devices  
960 by femtosecond laser pulses, *Journal of Optics A: Pure and Applied Optics*  
11 (1) (2008) 013001. doi:10.1088/1464-4258/11/1/013001.  
URL <http://dx.doi.org/10.1088/1464-4258/11/1/013001>
- [80] E. Gamaly, A. Rode, Physics of ultra-short laser interaction with matter:  
From phonon excitation to ultimate transformations, *Progress in Quantum*  
965 *Electronics* 37 (5) (2013) 215–323. doi:10.1016/j.pquantelec.  
2013.05.001.  
URL <http://dx.doi.org/10.1016/j.pquantelec.2013.05.001>
- [81] L. Sudrie, A. Couairon, M. Franco, B. Lamouroux, B. Prade, S. Tzortzakis,  
A. Mysyrowicz, Femtosecond laser-induced damage and filamentary  
970 propagation in fused silica, *Phys. Rev. Lett.* 89 (18) (2002) 4135.  
doi:10.1103/physrevlett.89.186601.  
URL <http://dx.doi.org/10.1103/physrevlett.89.186601>
- [82] A. Couairon, A. Mysyrowicz, Femtosecond filamentation in transparent  
media, *Physics Reports* 441 (2-4) (2007) 47–189. doi:10.1016/j.  
975 *physrep*.2006.12.005.  
URL <http://dx.doi.org/10.1016/j.physrep.2006.12.005>
- [83] A. Couairon, L. Sudrie, M. Franco, B. Prade, A. Mysyrowicz, Filamentation  
and damage in fused silica induced by tightly focused femtosecond  
laser pulses, *Phys. Rev. B* 71 (12) (2005) 125435. doi:10.1103/physrevb.  
980 *71.125435*.  
URL <http://dx.doi.org/10.1103/PhysRevB.71.125435>



- [84] S. Winkler, I. Burakov, R. Stoian, N. Bulgakova, A. Husakou, A. Mermillod-Blondin, A. Rosenfeld, D. Ashkenasi, I. Hertel, Transient response of dielectric materials exposed to ultrafast laser radiation, Appl. Phys. A 84 (4) (2006) 413–422. doi:10.1007/s00339-006-3644-7.  
985 URL <http://dx.doi.org/10.1007/s00339-006-3644-7>
- [85] N. M. Bulgakova, V. P. Zhukov, Y. P. Meshcheryakov, L. Gemini, J. Brajer, D. Rostohar, T. Mocek, Pulsed laser modification of transparent dielectrics: what can be foreseen and predicted by numerical simulations?, Journal of the Optical Society of America B 31 (11) (2014) C8.  
990 doi:10.1364/josab.31.0000c8.  
URL <http://dx.doi.org/10.1364/JOSAB.31.0000C8>
- [86] E. G. Gamaly, A. V. Rode, Transient optical properties of dielectrics and semiconductors excited by an ultrashort laser pulse, Journal of the Optical Society of America B 31 (11) (2014) C36. doi:10.1364/josab.31.0000c36.  
995 URL <http://dx.doi.org/10.1364/JOSAB.31.0000C36>
- [87] P. Balling, J. Schou, Femtosecond-laser ablation dynamics of dielectrics: basics and applications for thin films, Rep. Prog. Phys. 76 (3) (2013) 036502. doi:10.1088/0034-4885/76/3/036502.  
1000 URL <http://dx.doi.org/10.1088/0034-4885/76/3/036502>
- [88] B. Chimier, O. Utéza, N. Sanner, M. Sentis, T. Itina, P. Lassonde, F. Légaré, F. Vidal, J. C. Kieffer, Damage and ablation thresholds of fused-silica in femtosecond regime, Phys. Rev. B 84 (9) (2011) 094104.  
1005 doi:10.1103/physrevb.84.094104.  
URL <http://dx.doi.org/10.1103/PhysRevB.84.094104>
- [89] A. Braun, G. Korn, X. Liu, D. Du, J. Squier, G. Mourou, Self-channeling of high-peak-power femtosecond laser pulses in air, Optics Letters 20 (1) (1995) 73. doi:10.1364/ol.20.000073.  
1010 URL <http://dx.doi.org/10.1364/OL.20.000073>

