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Yue Su, Xiaobo Liu, Guo Lu, Wenbo Fan

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Transportation Research Record

OPTIMAL DESIGN OF BUS ROUTES FOR DIFFERENT VEHICLE TYPES CONSIDERING VARIOUS DRIVING REGIMES AND ENVIRONMENTAL FACTORS --Manuscript Draft--

Full Title:	OPTIMAL DESIGN OF BUS ROUTES FOR DIFFERENT VEHICLE TYPES CONSIDERING VARIOUS DRIVING REGIMES AND ENVIRONMENTAL FACTORS
Abstract:	<p>As a major part of public transportation system, bus transit has been regarded as an effective mode to alleviate the traffic congestion and solve vehicle emission problem. The performance of bus transit system depends largely on its design of proper stop locations. In this reasearch, we proposed a multi-period continuum model (peak hour and off-peak hour) to optimize the design of a bus route for four different vehicle types (i.e., supercharge bus, Compressed Natural Gas (CNG) bus, Lithium-ion battery bus, and diesel bus) considering driving regimes and the pollutant cost. Inter-stop driving regimes, including acceleration, cruising, coasting, and deceleration, are explicitly introduced into the optimization to determine whether and how the coasting regime should be undertaken in the tradeoff between vehicle's commercial speed and the operating cost. The comparison for the cost effectiveness of each alternative has been investigated in a life span with respect to different vehicle types. The method has been implemented in the real-word bus route 7 in Yaan City (China). The numerical experiments suggest that through optimization, the total system cost has been saved by more than 50%. The results of continuum model are validated by the comparison with the discretized results, and the outcomes are closely located in neighborhood (with error less than 3%). The life-cycle cost of four vehicle types is finally analyzed, and the result indicates that due to the high purchase prices, it's difficult for clean-energy buses to outperform conventional buses in a life cycle (normally 8 years), unless with subsidies provided.</p>
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1 **OPTIMAL DESIGN OF BUS ROUTES FOR DIFFERENT VEHICLE TYPES**
2 **CONSIDERING VARIOUS DRIVING REGIMES AND ENVIRONMENTAL FACTORS**

3
4
5
6 **Yue Su**

7 School of Transportation and Logistic, Southwest Jiaotong University
8 No.999, Xi'an Street, ChengDu, SiChuan, China, 611756
9 Tel: +86-13908214147 Email: yuesu@my.swjtu.edu.cn

10
11 **Xiaobo Liu, Ph.D.**

12 School of Transportation and Logistic, Southwest Jiaotong University
13 No.999, Xi'an Street, ChengDu, SiChuan, China, 611756
14 Tel: +86-13688002461 Email: xiaobo.liu@swjtu.cn

15
16 **Guo Lu**

17 School of Transportation and Logistic, Southwest Jiaotong University
18 No.999, Xi'an Street, ChengDu, SiChuan, China, 611756
19 Tel: +86-18620366066 Email: 496069232@qq.com

20
21 **Wenbo Fan, Ph.D., Corresponding Author**

22 School of Transportation and Logistic, Southwest Jiaotong University
23 No.999, Xi'an Street, ChengDu, SiChuan, China, 611756
24 Tel: +86-13658082981 Email: wbfan@swjtu.edu.cn

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1 ABSTRACT

2 As a major part of public transportation system, bus transit has been regarded as an effective mode
3 to alleviate the traffic congestion and solve vehicle emission problem. The performance of bus
4 transit system depends largely on its design of proper stop locations. In this reasearch, we proposed
5 a multi-period continuum model (peak hour and off-peak hour) to optimize the design of a bus
6 route for four different vehicle types (i.e., supercharge bus, Compressed Natural Gas (CNG) bus,
7 Lithium-ion battery bus, and diesel bus) considering driving regimes and the pollutant cost.
8 Inter-stop driving regimes, including acceleration, cruising, coasting, and deceleration, are
9 explicitly introduced into the optimization to determine whether and how the coasting regime
10 should be undertaken in the tradeoff between vehicle's commercial speed and the operating cost.
11 The comparison for the cost effectiveness of each alternative has been investigated in a life span
12 with respect to different vehicle types. The method has been implemented in the real-word bus
13 route 7 in Yaan City (China). The numerical experiments suggest that through optimization, the
14 total system cost has been saved by more than 50%. The results of continuum model are validated
15 by the comparison with the discretized results, and the outcomes are closely located in
16 neighborhood (with error less than 3%). The life-cycle cost of four vehicle types is finally
17 analyzed, and the result indicates that due to the high purchase prices, it's difficult for clean-energy
18 buses to outperform conventional buses in a life cycle (normally 8 years), unless with subsidies
19 provided.

