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Solving Security Constraints for 5G Slice Embedding: A Proof of Concept

François Boutigny^{2,*}, Stéphane Betgé-Brezetz¹, Gregory Blanc², Antoine Lavignotte², Hervé Debar², Houda Jmila²

Abstract

Network slicing is a prominent feature of 5G, which allow tenants to rent network and computing virtual resources from one or more Infrastructure Providers (InPs). Those resources are allocated according to tenants requirements, not only in terms of QoS but also in terms of security. In this paper, we build on our previous work to propose and evaluate a security-aware slice embedding implementation which enables tenants to declare security-oriented requirements, while limiting InP network information disclosure. To do so we improve our requirement model so that it becomes compatible with an Satisfiability Modulo Theories (SMT) formulation. Our implementation distinguishes two sub-problems, one for the intra-domain level (inside each InP) and one for the inter-domain level (in between the InPs). We leverage those sub-problems in a multi-level resolution algorithm to generate all multi-domain slice embeddings.

Keywords: virtual network embedding, network slicing, 5G security, satisfiability modulo theories, network virtualization

1. Introduction

Network operators anticipate an ever-growing number of more diverse connected devices, be them vehicles on roads, traffic lights in cities, robots in factories, or healthcare devices in human bodies. However, the companies producing those devices and operating them know little about network operators' business. For instance, they are not expected to own nor manage mobile network equipment.

The Mobile Virtual Network Operator (MVNO) business model fortunately shows that this is not mandatory. MVNOs already provide Internet access contracts to subscribers, like other mobile network operators, while owning no or a few equipment, contrary to mobile network operators. MVNOs rent equipment instead, after negotiating specific agreements on specific resources with the mobile network operators. 5G proposes to generalize the MVNO business model so that the aforementioned companies can operate their devices on top of network operators' infrastructures, as if they had their own.

In fact, they get a dedicated, customized, virtual, end-to-end network on top of a shared infrastructure, which

is called "slice". Companies renting slices are called "tenants". And the shared infrastructure does not only rely on network operators, but more broadly on a subset of all possible Infrastructure Providers (InPs).

In this paper, we focus on slice customization. It is crucial: as tenants know little about network operators' business, network operators know little about tenants' business. Tenants have their own requirements to make their services operational, or regulation-compliant. For instance, robots need low latency, while a remote radiography needs high bandwidth.

We are already used to define such QoS-oriented requirements, and to pay for it, in the cloud model. But 5G invites us to go beyond, by considering more general requirements, like security requirements. For instance, to comply with a certain regulation in a certain country, tenants may require that their slices must rent resources in certain other countries, from certain InPs, with a certain level of certification.

There is an obstacle, though. As of today, MVNOs get their infrastructures through manually configured resources. With 5G and thousands of tenants, such configuration will be unpractical. We then need to automate the slice deployment. In this paper, we present how we solve this problem. We borrow from the literature the concept of Virtual Network Embedding (VNE), which refers to the allocation of resources from the physical network to fulfill requirements of a virtual network request (Fischer et al., a). However, it was shown to be an NP-hard problem (Fischer et al., a). Besides, we identify two aspects of the slice embedding problem that previous works in VNE only covered partially: the multi-InP aspect, and the security requirement aspect. The algorithm we propose in this paper successfully covers both.

*Corresponding author

Email addresses: francois.boutigny@telecom-sudparis.org (François Boutigny), stephane.betge-brezetz@nokia-bell-labs.com (Stéphane Betgé Brezetz), gregory.blanc@telecom-sudparis.eu (Gregory Blanc), antoine.lavignotte@telecom-sudparis.eu (Antoine Lavignotte), herve.debar@telecom-sudparis.eu (Hervé Debar), houda.jmila@telecom-sudparis.eu (Houda Jmila)

¹Nokia Bell Labs France

²SAMOVAR, Télécom SudParis, Institut Mines Télécom, CNRS, Institut Polytechnique de Paris

1 Throughout this paper, the term domain refers to the
2 infrastructure owned by a certain InP. A given InP may
3 own different domains, which are not directly intercon-
4 nected together. When we use the term multi-domain,
5 we mean that there are both multiple InPs and multiple
6 domains operated by different InPs.

7 For the **multi-InP aspect**, Mano et al.; Dietrich et al.
8 have identified one key property: InPs are reluctant to
9 share information about their topologies to others. This
10 assumption still holds for slices. In addition, they assume
11 that InPs are interconnected in a full-mesh way. This as-
12 sumption does not hold for slices: some InPs may be net-
13 work operators, and others may be data-centers, or tenant-
14 owned networks. For this reason, we design our solution
15 so that it is able to use an InP to interconnect slice subsets
16 located in two distant InPs.

17 As for **security requirements**, state-of-the-art works
18 from Bays et al.; Liu et al.; Alaluna et al. mainly use “se-
19 curity levels”. The advantage of this approach is that it
20 directly fits into Integer Linear Programming (ILP) mod-
21 els (as levels can be mapped to integers). While ILP (or
22 its variants) is the main tool for describing VNE as an op-
23 timization problem so far, yet, from a security viewpoint,
24 we cannot define such an absolute scale: security levels
25 only make sense under a certain security policy, and we
26 cannot expect that our diverse tenants and InPs will agree
27 on a common security level reference.

28 Instead, our solution can express exclusion constraints,
29 which makes it possible to avoid untrusted resources based
30 on their location, vendors, certifications, or installed soft-
31 ware programs, for instance. The solution is extensible to
32 other requirements. In addition, we are aware that not all
33 requirements may be appropriate for a given tenant, and
34 that InPs may not support all of them either. For this
35 reason, our solution tolerates incomplete requirement and
36 resource descriptions. A basic rule is as follows: if an InP
37 does not support a certain requirement, then it is eligible
38 only if the tenant does not require it either. This rule is
39 enforced through the use of a default value.

40 Besides, we support an exclusion requirement, which
41 enable tenants to avoid sharing physical resources with
42 other tenants. As such, they can still get dedicated re-
43 sources if they mistrust virtualization technologies and
44 their isolation.

45 Our previous work (Boutigny et al.) had the following
46 contributions.

- 47 *i)* We proposed a naive slice generation algorithm, en-
48 compassing the following contributions.
- 49 *ii)* We proposed a heuristic for multi-domain embed-
50 ding. This heuristic enables domains to embed more
51 general slice subsets than in the state of the art. If
52 we consider that the slices are requested by tenants
53 in the form of graphs, state-of-the-art papers limited
54 those slice subsets to be connected subgraphs, while
55 we considered any subgraph and any subset of edges.

- 56 *iii)* We enabled set-like requirements (useful in security
57 for blacklisting).
- 58 *iv)* We enabled the extension of our work to other re-
59 quirements by identifying the necessary building blocks.

In this paper, we address different challenges raised by
the implementation of our work (Boutigny et al.).

- 60 *i)* We improve our **slice generation algorithm** by
61 leveraging Satisfiability Modulo Theories (SMT).
- 62 *ii)* We revise our **heuristic**, to give domains more ways
63 to assign resources to our slice subsets.
- 64 *iii)* We revise our **requirement model** with a more
65 algebraic-oriented description.
- 66 *iv)* We solve a **requirement consistency problem** which
67 appears when enabling tenants to process slice re-
68 quest declaration and slice embedding at distinct times.

This paper is organized as follows. Section 2 presents
the state of the art in VNE with multi-InP support and se-
curity requirement support. Section 3 describes VNE for a
single domain case, as a set of rules. Section 4 presents how
we model requirements. Section 5 formulates the single
domain slice embedding problem as detailed in Section 3
by using our requirement model as detailed in Section 4.
Then, we build upon Section 5 to formulate our multi-level
slice embedding algorithm. Section 6 adapts the single
domain formulation to the intra-domain level. Section 7
adapts the single domain formulation to the inter-domain
level. Section 8 presents our actual algorithm. Section 9
presents our implementation as well as a use case infras-
tructure. Section 10 shows our experiments and results
on the use case infrastructure. We conclude this work in
Section 11.

2. State of the Art

VNE is a long-studied problem. As explained by Fis-
cher et al. (a), even if we are told where to map virtual
nodes, finding the right link allocations corresponds to the
unsplittable flow problem, which is NP-hard (Kolliopoulos
and Stein). For this reason, the slice embedding problem
is also NP-hard.

This section presents works that address security as-
pects of the VNE problem. We identify mainly two as-
pects. Works in Subsection 2.1 enable the tenants to re-
quire security within their virtual network requests. Works
in Subsection 2.2 enable the InP to protect themselves.

2.1. Tenant-oriented Security

From the state-of-the-art, we identify two primitives
to provide security requirements into the VNE problem.
The first primitive uses a level to define a security require-
ment. When a tenant requires a certain level for a given
virtual resource, only substrate resources with a greater

1 level can be candidates to embed this virtual resource. In
2 other words, there is a direct placement relation between
3 the virtual resource and the substrate resource. Overall,
4 this primitive is used to define security levels based on an
5 absolute scale.

6 The second primitive defines a security requirement on
7 an exclusion or a collocation principle on the virtual re-
8 sources themselves. Contrary to the first primitive, this
9 means that there is a direct placement relation between
10 two or more virtual resources.

11 The reviewed papers provide a composition of the dif-
12 ferent primitives.

13 Liu et al. allow tenants to require a security level for
14 nodes and links, then following the first primitive. The
15 InP must provide a resource with higher security level.

16 Bays et al. focus on privacy issues for a tenant in an In-
17 ternet Service Provider (ISP) infrastructure context. The
18 tenants can enforce their security through two ways. First
19 way, by demanding a *cryptographic support* for any vir-
20 tual router. This translates into a binary attribute, fol-
21 lowing the first primitive. Second way, by enumerating
22 the other requests with which they do not want to share
23 any resource, following the second primitive. In addition,
24 tenants can request a virtual router to be in a specified
25 *location*, following the first primitive.

26 Alaluna et al. focus on tenant protection in the cloud
27 context. They propose four security attributes: cloud
28 provider *trustworthiness*, node *backups*, node *security level*,
29 and link *security level*. Node and link security levels follow
30 the first primitive, as well as cloud trustworthiness level.
31 In essence, the node and link backups are duplicate of
32 other virtual resources, as if the tenants require the same
33 node or link twice. What Alaluna et al. add is that a level
34 determines where to place the duplicate relatively to its
35 originator. The duplicate may then be in the same cloud
36 as the originator, or in another cloud. For this reason, the
37 node and link backups follow the second primitive.

38 Wang et al. allow tenants to require a security plan on
39 three categories: virtual network wise, virtual node wise,
40 and virtual link wise. Each plan further propose three
41 levels. The high (resp. medium, resp. low) level of the
42 virtual network wise security plan allows the virtual net-
43 work to be collocated with no (resp. trusted, resp. any)
44 other virtual network, in the same datacenter. The high
45 (resp. medium, resp. low) level of the virtual node wise
46 security plan allows the virtual node to be collocated with
47 any virtual node of no (resp. trusted, resp. any) other vir-
48 tual network, in the same host. The high (resp. medium,
49 resp. low) level of the virtual link wise security plan means
50 that there is point-to-point encryption (resp. end-to-end
51 encryption, resp. no encryption). In essence, the virtual
52 network wise security plan and the virtual node wise secu-
53 rity plan are based on the second primitive, and the virtual
54 link wise security plan is based on the first primitive.

55 Fischer et al. (c) propose to implement security re-
56 quirements on the ALEVIN simulator (see Fischer et al.
57 (b)). They classify security requirements into three types:
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node requirements, such as encryption support; link re-
quirements, such as data encryption; and topological re-
quirements, such as separating the virtual network request
into two administrative domains, one public, and one pri-
vate, this latter protected by a firewall. Their node and
link requirements are then based on the first primitive,
while the topological requirements are based on the sec-
ond primitive.

In this paper, and like Fischer et al. (c), we consider
that the notion of security or trustworthiness level is too
abstract, as it may cover different security properties in a
way which do not correspond to the various tenant needs.
InPs and tenants may not have the same vision. For this
reason, we want to enable tenants to express their security
needs with appropriate attributes. To do so, we want to
provide an attribute model general enough to cover every
security use case.

Our work is distinct to the aforementioned works as
follows.

Liu et al. provide a single security level for the tenant.
The definition of such a security level is however only rel-
evant when all tenants have the same security policy, and
the same security priorities. In a multi-tenant context as
with the 5G network slices, we cannot expect such a situ-
ation. Tenants may use slice for a large variety of services:
health care, vehicles, smart cities, governmental services,
and industrial automation.

Bays et al. investigate the VNE in the ISP infrastruc-
ture context, which is slightly different from slice embed-
ding in the sense that the tenants are requesting (virtual)
routers only, and virtual routers cannot be collocated on
the same substrate router.

Besides, Bays et al. propose a location requirement
such that the tenant can require a single location for each
virtual resource. We consider it to be too limited, as it can
only be applied for a geographical scale, be it continents,
countries, or cities, but not a mix of different scales: this
model makes them exclusive to each other. For instance,
if the location of every substrate resource is identified at
a country level, we have no way to support a city-level
requirement; and to support a continent-level requirement,
the only solution would be to enable the tenant to require
a *list* of countries. We consider that enabling tenants to
require a list of authorized locations is a plus, and that
requiring a list of values can also be generalized to other
requirements.

Another limit of Bays et al. is that they leverage the
second primitive at the request level, while we consider
three levels of isolation (exclude tenant, exclude request,
exclude resource), for the nodes as well as the edges of the
tenant requests.

For Alaluna et al., the cloud provider trustworthiness is
absolute, and does not vary for different tenants, while we
think it should be up to the opinion of the tenant. In that
latter case, it would mean that tenants can control in which
cloud provider their virtual resources are hosted. Besides,
the security levels are abstract, and the semantics of each

1 level may vary among tenants, among InPs, and also be-
2 tween tenants and InPs. We note also that, although they
3 consider multiple cloud providers, they consider that they
4 have full knowledge over their resources, while we follow a
5 limited information disclosure principle.

6 Wang et al. distinguish two steps. The first step is to
7 apply their three security plans (per virtual network, per
8 virtual node, per virtual edge), and is referred to as the
9 admission process. The second step is the execution of the
10 VNE algorithm itself. The idea is that the admission pro-
11 cess serves as a guide to the VNE algorithm, because the
12 admission process produces an auxiliary graph where each
13 substrate node is associated with a set of candidate virtual
14 nodes. Those candidates are obtained by computing the
15 exclusion constraints according to the security plan of the
16 current virtual network request.