- [90] E. T. J. Nibbering, P. F. Curley, G. Grillon, B. S. Prade, M. A. Franco, F. Salin, A. Mysyrowicz, Conical emission from self-guided femtosecond pulses in air, *Optics Letters* 21 (1) (1996) 62. doi:10.1364/ol.21.000062.  
1015 URL <http://dx.doi.org/10.1364/OL.21.000062>
- [91] J. Kasparian, J.-P. Wolf, Physics and applications of atmospheric non-linear optics and filamentation, *Opt. Express* 16 (1) (2008) 466. doi:10.1364/oe.16.000466.  
URL <http://dx.doi.org/10.1364/OE.16.000466>
- 1020 [92] S. Tzortzakis, L. Sudrie, M. Franco, B. Prade, A. Mysyrowicz, A. Coua-iron, L. Bergé, Self-guided propagation of ultrashort IR laser pulses in fused silica, *Phys. Rev. Lett.* 87 (21) (2001) 213902. doi:10.1103/physrevlett.87.213902.  
URL <http://dx.doi.org/10.1103/PhysRevLett.87.213902>
- 1025 [93] Z. Wu, H. Jiang, L. Luo, H. Guo, H. Yang, Q. Gong, Multiple foci and a long filament observed with focused femtosecond pulse propagation in fused silica, *Optics Letters* 27 (6) (2002) 448. doi:10.1364/ol.27.000448.  
URL <http://dx.doi.org/10.1364/OL.27.000448>
- 1030 [94] A. Zoubir, L. Shah, K. Richardson, M. Richardson, Practical uses of femtosecond laser micro-materials processing, *Applied Physics A* 77 (2) (2003) 311–315. doi:10.1007/s00339-003-2121-9.  
URL <http://dx.doi.org/10.1007/s00339-003-2121-9>
- 1035 [95] S. Tzortzakis, D. G. Papazoglou, I. Zergioti, Long-range filamentary propagation of subpicosecond ultraviolet laser pulses in fused silica, *Optics Letters* 31 (6) (2006) 796. doi:10.1364/ol.31.000796.  
URL <http://dx.doi.org/10.1364/OL.31.000796>
- [96] I. Blonskyi, V. Kadan, O. Shpotyuk, M. Iovu, P. Korenyuk, I. Dmitruk, Filament-induced self-written waveguides in glassy as4ge30s66, *Applied*

- 1040 Physics B 104 (4) (2011) 951–956. doi:10.1007/s00340-011-4390-x.  
URL <http://dx.doi.org/10.1007/s00340-011-4390-x>
- [97] D. G. Papazoglou, S. Tzortzakis, Physical mechanisms of fused silica re-  
structuring and densification after femtosecond laser excitation [invited],  
Optical Materials Express 1 (4) (2011) 625. doi:10.1364/ome.1.000625.  
1045 URL <http://dx.doi.org/10.1364/OME.1.000625>
- [98] Y. V. White, X. Li, Z. Sikorski, L. M. Davis, W. Hofmeister, Single-pulse  
ultrafast-laser machining of high aspect nano-holes at the surface of SiO<sub>2</sub>,  
Opt. Express 16 (19) (2008) 14411. doi:10.1364/oe.16.014411.  
URL <http://dx.doi.org/10.1364/OE.16.014411>
- 1050 [99] Z. Wu, H. Jiang, Q. Sun, H. Yang, Q. Gong, Filamentation and temporal  
reshaping of a femtosecond pulse in fused silica, Phys. Rev. A 68 (6) (2003)  
063820. doi:10.1103/physreva.68.063820.  
URL <http://dx.doi.org/10.1103/PhysRevA.68.063820>
- [100] S. Onda, W. Watanabe, K. Yamada, K. Itoh, J. Nishii, Study of filamen-  
1055 tary damage in synthesized silica induced by chirped femtosecond laser  
pulses, Journal of the Optical Society of America B 22 (11) (2005) 2437.  
doi:10.1364/josab.22.002437.  
URL <http://dx.doi.org/10.1364/JOSAB.22.002437>
- [101] S. Sowa, W. Watanabe, J. Nishii, K. Itoh, Filamentary cavity formation in  
1060 poly(methyl methacrylate) by single femtosecond pulse, Applied Physics  
A 81 (8) (2005) 1587–1590. doi:10.1007/s00339-005-3325-y.  
URL <http://dx.doi.org/10.1007/s00339-005-3325-y>
- [102] F. Ahmed, M. Lee, H. Sekita, T. Sumiyoshi, M. Kamata, Display glass cut-  
ting by femtosecond laser induced single shot periodic void array, Applied  
1065 Physics A 93 (1) (2008) 189–192. doi:10.1007/s00339-008-4672-2.  
URL <http://dx.doi.org/10.1007/s00339-008-4672-2>