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Keywords: Bus Route Design, Pollution, Driving Regimes, Continuum Model, Different Vehicle
Types

1 INTRODUCTION

2 With the astounding growth in automobile ownership, most cities in China have to face a number
3 of transportation related issues such as rapid environmental deterioration and serious congestion
4 problem in urban area. The pollutants emitted such as CO, VOC, and NO_x, bring unpleasant
5 impacts on the air quality as well as the public health. Zero-emission buses, such as battery electric
6 buses and supercharge buses, entail environmental friendliness, and have been recognized as a
7 new solution of environmental problem and traffic congestion. Supercharge bus is the newest bus
8 type in the market and it utilizes super capacitor as power source, which has the advantages of fast
9 charging and discharging ability. Compared with battery electric bus, supercharge bus takes far
10 less time to charge, and it doesn't need to replace batteries in a life cycle. Many cities in China
11 have spared no effort in developing a new transit network with clean-energy buses. For instance,
12 all the conventional buses (i.e., 16,000 CNG and diesel buses) in Shenzhen will be replaced with
13 electric buses by the end of 2018 (1). Despite the environmental advantages of the new-energy
14 buses, the expensive purchase price is an obstacle that impedes the progress of the shifting to clean
15 energy. For example, the unit vehicle price of an electrical bus is almost three times more
16 expensive than conventional bus (2). In order to yield a scientific assessment between
17 clean-energy buses and conventional buses, it is necessary to take the technical and economical
18 characteristics of different bus types into consideration to assess their cost competitiveness.

19 To make the transit system more efficient, it is necessary for designers to provide a
20 delicated transit system with stops being well located. There are two basic approaches for tackling
21 Transit Route Design (TRD) problem. One is discrete approach which has been frequently used in
22 the TRD problem. Studies (3-6) applied discrete approach to decide the optimal stop locations.
23 The basic idea of discrete model is to determine optimal set of stops for a given OD matrix and
24 predict travel time per link on the route (7). Dynamic programming is largely used in the
25 optimization. The disadvantage of this approach is the computational inefficiency because of the
26 numerous decision variables.

27 Another approach is the Continuum Approximation (CA) approach. Previous studies (8-18)
28 employed CA approach to obtain the optimal design of a transit system, where a small number of
29 continuous functions are inputted; e.g., lines and stations are specified in terms of the spacing
30 between them (19). Some other parameters including headway, bus capacity, fleet size, have also
31 been used in the optimization model (20-21). The pioneering work of CA approach employed in
32 transit route design seems to be the study of Newell (8), in which the mechanics of CA approach
33 was elaborated. After that, Vaughan and Cousins (14) took advantage of a continuous stop density
34 function to determine the number of stops so as to minimize user's travel time on a single corridor.
35 The demand pattern is a "many-to-many" demand pattern to reflect the spatial heterogeneity of
36 demand and is also presented in a continuous form. Later, Wirashinghe and Ghoneim (12)
37 proposed a more general continuous model to minimize the total system cost which consists of
38 user cost and operator cost with respect to a "many-to-many" demand pattern. The stop density is
39 expressed as a function of location and the integral of stop density function is firstly used as the
40 method to find optimal stop locations. Recently, Medina-Tapia et al. (16) employed CA-based
41 transit design model to a single bus corridor considering multiple periods (peak hour and off-peak
42 hour). A bi-directional stop density functions and multi-period headways are obtained. Further,
43 Amirgholy et al. (22) proposed a CA model to minimize user cost, agency cost, and pollutant cost
44 in a congested network. It seems to be the only CA-based work that considering environmental
45 impacts into transit route design.

46 The theme of this study is to optimize the bus route design for a single corridor considering

1 different bus types of which the inter-stop driving regimes are explicitly modeled. In the proposed
 2 model, the objective function is to minimize the total system cost, as a sum of user cost, operator
 3 cost, as well as pollutant cost. The model is based on continuum approximation and its accuracy is
 4 verified by a discrete approach. A case study is furnished on a bus route in Yaan City (China). The
 5 contributions of this work include: the environmental factors and inter-stop driving regimes are
 6 considered in transit route design model to analyze the system cost of different vehicle types. By
 7 intergrating the optimized system cost (on a per day basis) with life-cycle cost analysis, it can
 8 provide operator insight to choose appropriate vehicle type. Furthermore, the optimized coasting
 9 speeds for different bus types have been analyzed as well as the effect of coasting regimes on
 10 operation cost, which can be implemented in practical selection of driving regimes for different
 11 bus types.