17 These latter words are important, because we can imag-
18 ine that a past request refuse to be collocated with any
19 other request and that the current one accept it; with the
20 algorithm proposed by Wang et al., only the plan of the
21 current request is considered, leading to a break in the se-
22 curity plan of the past request. We consider this flaw to
23 be due to a lack of a single requirement model, from which
24 the algorithm of the admission process should be derived.

25 Last but not least, the attribute model proposed by
26 Fischer et al. (c), although general, is implementation de-
27 pendent. To support the second primitive in their topo-
28 logical requirements for instance, they add another em-
29 bedding algorithm to their system, the cross-domain link
30 embedding. As we focus on a security-oriented VNE, we
31 think it is better to have at least a sound mathematical
32 formulation, which can be verified and audited.

33 2.2. InP-oriented Security

34 Liu et al. consider not only tenant protection (as shown
35 in previous section), but also InP protection. This latter
36 is modelled with a level, like the tenant protection, but it
37 is a requirement *from* the InP. This means that tenants
38 must declare the security level of their virtual resources,
39 and that this level should be higher than the security level
40 of the substrate resource of the InP. Regarding the 5G
41 slice embedding, we may not be able to make the same
42 criticism than in the previous section hold, that is, such
43 a security level is too abstract, because we may expect
44 that in a multi-InP scenario, the different InPs share some
45 common priorities, as they are in a common business field.

46 Liu et al. is, to the best of our knowledge, the only work
47 requiring information from the tenants. The other works
48 providing security to the InPs rather focus on different
49 aspects of limited information disclosure.

50 Limited information disclosure is part of the multi-
51 provider VNE problem, as illustrated by Mano et al.; Di-
52 etrich et al., where InPs do not disclose their complete
53 topologies, but only some nodes. In this variant of the
54 VNE problem, the substrate is split into domains, each
55 domain being owned by an InP. We can divide works

supporting limited information disclosure into three cat-
egories.

The first category contains the work from Chowdhury
et al. In this work, the authors provide a *distributed* al-
gorithm that enable the domains to cooperate, and the
tenants to sent their requests to any of those domains. In
other words, there are multiple entry points for the VNE
algorithm.

The second and the third category only use a single
entry point for the tenant. It is in that case that we may
have a Virtual Network Provider (VNP). The second and
the third category distinguishes the inter-domain level and
the intra-domain level, but do differ in how the two levels
are related.

The second category follows a *top-down* approach. This
basically means that the virtual network request is pro-
cessed by an inter-domain level algorithm. This algorithm
aims at selecting the domains that will embed the corre-
sponding virtual network. Dietrich et al.; Houidi et al.
provide each an inter-domain level algorithm. This al-
gorithm leverages the only information that the InPs are
not reluctant to share, like how the domains are intercon-
nected, and what general types of resources they support.
Then, it splits the virtual network request into subparts,
that can each be considered as virtual network requests
themselves to an intra-domain level algorithm. At this
stage, we come back to a VNE problem where the sub-
strate is the respective domain. Any suitable VNE algo-
rithm can be used at this point, and Houidi et al. provide
their own for instance.

For its part, the third category follows a *bottom-up* ap-
proach. This basically means that the virtual network re-
quest is directly sent to *each* participating InP. Those InPs
then run an intra-domain level algorithm whose purpose is
to generate candidates. A candidate is actually a subpart
of the virtual network request that the InP accepts and
is able to embed. Then, an inter-domain level algorithm
runs to select the candidates. The set of candidates that
are selected has the property of covering the whole original
virtual network request. Such an algorithm is proposed by
Mano et al.; Di et al.

Note that the information disclosure is limited, not
null, in all approaches. This follows the natural assump-
tion that InPs know how they are interconnected with their
neighbors. For Houidi et al., the inter-domain level algo-
rithm knows the full pricing function of each InP for each
given virtual resource. For Dietrich et al., the VNP knows
the InP's border nodes (i.e. nodes interconnecting InPs
with each other), as well as the types and prices of vir-
tual nodes each InP supports. For Mano et al.; Di et al.,
the inter-domain level algorithm knows the InP's border
nodes and the bandwidth of the inter-domain links. Be-
sides, for Di et al., the actual price of each candidate is
also fully known.

From all these works, two in particular focus on re-
stricting access to prices, that is, Dietrich et al.; Mano
et al., as price information is also sensitive.

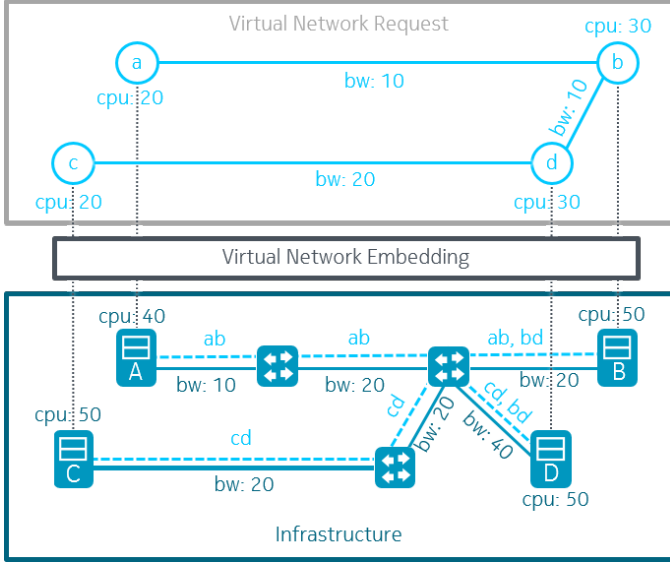


Figure 1: VNE illustration

Our work is distinct to the aforementioned works as follows.

For Dietrich et al., the capacity attributes (for nodes and for edges) are deeply inset into the matrix-oriented model, making it difficult to generalize to other attributes. Besides, the matrix-oriented model is not compatible with support for multiple attributes at once. Some topologies generated by the matrix model of the virtual network request may also be seen as undesirable by the tenant. It may be even uneasy for an InP to disclose node types when considering multiple attributes. In such a scenario, telling the external parties the features which are not available, or the combinations of features which are available, may be considered as very specific and then too sensitive.

For Mano et al., the partial embeddings are limited to connected subgraphs of the request. In other words, InPs cannot be used as a connection provider. It follows that the InP topology should be full-mesh, which does not correspond to the slice embedding scenario: we cannot expect every InP to be connected to all others.

Besides, Mano et al. do not make explicit the inter-domain nor the intra-domain algorithms, even if they rely on modified versions of other works of Yu et al.; Chowdhury et al.. Last but not least, they limit InP embedding propositions to be connected subparts, whereas we support more general pieces.

3. VNE Description for Single Domain

Let us describe the VNE problem for a single domain slice embedding. The situation itself is represented in Figure 1. Let the substrate be a weighted graph modelling the infrastructure. Substrate nodes model compute resources, and substrate edges model network links. Their weights correspond to some capacity. In Figure 1, such weights are labeled as *cpu* (node weight) or *bw* (edge weight). A tenant willing to rent resources from this infrastructure

must submit a virtual network request, which is also modelled as a weighted graph. The weights of virtual nodes and virtual links correspond to what the tenant demands. The goal of the problem is to embed this request into the substrate, under the following rules:

- R1 Node mapping rule.** Each virtual node is mapped to a *unique* substrate host.
- R2 Link mapping rule.** Each virtual link is mapped to a *unique* substrate path. A path is surely a set of contiguous links.
- R3 Eligible substrate hosts rule.** There are two kinds of substrate nodes: transit nodes (usually network devices) and end nodes, also called substrate hosts in node mapping rule (R1). Only substrate hosts can host a virtual node.
- R4 Eligible paths rule.** Paths in link mapping rule (R2) must be drawn between substrate hosts (as per eligible substrate hosts rule (R3)), which must be distinct.
- R5 Node-link mapping coordination rule.** The virtual nodes at the ends of each virtual link must be mapped to the hosts at the ends of the substrate path to which the virtual link is mapped.
- R6 Capacity rule.** The substrate node/links must accommodate all the demands of all the virtual node/links which are mapped to them.

In R1 and R2, we define as a *mapping* a relation between a tenant-desired virtual resource (a node or a link) and a physical resource (a *host* or a link). This mapping relation technically corresponds to different tasks. For instance, if virtual nodes represent virtual machines (VMs) and physical hosts represent servers, then the mapping represent the fact that the VM is installed in a server through a hypervisor. It is a 1-to-1 mapping, as illustrated by the association between the nodes *a* and *A* in Figure 1.

Other node mappings are possible. As mentioned in Section 2, Bays et al. consider virtual nodes to be virtual routers, that must be instantiated in physical routers. However, the development of network hypervisors (Blenk et al.) in the Software-Defined Network (SDN) context enables a single virtual SDN switch may be mapped to multiple physical SDN switches. As such, it is a 1-to-n mapping, which we decided to not address in this paper. For this reason, we follow R3.

Link mapping is another 1-to-n mapping, where the virtual link is mapped to a physical path, as illustrated by the association between the links *ab* and the links along the path between *A* and *B* in Figure 1. This raises the question of what happens when the path is empty, that is, when, technically, we are trying to emulate a link within a node. In this paper, we consider that this is still a tedious task, and we will avoid it at the physical level. This is why, in eligible paths rule (R4), we explicitly mention distinct hosts.

Besides, there is an important relation between the link and the node mapping, which is described in node-link mapping coordination rule (R5). It is typically this rule that becomes difficult to formulate when enabling a 1-to-n node mapping. With this rule, we consider that virtual links represent an authorized traffic from a source node to a destination node, and that it must flow through a continuous set of links from this source to that destination.

The concept of demand (and offer) from capacity rule (R6) will be described in the next section. In Figure 1, the demands are formulated by the weights within the virtual network request, and the offers by the weights within the infrastructure graph. There are some relations indeed between them. For instance, ab and bd can be mapped on the link between the rightmost switch and the server B because the sum of their requested bandwidths (bw) is lower than what the physical link provides.

4. Attribute-based Requirement Model

Requirements are expressed in natural language. They can be technical, but also legal or security-based. We propose here a model to translate requirements into mathematical constraints. This requirement model is built around the concept of *attribute*.

Attributes describe resources, either demanded by the tenants or owned by the InPs. For instance, when a tenant expresses the requirement “my server must be in the US or in the EU”, it translates as the virtual resource *tenant’s server* having a *location* attribute, which value is *US* or *EU*. This value is the tenant’s server demand (in location). Likewise, when an InP declares that “my server is in the US”, it translates as the physical resource *InP’s server* having a *location* attribute, which value is *US*. This value is the InP’s server offer (in location).

Attributes enables resources to be offered by providers to satisfy a demand from a tenant. We do so by exhibiting the mathematical constraints that each attribute should follow. Such constraints indicate that whether the mapping of the virtual resource to the physical resource fulfills the tenant’s requirements, or not. For our location attribute example, it is clear that offer and demand are related through the subset relation: the physical resource location must be in the authorized locations of the virtual resource.

Before introducing all the mathematical constraints, we now give a general overview of the notations used in this section. We denote as i a resource as required by a tenant, and as j a resource as provided by an InP. We denote as d_i all the demands for the resource i , and as o_j all the offers for the resource j . We denote as d_i^a (resp. o_j^a) a term in d_i (resp. o_j), corresponding to the attribute a . We denote as $x_{ij} \in \{\perp, \top\}$ an unknown which equals \top (meaning “true”) if resource i is allocated to resource j , and \perp (meaning “false”) otherwise. The notations used in this paper are detailed in Table A.3, given in appendix.

In this paper, we simplify the attribute classification presented in (Boutigny et al.), to only distinguish what we call tolerant and exclusion attributes. This distinction is a compromise between providing a unified attribute model and performances.

4.1. Exclusion Attributes: Semantics and Typology

Exclusion attributes enable tenants to forbid their virtual resources to share physical resources with some other virtual resources, be them owned by the same tenant or by other tenants. These attributes are special in the sense that their values directly refer to the resources of other tenants. Besides, exclusion by itself is a symmetric relation, as shown in Demonstration 1.

Demonstration 1 (Exclusion symmetry). *Let X^V be the set of virtual resources, and X^P the set of physical resources. Let $Exclude$ be the exclusion predicate, as a binary relation. It can be expressed as:*

$$\forall (i, k) \in X^V \times X^V, i \neq k, \\ Exclude(i, k) = (\forall j \in X^P, x_{ij} \Rightarrow \neg x_{kj})$$

By contraposition, $x_{ij} \Rightarrow \neg x_{kj}$ is equivalent to $x_{kj} \Rightarrow \neg x_{ij}$, hence $Exclude(i, k) = Exclude(k, i)$. \square

We exhibit three exclusion attributes, namely *xres* (resource exclusion), *xreq* (request exclusion), and *xten* (tenant exclusion). They follow respectively Equation (1), Equation (2) and Equation (3). These three equations only differ in what the tenant expresses as the elements to be excluded, respectively resources, requests, and tenants. Each equation then derives exclusion relationships from those elements. An excluded tenant will lead its requests to be excluded, and an excluded request will lead its resources to be excluded. In other words, Equation (1) treats more fine-grained values than Equation (2), which treats more fine-grained values than Equation (3).

$$x_{ij} \Rightarrow \bigwedge_{k \in d_i^{xres}} \neg x_{kj} \quad (1)$$

Equation (1) means that if i is allocated to j , then no excluded resource k can be allocated to j , where d_i^{xres} is the set of all resources k which are excluded by the requester.

$$x_{ij} \Rightarrow \bigwedge_{\substack{R \in d_i^{xreq} \\ k \in X^R}} \neg x_{kj} \quad (2)$$

Equation (2) means that if i is allocated to j , then no virtual resource k from any excluded request R can be allocated to j , where d_i^{xreq} is the set of all requests R which are excluded by the requester. The set X^R represents the set of resources in a request R .

$$x_{ij} \Rightarrow \bigwedge_{\substack{T \in d_i^{xten} \\ R \in \mathcal{R}_T \\ k \in X^R}} \neg x_{kj} \quad (3)$$

Equation (3) means that if i is allocated to j , then no virtual resource k from any request $R \in \mathcal{R}_T$ from any excluded tenant T can be allocated to j , where d_i^{xten} is the set of all tenants T which are excluded by the requester. The set \mathcal{R}_T represents the set of requests owned by a tenant T .

