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Unconventional magnification behaviour in microsphere-assisted microscopy

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Abstract

Microsphere-assisted microscopy is an original sub-diffraction-limit imaging technique allowing to reach a few hundred nanometres of lateral resolution in air only by placing a microbead in a classical optical microscope. This work aims to highlight the magnification process in microsphere-assisted microscopy which behaves differently from the case in optical microscopy. As a matter of fact, the lateral magnification of an optical microscope does not change according to focus plane positions. Experiments on the super-resolution imaging technique, performed in air through soda-lime-glass microspheres and attested by simulations, demonstrate a significant increase in the magnification factor along the microsphere imaging depth, *i.e.* at different object-focal-plane position of the objective. Moreover, it is shown that the magnification range, as well as its slope, depend on the size of the microsphere. Additionally, the influence of the spectral width of the illumination light source on the magnification range is highlighted.

Keywords: Microscopy; Super-resolution; Microsphere; Magnification; Image formation

1. Introduction

Resolving power in classical optical microscopy is limited by diffraction of light, resulting in a minimal distance between two distinct object details of half of the wavelength in air using a low-coherent 5 illumination [1]. The idea of sub-diffraction-limit imaging techniques appeared with the need to visualize increasingly smaller elements [2], and with the notion of *ultra-microscopy* [3]. Several superresolution imaging techniques have then been devel-10 oped in the last century such as confocal microscopy [4, 5] (enhanced further using a double-pass configuration [6] and the photonic jet phenomenon [7]), scanning near-field optical microscopy [8, 9], structured illumination microscopy [10, 11] and

- ¹⁵ structured illumination microscopy [10, 11] and metamaterials-based superlenses [12, 13]. They contributed to bringing optical nanoscopy to the forefront, reinforced by the Nobel Prize for Chemistry in 2014 with super-resolved fluorescence mi-
- ²⁰ croscopy [14] and single molecule localization microscopy [15]. Nevertheless, these super-resolution imaging techniques require high stability systems, complex alignment or long acquisition time.

In 2011, Z. Wang et al. experimentally demonstrated full-field label-free super-resolution microscopy using low-refractive-index microspheres (silica microspheres with n ~ 1.46) [16]. Relatively easy-to-implement, this approach consists in introducing a transparent microsphere in a classical white-light microscope, *i.e.* between the sample and the microscope objective. A magnified image, providing sub-diffraction-limit information, is then generated and collected by the microscope. In the past years, several papers have focused on the performance of the super-resolution imaging technique. In 2012, A. Darafsheh et al. suggested placing high-refractive-index microspheres (barium-titanate-glass with $n \sim 2.0$) in an immersion liquid [17] and, more recently, embedding in an elastomer layer [18]. Studies of the immersion medium influence on the image contrast [19, 20] and on the image nature [21, 22], as well as of the role of the coherence of light on the lateral resolution [23, 24], lead to a better understanding of the imaging technique. In addition, the manipulation of the microspheres has been studied, making it possible to perform contact-less super-resolution measurements [25, 26, 27, 28]; methods which could, for example, avoid damaging biological samples during

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- ⁵⁰ the acquisitions [29, 30, 31]. These contributions are promising as regards the fabrication of novel optical devices (*e.g.*, microsphere-embedded microscope accessories), enabling to improve optical microscopy.
- Experimentally, microsphere-assisted microscopy is able to reach a lateral resolution higher than confocal microscopy and solid immersion lens [32], and similar to structured illumination microscopy and negative-index superlenses [33]. However, being a
- recent imaging technique, some phenomena are still not fully explained such as the origin of the superresolution resolving power. Due to the micron-scale size of the spheres, their imaging properties do not exactly follow geometrical optics [34, 35] and theo-
- ⁶⁵ retical analyses thus remain complex. A possible collection of the evanescent waves by the microsphere followed by their conversion in the far field yet appears relevant to explain the super-resolving power [36, 37]. In microsphere-based nanoscopy,
- ⁷⁰ it is not only the super-resolution phenomenon is still not fully explained, but also the image process investigated in this work. Unlike optical microscopy, microsphere-assisted microscopy provides long axial focusing ranges, therefore the imaging
- ⁷⁵ factor within the imaging depth behaves in an unconventional way. As a matter of fact, the lateral ¹⁰⁵ magnification differs according to the axial position of the microscope objective. As an example, Supplementary Movie illustrates the change in the
- ⁸⁰ magnification along the optical axis using a sodalime-glass microsphere with a diameter of 29 μ m. ¹¹⁰ This effect could lead to divergent interpretations of results (*e.g.*, the magnification from a 4.7- μ mdiameter microsphere equals ×4.1 and even ×8 in
- Ref [16], and ×2.8, in Ref [28]). Therefore, a range of magnification factor is sometimes preferred in order to avoid any position-dependent-magnification confusions [29, 38].
- This work exposes a fundamental property in ⁹⁰ microsphere-assisted microscopy by highlighting the non-classical evolution of the lateral magnification along the imaging depth. Through experimental measurements and numerical analyses, the influence of the microsphere diameter, as well as the here devide the light expression on the measuring setting
- bandwidth of the light source, on the magnification behaviour is shown in air.