12 The remainder of this paper is organized as follows. Next section introduces the
 13 optimization model, with variables' definitions and formulations of the CA model, respectively. In
 14 the section, a non-linear total cost minimization problem is introduced. After that, the numerical
 15 application is conducted by using the real demand data, and the obtained results are analyzed. In
 16 the end of section 3, a cost-effectiveness analysis among different vehicle types has been
 17 elaborated. Last section concludes the findings and indicates further extensions.

18 **METHODOLOGY**

19 The proposed model contains a bus corridor of length L , operating in two directions (denoted by
 20 $r \in \{eb, wb\}$, indicating eastbound and westbound, respectively). The bus travelling in each
 21 direction of the corridor stops at each stop. The studied periods are: peak hour period and off-peak
 22 hour period (denoted by $i \in \{pk, op\}$, indicating peak-hour period and off-peak hour period,
 23 respectively) whose service headways are different to reflect the demand variation but equal in
 24 both directions. To facilitate organization, the developed model is based on a few assumptions as
 25 follows:
 26

- 27 1. The value of time for all passengers is assumed to be the same, regardless the citizen's
 28 status, income, etc.
- 29 2. Passengers choose the nearest stop to board or alight bus.
- 30 3. There is no congestion on the corridor, so bus acceleration or deceleration results from
 31 whether it has to stop or start at bus stop. For the segment between two stops, different driving
 32 regimes could be adopted regarding whether coasting occurs, for instance. In addition, the
 33 acceleration and deceleration rates are constant in this study.
- 34 4. No bus congestion is assumed to occur at bus stops, so each bus opens its door as soon as it
 35 arrives at a stop.
 36

37 In this section, we will first introduce the objective function TC_m , which represents the
 38 total system cost (on a per day basis), with $m \in \{SC, CNG, Li, Di\}$, representing supercharge,
 39 CNG, Lithium-ion, and diesel buses, respectively. TC_m contains user cost $C_{u,m}$, operator cost $C_{o,m}$,
 40 and pollutant cost $C_{p,m}$:
 41

$$42 \quad TC_m = C_{u,m} + C_{o,m} + C_{p,m}, \quad (1)$$

$$43 \quad m \in \{SC, CNG, Li, Di\}$$

The user cost includes three parts: the cost that passengers have to access or egress the stop ($C_{a,m}$), the cost that passengers spent waiting at the stop ($C_{w,m}$), and the in-vehicle cost ($C_{v,m}$).

$$C_{u,m} = C_{a,m} + C_{w,m} + C_{v,m}, \quad (1a)$$

$$m \in \{SC, CNG, Li, Di\}$$

The operator cost entails those of: the stop construction and maintenance cost $C_{s,m}$; the Vehicle Hour Traveled (VHT) related cost (i.e., the cost associated with fleet size $C_{f,m}^f$, the cost associated with driver salary $C_{f,m}^h$, and idling cost $C_{f,m}^d$); Vehicle Kilometer Traveled (VKT) related cost (i.e., the cost associated with distance under cruising regime $C_{f,m}^c$, coasting regime $C_{f,m}^{co}$, and acceleration and deceleration regimes $C_{f,m}^{ad}$). The operator cost is thus given by:

$$C_{o,m} = C_{s,m} + (C_{f,m}^f + C_{f,m}^h + C_{f,m}^d) + (C_{f,m}^c + C_{f,m}^{co} + C_{f,m}^{ad}), \quad (1b)$$

$$m \in \{SC, CNG, Li, Di\}$$

The expression of pollutant cost $C_{p,m}$ will be formulated later in Equation 30 and 31.

Continuum Approximation Models

User Cost

Seeing in Equation 1a, each cost item will be explained in brief expression in this part.