The reason why we define three distinct exclusion attributes is because the needs of the tenants may evolve over time. They may need new slices, new virtual resources. And of course, new tenants may appear. Our exclusion attributes capture those three layers. In practice, we apply these exclusion attributes for different cases.

Excluding resources from each other in a tenant's own request is useful if they carry data that are sensitive (1). Limiting co-locations in between the tenant's requests is useful when the tenant is a company, as one request may represent the company's intellectual property department slice, another the financial department slice (2). Excluding tenants is useful when the other tenants are identified as rivals (3). All those cases are driven by mistrust in slice isolation at the physical level.

To be more precise, a tenant may trust the resource isolation as enforced by the InPs for some resources, but not for some critical ones. In that case, this tenant may desire more control over the physical resources in which those critical resources are hosted, perhaps even to the point of allocating a dedicated physical resource for them.

4.2. Tolerant Attributes: Semantics and Typology

Aside the exclusion attribute, we have *tolerant* attributes. We call them as such because they tolerate unspecified values for both InPs and tenants. We want the list of tolerant attributes to be extended freely by InPs. In this paper, we provide a first list of tolerant attributes in Table 1. More specifically, Table 1a shows how tenants and InPs may view them, and Table 1b shows the building blocks of the corresponding mathematical structure. Besides, tolerant attributes are divided into binding and non-binding attributes.

$$\sum_{i \in X^R, x_{ij} = \top} d_i^a \leq o_j^a \quad (4)$$

Binding attributes result in constraints where the acceptance of a demand can impact the acceptance of another one. It is typically the case when we model an attribute where demands will consume an offer. Let *mem* be the memory attribute, which is a binding attribute. Let d_1 be a demand allocated to an offer o_j , that is, the variable x_{1j} is true. This means that the demand d_1 in *mem* is lower than the offer o_j in *mem* (that is, $d_1^{mem} \leq o_j^{mem}$). Now, let d_2 be another demand such that the demand d_2 in *mem* is lower than the offer o_j in *mem*. The condition allowing to allocate d_2 along d_1 to o_j (that is, both variables x_{1j} and x_{2j} are true), is *stricter* than only having $d_1^{mem} \leq o_j^{mem}$ and $d_2^{mem} \leq o_j^{mem}$, because the demand d_1 in *mem* already consumes the offer o_j in *mem*. In other

Table 1: Extensible List of Tolerant Attributes

a Classification and Description					
Attribute	InP	Description	Tenant	Description	Type
<i>mem</i>		Available memory in physical node		Required memory for virtual node	B_N
<i>bw</i>		Available bandwidth of physical link		Required bandwidth for virtual link	B_L
<i>loc</i>		The location of physical node		Authorized locations for virtual node	NB_N
<i>ven</i>		The vendor of physical node		Authorized vendors along virtual link	NB_L , with aggregation function
<i>ber</i>		Bit error rate (BER) of physical link		Maximal authorized BER of virtual link	NB_L
b Building Blocks					
Attribute	V_a	Ordering operator (demand \leq offer)	+	m_a	M_a
<i>mem</i>	$\mathbb{N} \cup \{\infty\}$	Natural order	Standard addition	0	∞
<i>bw</i>	$\mathbb{N} \cup \{\infty\}$	Natural order	Standard addition	0	∞
<i>loc</i>	Powerset of <i>Locations</i>	\supseteq (note the direction: offer must be in demand)	\cap	<i>Locations</i>	\emptyset
<i>ven</i>	Powerset of <i>Vendors</i>	\supseteq (note the direction: offer must be in demand)	\cap	<i>Vendors</i>	\emptyset
<i>ber</i>	Percentage	\geq (note the direction)	min	100%	0%

words, the demand d_2 in *mem* can only consume what remains of the offer o_j in *mem*, that is, $o_j^{mem} - d_1^{mem}$. The aforementioned condition is then that the demand d_2 in *mem* should be lower than the remaining *mem* of o_j , that is: $d_2^{mem} \leq o_j^{mem} - d_1^{mem}$, better rewritten as: $d_1^{mem} + d_2^{mem} \leq o_j^{mem}$.

Binding attributes follow Equation (4). This latter means that the sum of the demands d_i^a of all the virtual resources $i \in X^V$ that have been allocated to j must be lower than the offer o_j^a . There is one such constraint per offer o_j and per attribute a .

$$x_{ij} \Rightarrow d_i^a \leq o_j^a \quad (5)$$

On the contrary, *non-binding* attributes result in constraints where each demand acceptance can be checked separately. It is typically the case of the BER attribute. Let *ber* be the BER attribute, which is a non-binding attribute. Let d_1 be a demand allocated to an offer o_j , that is, the variable x_{1j} is true. This means that the demand d_1 in *ber* is lower than the offer o_j in *ber*, that is: $d_1^{ber} \leq o_j^{ber}$ (note the orientation of \leq : d_i^a here is the maximal accepted BER, as shown in Table 1a). Now, let d_2 be another demand such that the demand d_2 in *ber* is lower than the offer o_j in *ber*. The condition allowing to allocate d_2 along d_1 to o_j (that is, both variables x_{1j} and x_{2j} are true), is

1 equivalent to have $d_1^{ber} \geq o_j^{ber}$ and $d_2^{ber} \geq o_j^{ber}$. The afore-
 2 mentioned condition is then that any demand allocated
 3 to an offer o_j should be lower than the offer in *ber*, bet-
 4 ter rewritten as: $\min\{d_1^{mem}, d_2^{mem}\} \geq o_j^{mem}$ (again, see
 5 Table 1a regarding the orientation of \geq).

6 For non-binding attributes, it is better to use (5)-like
 7 constraints. There is one per offer o_j , per demand d_i and
 8 per attribute a .

10 4.3. Tolerant Attributes: Structure

11 More generally, we define an attribute a as a tuple
 12 $(V_a, \leq, +, m_a, M_a, \times)$. The building blocks of this struc-
 13 ture for different tolerant attributes is shown in Table 1b.
 14 Note that \times does not appear in this table. We actually
 15 introduce it later, and define it from the other building
 16 blocks.

17 We build this tuple from a commutative, partially or-
 18 dered monoid denoted $(W_a, \leq, +)$. Monoids are structures
 19 such that the law is associative, and supports a neutral el-
 20 ement. We denote m_a such neutral element. We suppose
 21 that there exists in W_a an element which is both its great-
 22 est and its absorbing element. This element is denoted
 23 M_a . Then V_a is the positive cone of W_a , that is, the set of
 24 all values which are greater than m_a . It follows that m_a
 25 is both the least and the neutral element in V_a .

26 We show in Demonstration 2 that we can always extend
 27 a monoid to get the M_a element, and the *mem* attribute in
 28 Table 1b, is an example of a monoid $(\mathbb{N}, \leq, +)$ we extend
 29 to get such M_a element, which is denoted as ∞ .

30 **Demonstration 2.** *Let $(W_a, \leq, +)$ be a commutative, par-*
 31 *tially ordered monoid. Let e be an ex nihilo element not in*
 32 *W_a . Let \leq' and $+'$ be extensions of \leq and $+$ such that:*

$$\begin{aligned} 36 \quad & \forall x \in W_a \cup \{e\}, x \leq' e \\ 37 \quad & e +' x = e +' e = e \\ 38 \quad & \forall x, y \in W_a, x \leq y \text{ is equivalent to } x \leq' y \\ 39 \quad & x +' y = x + y \end{aligned}$$

40 Then, we have stability of $+'$:

$$\begin{aligned} 44 \quad & \forall x, y \in W_a, \quad x +' y = x + y \quad \in W_a \subset W_a \cup \{e\} \\ 45 \quad & \forall x \in W_a, \quad e +' x = e + e = e \quad \in W_a \cup \{e\} \end{aligned}$$

46 We have associativity of $+'$ (demonstration is short-
 47 ened thanks to commutativity assumption):

$$\begin{aligned} 50 \quad & \forall x, y, z \in W_a, (x +' y) +' z = (x + y) + z \\ 51 \quad & \quad \quad \quad = x + (y + z) \\ 52 \quad & \quad \quad \quad = x +' (y +' z) \\ 53 \quad & \forall x, y \in W_a, (e +' x) +' y = (x +' y) +' e = e \end{aligned}$$

54 We have reflexivity of \leq' :

$$\begin{aligned} 56 \quad & \forall x \in W_a, x \leq' x \Rightarrow x \leq x \\ 57 \quad & e \leq' e \end{aligned}$$

We have antisymmetry of \leq' , as e is the greatest ele-
 ment by construction:

$$\begin{aligned} \forall x, y \in W_a, x \leq' y \wedge y \leq' x \Rightarrow x \leq y \wedge y \leq x \Rightarrow x = y \\ \forall x \in W_a, x \leq' e \wedge e \leq' x \Rightarrow e = x \end{aligned}$$

We have transitivity of \leq' . Let $x, y, z \in W_a \cup \{e\}$ such
 that $x \leq' y$ and $y \leq' z$.

- 1) if $z = e$, then $x \leq' z$ is true
- 2) if $z \neq e$, then $y \neq e$ hence $x \neq e$ then $x \leq z$ so $x \leq' z$

And we have compatibility between \leq' and $+'$. Let
 $x, y \in W_a \cup \{e\}$ such that $x \leq' y$.

We can show that: $\forall t \in W_a \cup \{e\}, x +' t \leq' y +' t$.

- 1) if $y = e$, we have $\forall t \in W_a \cup \{e\}, x +' t \leq' e +' t$
- 2) if $y \neq e$, then $x \neq e$
 - 2a) if $t = e$, we have $x +' e \leq' y +' e$
 - 2b) if $t \neq e$, we have $x + t \leq y + t$ hence $x +' t \leq' y +' t$

Consequently, $(W_a \cup \{e\}, \leq', +')$ is also a partially or-
 dered monoid. \square

We also define an operation \times as a function which takes
 a boolean (x_{ij}) and an attribute value $(d_i^a \in V_a)$ and re-
 turns another attribute value. It follows (6).

$$x_{ij} \times d_i^a = \begin{cases} d_i^a & \text{if } x_{ij} = \top \text{ (} x_{ij} \text{ is true)} \\ m_a & \text{otherwise} \end{cases} \quad (6)$$

As a general note, we can also define the \sum symbol
 in (4) with the law of the monoid, by applying recursively
 the associativity of this law, in the same way \sum is a re-
 currence over $+$. In this paper though, all the binding
 attributes we present are derived from integers.

Among those building blocks, m_a and M_a happen to
 play a convenient role for both tenants and InPs.

The m_a value of tolerant attributes as well as the value
 \emptyset of the exclusion attributes can be used as a *default* value.
 In other words, when tenants fill their requests, they can
 skip some attributes, and focus only on those which are
 meaningful to them. The m_a value guarantees through (4)
 and (5) that *all* offers will be considered. For the exclu-
 sion constraint, the \emptyset value guarantees through (1) that
 all offers *but* the ones we have excluded will be considered
 (since exclusion is a symmetric relation).

At the same time, m_a can be used by the InPs to in-
 dicate that the value for this attribute is unknown or that
 the attribute itself is unsupported. In that case, the m_a
 value guarantees through (4) and (5) that *only* demands
 which use m_a too will be considered. In other words, the
 equations follow the legitimate rule that demands with a
 meaningful value for the tenant (that is, different from m_a)
 will not be allocated to offers with an unknown value or
 not supporting the attribute (that is, equal to m_a).

For its part, M_a is a convenient way to implement a
 resource with an unlimited capacity (or a capacity which

cannot be exhausted), while still complying with capacity rule (R6). Indeed, by definition, when $o_j^a = M_a$, Eqs. (4) and (5) are true. In other words, when $o_j^a = M_a$, the offer can accept as many demands as desired. Note that unlike m_a , we do not have any equivalent of M_a for exclusion attribute. However, disabling the exclusion attribute for some offers happens to be useful in some cases. We will present how and why in Sections 6 and 7.

4.4. Node and Link Attributes: Semantics and Typology

Along the tolerant-exclusion distinction, we also distinguish *node* and *link* attributes. Node attributes are on tenant virtual nodes, and link attributes are on tenant virtual links. In other words, attributes are classified according to the tenant’s viewpoint.

As shown in appendix in Table A.3, we denote as B_N (resp. NB_N) the set of binding (resp. non-binding) node attributes, and as B_L (resp. NB_L) the set of binding (resp. non-binding) link attributes.

The semantics of some attributes may not involve a virtual node and a physical node, or a virtual link and a physical link, but a virtual link and physical nodes. In Table 1a, we present such a *link* attribute (from the tenant’s viewpoint), *ven*, whose offer is related to the nodes at the tips of the physical link. For such attribute, in (4) and (5), we consider that o_j^a is the result of some aggregation function denoted \cdot_a , as defined in Definition 1.

Definition 1 (Aggregation function). *A tolerant, link attribute a is a tuple $(V_a, \leq, +, m_a, M_a, \times, \cdot_a)$ where $(V_a, \leq, +, m_a, M_a, \times)$ is an attribute, and \cdot_a is a binary relation called the aggregation function. This latter is used to define the offer o_j^a for some link j , as follows. We denote as h and t the nodes at the tips of the link j .*

$$\forall j = (h, t) \in L^S, o_j^a := o_h^a \cdot_a o_t^a$$

The relation \cdot_a must have the following properties, with respect to the attribute $(V_a, \leq, +, m_a, M_a, \times)$.

- 1) neutral element: $\forall v \in V_a, M_a \cdot_a v = v \cdot_a M_a = v$
- 2) absorbing element: $\forall v \in V_a, m_a \cdot_a v = v \cdot_a m_a = m_a$
- 3) idempotence: $\forall x \in V_a, x \cdot_a x = x$
- 4) commutative: $\forall x, y \in V_a, x \cdot_a y = y \cdot_a x$
- 5) translation-invariance: $\forall x, y, t \in V_a, x \leq y \Rightarrow x \cdot_a t \leq y \cdot_a t$

Note that the neutral (resp. absorbing) element of \cdot_a is the absorbing (resp. neutral) element of $+$.

As an example, for the tolerant link attribute *ven*, we define \cdot_{ven} as the union (\cup) of the node offers, for instance. The reader can easily verify that the union follows the properties given in Definition 1 as M_{ven} is the empty set (\emptyset), m_{ven} is the set of all vendors, and the ordering operator is the containment (\supset), as shown in Table 1b.