2. Methods

A classical optical microscope in reflecting mode has thus been enhanced by introducing glass mi- 130

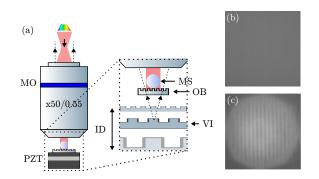


Figure 1: (a) Layout of optical head of the microsphere-based nanoscope. A microscope objective (MO) collects the virtual image (VI) of the object (OB) which was magnified before by a 25-µm-diameter glass microsphere (MS). A piezoelectric device (PZT) allows the MO to focus on different axial VI planes along the imaging depth (ID) by displacing OB and MS. Comparison of performance between (b) the MO alone and (c) the MS-based nanoscope, by imaging a 200-nm-line-width grating.

crospheres (Cospheric LLC, California) having different diameters. The soda-lime-glass microspheres have a refractive index of 1.52. The illumination part consists of a white-light source ($\lambda_0 = 650 \text{ nm}$, $\Delta \lambda = 400 \text{ nm}$) where the emitting beam passes through a Köhler arrangement to provide a more homogeneous intensity distribution. The field diaphragm was closed to improve the imaging contrast [39]. In addition, wavelength filters (a cyan filter with $\lambda_0 = 567$ nm and $\Delta \lambda = 90$ nm, and a blueline filter with $\lambda_0 = 445$ nm and $\Delta \lambda = 11$ nm) can be introduced in order to limit the spectral width of the light source. The object beam is then directed towards the imaging part and passes through the transparent microsphere and the microscope objective (Zeiss, $\times 50$, NA = 0.55), as shown in Fig. 1(a). Here, the object is a contrast grating target having a period of 400 nm (fabricated at the MIMENTO Technology Centre, FEMTO-ST Institute, France). It can be noted that the optical microscope alone is not able to visualize the 200-nm-groove-width features (Fig. 1(b)). Indeed, considering the transfer function of the whole white-light microscope, the resolving power equals only 840 nm. The microsphere is thus placed in air on the surface of the object, allowing the super-resolution phenomenon A camera finally records the superto occur. resolution image (Fig. 1(c)) from the objective lens and a relay lens. In order to capture the virtual images at different positions along the optical axis, a piezoelectric device is vertically displaced. The

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depth of field of the microscope objective ($\sim 1 \text{ µm}$) defines the axial sampling of the measurements. During the experiments, the measurements were repeated using ten similar microspheres. For ex-

ample, experimental curve represented in Fig. 2(a) results thus of an averaging of the magnification factors through ten 25-µm-diameter microspheres. Moreover, at each axial position, the magnification measurements were performed ten times.

¹⁴⁰ 3. Results and Discussion

The lateral magnification generated by the microsphere, defined as the ratio between the grating periods in the image plane and the object plane, was first measured according to the axial position where the objective focuses on (Fig. 2(a)). A soda-lime-glass microsphere having a diameter of 25 µm was used to demonstrate the influence of the axial image position on the magnification factor. A linear evolution along the microsphere imag-

- ¹⁵⁰ ing depth ($\sim 60 \ \mu m$) is shown, changing from $\times 4$ to $\times 8$ while maintaining the super-resolution phenomenon. This effect is significantly different to conventional full-field optical system. Indeed, the magnification of a conventional optical microscope
- remains constant over the clearest imaging depth. Obviously, the virtual images at positions deeper than 60 µm have a contrast decreased (Fig. 2(b.iii)). In order to confirm the experimental results, a rigorous 2D electromagnetic model was also imple-
- ¹⁶⁰ mented by using a finite element method (Comsol Multiphysics). The simulation of super-resolution virtual imaging consists in two steps [23, 40, 41]: firstly, the electric fields from the object, *i.e.* two point sources, interacts with a microsphere. Then,
- the transmitted electric field is time-reverse propagated in free-space, *i.e.* propagated in the opposite axial direction without the microsphere, in order to retrieve the two images. The broadband illumination of the light source is considered by
- ¹⁷⁰ repeating the two steps over the visible spectral range ($\Delta \lambda = 400$ nm, step = 50 nm) and then the wavelength-dependant irradiance distributions are summed (Fig. 2(c)). The simulated magnification factor, *i.e.* the transverse distance ratio between
- the two resulting images and the two point sources, is represented by the blue dotted line in Fig. 2(a) according to the longitudinal position of the image plane, showing magnification values in good agreement with the experiments.