For users who access and egress at point x on the corridor, the access/egress cost is computed by multiplying the number of users, the value of access time, and the average walking time. Assuming that the distribution of demand in the neighborhood of x is uniformed, the expected walking distance can be formulated as $\frac{1}{4 \cdot \delta_{m,r}(x) \cdot v_a}$, thus the access/egress cost is:

$$C_{a,m} = \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L \frac{(a l_{r,i}(x) + b o_{r,i}(x)) \cdot T_i \cdot \theta_a}{4 \cdot \delta_{m,r}(x) \cdot v_a} dx, \quad (2)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}$$

where

θ_a : value of access time (dollar per passenger hour);

1 $al_{r,i}(x)$: number of passengers who would like to alight at point x ($x \in [0, L]$). Noting that $r \in$
 2 $\{eb, wb\}$, $i \in \{pk, op\}$ (passenger per kilometer per hour);
 3 $bo_{r,i}(x)$: number of passengers who would like to board at point x ($x \in [0, L]$). Noting that $r \in$
 4 $\{eb, wb\}$, $i \in \{pk, op\}$ (passenger per kilometer per hour);
 5 T_i : duration of period i , with $i \in \{pk, op\}$ (hour);
 6 $\delta_{m,r}(x)$: stop density function, with $m \in \{SC, CNG, Li, Di\}$, $r \in \{eb, wb\}$ (number of stops per
 7 kilometer);
 8 v_a : average walking speed for passengers accessing and egressing bus stop; (kilometer per hour);

9 The daily waiting cost is considered as the product of average waiting time per passenger,
 10 the number of boarding passengers, and the value of waiting time. Considering that waiting
 11 passengers will get on the first bus that passes by, the waiting time of passengers will increases
 12 linearly with average headway between buses, h_i . The expected passenger's waiting time will be
 13 half of headway if bus's arrival is perfectly regular. Therefore, the expression for user waiting cost
 14 is given as follow:
 15
 16

$$17 \quad C_{w,m} = \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L bo_{r,i}(x) \cdot T_i \cdot \frac{h_i}{2} \cdot \theta_w dx, \quad (3)$$

$$18 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}$$

19
 20
 21 where:

22 θ_w : value of waiting time (dollar per passenger hour);

23 h_i : the average headway, with $i \in \{pk, op\}$ (hour);

24 The in-vehicle cost is the cost generated by all on board passengers when bus is operating
 25 between stops. It contains the cost accounted for bus travelling and for bus idling. Different driving
 26 regimes might be conducted while bus is moving to next stop and it is essential to fully understand
 27 the driving scenarios that could happen. Generally, there are four basic driving regimes (i.e.,
 28 acceleration, cruising, coasting, and deceleration).

29 The bus's travel time between stops depends on whether a transit vehicle can reach its
 30 maximum speed or not. Supposed S_c to be the critical distance for completing a perfect
 31 acceleration to cruising speed and a perfect deceleration from the cruising speed. For a stop
 32 spacing (represented as $\frac{1}{\delta_{m,r}(x)}$), here are four driving scenarios:

33 Scenario 1: No coasting and cruising, $\frac{1}{\delta_{m,r}(x)} \leq S_c$

34 In this case, the inter-stop travel time t_1 consist of two parts, the time for accelerating to
 35 some speed v_s ($v_s \leq v_{c,i}$, $v_{c,i}$ is the cruising speed, with $i \in \{pk, op\}$) and time for braking; The
 36 time items are expressed as follows:
 37
 38

$$39 \quad t_1 = t_{a1} + t_{b1} \quad (4)$$

40
 41

$$1 \quad t_{a1} = \frac{v_s}{a_v} \quad (4a)$$

$$2$$

$$3$$

$$4 \quad t_{b1} = \frac{v_s}{d_v} \quad (4b)$$

5
6
7 where, a_v , d_v indicate the acceleration and deceleration rates, respectively, in unit km/h²;

8 We can calculate v_s as following expression:

$$9$$

$$10$$

$$11 \quad v_s = \sqrt{\frac{2a_v d_v}{(a_v + d_v) \cdot \delta_{m,r}(x)}} \quad v_s \leq v_{c,i}, \frac{1}{\delta_{m,r}(x)} \leq S_c, \quad (5)$$

12 $m \in \{SC, CNG, Li, Di\}$, $i \in \{pk, op\}$, $r \in \{eb, wb\}$;