All the attributes in Table 1a can be formulated in Satisfiability Modulo Theories (SMT). As usual, the integer

attributes leverage integer theory, and the exclusion attribute leverage boolean theory. Our novelty is that the powerset attributes leverage bitvector theory. The bitvector theory is a convenient way to represent elements of a powerset $\mathcal{P}(S)$, where S is finite, because we can associate each bit to the presence/absence of a certain element of S . Then for $S = \{a, b, c\}$ we can associate a sequence (a, b, c) that tells which bit corresponds to which element, and then $\emptyset = 000$, $S = 111$, $\{a, c\} = 101$. The inclusion operation, $x \subset y$ (see Table 1b), is then equivalent to the predicate $(x = x \odot y)$ where \odot denotes the bitwise logical “and”.

5. Single Domain Slice Embedding Formulation

We leverage our attribute-based requirement model to express the slice embedding problem as a constraint satisfaction problem. In this section, we present these mathematical constraints for a limited case, with only one InP and only one domain, namely, the single domain scenario. We will build upon it in the next sections.

The constraint satisfaction problem formulation serves as an interface between the human language, and the slice embedding algorithm. We consider this interface as an important feature for InPs, tenants, and system auditors, as it enables them to verify what the algorithm means, and how the attributes are applied.

For this reason, we use an SMT formulation. SMT solvers take a constraint satisfaction problem, and return one solution at a time. This solution is not necessarily optimal. To get an optimal solution, we must add an optimality constraint to the solver.

In this paper, we do not focus on finding any optimal solution. Instead, we enumerate all of them, to verify that the constraints are applied as intended. The reason is that in VNE, the ILP formulations are cautiously including some optimization features. For instance, paths are expressed using a flow conservation constraint. If we consider this constraint alone, it allows very convoluted paths, which can loop and go through the source and destination many times. Yet, the associated optimality constraint keeps the ILP solver from generating such paths. In this paper, as we leverage more theories than the integer one, it is important to be sure, before any optimization, that our problem is correctly formulated.

We already semantically describe the single domain scenario in Section 3 as a set of rules to follow. We now translate those rules into mathematical constraints, as follows.

The substrate, or physical infrastructure, or single domain, is denoted S . It is a directed, connected graph: $S = (N^S, L^S)$. Some nodes are hosts: $H^S \subset N^S$. Only them can host slice nodes. Besides, we have a set of (slice) requests, denoted \mathcal{R} . Each request is owned by a tenant. Each request R is modeled as a directed graph $R = (N^R, L^R)$. The reader can refer to Table A.3 in appendix for all notations used in this paper. We treat one

request at a time, so we distinguish a request-to-embed, denoted NR , from already embedded slices.

$$\bigvee_{j \in H^S} \left(n_{ij} \wedge \bigwedge_{k \neq j} \neg n_{ik} \right) \quad \forall R \in \mathcal{R}, i \in N^R \quad (C1)$$

$$\bigvee_{j \in P^S} \left(p_{ij} \wedge \bigwedge_{k \neq j} \neg p_{ik} \right) \quad \forall R \in \mathcal{R}, i \in L^R \quad (C2)$$

Equation (C1) (resp. (C2)) implements node mapping rule (R1) (resp. link mapping rule (R2)), expressed as a one-and-only-one predicate over the n_{ij} (resp. p_{ij}). The n_{ij} and the p_{ij} correspond to the unknowns x_{ij} as used in Section 4. As such, n_{ij} (resp. p_{ij}) are Boolean variables which are true (denoted \top) when a virtual node i (resp. a virtual link i) is mapped to a substrate host j (resp. a path j), and false (denoted \perp) otherwise. Especially, $j \in H^S$ complies with eligible substrate hosts rule (R3), and $j \in P^S$ complies with eligible paths rule (R4). We will give further details about P^S after the description of all equations.

$$\left. \begin{aligned} p_{ij} = & (n_{ux} \wedge n_{vy}) \wedge \bigwedge_{k \in L_j} l_{ik} \wedge \bigwedge_{k \in L^S \setminus L_j} \neg l_{ik} \\ & \forall R \in \mathcal{R}, i \in L^R, j \in P^S, i = (u, v), E_j = (x, y) \end{aligned} \right\} \quad (C3)$$

The p_{ij} variables are derived from n_{ij} and l_{ij} through (C3). It defines the link-to-path variables as the conjunction of mapping link ends to path ends (cf. node-link mapping coordination rule (R5)), then mapping the link to the links along the path, and then not mapping the link to the other links in the infrastructure.

$$n_{i,m(i)} \quad \forall R \in \mathcal{R} \setminus \{NR\}, i \in N^R \quad (C4)$$

$$p_{i,m(i)} \quad \forall R \in \mathcal{R} \setminus \{NR\}, i \in L^R \quad (C5)$$

Equation (C4) (resp. (C5)) tells that each virtual node i (resp. each virtual link i) of the already embedded requests should be mapped to the physical node $m(i)$ (resp. physical path $m(i)$), where m is a function that memorizes the previous results of the embedding algorithm.

$$\sum_{\substack{i \in N^R \\ R \in \mathcal{R}}} n_{ij} \times d_i^a \leq o_j^a \quad \forall a \in B_N, j \in H^S \quad (C6)$$

$$n_{ij} \Rightarrow d_i^a \leq o_j^a \quad \forall a \in NB_N, R \in \mathcal{R}, i \in N^R, j \in H^S \quad (C7)$$

$$\sum_{\substack{i \in L^R \\ R \in \mathcal{R}}} l_{ij} \times d_i^a \leq o_j^a \quad \forall a \in B_L, j \in L^S \quad (C8)$$

$$l_{ij} \Rightarrow d_i^a \leq o_j^a \quad \forall a \in NB_L, R \in \mathcal{R}, i \in L^R, j \in L^S \quad (C9)$$

Equation (C6) (resp. (C7), (C8), (C9)) guarantee requirements based on tolerant attributes B_N (resp. NB_N ,

B_L , NB_L). They express that the offer o_j^a of a InP resource j in a given attribute a is limited, and that the demands d_i^a of the tenant resources i allocated to j are bounded by the offer. They comply with capacity rule (R6).

Equations (C6) and (C8) are based on (4), using n_{ij} and l_{ij} instead of x_{ij} . They describe binding attributes. For their part, equations (C7) and (C9) are based on (5), using n_{ij} and l_{ij} instead of x_{ij} . They describe non-binding attributes.

$$n_{ij} \Rightarrow \bigwedge_{k \in d_i^{xres}} \neg n_{kj} \quad \forall R \in \mathcal{R}, i \in N^R, j \in H^S \quad (C10)$$

$$n_{ij} \Rightarrow \bigwedge_{\substack{R' \in d_i^{xreq} \\ k \in N^{R'}}} \neg n_{kj} \quad \forall R \in \mathcal{R}, i \in N^R, j \in H^S \quad (C11)$$

$$n_{ij} \Rightarrow \bigwedge_{\substack{T \in d_i^{xten} \\ R' \in \mathcal{R}_T \\ k \in N^{R'}}} \neg n_{kj} \quad \forall R \in \mathcal{R}, i \in N^R, j \in H^S \quad (C12)$$

Equations (C10), (C11), and (C12) express node exclusion constraints. They are based respectively on (1), (2) and (3), using n_{ij} instead of x_{ij} . They also comply with R6.

$$l_{ij} \Rightarrow \bigwedge_{k \in d_i^{xres}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in L^S \quad (C13)$$

$$l_{ij} \Rightarrow \bigwedge_{\substack{R' \in d_i^{xreq} \\ k \in L^{R'}}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in L^S \quad (C14)$$

$$l_{ij} \Rightarrow \bigwedge_{\substack{T \in d_i^{xten} \\ R' \in \mathcal{R}_T \\ k \in L^{R'}}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in L^S \quad (C15)$$

Finally, Equations (C13), (C14), and (C15) express link exclusion constraints. Likewise, they are based respectively on (1), (2) and (3), using l_{ij} instead of x_{ij} . They also comply with R6.

To give more details, the R4 is enforced through the set P^S (see Table A.3 in appendix), which is the set of what we call ‘‘loop-free paths’’. Loop-free paths are defined by Definition 2. They are indeed a special case of simple paths. For this reason, we can easily design a loop-free path generating algorithm from a simple path generating algorithm.

Definition 2 (Loop-free path). *A loop-free path from a source to a target in a graph is a path that traverse each node of the graph at most once when source is different from target. When source equals target, and if there exists a self-loop (source, source), there is a unique loop-free path between source and source. It is the sequence (source, (source, source), source).*

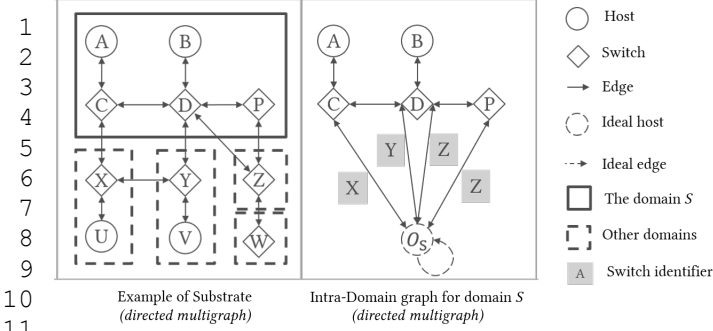


Figure 2: Example of Intra Domain Graph

6. Intra-Domain Level Slice Embedding Formulation

The constraint satisfaction problem as formulated in Section 5 for a single domain can be reused for a multi-domain scenario. An obvious way is to consider as the substrate the aggregation of all the resources from all the involved domains. Yet, for slices, the domains are the infrastructure networks and are owned by several InPs which are likely reluctant to share their information, as they are commercially sensitive.

Instead, we need to make the InPs cooperate while keeping their information private. In this paper, we consider two levels of cooperation, the inter-domain level and the intra-domain level. The inter-domain level is presented in Section 7.

Broadly speaking, the inter-domain level embeds a *whole* slice in the *domains* without any knowledge about what they contain. The constraints hold on how the domains are interconnected.

On the contrary, the intra-domain level embeds a *part* of a slice in the *resources* of a domain with full knowledge about their attributes. By a part, we mean that all requests must be embedded when considering *all* the domains, but that in the same time, some domains may not embed any request. Indeed, the resources allocated for a given request may belong to completely different domains.

Modeling partial mapping is not easy though, because node mapping and link mapping are interrelated. In our previous work (Boutigny et al.), we modeled the partial mapping with a heuristic, which allows to abstractly embed some tenant nodes in some border nodes, where border nodes are the nodes at the boundary of the given domain. This embedding is abstract in the sense that border nodes were virtually considered as host candidates, despite not being hosts. Besides, no attribute constraint was applied for them.

Such heuristic is very limited though, because we could have two different virtual links leading to the *same* virtual node exiting the domain through two *different* border nodes. This was not possible with our previous heuristic, by definition.

Instead, we use a topological heuristic. This heuristic enables us to reuse Section 5.

Before explaining the heuristic, let us model what a domain can know. We use Figure 2 for this purpose. On the left, we present an example of a substrate, with 5 different domains, among which one is denoted S . As in Section 5, it is a directed, connected graph.

On the right of Figure 2, we present the intra-domain level graph, which actually represents what we assume S can know, in the form of a directed, connected multigraph. Formally, we consider that any domain knows:

- i) its own resources, as well as their attributes (in Figure 2, this corresponds to A, B, C and D , as well as the links between them),
- ii) the inter-domain links, as well as their attributes (in Figure 2, this corresponds to (C, X) and (D, Y) , among others), and
- iii) the switches to which it is connected in its neighboring domains, but *not* their attributes (in Figure 2, this corresponds to X, Y and Z).

Under these limitations of what S knows, the inner topology of other domains ((X, U) in Figure 2 for instance), as well as if and how they could be interconnected ((X, Y) for instance), is not known.

The heuristic relies on a special *host* node, denoted O^S where S is the domain being considered. This node models the outside of the domain, which encompasses all other domains, being its neighbors or not. When we map a virtual node to the outside, we say that we have a partial mapping (or that the node is outsourced). In other words, we consider that such virtual node will be hosted in another domain. The outside host is connected to every border nodes (C, D and P for S in Figure 2). This follows a legitimate assumption, that there is no disjoint domain in our system.

Besides, we consider a self-loop (O^S, O^S) , which allows us to outsource a virtual link to the outside. The existence of (O^S, O^S) is required to enable and define the outsourcing of a virtual edge (x, y) , when both x and y must be outsourced too.

From this topology, we define OP^S the set of loop-free paths in P^S that start from a host inside the domain, go out through O^S and then end in another host inside the domain. The candidate paths we consider for our embedding are then in $P^S \setminus OP^S$.

By construction, there is a unique path in $P^S \setminus OP^S$ that contains (O^S, O^S) . It is the path $(O^S, (O^S, O^S), O^S)$, as explained in Definition 2. As such, as we have a unique way to outsource a virtual node, we have also a unique way to outsource a virtual link. This unicity is important to guarantee when generating solutions, otherwise we would generate the same solution multiple times.

Another property of the host O^S and the self-loop (O^S, O^S) is that they are ideal, which is defined in Definition 3.

Definition 3 (Ideal node and link). *An ideal node (or link) is a resource for which we do not apply any tolerant nor exclusion constraints. For a tolerant attribute a , this is done by using the value M_a . As exclusion attributes do not exhibit such a convenient value, we do not generate exclusion constraints from ideal nodes and links. In other words, they will accept whatever request they receive.*

By doing so, O^S and (O^S, O^S) limit the possible embeddings in the given intra-domain in terms of topology, rather than attribute values, as any partial embedding requires to go through an inter-domain link.

Inter-domain links are the most critical part of the intra-domain graph indeed, as each of them is known by two domains. For most attributes, the inter-domain link attribute values do not depend on the intra-domain being considered, as we assume that such values are shared between the two involved InPs. Consequently, a virtual link is allocated to an inter-domain link, only if both domains allocate this virtual link to the inter-domain link at their intra-domain level. Furthermore, if we had the full knowledge of the topology, this virtual link could also be allocated to the same inter-domain link, as we have the same attribute values.