- [103] J. F. Herbstman, A. J. Hunt, High-aspect ratio nanochannel formation by single femtosecond laser pulses, *Opt. Express* 18 (16) (2010) 16840. doi:10.1364/oe.18.016840.  
1070 URL <http://dx.doi.org/10.1364/OE.18.016840>
- [104] F. Ahmed, M. S. Ahsan, M. S. Lee, M. B. G. Jun, Near-field modification of femtosecond laser beam to enhance single-shot pulse filamentation in glass medium, *Appl. Phys. A* 114 (4) (2013) 1161–1165. doi:10.1007/s00339-013-7705-4.  
1075 URL <http://dx.doi.org/10.1007/s00339-013-7705-4>
- [105] F. Courvoisier, V. Boutou, J. Kasparian, E. Salmon, G. Mejean, J. Yu, J. Wolf, Ultraintense light filaments transmitted through clouds, *Appl. Phys. Lett.* 83 (2) (2003) 213. doi:10.1063/1.1592615.  
URL <http://dx.doi.org/10.1063/1.1592615>
- 1080 [106] M. Kolesik, J. V. Moloney, Self-healing femtosecond light filaments, *Optics Letters* 29 (6) (2004) 590. doi:10.1364/ol.29.000590.  
URL <http://dx.doi.org/10.1364/OL.29.000590>
- [107] A. Dubietis, E. Gaižauskas, G. Tamošauskas, P. D. Trapani, Light filaments without self-channeling, *Phys. Rev. Lett.* 92 (25) (2004) 253903.  
1085 doi:10.1103/physrevlett.92.253903.  
URL <http://dx.doi.org/10.1103/PhysRevLett.92.253903>
- [108] A. Dubietis, E. Kucinskas, G. Tamosauskas, E. Gaizauskas, M. A. Porras, P. D. Trapani, Self-reconstruction of light filaments, *Optics Letters* 29 (24) (2004) 2893. doi:10.1364/ol.29.002893.  
1090 URL <http://dx.doi.org/10.1364/OL.29.002893>
- [109] A. Couairon, E. Gaižauskas, D. Faccio, A. Dubietis, P. D. Trapani, Nonlinear x-wave formation by femtosecond filamentation in kerr media, *Physical Review E* 73 (1) (2006) 016608. doi:10.1103/physreve.73.016608.  
URL <http://dx.doi.org/10.1103/PhysRevE.73.016608>

- 1095 [110] M. A. Porras, A. Parola, Nonlinear unbalanced bessel beams in the col-  
lapse of gaussian beams arrested by nonlinear losses, *Optics Letters* 33 (15)  
(2008) 1738. doi:10.1364/ol.33.001738.  
URL <http://dx.doi.org/10.1364/OL.33.001738>
- [111] P. Polesana, M. Franco, A. Couairon, D. Faccio, P. D. Trapani, Filamen-  
1100 tation in kerr media from pulsed bessel beams, *Phys. Rev. A* 77 (4) (2008)  
043814. doi:10.1103/physreva.77.043814.  
URL <http://dx.doi.org/10.1103/PhysRevA.77.043814>
- [112] D. E. Roskey, M. Kolesik, J. V. Moloney, E. M. Wright, Self-action and  
regularized self-guiding of pulsed bessel-like beams in air, *Opt. Express*  
1105 15 (16) (2007) 9893. doi:10.1364/oe.15.009893.  
URL <http://dx.doi.org/10.1364/OE.15.009893>
- [113] E. Gaizauskas, E. Vanagas, V. Jarutis, S. Juodkazis, V. Mizeikis, H. Mi-  
sawa, Discrete damage traces from filamentation of gauss-bessel pulses,  
*Optics Letters* 31 (1) (2006) 80. doi:10.1364/ol.31.000080.  
1110 URL <http://dx.doi.org/10.1364/OL.31.000080>
- [114] M. A. Porras, A. Parola, D. Faccio, A. Dubietis, P. D. Trapani, Non-  
linear unbalanced bessel beams: Stationary conical waves supported by  
nonlinear losses, *Phys. Rev. Lett.* 93 (15) (2004) 153902. doi:10.1103/  
*physrevlett*.93.153902.  
1115 URL <http://dx.doi.org/10.1103/PhysRevLett.93.153902>
- [115] P. Polesana, A. Dubietis, M. A. Porras, E. Kučinskas, D. Faccio, A. Coua-  
iron, P. D. Trapani, Near-field dynamics of ultrashort pulsed bessel beams  
in media with kerr nonlinearity, *Physical Review E* 73 (5) (2006) 056612.  
doi:10.1103/physreve.73.056612.  
1120 URL <http://dx.doi.org/10.1103/PhysRevE.73.056612>
- [116] P. Polynkin, M. Kolesik, A. Roberts, D. Faccio, P. D. Trapani, J. Moloney,  
Generation of extended plasma channels in air using femtosecond bessel