13
14
15 Noting that the sum of acceleration and deceleration distance is $\frac{1}{\delta_{m,r}(x)}$, which represents
16 the stop spacing. The per-kilometer travel time $t_{m,r,i}^v(x)$ can be determined by substituting
17 Equation 5 in Equation 4 and then dividing the spacing. Thus, per-kilometer travel time (in unit of
18 hour) in scenario 1 can be formulated as:

$$19$$

$$20$$

$$21 \quad t_{m,r,i,1}^v(x) = \sqrt{\frac{2(a_v^2 + d_v^2) \cdot \delta_{m,r}(x)}{(a_v + d_v) \cdot a_v \cdot d_v}}, \frac{1}{\delta_{m,r}(x)} \leq S_c, \quad (6)$$

22 $m \in \{SC, CNG, Li, Di\}$, $i \in \{pk, op\}$, $r \in \{eb, wb\}$;

23
24
25 Scenario 2: No coasting, $\frac{1}{\delta_{m,r}(x)} \geq S_c$

26 In this scenario, there are three intervals in an inter-stop traveling: the time for accelerating
27 to cruising speed $v_{c,i}$ (t_{a2}); the time for cruising (t_{c2}); and the time for braking (t_{b2}). The travel
28 time for each regime and the total travel time in a stop spacing for this scenario (t_2) are:

$$29$$

$$30$$

$$31 \quad t_2 = t_{a2} + t_{c2} + t_{b2} \quad (7)$$

$$32$$

$$33$$

$$34 \quad t_{a2} = \frac{v_{c,i}}{a_v} \quad (7a)$$

$$t_{c2} = \frac{1}{\delta_{m,r}(x) \cdot v_{c,i}} - \frac{v_{c,i}}{2a_v} - \frac{v_{c,i}}{2d_v} \quad (7b)$$

$$t_{b2} = \frac{v_{c,i}}{d_v} \quad (7c)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

Thus, the per-kilometer travel time, denoted as $t_{m,r,i,2}^v(x)$, in unit of hour, is formulated as:

$$t_{m,r,i,2}^v(x) = \frac{v_{c,i} \cdot \delta_{m,r}(x)}{2} \cdot \left(\frac{1}{a_v} + \frac{1}{d_v} \right) + \frac{1}{v_{c,i}}, \frac{1}{\delta_{m,r}(x)} \geq S_c, \quad (8)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

$$\text{Scenario 3: No cruising, } \frac{1}{\delta_{m,r}(x)} \geq S_c$$

Similarly, there are three intervals composed in this scenario: accelerating, coasting to speed v_{co} with deceleration rate c_v (km/h²), and then braking regime is applied to reach the stop. Thus, we can determine the inter-stop travel time for each regime as follows:

$$t_3 = t_{a3} + t_{co3} + t_{b3} \quad (9)$$

$$t_{a3} = \frac{v_{c,i}}{a_v} \quad (9a)$$

$$t_{co3} = \frac{v_{c,i} - v_{co}}{c_v} \quad (9b)$$

$$t_{b3} = \frac{v_{co}}{d_v} \quad (9c)$$

$$\text{with } i \in \{pk, op\};$$

Thus, the per-kilometer travel time (hour) in this scenario is formulated as:

$$t_{m,r,i,3}^v(x) = \left(v_{c,i} \cdot \left(\frac{1}{a_v} + \frac{1}{c_v} \right) + v_{co,m,r,i}(x) \cdot \left(\frac{1}{d_v} - \frac{1}{c_v} \right) \right) \cdot \delta_{m,r}(x), \frac{1}{\delta_{m,r}(x)} \geq S_c, \quad (10)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

With $v_{co,m,r,i}(x)$ (indicating the speed at the end of coasting with $i \in \{pk, op\}, r \in \{eb, wb\}, m \in \{SC, CNG, Li, Di\}$, at point x) expressed as below, which is derived from the work of Vuchic (23):

$$v_{co,m,r,i}(x) = \sqrt{\frac{2a_v \cdot d_v \cdot c_v - (a_v + c_v) \cdot v_{c,i}^2 \cdot d_v \cdot \delta_{m,r}(x)}{a_v \cdot (d_v - c_v) \cdot \delta_{m,r}(x)}}, \quad (11)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

Scenario 4: With coasting and cruising, $\frac{1}{\delta_{m,r}(x)} \geq S_c$

In this scenario, buses can accelerate to cruising speed, then keep constant speed, coast from cruising speed, and brake. Thus, four intervals are included in the inter-stop travel time.

$$t_4 = t_{a4} + t_{c4} + t_{co4} + t_{b4} \quad (12)$$

The distance for an inter-stop cruising is:

$$S_m = \frac{1}{\delta_{m,r}(x)} - \left(\frac{v_{c,i}^2}{2a_v} + \frac{v_{c,i}^2 - v_{co}^2}{2c_v} + \frac{v_{co}^2}{2d_v} \right), \quad (13)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