However, the reader may be interested by the case of tolerant attributes which rely on aggregation functions, as defined in Section 4, like the *ven* attribute. In that case, the inter-domain link attribute values are *different*, due to the presence of O^S . They are also different from the inter-domain link attribute value under the fully disclosed infrastructure. Thanks to the properties of aggregation function (cf. Definition 1), this difference does not impact the validity of the embedding, as we show in Demonstration 3. More precisely, we show that if an embedding is rejected with full knowledge of the attributes, then it is also rejected by our solution.

Demonstration 3. *Let d and e be two domains. Let S be the infrastructure with full knowledge about d and e . Let $(h, t) \in L^S$ be an inter-domain link between d and e . Let a be a tolerant link attribute with an aggregation function \cdot_a .*

Thanks to the aggregation function properties, as given in Def. 1, we can show that having both domains separately accepting the demand on abstract links (h, O^d) and (O^e, t) respectively implies to accept the demand with full knowledge on the physical link (h, t) .

By definition of outside hosts O^d and O^e , as well as per Def. 1, we have:

$$\begin{aligned} o_{(h, O^d)}^a &= o_h^a \cdot_a M_a = o_h^a \\ o_{(O^e, t)}^a &= M_a \cdot_a o_t^a = o_t^a \\ o_{(h, t)}^a &= o_h^a \cdot_a o_t^a \end{aligned}$$

Let R be a request and $i \in L^R$ a link. Let assume that each domain d and e separately accepts the demand d_i^a for o_h^a

and o_t^a respectively. In other words:

$$\begin{aligned} 1) \quad & d_i^a \leq o_h^a \\ 2) \quad & d_i^a \leq o_t^a \end{aligned}$$

It follows:

$$\begin{aligned} \text{translation-invariance of 1): } & d_i^a \cdot_a o_t^a \leq o_h^a \cdot_a o_t^a \\ \text{translation-invariance of 2): } & d_i^a \cdot_a d_i^a \leq o_t^a \cdot_a d_i^a \\ \text{idempotence of } \cdot_a: & d_i^a \cdot_a d_i^a = d_i^a \\ \text{commutativity of } \cdot_a: & d_i^a \leq d_i^a \cdot_a o_t^a \\ \text{transitivity of } \leq: & d_i^a \leq o_h^a \cdot_a o_t^a \end{aligned}$$

As we have $d_i^a \leq o_h^a \cdot_a o_t^a$, we know that S will also accept the demand d_i^a for the link (h, t) . \square

We now present how this heuristic helps us to adapt the equations from Section 5. Every rule from Section 3 which is modified is explicitly cited. Those which are not cited are not modified.

Eq. (C1) still holds because O^S is a host. Eqs. (C2) and (C3) only hold for $P^S \setminus OP^S$ and not P^S . This is to reduce recursivity: otherwise, if we enable paths going out through O^S , it means that the other domains would have to also validate the link mapping. In other words, we alter eligible paths rule (R4) by using a subset of the available paths. See (C2b), (C3b).

Eqs. (C4) and (C5) still hold because O^S is a host. Eqs. (C6) and (C7) still hold because O^S is an ideal node (cf. Def. 3). Eqs. (C8) and (C9) still hold because (O^S, O^S) is an ideal link (cf. Def. 3). Eqs. (C10), (C11) and (C12) only hold for host $j \neq O^S$, because O^S is an ideal node (cf. Def. 3). In other words, we alter capacity rule (R6) to make an exception for exclusion attributes on O^S , which is consistent with its abstract nature. See (C10b), (C11b), (C12b).

Equations (C13), (C14), and (C15) only hold for link $j \neq (O^S, O^S)$ because (O^S, O^S) is an ideal link (cf. Def. 3). In other words, we alter R6 to make an exception for exclusion attributes on (O^S, O^S) , which is consistent with its abstract nature. See (C13b), (C14b), and (C15b).

$$\bigvee_{j \in P^S \setminus OP^S} \left(p_{ij} \wedge \bigwedge_{k \neq j} \neg p_{ik} \right) \quad \forall R \in \mathcal{R}, i \in L^R \quad (C2b)$$

$$\left. \begin{aligned} p_{ij} &= (n_{ux} \wedge n_{vy}) \wedge \bigwedge_{k \in L_j} l_{ik} \wedge \bigwedge_{k \in L^S \setminus L_j} \neg l_{ik} \\ \forall R \in \mathcal{R}, i \in L^R, & j \in P^S \setminus OP^S, i = (u, v), E_j = (x, y) \end{aligned} \right\} \quad (C3b)$$

$$n_{ij} \Rightarrow \bigwedge_{k \in d_i^{xres}} \neg n_{kj} \quad \forall R \in \mathcal{R}, i \in N^R, j \in DH^S \quad (C10b)$$

$$n_{ij} \Rightarrow \bigwedge_{\substack{R' \in d_i^{xreq} \\ k \in N^{R'}}} \neg n_{kj} \quad \forall R \in \mathcal{R}, i \in N^R, j \in DH^S \quad (C11b)$$

$$n_{ij} \Rightarrow \bigwedge_{\substack{T \in d_i^{xten} \\ R' \in \mathcal{R}_T \\ k \in N^{R'}}} \neg n_{kj} \quad \forall R \in \mathcal{R}, i \in N^R, j \in DH^S \quad (C12b)$$

$$l_{ij} \Rightarrow \bigwedge_{k \in d_i^{xres}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in DL^S \quad (C13b)$$

$$l_{ij} \Rightarrow \bigwedge_{\substack{R' \in d_i^{xreq} \\ k \in L^{R'}}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in DL^S \quad (C14b)$$

$$l_{ij} \Rightarrow \bigwedge_{\substack{T \in d_i^{xten} \\ R' \in \mathcal{R}_T \\ k \in L^{R'}}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in DL^S \quad (C15b)$$

7. Inter-Domain Level Slice Embedding Formulation

At the inter-domain level, we only know the different domains and the inter-domain links. The inner domain topologies are unknown. Yet, we can still reuse the rules in Section 3, and use them to adapt the equations from Section 5, as follows.

We model the inter-domain level as a directed, connected, multigraph. Each node represents a domain, and each link represents an inter-domain link. Domains may be connected through more than one pair of border routers.

We leverage the multigraph to know which domains can be used for our slice, and which inter-domain links will be used. The inner details of the embedding are left to the domains.

In such graph, according to node mapping rule (R1) and eligible substrate hosts rule (R3), we need to consider that each domain is a host. Then, we must relax eligible paths rule (R4) to enable two virtual nodes to be embedded in the same domain. To do so, we will use loop-free paths (cf. Def. 2), and benefit from the behavior with self-loops. Then, we keep node-link mapping coordination rule (R5) as is. Finally, we must relax capacity rule (R6), because we cannot treat neither exclusion nor tolerant attribute constraint at this level due to our lack of knowledge. This is also true for the self-loops on the domains.

From this update of the rules, it follows that we only need to consider domains as hosts, with a self-loop edge. As we do not consider exclusion nor tolerant attributes on domains nor on their self-loops, we can make them all ideal, as per Definition 3.

We now present how this heuristic helps us to adapt the equations from Section 5.

Eqs. (C1), (C2), (C3), (C4) and (C5) still hold because domains are hosts. Eqs. (C6) and (C7) still hold because domains are ideal nodes. Eqs. (C8) and (C9) still hold because domain self-loops are ideal links. Eqs. (C10), (C11) and (C12) are removed because domains are ideal nodes. Eqs. (C13), (C14), and (C15) only hold for inter-domain links, as domain self-loops are ideal links. See (C13c), (C14c), and (C15c).

$$l_{ij} \Rightarrow \bigwedge_{k \in d_i^{xres}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in L^S \setminus SL^S \quad (C13c)$$

$$l_{ij} \Rightarrow \bigwedge_{\substack{R' \in d_i^{xreq} \\ k \in L^{R'}}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in L^S \setminus SL^S \quad (C14c)$$

$$l_{ij} \Rightarrow \bigwedge_{\substack{T \in d_i^{xten} \\ R' \in \mathcal{R}_T \\ k \in L^{R'}}} \neg l_{kj} \quad \forall R \in \mathcal{R}, i \in L^R, j \in L^S \setminus SL^S \quad (C15c)$$

8. Multi-level Resolution Algorithm

Our algorithm is presented in Figure 3. It follows a delegation system, where an *InterDomainSolver* serves as the unique interface between the tenant and the InPs. The *InterDomainSolver* views only the interconnections between the different domains. It is the only solver belonging to what is called the “inter-domain level”. Each *IntraDomainSolver* corresponds to a domain, and is the only entity being able to access the data of that domain. They belong to what is called the “intra-domain level”. The delegation system appears in how the two levels interact. Indeed, the *InterDomainSolver* acts as an initiator, which queries each *IntraDomainSolver*. It uses their replies to generate the solutions.

The system involves a tenant T submitting a new request NR to the unique *InterDomainSolver* (denoted U), which collaborates with the multiple *IntraDomainSolvers* (denoted d). From now, the inter-domain level must be distinguished from the intra-domain level. To do so, the letters U and d will be used in *superscript*. For instance, x_{ij}^d is an allocation variable x_{ij} from the intra-domain level problem (any n_{ij} , p_{ij} or l_{ij} in Section 6), while x_{ij}^U is an allocation variable x_{ij} from the inter-domain level problem (any n_{ij} , p_{ij} or l_{ij} in Section 7).

The generation algorithm, presented in Fig. 3 starts with an **initialization** step, where the tenant T sends the new request NR (step 1) to the inter-domain solver U , which relays it to the different intra-domain solver instances d (step 3). When receiving NR , each solver runs independently a *genCstr* (for “generate constraints”) function. The function *genCstr* ^{d} (resp. *genCstr* ^{U}) is the function which generates the intra-domain level (resp. inter-domain level) mathematical constraints as per the formulation in Section 6 (resp. Section 7).

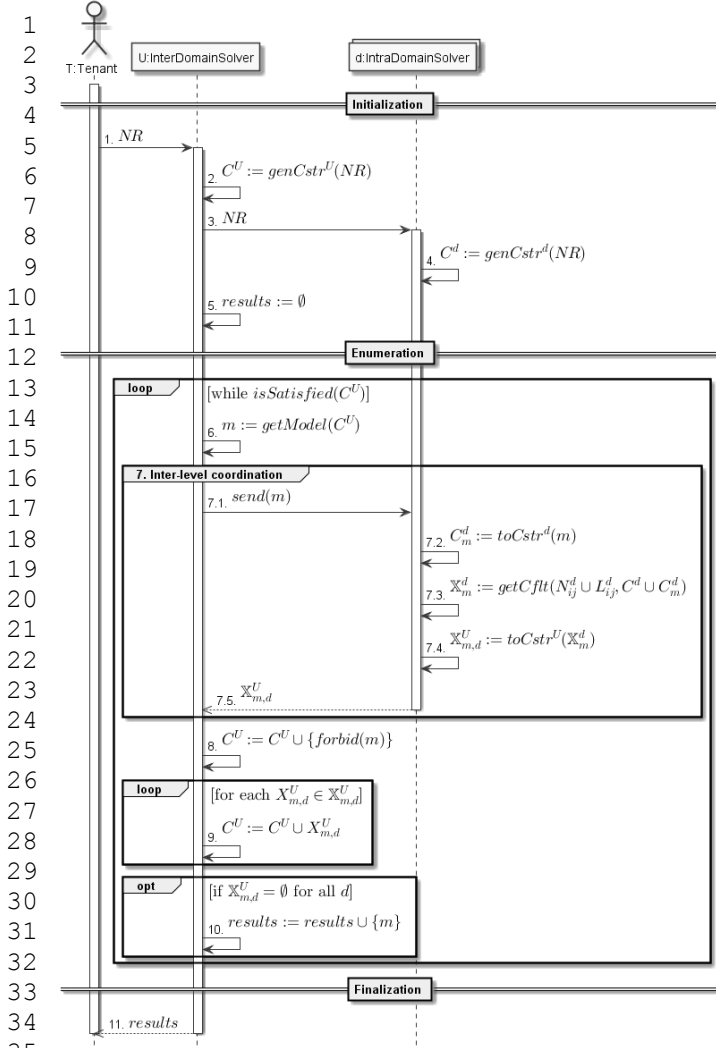


Figure 3: Embeddings Generation Sequence Diagram

Then at the **enumeration** step, all solvers participate in a *while* loop. The embeddings are outputted in the variable *results*. We will describe how the enumeration works shortly after. The algorithm ends with the **finalization** step, which sends the *results* back to the tenant *T*.

8.1. Solution Enumeration

Enumerating solutions with a SMT solver is a known algorithm: while the constraints are satisfied (through an *isSatisfied* function), the solver gets a model *m* (through the *getModel* function, see step 6), stores it in the results, and adds a new constraint to the problem which prevents itself from re-generating *m*. This new constraint is returned by the function *forbid* (step 8).

$$\bigvee_{x \in X} x \neq m(x) \quad (7)$$

$$\bigvee_{x \in N_{ij} \cup P_{ij}, m(x) = \top} \neg x \quad (8)$$

By definition, a model *m* is a function that assigns a value *m(x)* to each variable $x \in X$ of the original problem. Hence forbidding a model *m* can be done with (7). This expression is for the general case. If we adapt it to our slice embedding formulation, we can exhibit a shorter (in terms) forbidding constraint, represented in (8).

The reason why we can do so is that knowing, for each virtual resource $i \in N^R \cup L^R$ of a request *R*, the resource $j \in H^d \cup OP^d$ (resp. $j \in H^U \cup P^U$) to which it is mapped at the intra-domain level (resp. the inter-domain level), *fully determines* the values of all the variables N_{ij} , P_{ij} and L_{ij} (for the respective level), as we show in Demonstration 4. Thus (8) is the chosen expression for *forbid*.

Demonstration 4. *For the sake of simplicity, this demonstration ignores the difference between the inter-domain level and the intra-domain level. We rely on the embedding formulation in Section 5. Considering inter-domain level equations or intra-domain level equations does not alter the reasoning.*

Let suppose we have a solution. It follows:

1) Equation (C1) is true:

$$\forall i \in N^R, \bigvee_{j \in H^S} \left(n_{ij} \wedge \bigwedge_{k \neq j} \neg n_{ik} \right) = \top$$

2) \vee -split:

$$\forall i \in N^R, \exists j \in H^S, n_{ij} \wedge \bigwedge_{k \neq j} \neg n_{ik} = \top$$

3) \wedge -split:

$$\forall i \in N^R, \exists j \in H^S, (n_{ij} = \top) \wedge (\forall k \neq j, n_{ik} = \perp)$$

4) *j* is indeed unique:

$$\forall i \in N^R, \exists! j \in H^S, (n_{ij} = \top) \wedge (\forall k \neq j, n_{ik} = \perp)$$

As such, knowing a resource *j* such that n_{ij} is true means that it is the *j* such that n_{ij} is true and that for all $k \neq j$, the n_{ik} are false.