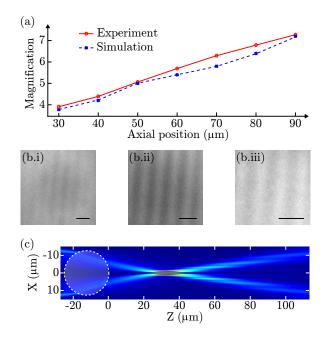


Figure 2: (a) Experimental (red solid line) and numerical (blue dotted line) evolutions of the lateral magnification from a 25-µm-diameter microsphere placed in air using a whitelight source, along the microbead imaging depth (Z axial position). (b.i), (b.ii) and (b.iii) are the virtual images of the 400 nm periodic object at Z positions of 30 µm, 55 µm and 90 µm, respectively. Scale bars represent 1 µm. (c) Simulation of the virtual image formation. The object is two point sources placed against the microsphere. The white dotted line illustrates the initial microsphere location.

Afterwards, the influence of the microsphere diameter on the position-dependent magnification were studied. Eight glass microspheres having different diameters were hence placed on the 400-nmperiod grating. In order to be readable, Fig. 3(a)represents the linear evolution of the magnification factor for only three microsphere diameters, *i.e.* 7 µm, 46 µm and 125 µm. The curves allow to affirm that, at a given axial image position, the large microspheres provide a magnification factor lower than when using small microspheres. Moreover, the microsphere-diameter dependence on the magnification curve slope α has been estimated and is represented in Fig. 3(b) (red-color curve). A fitting curve is juxtaposed, highlighting the inverse proportionality of the slope α according to the diameter D of the soda-lime-glass microspheres. In air and using soda-lime-glass microspheres, this relation is expressed as:

$$\alpha[\text{in }\mu\text{m}^{-1}] = \frac{1.83}{D[\text{in }\mu\text{m}]}$$
(1)

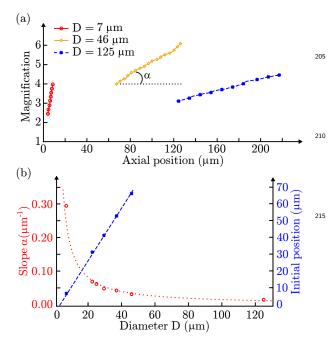


Figure 3: (a) Evolution of the lateral magnification in air from microspheres having diameters of 7 µm (in red), 46 µm (in orange) and 125 µm (in blue), as a function of axial position Z. The microspheres are illuminated by a whitelight source. (b) Evolution of the magnification inclination α (in red) and the initial image position (in blue) for different diameters D. A curve fitting 1/D function is superimposed to the calculated values α .

In addition, Fig. 3(a) shows that the smaller the 180 diameter, the narrower the imaging depth. Indeed, not only the magnification factor but also the imaging depth are highly affected by the size of the 220 microsphere. Large microspheres are able to form virtual images at deeper positions, requiring focus with the microscope objective at further positions. The linear proportionality of the initial position of the imaging depth as a function of the diameter of soda-lime-glass microspheres is shown in Fig. 3(b)(blue-color curve). 190

Finally, the role of the spectral bandwidth of the light source on the magnification was investigated. For this purpose, the performance of the 25-µmdiameter microsphere using the white light source 230 (represented by the red line in Fig 4) is compared with two spectrally-gated illuminations, *i.e.* using the cyan filter (cyan line) and the blue-line-filter (blue line). The magnification factor remains linearly proportional according to the axial position 235

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200 and the magnification values do not differ, regardless of the spectral bandwidth of the light source.

However, only the magnification range appears limited. As a matter of fact, the smaller the spectral bandwidth is, the narrower the microsphere imaging depth is. This spectral width dependence shows that the unconventional magnification behaviour is mainly due to a significant contribution of the chromatic aberrations. Wavelength-dependent beams are focused in different image planes, each having a different magnification. This leads to difficulties to define a unique magnification. Nevertheless, this yields a degree of freedom during the measurement, *i.e.* the possibility to adjust the required magnification. This assumption can be retrieved through glass microspheres having another diameter.

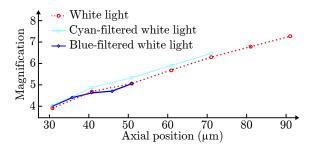


Figure 4: Evolution of the lateral magnification from a 25µm-diameter microsphere placed in air as a function of axial position under three types of light source having 400 nm (in red line), 90 nm (in cyan line) and 11 nm (in blue line) of bandwidth.

In this work, the unconventional behaviour of the lateral magnification in microsphere-assisted microscopy has been achieved in air using soda-limeglass microspheres. It should however be mentioned that this effect occurs also using microspheres having a higher refractive index and in a different immersion configuration [35, 42].

4. Conclusions

This work exposes a fundamental property in microsphere-assisted microscopy: the non-classical behaviour of the lateral image magnification. Contrarily to classical optical microscopy, the magnification factor is subject to a linear evolution along the microsphere imaging depth, and both geometrical and optical parameters have an influence on the position-dependent magnification slope and range. Indeed, through experiments and numerical simulations, it was shown that the microsphere size is inversely proportional to the magnification growth and is linearly proportional to the initial position of the axial field of view. Furthermore, the reduction

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of the spectral bandwidth of the illumination light ²⁹⁵ source indeed limits the magnification range.

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