We denote the inter-stop cruising time as t_{c4} , and present it in the following equation:

$$1 \quad t_{c4} = \frac{1}{\delta_{m,r}(x) \cdot v_{c,i}} - \frac{v_{c,i}}{2} \left(\frac{1}{a_v} + \frac{1}{c_v} \right) - \frac{v_{co}^2}{2v_{c,i}} \left(\frac{1}{d_v} - \frac{1}{c_v} \right), \quad (14)$$

$$2 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

3

4

5 Thus, the inter-stop travel time is:

6

7

$$8 \quad t_4 = \frac{1}{\delta_{m,r}(x) \cdot v_{c,i}} + \left(\frac{1}{a_v} + \frac{1}{c_v} \right) \cdot \frac{v_{c,i}}{2} + \left(\frac{1}{c_v} - \frac{1}{d_v} \right) \cdot \left(\frac{v_{co}^2}{2v_{c,i}} - v_{co} \right), \quad (15)$$

$$9 \quad \text{with } i \in \{pk, op\};$$

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It should be mentioned that, even though it exists four different operation scenarios, the scenario 1 to 3 can be regarded as the special situations of scenario 4 (explained later), which the per-kilometer travel time $t_{m,r,i}^v(x)$ and the time covered by each driving regime in per-kilometer distance are:

$$19 \quad t_{m,r,i}^v(x) = t_{a,m,r,i}^v(x) + t_{c,m,r,i}^v(x) + t_{co,m,r,i}^v(x) + t_{b,m,r,i}^v(x) \quad (16)$$

19

20

21

22

23

24

Where

$$25 \quad t_{a,m,r,i}^v(x) = \frac{v_{c,i} \cdot \delta_{m,r}(x)}{a_v} \quad (16a)$$

25

26

27

$$28 \quad t_{co,m,r,i}^v(x) = \frac{v_{c,i} - v_{co,m,r,i}(x)}{c_v} \cdot \delta_{m,r}(x) \quad (16b)$$

28

29

30

$$31 \quad t_{b,m,r,i}^v(x) = \frac{v_{co,m,r,i}(x) \cdot \delta_{m,r}(x)}{d_v} \quad (16c)$$

31

32

33

$$34 \quad t_{c,m,r,i}^v(x) = \left[\frac{1}{\delta_{m,r}(x) \cdot v_{c,i}} - \frac{v_{c,i}}{2} \left(\frac{1}{a_v} + \frac{1}{c_v} \right) + \frac{v_{co,m,r,i}(x)}{2v_{c,i}} \left(\frac{1}{d_v} - \frac{1}{c_v} \right) \right] \cdot \delta_{m,r}(x) \quad (16d)$$

$$t_{m,r,i}^v(x) = \left[\left(\frac{1}{c_v} - \frac{1}{d_v} \right) \cdot \frac{v_{co,m,r,i}^2(x)}{2v_{c,i}} - \left(\frac{1}{c_v} - \frac{1}{d_v} \right) \cdot v_{co,m,r,i}(x) + \frac{1}{v_{c,i} \cdot \delta_{m,r}(x)} + \frac{v_{c,i}}{2a_v} + \frac{v_{c,i}}{2c_v} \right] \cdot \delta_{m,r}(x), \quad (17)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

In scenario 2, which doesn't contain coasting regime, it means that the coasting speed equals to the cruising speed, $v_{co,m,r,i}(x) = v_{c,i}$. Thus, Equation 17 can be converted into Equation 8. As for scenario 3, which has no cruising regime, the cruising time is zero ($t_{c,m,r,i}^v(x) = 0$), and we can find the expression of $v_{co,m,r,i}(x)$ the same as Equation 11. Scenario 1 is the combination of two conditions ($v_{co,m,r,i}(x) = v_{c,i}$ and $t_{c,m,r,i}^v(x) = 0$) and the Equation 6 can be yielded.

Therefore, Equation 17 is the generalized formula with four scenarios included. We can make an assumption that scenario 4 is conducted in every inter-stop trip. By plugging $v_{co,m,r,i}(x)$ into optimization, we can finally yield the profile of $v_{co,m,r,i}(x)$. Once we get the $v_{co,m,r,i}(x)$ profile, the effect of coasting regime on operation cost can be explored.