- beams, *Opt. Express* 16 (20) (2008) 15733. doi:10.1364/oe.16.015733.  
URL <http://dx.doi.org/10.1364/OE.16.015733>
- 1125 [117] S. Akturk, B. Zhou, M. Franco, A. Couairon, A. Mysyrowicz, Generation of long plasma channels in air by focusing ultrashort laser pulses with an axicon, *Optics Communications* 282 (1) (2009) 129–134. doi:10.1016/j.optcom.2008.09.048.  
URL <http://dx.doi.org/10.1016/j.optcom.2008.09.048>
- 1130 [118] P. Polesana, D. Faccio, P. D. Trapani, A. Dubietis, A. Piskarskas, A. Couairon, M. A. Porras, High localization, focal depth and contrast by means of nonlinear bessel beams, *Opt. Express* 13 (16) (2005) 6160. doi:10.1364/opex.13.006160.  
URL <http://dx.doi.org/10.1364/OPEX.13.006160>
- 1135 [119] D. Faccio, E. Rubino, A. Lotti, A. Couairon, A. Dubietis, G. Tamošauskas, D. G. Papazoglou, S. Tzortzakis, Nonlinear light-matter interaction with femtosecond high-angle bessel beams, *Phys. Rev. A* 85 (3) (2012) 033829. doi:10.1103/physreva.85.033829.  
URL <http://dx.doi.org/10.1103/PhysRevA.85.033829>
- 1140 [120] P. Polesana, A. Couairon, D. Faccio, A. Parola, M. A. Porras, A. Dubietis, A. Piskarskas, P. D. Trapani, Observation of conical waves in focusing, dispersive, and dissipative kerr media, *Phys. Rev. Lett.* 99 (22) (2007) 223902. doi:10.1103/physrevlett.99.223902.  
URL <http://dx.doi.org/10.1103/PhysRevLett.99.223902>
- 1145 [121] L. Z. L. Zhao, F. W. F. Wang, L. J. L. Jiang, Y. L. Y. Lu, W. Z. W. Zhao, J. X. J. Xie, X. L. X. Li, Femtosecond bessel-beam-assisted high-aspect-ratio microgroove fabrication in fused silica, *Chinese Optics Letters* 13 (4) (2015) 041405–41408. doi:10.3788/col201513.041405.  
URL <http://dx.doi.org/10.3788/COL201513.041405>
- 1150 [122] M. Mikutis, T. Kudrius, G. Šlekys, D. Paipulas, S. Juodkazis, High 90% efficiency bragg gratings formed in fused silica by femtosecond gauss-bessel

