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► **To cite this version:**

José Geraldo Telles, Americo Cunha Jr, Tiago Roux Oliveira. On the classical and fractional control of a nonlinear inverted cart-pendulum system: a comparative analysis. 15th International Conference on Vibration Engineering and Technology of Machinery (VETOMAC 2019), Nov 2019, Curitiba, Brazil. <hal-02388465>

HAL Id: hal-02388465

<https://hal.science/hal-02388465v1>

Submitted on 1 Dec 2019

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On the Classical and Fractional Control of a Nonlinear Inverted Cart-Pendulum System: a comparative analysis

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Abstract. Fractional-order control is based on the fractional calculus and its use is being explored for many researchers in order to improve the performance of control systems. In this paper, fractional integrators are employed in a state-feedback controller and applied to an inverted cart-pendulum system. Faster transient responses and increased (local) attraction domains are achieved when compared to the integer integrator based implementation of the proposed control law.

Keywords: *fractional calculus, state feedback, fractional-order control, cart-pendulum system*

1 Introduction

The use of the traditional PID-controller in nonlinear dynamical systems is widespread in the literature, despite its well known limitations to minimize response/settling time, as well as to prevent (or reduce) overshoot. In this nonlinear context, fractional-order PID-controllers are proving to be a well-liked solution, as they have the ability to circumvent some of these classical limitations [1, 2].

In this sense, the present work aims to evaluate the performance of a fractional-order PID-controller in comparison with a integer-order PID-controller. For this purpose, both controllers are applied to stabilize the nonlinear dynamics of an inverted cart-pendulum system, Figure 1 (left).

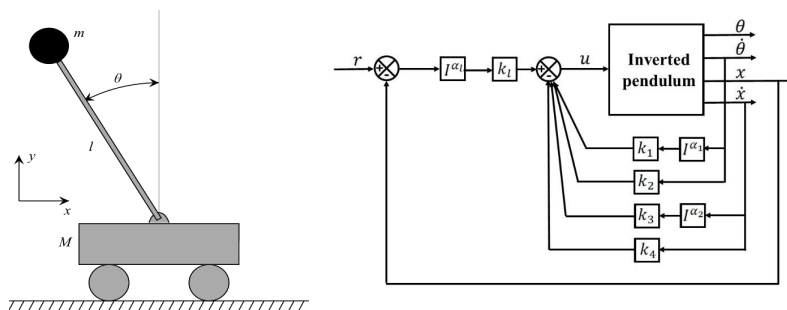


Fig. 1 Illustration of the inverted cart-pendulum system (left) and the proposed fractional-order PID-controller (right).

2 Controlled inverted cart-pendulum

The dynamic behavior of the inverted cart-pendulum is governed by

$$\begin{aligned} ml \cos \theta \ddot{x}(t) + (I + ml^2) \ddot{\theta}(t) - mgl \sin \theta(t) &= 0, \\ (m + M) \ddot{x}(t) + ml \cos \theta(t) \ddot{\theta}(t) - ml \sin \theta(t) \dot{\theta}^2(t) &= 0, \end{aligned} \quad (2.1)$$

where g is the gravity acceleration; m , l , I represent the pendulum mass, length, and moment of inertia, respectively; M is the cart mass; t is the time; x is the cart horizontal displacement; and θ is the pendulum angular displacement; the upper dot is an abbreviation for time-derivative.

The fractional controller employed here, Figure 1 (right), consider the system state (not the error) and its time-derivative in the feedback law, that uses a Riemann-Liouville fractional integral

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f(\tau) d\tau, \quad (2.2)$$

where Γ represents the Gamma function, and the order α is a positive parameter [3, 4]. This approach is very appealing for systems in which state-derivatives are easier to be obtained than the state signals.

The simulations are performed with aid of FOMCON Matlab toolbox. Figure 2 shows the results for both controllers. Although larger initial angles are achieved with the proposed fractional controller, this value has been chosen for comparisons purposes because it is the maximum possible angle allowed for the classical controller.

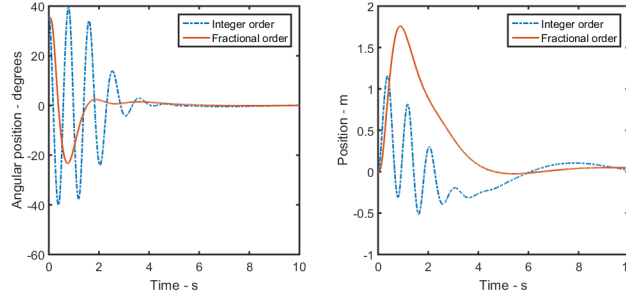


Fig. 2 Comparison between pendulum angular displacement (left) and cart horizontal displacement (right) stabilized with the integer-order (dashed blue) and the fractional-order (continuous red) controller. The controller parameters are $r = 0$, $k_1 = -157$, $k_2 = -65$, $k_3 = -35$, $k_4 = -70$, $k_l = -36$, $\alpha_1 = 0.8$, $\alpha_2 = 1$ and $\alpha_l = 0.7$. The initial conditions are $\theta_0 = 36.5^\circ$, $\dot{\theta}_0 = 0$ rad/s, and $x_0 = 0$ m.

3 Conclusions

State-feedback control implementation using fractional integrators can enhance the closed-loop control system performance in terms of improved transient and larger attraction domains.

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