We apply the same reasoning to Equation (C2). In other words, knowing all the n_{ij} which are true and all the p_{ij} which are true is equivalent to know the values of all the n_{ij} and p_{ij} .

Besides, let $i = (u, v) \in L^R$. Let $j \in P^S$ such that $p_{ij} = \top$. Let $E_j = (x, y)$. It follows:

1) Equation (C3) is true:

$$p_{ij} = (n_{ux} \wedge n_{vy}) \wedge \bigwedge_{k \in L_j} l_{ik} \wedge \bigwedge_{k \in L^S \setminus L_j} \neg l_{ik} = \top$$

2) \wedge -split:

$$(n_{ux} = \top) \wedge (n_{vy} = \top) \wedge (\forall k \in L_j, l_{ik} = \top) \wedge (\forall k \in L^S \setminus L_j, l_{ik} = \perp)$$

In other words, for a given *i*, knowing a *j* such that p_{ij} is true implies that we know the values of all the l_{ik} for $k \neq j$. It follows that knowing all the p_{ij} which are true implies to know the values of all l_{ij} .

Knowing all the n_{ij} which are true and all the p_{ij} which are true is then sufficient to describe the solution. \square

1 In our algorithm, the **enumeration** stage features three
2 steps, numbered 7, 9 and 10. They are all related to
3 the **inter-level coordination**. At step 7.1, it sends the
4 inter-domain level model m to the solver d . At step 7.2,
5 this model is translated into intra-domain level constraints
6 with $toCstr^d$ (for “translate to constraints”). At step 7.3,
7 it generates the set of minimal unsatisfiable subsets. We
8 will explain its exact meaning later, but the reader can
9 interpret it globally as the set of conflicting variables. At
10 step 7.4, those minimal unsatisfiable subsets are translated
11 back into inter-domain level constraints with $toCstr^U$, which
12 are sent to U at step 7.5.

13 At step 9, the algorithm uses the minimal unsatisfi-
14 cable subsets as new forbidding constraints, similarly to
15 the function *forbid* at step 8. At step 10, the algorithm
16 determines if the model m is a solution. It is the case if
17 every domain d returns an empty set of minimal unsatis-
18 fiable subsets. Naturally, if m is a solution, no forbidding
19 constraint is added at step 9, but as the *forbid* function is
20 called in step 8, the solver never solves twice the same con-
21 straints. Consequently, the enumeration loop will always
22 terminate.

24 8.2. Inter-level Coordination

25 Let us now talk about the **inter-level coordination**.
26 Above all the algorithm uses *getCflt* (for “get conflicts”), a
27 function relying on another known SMT algorithm, called
28 MARCO (Liffiton et al.), which produces all minimal unsat-
29 isfiable subsets. Given a problem as a set of constraints
30 (like $\{a, \neg a, b, \neg b\}$), an unsatisfiable subset is a subset of
31 the problem constraints which are unsatisfiable. For in-
32 stance, $\{a, \neg a, b\}$ is such an unsatisfiable subset, as well
33 as $\{a, \neg a\}$. A *minimal* unsatisfiable subset is then an un-
34 satisfiable subset which does not contain any other unsat-
35 isfiable subset. As $\{a, \neg a, b\}$ contains $\{a, \neg a\}$, it is not a
36 minimal unsatisfiable subset, but $\{a, \neg a\}$ is. Minimal un-
37 satisfiable subsets are not necessarily unique. For instance,
38 $\{b, \neg b\}$ is also a minimal unsatisfiable subset of the same
39 set of constraints. The MARCO algorithm can be directly
40 interconnected with z3 (Bjørner).

41 In our case, *getCflt* use a slight modification of the
42 MARCO algorithm so that the minimal unsatisfiable sub-
43 sets are only composed of our allocation variables. In
44 other words, $getCflt(X_{ij}, C)$ enumerates all variables in
45 X_{ij} that make the problem C unsatisfiable. To do so, the
46 algorithm first translates the inter-domain level mode m in
47 terms of intra-domain level constraints C_m^d with $toCstr^d$.
48 The problem given to *getCflt* is then $C := C^d \cup C_m^d$
49 where C^d is the initial intra-domain level problem (from
50 $genCstr^d$). The result is denoted as \mathbb{X}_m^d and is a set of
51 *sets* of intra-domain level allocation variables, which can-
52 not be true together. We then translate back the intra-
53 domain level allocation variables in \mathbb{X}_m^d into inter-domain
54 level allocation variables thanks to $toCstr^U$. The result is
55 denoted $\mathbb{X}_{m,d}^U$.

56 The translation from the inter-domain level to the intra-
57 domain level realized by $toCstr^d$ is given in Eqs. (9), (10),

and (11). Equation (9) means that mapping a tenant node
 i to a domain d is translated to the constraint “do not map
 i to the outside” within the domain d . On the contrary,
(9) means that mapping a tenant node i to a domain dif-
ferent from d is translated to the constraint “map i to the
outside” within the domain d . Equation (11) means that
mapping a tenant link i on a inter-domain link from/to
the domain d is kept as is within the domain d .

$$\forall i \in N^{NR}, \quad \exists n_{ij}^U, j = d \Rightarrow \neg n_{i,O^a}^d \quad (9)$$

$$\forall i \in N^{NR}, \quad \exists n_{ij}^U, j \neq d \Rightarrow n_{i,O^a}^d \quad (10)$$

$$\forall i \in L^{NR}, \quad \exists l_{ij}^U, j \in IL_d^U \Rightarrow l_{ij}^d \quad (11)$$

When generating constraints with the function $genCons^U$
(or $genCons^d$), the solver of course need to know the set
of our loop-free paths P^U (or P^d , see Table A.3 in ap-
pendix). If we assume that the infrastructure topology
does not change, then this set can be considered as con-
stant, like Yu et al. does, and produced with a depth-first
search algorithm.

8.3. Solution Selection

Figure 4 presents what happens after the tenant chooses
which embedding is the most suitable. Those steps are
run after the generation algorithm. First, at step 12,
the tenant chooses a model m in the *results* the tenant
got from the generation algorithm. Then at step 13, the
inter-domain solver runs the $embed^U$ function, which will
memorize, for the future executions, which mapping vari-
ables n_{ij} and p_{ij} must be set to true. It is actually how
Eqs. C4 and C5 are filled. Steps 14 and 15 are the same
as 7.1 and 7.2. The reason why they are repeated is be-
cause we consider that d does not remember C_m^d for every
 m . Step 16 generates a new model, m^d , which represents
the partial mapping of the newcoming request NR at the
intra-domain level. Model m^d is only known by d . It
is not unique, but it is guaranteed to exist, because if
 $m \in results$, then neither d nor any other intra-domain
solver sent any minimal unsatisfaction subset at step 7.5.
At step 17, the intra-domain solver runs its counterpart of
 $embed^U$, namely $embed^d$, to fill Eq. C4 and Eq. C5 at the
intra-domain level.

The tenant T now has a slice. Note however that how
the physical resources are technically reserved and con-
figured is out of the scope of this paper, therefore not
presented in Figure 4.

9. Prototype and Use Case

Figure 5 shows an exemplary use case. Here, the ten-
ant wants to outsource some IT resources. The request is
composed from an access node, x , two remote workspaces
for two users a and b , a server s and a database d . Apart
from the access part, the tenant wants to explicitly avoid

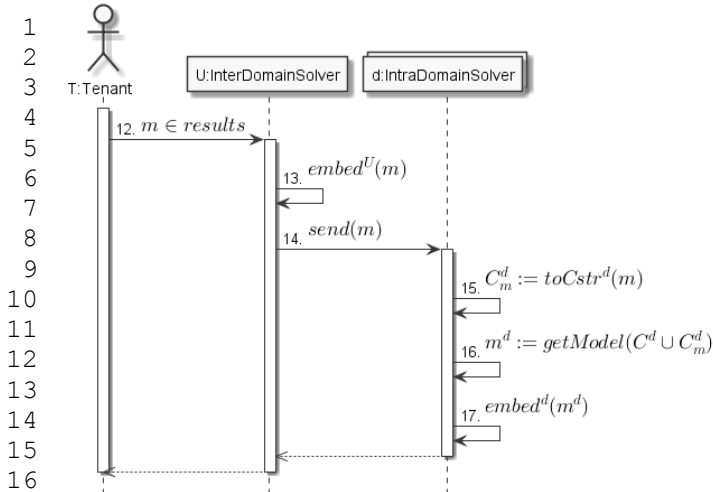


Figure 4: Embedding Selection Sequence Diagram

switch vendor $V3$. The database containing some sensitive or regulation-protected information cannot be stored in $Loc1$. Using the exclusion attribute, the tenant explicitly avoids the co-location of some virtual nodes and virtual edges. This is to prevent a and b from spying on each other, and also from acquiring protected data from d .

Our prototype runs on a server. The tenant connects to it and describes the request on a form page. Figure 6 shows the specific form for the resource d . In our use case, the tenant only modifies the fields for *Requested memory* (mem attribute), the *Authorized Locations* (loc attribute) and *Forbidden Resources* ($xres$ attribute). Others are left to their default value.

The tenant then saves the request and can now find it on a dashboard page, which encompasses all already submitted requests and their states (embedded, not embedded). This dashboard is displayed in Figure 7, along with a summary of the demands for each resource. The table on the right presents how each value given by the tenant is translated by the server. The fields *Requested memory*, *Authorized Locations* and *Forbidden Resources* semantically describe what the tenant asked for.

The tenant now executes our embedding generation algorithm (see Figure 3). The tenant is then redirected to an interface similar to the one shown on Figure 8. This interface shows every found embedding, depicted as a graph. The embeddings are numbered by the order in which they were found. The bullet on top of each embedding enables the tenant to select the desired embedding. Once an embedding is selected, we run the embedding selection algorithm (see Figure 4).

The core language of this prototype is Python 3. In particular, we use the following Python 3 packages.

- The `z3` SMT solver (de Moura and Bjørner; Bjørner and de Moura) which is available through the `z3-solver` Python package.
- The `networkx` package (Hagberg) provides graph manipulation and modelling. Particularly, we use their

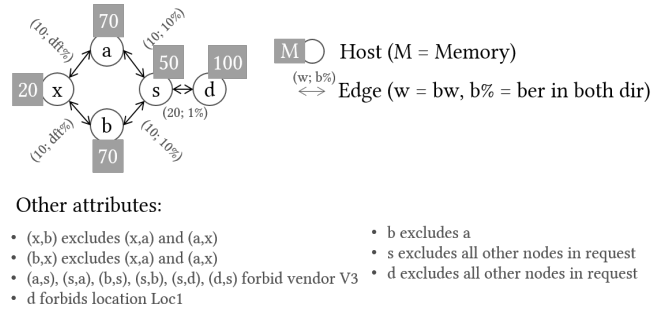


Figure 5: Use case request

simple path generation algorithm, which is based on a breadth-first search.

- The `multiprocessing` standard Python package provides domain isolation by modelling them as processes.
- The `bottle` package (Hellkamp) provides a web-based interface to tenants.
- The `ZODB` package (Fulton) provides a database for storing Python objects, especially graphs from `networkx`.

In this prototype, we decouple request declaration from request embedding. As such, tenants may declare requests and embed them later, or re-embed them after freeing resources, for instance. Consequently, we have to store requests in the database.

Yet, this raises a technical problem for the powerset-based attributes. The very reason of this problem is that in Section 4 we do not provide any notion of mutability in our mathematical model. The following paragraphs describe how ignoring mutability may affect the data, and how we technically implement this mutability.

We want to enable *Vendors* to be a mutable set, evolving over time. Let $Vendors(t)$ denote such time-dependent variable. At the initialization time (t_0), we assume that $Vendors(t_0) = \{v_X, v_Y\}$. At another (later) instant, t_1 , we assume that $Vendors(t_1) = \{v_X, v_Y, v_T\}$.

Consider the requirement “vendor v_X is not authorized” as expressed by the tenant. The tenant creates a request with such requirement at t_0 . We know that $Vendors(t_0) = \{v_X, v_Y\}$, so we know that the vendors authorized by the tenant are in $\{v_Y\}$.

At this stage, it seems legitimate to exactly memorize only the information $\{v_Y\}$ to represent the requirement given by the tenant. However, this becomes wrong with the mutability of *Vendors*. Indeed, at t_1 , as $Vendors(t_1) = \{v_X, v_Y, v_T\}$, the value $\{v_Y\}$ itself does *not* represent “vendor v_X is not authorized” but “vendor v_X and vendor v_T are not authorized”. In other words, we have semantically changed the meaning of the requirement.

We solve this issue by explicitly storing in the database a tuple $(Vendors, \{v_X\})$, where *Vendors* is a reference to the vendor set as actually stored in the database, thanks to the object-oriented nature of `ZODB`, and $\{v_X\}$ is the set of

Save
Cancel
Design your node

Node name

Requested Memory

Authorized Locations:
 except
 Loc1
 Loc2
 Loc34
 all others

Forbidden Tenants:
 except
 Competitor
 MyCompany
 Tenant
 Tenant1

Forbidden Requests:
 except

 all others

Forbidden Resources:
 except

 all others

Figure 6: Declaring resource d on prototype

Select	Date	Request	Hide	Node d
<input type="checkbox"/>	3/15/2019		<input type="checkbox"/>	Requested Memory: 100 Authorized Locations: LOCATIONS EXCEPT Loc1 Forbidden Resources: ANY OTHER NODE model_mem: 100 model_loc: 3 model_x_ten: NONE model_x_req: NONE model_x_res: s-Request-Tenant, AND a-Request-Tenant, AND b-Request-Tenant, AND x-Request-Tenant

Figure 7: Description of resource d on prototype

values which are explicitly given by the tenant. The exact value as used by the embedding algorithm is then recomputed each time. As such, in our example, the algorithm still interprets “vendor v_X is not authorized” from the data at t_1 . Obviously, this mechanism is even more necessary for exclusion attributes, as their value sets change whenever a new object appears.