- laser beams, *Optical Materials Express* 3 (11) (2013) 1862. doi:10.1364/ome.3.001862.  
URL <http://dx.doi.org/10.1364/OME.3.001862>
- 1155 [123] M. K. Bhuyan, F. Courvoisier, P. A. Lacourt, M. Jacquot, R. Salut, L. Furfaro, J. M. Dudley, High aspect ratio nanochannel machining using single shot femtosecond bessel beams, *Appl. Phys. Lett.* 97 (8) (2010) 081102. doi:10.1063/1.3479419.  
URL <http://dx.doi.org/10.1063/1.3479419>
- 1160 [124] M. K. Bhuyan, F. Courvoisier, P.-A. Lacourt, M. Jacquot, L. Furfaro, M. J. Withford, J. M. Dudley, High aspect ratio taper-free microchannel fabrication using femtosecond bessel beams, *Opt. Express* 18 (2) (2010) 566. doi:10.1364/oe.18.000566.  
URL <http://dx.doi.org/10.1364/OE.18.000566>
- 1165 [125] O. Jedrkiewicz, S. Minardi, A. Couairon, V. Jukna, M. Selva, P. D. Trapani, Plasma absorption evidence via chirped pulse spectral transmission measurements, *Appl. Phys. Lett.* 106 (23) (2015) 231101. doi:10.1063/1.4922371.  
URL <http://dx.doi.org/10.1063/1.4922371>
- 1170 [126] S. Juodkazis, K. Nishimura, S. Tanaka, H. Misawa, E. G. Gamaly, B. Luther-Davies, L. Hallo, P. Nicolai, V. T. Tikhonchuk, Laser-induced microexplosion confined in the bulk of a sapphire crystal: Evidence of multimegabar pressures, *Phys. Rev. Lett.* 96 (16) (2006) 166101. doi:10.1103/physrevlett.96.166101.  
1175 URL <http://dx.doi.org/10.1103/PhysRevLett.96.166101>
- [127] M. K. Bhuyan, P. K. Velpula, J. P. Colombier, T. Olivier, N. Faure, R. Stoian, Single-shot high aspect ratio bulk nanostructuring of fused silica using chirp-controlled ultrafast laser bessel beams, *Appl. Phys. Lett.* 104 (2) (2014) 021107. doi:10.1063/1.4861899.  
1180 URL <http://dx.doi.org/10.1063/1.4861899>

- [128] M. K. Bhuyan, O. Jedrkiewicz, V. Sabonis, M. Mikutis, S. Recchia, A. Aprea, M. Bollani, P. D. Trapani, High-speed laser-assisted cutting of strong transparent materials using picosecond bessel beams, *Appl. Phys. A* 120 (2) (2015) 443–446. doi:10.1007/s00339-015-9289-7.  
1185 URL <http://dx.doi.org/10.1007/s00339-015-9289-7>
- [129] P. K. Velpula, M. K. Bhuyan, C. Mauchair, J.-P. Colombier, R. Stoian, Role of free carriers excited by ultrafast bessel beams for submicron structuring applications, *Optical Engineering* 53 (7) (2014) 076108. doi:10.1117/1.oe.53.7.076108.  
1190 URL <http://dx.doi.org/10.1117/1.OE.53.7.076108>
- [130] S. Mitra, M. Chanal, R. Clady, A. Mouskeftaras, D. Grojo, Millijoule femtosecond micro-bessel beams for ultra-high aspect ratio machining, *Appl. Opt.* 54 (24) (2015) 7358. doi:10.1364/ao.54.007358.  
URL <http://dx.doi.org/10.1364/AO.54.007358>
- 1195 [131] N. S. Grigoryan, T. Zier, M. E. Garcia, E. S. Zijlstra, Ultrafast structural phenomena: theory of phonon frequency changes and simulations with code for highly excited valence electron systems, *Journal of the Optical Society of America B* 31 (11) (2014) C22. doi:10.1364/josab.31.000c22.  
URL <http://dx.doi.org/10.1364/JOSAB.31.000C22>
- 1200 [132] P. P. Rajeev, M. Gertsvolf, P. B. Corkum, D. M. Rayner, Field dependent avalanche ionization rates in dielectrics, *Phys. Rev. Lett.* 102 (8) (2009) 083001. doi:10.1103/physrevlett.102.083001.  
URL <http://dx.doi.org/10.1103/PhysRevLett.102.083001>
- 1205 [133] W.-J. Tsai, C.-J. Gu, C.-W. Cheng, J.-B. Horng, Internal modification for cutting transparent glass using femtosecond bessel beams, *Optical Engineering* 53 (5) (2013) 051503. doi:10.1117/1.oe.53.5.051503.  
URL <http://dx.doi.org/10.1117/1.OE.53.5.051503>
- [134] D. Grojo, A. Mouskeftaras, P. Delaporte, S. Lei, Limitations to laser machining of silicon using femtosecond micro-bessel beams in the infrared,



