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# A Data-driven Approach for Inference of the Evolution Equation of a Duffing Oscillator

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**Abstract.** As more and more data is available on a daily basis, it is natural to seek to infer relevant information embedded in these datasets. In the context of dynamical systems, this idea translates into the use of observations (data) to infer the evolution law. In this sense, machine learning techniques are getting more space in the analysis and synthesis of dynamical systems. This paper uses regularized data-driven regression algorithm to infer the evolution law of a Duffing oscillator, using a library of mathematical functions obtained from a dataset generated from the underlying physical system.

**Keywords:** *Duffing oscillator, machine learning, inverse problem, data-driven inference, sequential threshold least-squares*

## 1 Introduction

A dataset obtained from the sampling of a dynamic system can be used to develop a library of testing functions from which one can perform an sequential threshold least-squares that allow to infer the system dynamic evolution law [1]. In this sense, this paper illustrates the basic principle behind the sequential threshold least-squares in the reconstruction of the evolution law of a Duffing oscillator.

## 2 Data-driven identification procedure

If a dynamical systems have a small amount of terms in its evolution law, it is very likely it can be parametrized by a sparse linear combination of elementary functions. In this sense, the key idea here is to construct, with aid of available observations (data) from the original system, a library of polynomial functions in a way that a sparse linear combination of them may be enough to approximate the evolution law.

For sake of identification of the coefficients (coordinates) of this functional representation a machine learning approach, based on a sequential threshold least-squares regression, is employed. In this method a threshold is defined so that the regression coefficients that have their absolute value below this parameter are considered zero, being the regression computed again without these terms.

Considering the library of elementary functions defined by the matrix

$$\Theta(\mathbf{X}) = [\mathbf{1} \quad \mathbf{X} \quad \mathbf{X}^{P_2} \quad \dots \quad \mathbf{X}^{P_n}], \quad (2.1)$$

where  $\mathbf{X}^{P_n}$  represents all dimension combinations that form a  $n$ -degree polynomial, the evolution law for the dynamical system state  $\mathbf{X}$  is obtained from

$$\dot{\mathbf{X}} = \Theta(\mathbf{X})\Xi + \eta\mathbf{Z}, \quad (2.2)$$

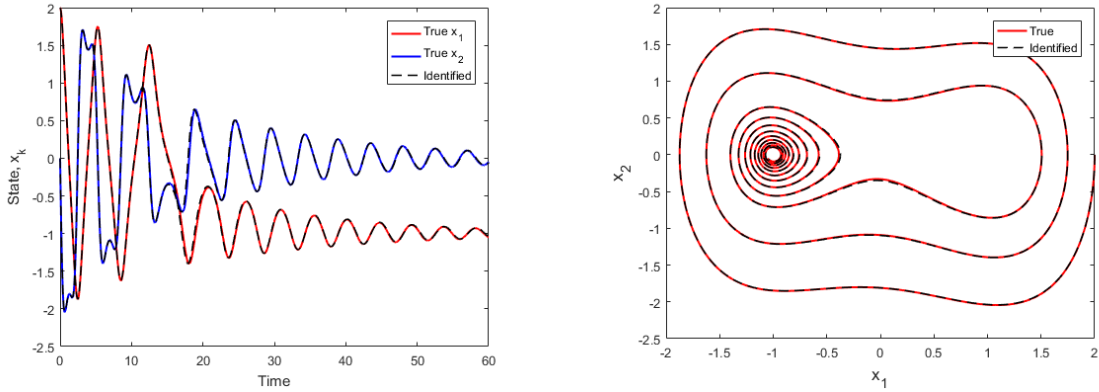
where  $\Xi$  determines which functions are activated and their weights,  $\mathbf{Z}$  is a matrix with zero-mean normally distributed random values, and  $\eta$  is the corresponding standard deviation.

### 3 Numerical experiment

For sake of illustration of the machine learning identification procedure, a Duffing like damped dynamical system is considered

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -0.1x_2 + x_1 - x_1^3. \quad (3.1)$$

A comparison between the “exact” (reference) response of this dynamic system and data-driven time series and phase space trajectory, inferred by the identification procedure, can be seen in Figure 1.



(a) “Exact” and data-driven time series for  $x_1$  and  $x_2$ .

(b) “Exact” and data-driven phase space trajectory.

**Fig. 1** “Exact” and data-driven identified dynamics with initial conditions  $x_1(0) = 2$  and  $x_2(0) = 0$ . The “exact” solution is obtained by RK45 method with 6000 equally spaced time-steps between 0 and 60 seconds. The dataset has 1200 points randomly taken from the reference time series. The data-driven solution is obtained from a mathematical functions library with polynomials functions up to fifth degree, standard deviation 0.1 and threshold value 0.075.

### 4 Conclusions

The machine learning approach employed is able to recognize with incredible precision the law of evolution of the Duffing oscillator, even with high intensity of noise polluting the database, demonstrating to be an effective method for inferring the evolution law of dynamic systems.

### References

- [1] S. L. Brunton, J. L. Proctor, and J. N. Kutz. Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *Proceedings of the National Academy of Sciences of the United States of America*, 113:3932–3937, 2016.