The same problem appears when dealing with the m_a value of the powerset-based attributes. For instance in Table 1b, $m_{ven} = Vendors$, where $Vendors$ is the set of all supported vendors (indeed, V_{ven} is the powerset of $Vendors$). This set may change over time, leading to an ambiguity when interpreting the value as it is stored later.

Consider $Vendors' := Vendors \cup \{v_Z\}$. We do not know what better suits the tenant intent. We can still consider m_{ven} as the default value and allow v_Z , or we can stick to allowing v_X and v_Y .

To avoid this ambiguity, we also enable tenants to declare if they authorize future values, thanks to the *and others* keyword.

10. Experimentations

In this section, we evaluate the correctness of our algorithm, and provide some measures about its time computation. We conduct the following experiments: *i*) we show that computation time increases faster with the number of embeddings, and that all embeddings are correctly generated; *ii*) we show how time computation evolves when no suitable embedding exists.

We implement our work under Ubuntu 16.04 on a i7-6700HQ @ 2.60GHz machine with 16 GB RAM and 300 GB of storage. Domains are modeled as separate processes. We do not enforce any delay between calling a domain server and getting its reply, even though we expect

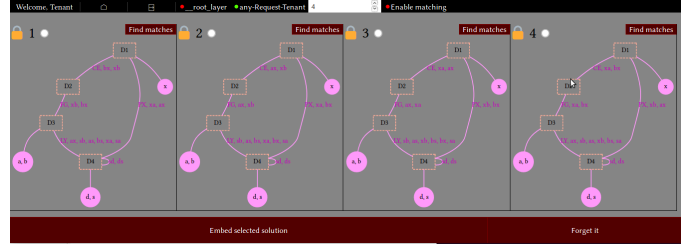


Figure 8: Proposed embeddings

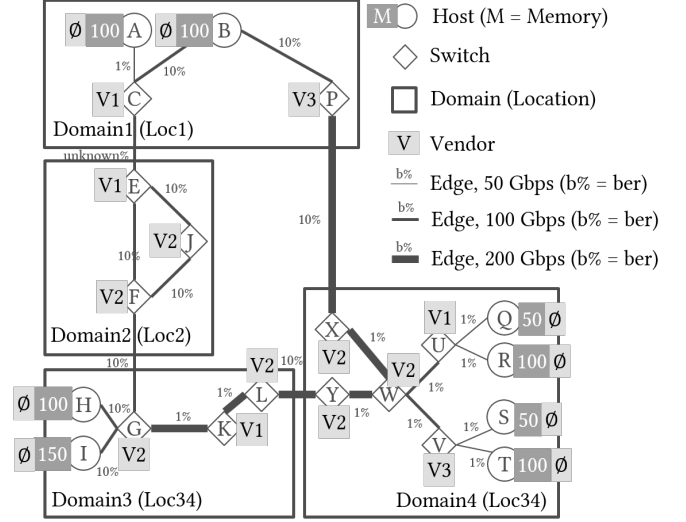


Figure 9: The use case substrate, with its sub domains

such delay to have an impact on the time performances of our algorithm when deployed in a real system. We leave such optimization for future works and here evaluate our algorithm as a proof of concept.

The substrate which is used for all our evaluations is described in Figure 9. This figure represents the substrate as a set of interconnected boxes (the domains), with their hosts, switches and links inside the boxes. The substrate is divided into four domains ($Domain1$ to $Domain4$). Domains are interconnected following a ring topology. The $Domain2$ is distinct from the others in that there are no host inside. The attribute values for each host, switch and link are also represented. The node location is applied domain-wise, hence the location value is indicated next to the domain name, between brackets. Host memory and vendor are indicated next to the host name. For their part, edges come in three bandwidth flavors (50 Gbps, 100 Gbps, 200 Gbps). The bit error rate is represented as a percentage. Note that for the edge (C, E) , the value is *unknown%*. This corresponds to the case when the InPs are unable to provide attribute information. As described in Section 4, the actual value is set to m_{ber} , which is equal to 100%.

Figure 10 evaluates our algorithm for different requests in time and in correctness. The requests come with three different topologies, in which we increase the number of nodes. Those topologies are chain, full-mesh, and star. They do not enforce any requirement: all attributes are

1 set to their default value. Consequently, we generate the
 2 maximal number of embeddings for each request.

3 More specifically, Figure 10a shows the measured number
 4 of embeddings to be generated for each request. This
 5 measured number of embeddings is then compared to the
 6 theoretical number of embeddings in Figure 10b. For a
 7 chain topology, the theoretical number of embeddings is
 8 3×5^n where n is the number of nodes (see Demonstration
 9 5). For a star topology, the theoretical number of
 10 embeddings is 3×5^n where n is the number of nodes (see
 11 Demonstration 6 in appendix). For a full-mesh topology,
 12 the theoretical number of embeddings for $n \in (1, 2, 3, 4)$ is
 13 given in Demonstration 7 in appendix, and is respectively
 14 3, 27, 673, 41985.
 15

16 **Demonstration 5.** Let $f(n)$ be the number of solutions
 17 for a chain of n nodes, with the substrate given in Figure 9.
 18 We show by induction that $f(n) = 3 \times 5^n$. We denote as N_i
 19 where $i \in (1, \dots, n)$ the nodes in the graph. We denote as
 20 m_k^n a solution for a chain of n nodes where k is an index
 21 identifying the solution.

22 **Basis.** When $n = 1$, the request is a single node, N_1 .
 23 Among the four domains in our substrate, only three contain
 24 hosts: Domain1, Domain3, and Domain4. Then we
 25 have 3 ways of embedding our request: $m_1^0(N_1) = \text{Domain1}$,
 26 $m_2^0(N_1) = \text{Domain3}$ and $m_3^0(N_1) = \text{Domain4}$. Then
 27 $f(1) = 3$.

28 **Induction step.** Let m_k^n be a solution for a chain
 29 of n nodes. Let m_q^{n+1} be a solution for a chain of $n + 1$
 30 nodes such that:
 31

$$\forall i \in (1, \dots, n), \quad m_q^{n+1}(N_i) = m_k^n(N_i)$$

$$\forall i \in (1, \dots, n - 1), \quad m_q^{n+1}((N_i, N_{i+1})) = m_k^n((N_i, N_{i+1}))$$

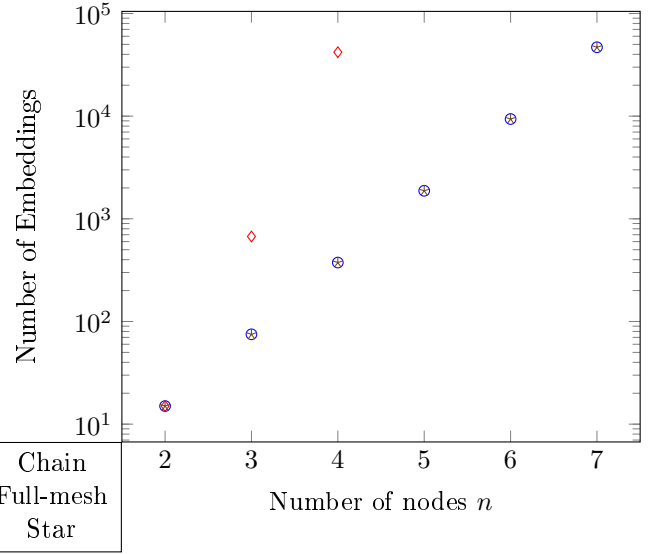
32 Then we need to embed N_{n+1} and (N_n, N_{n+1}) . Let
 33 $d = m_q^{n+1}(N_n)$ be the domain hosting N_n .

34 If $m_q^{n+1}(N_{n+1}) = d$, then by Definition 2 we have
 35 $m_q^{n+1}((N_n, N_{n+1})) = (d, (d, d), d)$. We then have described
 36 one (1) solution from m_k^n .

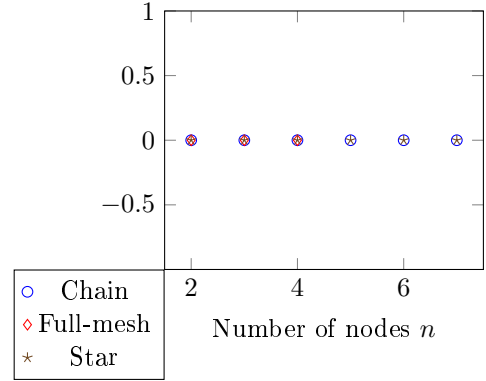
37 If $m_q^{n+1}(N_{n+1}) \neq d$, then we can choose among the
 38 two (2) other domains. Let $d' = m_q^{n+1}(N_{n+1})$, $d' \neq d$.
 39 Let d'' be the remaining domain, $d'' \neq d'$, $d'' \neq d$. As
 40 the substrate topology is a ring, we have two (2) paths that
 41 join d to d' . The first path is $(d, (d, d'), d')$ and the second
 42 path is $(d, (d, d''), d'', (d'', d'), d')$. They can be chosen for
 43 the mapping of (N_n, N_{n+1}) . We then have described four
 44 $(4 = 2 \times 2)$ other solutions from m_k^n .

45 Then $f(n + 1) = (1 + 4) \times f(n) = 5 \times f(n)$. \square

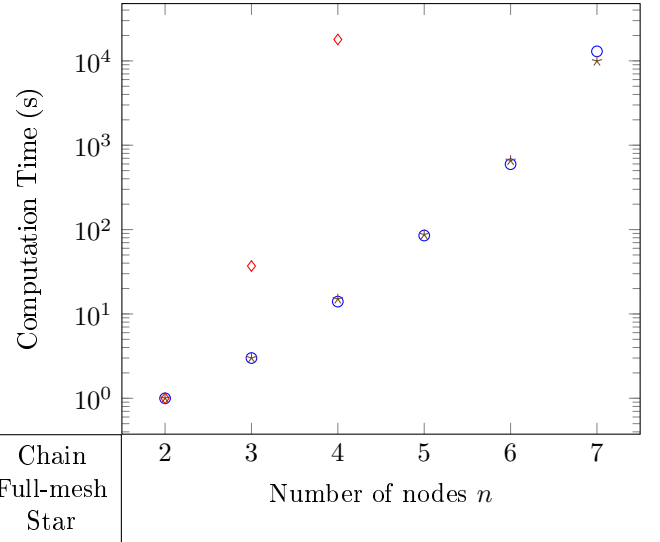
46 As shown in Figure 10b, there is no deviation from the
 47 expected number of embeddings for any tested request.
 48 For star and chain topologies, the experiment was
 49 conducted from $n = 2$ to $n = 7$ and repeated 50 times until
 50 $n = 5$ and 5 times for $n = 6$ and $n = 7$ due to compu-
 51 tation time. For full-mesh topology, the experiment was
 52 conducted from $n = 2$ to $n = 4$ and repeated 5 times due
 53 to computation time.
 54
 55
 56
 57
 58
 59
 60



a Number of Embeddings per Number of Nodes n for Chain, Full-mesh and Star Topology



b Difference between Theoretical and Measured Numbers of Embeddings per Number of Nodes n for Chain, Full-mesh and Star Topology



c Computation Time per Number of Nodes n for Chain, Full-mesh and Star Topology

Figure 10: Computation Time Trend for Chain, Full mesh and Star Topology

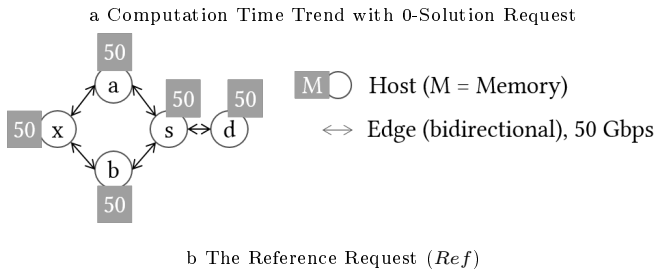
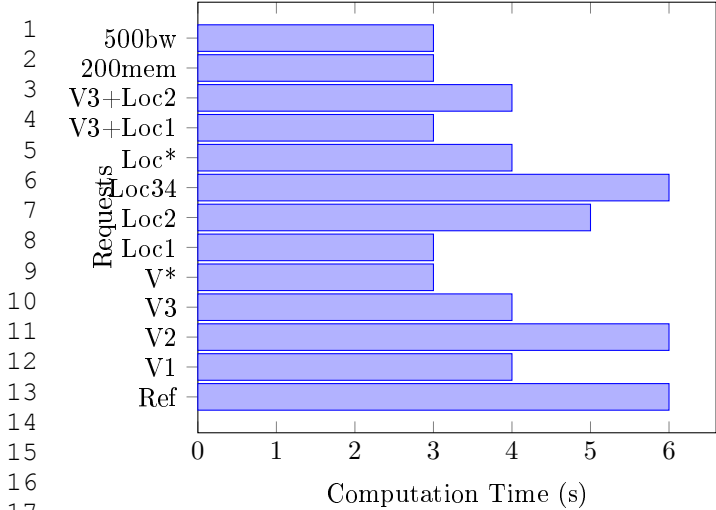


Figure 11: Computation Time Trend with 0 Solution Request

In Figure 10c, we show the computation time for each execution. As explained before, this computation time is not expected to represent real time deployment, with communication delays between the domains. We can notice that the computation time grows faster than the number of embeddings between $n = 6$ and $n = 7$. These computation times are way shorter than with the implementation (not disclosed) of our previous algorithm (Boutigny et al.), which took at least several minutes for the $n = 3$ case.

Figure 11a shows computation time for different requests. The idea of this experiment is to evaluate how much time the algorithm needs to find out that there is no solution. For this experiment, we have a reference request (cf. Figure 11b) whose requirements are strong enough to not be met, thus there are 0 embeddings. We then strengthen the requirements of this reference request and measure the computation time. Each time, we reset the substrate. The different requests are derived from the reference request following Table 2.