- 1210 J. Appl. Phys. 117 (15) (2015) 153105. doi:10.1063/1.4918669.  
URL <http://dx.doi.org/10.1063/1.4918669>
- [135] A. Vinçotte, L. Bergé, Femtosecond optical vortices in air, Phys. Rev. Lett. 95 (19) (2005) 193901. doi:10.1103/physrevlett.95.193901.  
URL <http://dx.doi.org/10.1103/PhysRevLett.95.193901>
- 1215 [136] L. T. Vuong, T. D. Grow, A. Ishaaya, A. L. Gaeta, G. W. 't Hooft, E. R. Eliel, G. Fibich, Collapse of optical vortices, Phys. Rev. Lett. 96 (13) (2006) 133901. doi:10.1103/physrevlett.96.133901.  
URL <http://dx.doi.org/10.1103/PhysRevLett.96.133901>
- [137] P. Polynkin, C. Ament, J. V. Moloney, Self-focusing of ultraintense femtosecond optical vortices in air, Phys. Rev. Lett. 111 (2) (2013) 023901. doi:10.1103/physrevlett.111.023901.  
1220 URL <http://dx.doi.org/10.1103/PhysRevLett.111.023901>
- [138] V. Jukna, C. Milián, C. Xie, T. Itina, J. Dudley, F. Courvoisier, A. Coua-iron, Filamentation with nonlinear bessel vortices, Opt. Express 22 (21) (2014) 25410. doi:10.1364/oe.22.025410.  
1225 URL <http://dx.doi.org/10.1364/OE.22.025410>
- [139] M. A. Porras, C. Ruiz-Jiménez, Nondiffracting and nonattenuating vortex light beams in media with nonlinear absorption of orbital angular momentum, Journal of the Optical Society of America B 31 (11) (2014) 2657. doi:10.1364/josab.31.002657.  
1230 URL <http://dx.doi.org/10.1364/JOSAB.31.002657>
- [140] J. Fan, E. Parra, I. Alexeev, K. Y. Kim, H. M. Milchberg, L. Y. Margolin, L. N. Pyatnitskii, Tubular plasma generation with a high-power hollow bessel beam, Physical Review E 62 (6) (2000) R7603–R7606. doi:10.1103/physreve.62.r7603.  
1235 URL <http://dx.doi.org/10.1103/physreve.62.r7603>

- [141] C. Xie, V. Jukna, C. Milián, R. Giust, I. Ouadghiri-Idrissi, T. Itina, J. M. Dudley, A. Couairon, F. Courvoisier, Tubular filamentation for laser material processing, *Sci. Rep.* 5 (2015) 8914. doi:10.1038/srep08914.  
1240 URL <http://dx.doi.org/10.1038/srep08914>
- [142] S. Shiffler, P. Polynkin, J. Moloney, Self-focusing of femtosecond diffraction-resistant vortex beams in water, *Optics Letters* 36 (19) (2011) 3834. doi:10.1364/ol.36.003834.  
URL <http://dx.doi.org/10.1364/OL.36.003834>
- 1245 [143] C. L. Arnold, S. Akturk, A. Mysyrowicz, V. Jukna, A. Couairon, T. Itina, R. Stoian, C. Xie, J. M. Dudley, F. Courvoisier, S. Bonanomi, O. Jedrkiewicz, P. D. Trapani, Nonlinear bessel vortex beams for applications, *J. Phys. B: At. Mol. Opt. Phys.* 48 (9) (2015) 094006. doi:10.1088/0953-4075/48/9/094006.  
1250 URL <http://dx.doi.org/10.1088/0953-4075/48/9/094006>
- [144] O. Jedrkiewicz, S. Bonanomi, M. Selva, P. Di Trapani, Experimental investigation of high aspect ratio tubular microstructuring of glass by means of picosecond bessel vortices, *Applied Physics A* 120 (1) (2015) 385–391. doi:10.1007/s00339-015-9200-6.  
1255 URL <http://dx.doi.org/10.1007/s00339-015-9200-6>
- [145] A. Mathis, F. Courvoisier, L. Froehly, L. Furfaro, M. Jacquot, P. A. Lacourt, J. M. Dudley, Micromachining along a curve: Femtosecond laser micromachining of curved profiles in diamond and silicon using accelerating beams, *Appl. Phys. Lett.* 101 (7) (2012) 071110. doi:10.1063/1.4745925.  
1260 URL <http://dx.doi.org/10.1063/1.4745925>
- [146] M. V. Berry, Nonspreading wave packets, *Am. J. Phys.* 47 (3) (1979) 264. doi:10.1119/1.11855.  
URL <http://dx.doi.org/10.1119/1.11855>

- 1265 [147] M. A. Bandres, Accelerating beams, *Optics Letters* 34 (24) (2009) 3791.  
doi:10.1364/ol.34.003791.  
URL <http://dx.doi.org/10.1364/OL.34.003791>
- [148] M. A. Bandres, I. Kaminer, M. Mills, B. Rodríguez-Lara, E. Greenfield,  
M. Segev, D. N. Christodoulides, Accelerating optical beams, *Optics and*  
1270 *Photonics News* 24 (6) (2013) 30. doi:10.1364/opn.24.6.000030.  
URL <http://dx.doi.org/10.1364/OPN.24.6.000030>
- [149] J. Baumgartl, M. Mazilu, K. Dholakia, Optically mediated particle clear-  
ing using airy wavepackets, *Nature Photonics* 2 (11) (2008) 675–678.  
doi:10.1038/nphoton.2008.201.  
1275 URL <http://dx.doi.org/10.1038/nphoton.2008.201>
- [150] T. Vettenburg, H. I. C. Dalgarno, J. Nylk, C. Coll-Lladó, D. E. K. Ferrier,  
T. Čižmár, F. J. Gunn-Moore, K. Dholakia, Light-sheet microscopy using  
an airy beam, *Nature Methods* 11 (5) (2014) 541–544. doi:10.1038/  
nmeth.2922.  
1280 URL <http://dx.doi.org/10.1038/nmeth.2922>
- [151] D. Abdollahpour, S. Suntsov, D. G. Papazoglou, S. Tzortzakis, Spatiotem-  
poral airy light bullets in the linear and nonlinear regimes, *Phys. Rev.*  
*Lett.* 105 (25) (2010) 253901. doi:10.1103/physrevlett.105.253901.  
URL <http://dx.doi.org/10.1103/PhysRevLett.105.253901>
- 1285 [152] I. Kaminer, M. Segev, D. N. Christodoulides, Self-accelerating self-trapped  
optical beams, *Phys. Rev. Lett.* 106 (21) (2011) 213903. doi:10.1103/  
*physrevlett.106.213903*.  
URL <http://dx.doi.org/10.1103/PhysRevLett.106.213903>
- [153] A. Bahabad, M. M. Murnane, H. C. Kapteyn, Manipulating nonlinear  
1290 optical processes with accelerating light beams, *Phys. Rev. A* 84 (3) (2011)  
033819. doi:10.1103/physreva.84.033819.  
URL <http://dx.doi.org/10.1103/PhysRevA.84.033819>

- [154] A. Libster-Hershko, I. Epstein, A. Arie, Rapidly accelerating mathieu and weber surface plasmon beams, *Phys. Rev. Lett.* 113 (12) (2014) 123902. doi:10.1103/physrevlett.113.123902. URL <http://dx.doi.org/10.1103/PhysRevLett.113.123902>
- [155] N. Voloch-Bloch, Y. Lereah, Y. Lilach, A. Gover, A. Arie, Generation of electron airy beams, *Nature* 494 (7437) (2013) 331–335. doi:10.1038/nature11840. URL <http://dx.doi.org/10.1038/nature11840>
- [156] A. Lotti, D. Faccio, A. Couairon, D. G. Papazoglou, P. Panagiotopoulos, D. Abdollahpour, S. Tzortzakis, Stationary nonlinear airy beams, *Phys. Rev. A* 84 (2) (2011) 021807. doi:10.1103/physreva.84.021807. URL <http://dx.doi.org/10.1103/PhysRevA.84.021807>
- [157] P. Polynkin, M. Kolesik, J. V. Moloney, G. A. Siviloglou, D. N. Christodoulides, Curved plasma channel generation using ultraintense airy beams, *Science* 324 (5924) (2009) 229–232. doi:10.1126/science.1169544. URL <http://dx.doi.org/10.1126/science.1169544>
- [158] L. Froehly, F. Courvoisier, A. Mathis, M. Jacquot, L. Furfaro, R. Giust, P. A. Lacourt, J. M. Dudley, Arbitrary accelerating micron-scale caustic beams in two and three dimensions, *Opt. Express* 19 (17) (2011) 16455. doi:10.1364/oe.19.016455. URL <http://dx.doi.org/10.1364/OE.19.016455>
- [159] F. Courvoisier, A. Mathis, L. Froehly, R. Giust, L. Furfaro, P. A. Lacourt, M. Jacquot, J. M. Dudley, Sending femtosecond pulses in circles: highly nonparaxial accelerating beams, *Optics Letters* 37 (10) (2012) 1736. doi:10.1364/ol.37.001736. URL <http://dx.doi.org/10.1364/OL.37.001736>
- [160] Y. Kaganovsky, E. Heyman, Wave analysis of airy beams, *Opt. Express*