We interpret our results as follows. First, as expected, the time taken by the algorithm to ensure there is no solution is finite, and varies depending on the requirements. Second, strengthening or adding requirements to a request does not increase the computation time, but keeps it as-is or lowers it. Third, we cannot conclude that there is a relation between computation time and the requirement values, for a given attribute. For instance, we expected the time to discover that there is no solution to be lower for V^* than for $V1$, $V2$ and $V3$ respectively, but we did not expect the time to discover that there is no solution

Request name	Difference with <i>Ref</i> request
500bw	$d_{(s,d)}^{bw} = d_{(d,s)}^{bw} = 500$
200mem	$d_s^{mem} = 200$
$V3 + Loc2$	$d_{(s,d)}^{ven} = d_{(d,s)}^{ven} = Vendors \setminus \{V3\}$ $\wedge d_s^{loc} = Locations \setminus \{Loc2\}$
$V3 + Loc1$	$d_{(s,d)}^{ven} = d_{(d,s)}^{ven} = Vendors \setminus \{V3\}$ $\wedge d_s^{loc} = Locations \setminus \{Loc1\}$
Loc*	$d_s^{loc} = \emptyset$
Loc34	$d_s^{loc} = Locations \setminus \{Loc34\}$
Loc2	$d_s^{loc} = Locations \setminus \{Loc2\}$
Loc1	$d_s^{loc} = Locations \setminus \{Loc1\}$
V^*	$d_{(s,d)}^{ven} = d_{(d,s)}^{ven} = \emptyset$
V3	$d_{(s,d)}^{ven} = d_{(d,s)}^{ven} = Vendors \setminus \{V3\}$
V2	$d_{(s,d)}^{ven} = d_{(d,s)}^{ven} = Vendors \setminus \{V2\}$
V1	$d_{(s,d)}^{ven} = d_{(d,s)}^{ven} = Vendors \setminus \{V1\}$

Table 2: The Other Requests

to be greater for Loc^* than for $Loc1$. In the same time, when combining requirements, such as in $V3 + Loc2$ and $V3 + Loc1$ cases, we get a lower time than when considering $V3$, $Loc2$ or $Loc1$ independently.

This last interpretation leads us to draw the conclusion that our algorithm is dependent on the attribute values of the requests and of the domains. Consequently, the algorithm displays varying computation times depending on the attributes in use.

In general, bidding attributes are expected to be more computationally expensive than non-bidding attributes, because the algorithm may have to try several variable assignments before satisfying Equation (C6) (or Equation (C8)), while one variable assignment is required in Equation (C7) (or Equation (C9)).

In fact, Equation (C6) and Equation (C8) are very easy to *un-satisfy*, which is illustrated by the short times for $500bw$ and $200mem$ in Figure 11a.

As a last remark, when the aforementioned equations rely on an offer in an attribute a whose value is not set by the InP, the default value (see Subsection 4.3), m_a , is used by the algorithm. By design, this default value drastically reduces the number of variable assignments that the algorithm may perform to satisfy Equation (C6) (or Equation (C8)), enabling us to support missing attribute data without impacting performances.

11. Conclusion

In this paper, we present how to solve the slice embedding problem in a multi-domain context where domain owners are reluctant to expose their resource attributes or topology. Our solution distinguishes two levels, the intra-domain and the inter-domain, where the inter-domain is known by a trusted third party and the intra-domain is only known by the Infrastructure Provider (InP) owning

1 it. We design two Satisfiability Modulo Theories (SMT)
2 problems for those levels, and we propose an algorithm to
3 coordinate them in order to generate all possible solutions
4 in an efficient manner. Besides, we support an extensi-
5 ble and tolerant requirement model, which enables InPs
6 to guarantee new requirements and even blacklist-based
7 ones, which are useful for security purpose.

8 Our work has some downsides though. First, even
9 though we want InPs to limit information disclosure, the
10 third party may end by having some knowledge about it,
11 if it records the replies from the InPs. Second, we do not
12 support all known requirements, like latency. The reason
13 holds in the information disclosure limitation. Latency
14 cannot be modelled within our requirement model, because
15 we must validate the latency for a whole path, which may
16 go through multiple domains.

19 Acknowledgments

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Appendix A. Appendices

Demonstration 6. Let $f(n)$ be the number of solutions for a star of n nodes, with the substrate given in Figure 9. We show by induction that $f(n) = 3 \times 5^n$. We denote as N_i where $i \in (1, \dots, n)$ the nodes in the graph. The node N_1 is the center of the star. We denote as m_k^n a solution for a star of n nodes where k is an index identifying the solution.

Basis. Same as Demonstration 5.

Induction step. Let m_k^n be a solution for a star of n nodes. Let m_q^{n+1} be a solution for a star of $n+1$ nodes such that:

$$\begin{aligned} \forall i \in (1, \dots, n), \quad m_q^{n+1}(N_i) &= m_k^n(N_i) \\ \forall i \in (2, \dots, n), \quad m_q^{n+1}((N_1, N_i)) &= m_k^n((N_1, N_i)) \end{aligned}$$

Then we need to embed N_{n+1} and (N_1, N_{n+1}) . Let $d = m_q^{n+1}(N_1)$ be the domain hosting N_1 .

If $m_q^{n+1}(N_{n+1}) = d$, then by Definition 2 we have $m_q^{n+1}((N_1, N_{n+1})) = (d, (d, d), d)$. We then have described one (1) solution from m_k^n .

If $m_q^{n+1}(N_{n+1}) \neq d$, then we can choose among the two (2) other domains. Let $d' = m_q^{n+1}(N_{n+1})$, $d' \neq d$. Let d'' be the remaining domain, $d'' \neq d'$, $d'' \neq d$. As the substrate topology is a ring, we have two (2) paths that join d to d' . The first path is $(d, (d, d'), d')$, and the second path is $(d, (d, d''), d'')$, (d'', d') , d' . They can be chosen for the mapping of (N_1, N_{n+1}) . We then have described four ($4 = 2 \times 2$) other solutions from m_k^n .

Then $f(n+1) = (1+4) \times f(n) = 5 \times f(n)$. \square

Demonstration 7. Let $f(n)$ be the number of solutions for a complete directed graph of n nodes, with the substrate given in Figure 9. We show that $f(1) = 3$, $f(2) = 27$, $f(3) = 673$ and $f(4) = 41985$. We denote as N_i where $i \in (1, \dots, n)$ the nodes in the graph. We denote as m_k^n a solution for a complete directed graph of n nodes where k is an index identifying the solution.

Case $n = 1$. Same as Demonstration 5.

Case $n = 2$. Let m_k^1 be a solution for a complete directed graph of 1 node. Let m_q^2 be a solution for a complete directed graph of 2 nodes such that:

$$m_q^2(N_1) = m_k^1(N_1)$$

Then we need to embed N_2 , (N_1, N_2) and (N_2, N_1) . Let $d = m_q^2(N_1)$ be the domain hosting N_1 .

If $m_q^2(N_2) = d$, then $m_q^2((N_1, N_2)) = m_q^2((N_2, N_1)) = (d, (d, d), d)$ by Definition 2. We then have described one (1) solution from m_k^1 .

If $m_q^2(N_2) \neq d$, then we can choose among the two (2) other domains. Let $d' = m_q^2(N_2)$, $d' \neq d$. Let d'' be the remaining domain, $d'' \neq d'$, $d'' \neq d$. As the substrate topology is a ring, we have two (2) paths that join d to d' . Those paths are $(d, (d, d'), d')$ and $(d, (d, d''), d'')$, (d'', d') , d' . They

Table A.3: All Notations

Symbol	Meaning
\mathcal{R}	The request set. We embed one request at a time.
NR	The new coming request, which we try to embed
$\mathcal{R} \setminus \{NR\}$	The already embedded requests
$R \in \mathcal{R}$	A request, as a directed graph
T	A tenant
\mathcal{R}_T	The requests from tenant T
S	The substrate, as a directed graph
N^R, N^S	Nodes in request R or substrate S
$H^S \subset N^S$	Hosts in substrate S
L^R, L^S	Links in request R or substrate S
P^S	Loop free host to host paths (cf. Definition 2)
$E_j = (x, y), j \in P^S$	The nodes at the tips of path j . Nodes $x, y \in N^S$
$L_j, j \in P^S$	The set of links along path j
$n_{ij} \in \{\perp, \top\}$	Allocating $j \in H^S$ for $i \in N^R$
$l_{ij} \in \{\perp, \top\}$	Allocating $j \in L^S$ for $i \in L^R$
$p_{ij} \in \{\perp, \top\}$	Allocating $j \in P^S$ for $i \in L^R$
$m(i)$ s.t. $R \neq NR$	The host/path to which node/link i is mapped
NB_N, B_N, NB_L, B_L	Non binding/Binding Node/Link attributes
$d_i^a \in V_a$	Demand in attribute a of $i \in N^R \cup L^R$
$o_j^a \in V_a$	Offer in attribute a of $j \in N^S \cup L^S$
Section 4	
<i>Locations</i>	The set of all physical locations
<i>Vendors</i>	The set of all switch vendors
X^V, X^P	Virtual and physical resources
X^R	Resources in request R
$x_{ij} \in \{\perp, \top\}$	Allocating $j \in X^S$ for $i \in X^R$
Section 6	
$O^S \in H^S$	The outside node
$DH^S = H^S \setminus \{O^S\}$	The inner hosts
$DL^S = L^S \setminus \{(O^S, O^S)\}$	The real links (intra + inter domain)
$N_j, j \in P^S$	Nodes along path j
$OP^S = \{j \in P^S \mid$ $E_j \in (DH^S)^2, O^S \in N_j\}$	Loop free paths going out through O^S
Section 7	
$SL^S \subset L^S$	The domain self loops
Section 8	
$x \in X$	A variable x
$m(x)$	The value assigned to variable x by a model m
N_{ij}, P_{ij}	The set of node/path allocation variables n_{ij}/p_{ij}
U	Refer to the inter domain level
d	Refer to the domain d at the intra domain level
IL_d^U	Inter domain links connected to domain d

1 can be chosen for the mapping of the two (2) edges (N_1, N_2)
 2 and (N_2, N_1). We then have described eight ($8 = 2 \times 2 \times 2$)
 3 other solutions from m_k^1 .

4 Then $f(2) = (1 + 8) \times f(1) = 9 \times 3 = 27$.

5 **Case $n = 3$.** We have 3 nodes to embed in 3 do-
 6 mains. Some nodes can be hosted in the same domain.

7 We enumerate the different cases as follows:

- 8 • If all nodes are hosted in distinct domains.

9 We have 1 way to map 3 nodes into 3 clusters.

10 The number of node mappings is an arrange-
 11 ment of 3 clusters in 3 domains: $3 \times 2 \times 1$.

12 The number of link mappings per node mapping
 13 is 2^6 as 6 directed edges are not mapped to any
 14 self-loops.

15 Result: $1 \times (3 \times 2 \times 1) \times 2^6 = 384$ solutions.

- 16 • If two nodes are hosted in the same domain.

17 We have 3 way to map 3 nodes into 2 clusters.

18 The number of node mappings is an arrange-
 19 ment of 2 clusters in 3 domains: 3×2 .

20 The number of link mappings per node mapping
 21 is 2^4 as 4 directed edges are not mapped to any
 22 self-loops.

23 Result: $3 \times (3 \times 2) \times 2^4 = 288$ solutions.

- 24 • If all nodes are hosted in the same domain.

25 We have 1 way to map 3 nodes into 1 cluster.

26 The number of node mappings is an arrange-
 27 ment of 1 cluster in 3 domains: 3.

28 But neither Domain1 nor Domain3 can host
 29 the whole 3-complete directed graph, so the ar-
 30 rangement is 1.

31 The number of link mappings per node mapping
 32 is 2^0 as 0 directed edges are not mapped to any
 33 self-loops.

34 Result: $1 \times (3 - 2) \times 2^0 = 1$ solution.

35 The number of link mappings per node mapping is 2^k where
 36 k is the number of directed edges that are not mapped to
 37 any self-loops because by Definition 2 an edge mapped to a
 38 self-loop has one candidate and the substrate topology is a
 39 ring, so we have two candidate paths otherwise.

40 At the intra-domain level, contrary to chains in Demon-
 41 stration 5, a 3-complete directed graph is not bipartite, so
 42 we cannot respect eligible paths rule (R4) with only 2 sub-
 43 strate hosts, which is the number of hosts in Domain1 and
 44 Domain3. Then, some cases must be rejected.

45 Then $f(3) = 384 + 288 + 1 = 673$.

46 **Case $n = 4$.** We have 4 nodes to embed in 3 do-
 47 mains. Some nodes can be hosted in the same domain.

48 We enumerate the different cases as follows:

- 49 • If all nodes are hosted in distinct domains.

Less domains than nodes: not applicable.

- If two nodes are hosted in the same domain and the others in distinct domains.

We have 6 ways to map 4 nodes into 3 clusters with one cluster of two nodes.

The number of node mappings is an arrangement of 3 clusters in 3 domains: $3 \times 2 \times 1$.

The number of link mappings per node mapping is 2^{10} as 10 directed edges are not mapped to any self-loops.

Result: $6 \times (3 \times 2 \times 1) \times 2^{10} = 36864$ solutions.

- If we group nodes into two clusters of two nodes each.

We have 3 way to map 4 nodes into 2 clusters of two nodes each.

The number of node mappings is an arrangement of 2 cluster in 3 domains: 3×2 .

The number of link mappings per node mapping is 2^8 as 8 directed edges are not mapped to any self-loops.

Result: $3 \times (3 \times 2) \times 2^8 = 4608$ solutions.

- If three nodes are hosted in the same domain and the other in distinct domain.

We have 4 way to map 4 nodes into 2 clusters with one cluster of three nodes.

The number of node mappings is an arrangement of 2 clusters in 3 domains: 3×2 .

But neither Domain1 nor Domain3 can host a 3-complete directed graph, so the arrangement is 2×1 .

The number of link mappings per node mapping is 2^6 as 6 directed edges are not mapped to any self-loops.

Result: $4 \times (2 \times 1) \times 2^6 = 512$ solutions.

- If all nodes are hosted in the same domain.

We have 1 way to map 4 nodes into 1 cluster.

The number of node mappings is an arrangement of 1 cluster in 3 domains: 3.

But neither Domain1 nor Domain3 can host the whole 4-complete directed graph, so the arrangement is 1.

The number of link mappings per node mapping is 2^0 as 0 directed edges are not mapped to any self-loops.

Result: $1 \times (3 - 2) \times 2^0 = 1$ solution.

1 *At the intra-domain level, contrary to chains in Demon-*
2 *stration 5, a 3-complete directed graph is not bipartite, so*
3 *we cannot respect R4 with only 2 substrate hosts, which*
4 *is the number of hosts in Domain1 and Domain3. Then,*
5 *some cases must be rejected.*

6 *Then $f(4) = 36864 + 4608 + 512 + 1 = 41985$. □*

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