18 (8) (2010) 8440. doi:10.1364/oe.18.008440.

URL <http://dx.doi.org/10.1364/OE.18.008440>

[161] E. Greenfield, M. Segev, W. Walasik, O. Raz, Accelerating light beams along arbitrary convex trajectories, Phys. Rev. Lett. 106 (21) (2011) 213902. doi:10.1103/physrevlett.106.213902.

1325

URL <http://dx.doi.org/10.1103/PhysRevLett.106.213902>

[162] A. Mathis, F. Courvoisier, R. Giust, L. Furfaro, M. Jacquot, L. Froehly, J. M. Dudley, Arbitrary nonparaxial accelerating periodic beams and spherical shaping of light, Optics Letters 38 (13) (2013) 2218. doi:10.1364/ol.38.002218.

1330

URL <http://dx.doi.org/10.1364/OL.38.002218>

[163] P. Aleahmad, M.-A. Miri, M. S. Mills, I. Kaminer, M. Segev, D. N. Christodoulides, Fully vectorial accelerating diffraction-free helmholtz beams, Phys. Rev. Lett. 109 (20) (2012) 203902. doi:10.1103/physrevlett.109.203902.

1335

URL <http://dx.doi.org/10.1103/PhysRevLett.109.203902>

[164] M. A. Alonso, M. A. Bandres, Spherical fields as nonparaxial accelerating waves, Optics Letters 37 (24) (2012) 5175. doi:10.1364/ol.37.005175.

URL <http://dx.doi.org/10.1364/OL.37.005175>

[165] M. Gecevičius, M. Beresna, R. Drevinskas, P. G. Kazansky, Airy beams generated by ultrafast laser-imprinted space-variant nanostructures in glass, Optics Letters 39 (24) (2014) 6791. doi:10.1364/ol.39.006791.

1340

URL <http://dx.doi.org/10.1364/OL.39.006791>

[166] Y. Kaganovsky, E. Heyman, Nonparaxial wave analysis of three-dimensional airy beams, Journal of the Optical Society of America A 29 (5) (2012) 671. doi:10.1364/josaa.29.000671.

1345

URL <http://dx.doi.org/10.1364/JOSAA.29.000671>

- [167] A. Mathis, L. Froehly, L. Furfaro, M. Jacquot, J. M. Dudley, F. Courvoisier, Direct machining of curved trenches in silicon with femtosecond accelerating beams, JEOS:RP 8. doi:10.2971/jeos.2013.13019.  
1350 URL <http://dx.doi.org/10.2971/jeos.2013.13019>
- [168] I. D. Chremmos, Z. Chen, D. N. Christodoulides, N. K. Efremidis, Abruptly autofocusing and autodefocusing optical beams with arbitrary caustics, Phys. Rev. A 85 (2) (2012) 023828. doi:10.1103/physreva.85.023828.  
1355 URL <http://dx.doi.org/10.1103/PhysRevA.85.023828>
- [169] P. Panagiotopoulos, D. Papazoglou, A. Couairon, S. Tzortzakis, Sharply autofocused ring-airy beams transforming into non-linear intense light bullets, Nature Communications 4. doi:10.1038/ncomms3622.  
1360 URL <http://dx.doi.org/10.1038/ncomms3622>
- [170] D. G. Papazoglou, N. K. Efremidis, D. N. Christodoulides, S. Tzortzakis, Observation of abruptly autofocusing waves, Optics Letters 36 (10) (2011) 1842. doi:10.1364/ol.36.001842.  
URL <http://dx.doi.org/10.1364/OL.36.001842>