After bus finishes its inter-stop travel, the bus dwell time will generate for passengers boarding and alighting and for opening and closing doors. Supposing that an onboard fare payment method is applied and one door is used for boarding and one or more doors are used for alighting, the dwell time at each stop is dominated by the process that takes longer. To facilitate modeling, we convert dwell time at each stop to dwell time on per kilometer basis, denoted as $t_{m,r,i}^d(x)$. The expression of $t_{m,r,i}^d(x)$ is:

$$t_{m,r,i}^d(x) = t_d(x) \cdot \delta_{m,r}(x) \quad (18)$$

Where

$$t_d(x) = t_0 + \max \left(bo_{r,i}(x) \cdot h_i \cdot \frac{1}{\delta_{m,r}(x)} \cdot t_b, al_{r,i}(x) \cdot h_i \cdot \frac{1}{\delta_{m,r}(x)} \cdot t_a \right), \quad (19)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

t_0 is the dead time for opening and closing doors, t_b and t_a are the average boarding and alighting time (hour) per passenger, respectively.

To summarize, the expression for in-vehicle cost is presented as follow:

$$1 \quad C_{v,m} = \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L P_{r,i}(x) \cdot T_i \cdot (t_{m,r,i}^v(x) + t_{m,r,i}^d(x)) \cdot \theta_v dx, \quad (20)$$

$$2 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

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4

5 Where

6 $P_{r,i}(x)$: the number of passenger load at point x in direction r , in period i (passengers per hour);

7 $t_{m,r,i}^v(x)$: bus's per-kilometer travel time (hour) with $r \in \{eb, wb\}, i \in \{pk, op\}, m \in$
8 $\{SC, CNG, Li, Di\}$;

9 $t_{m,r,i}^d(x)$: bus's per-kilometer dwell time (hour) with $r \in \{eb, wb\}, i \in \{pk, op\}, m \in$
10 $\{SC, CNG, Li, Di\}$;

11 θ_v : the value of in-vehicle time (dollar per hour);

12

13 *Operator Cost*

14 Each cost item in Equation 1b will be discussed in this part. The cost for stop construction and
15 maintenance is determined by θ_f and θ_0 , which represent the daily cost for stop construction
16 (\$/day) and the hourly cost for stop maintenance (\$/hour), respectively. Thus, the per-day cost for
17 stops on the corridor is:

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19

$$20 \quad C_{s,m} = \sum_{r=eb,wb} \int_0^L (\theta_f + \theta_0 T) \cdot \delta_{m,r}(x) dx, \quad (21)$$

$$21 \quad m \in \{SC, CNG, Li, Di\}, r \in \{eb, wb\}$$

22
23

24 Where, T presents the total operating time of a stop (hour).

25 The cost associated with fleet size is closely related to the maximum fleet size required,
26 which is correspondent to peak hour fleet size, denoted as B_h . B_h is determined as the quotient of
27 peak hour cycle time t_{peak}^c and peak hour headway h_{peak} . Thus:

28
29

$$30 \quad C_{f,m}^f = \theta_b \cdot B_h \quad (22)$$

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32

$$33 \quad B_h = \frac{t_{peak}^c}{h_{peak}} \quad (23)$$

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$$36 \quad t_i^c = \sum_{r=eb,wb} \int_0^L (t_{m,r,i}^v(x) + t_{m,r,i}^d(x)) dx, \quad (24)$$

$$37 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

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Where, θ_b is the fixed cost per bus (dollars per vehicle per day).

To determine the cost associated with driver wage, we suppose that a fixed wage θ_h is paid hourly for each on-duty hour, and the cost item for labor is expressed as:

$$8 \quad C_{f,m}^h = \theta_h \sum_{i=pk,op} \frac{T_i}{h_i} \cdot t_i^c, \quad (25)$$

$$9 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\};$$

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Where θ_h is the hourly salary for driver (dollar per hour).

The cost associated with idling C_{mf}^d is presented as:

$$16 \quad C_{f,m}^d = \theta_d \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L \frac{T_i}{h_i} \cdot t_{m,r,i}^d(x) dx, \quad (26)$$

$$17 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

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Where θ_d is the per hour cost of idling per vehicle(dollar per vehicle per hour);

Then, we discuss the VKT related cost:

The cost associated with distance travelled by buses at cruising speed for daily bus flow is presented as the integral of the product of vehicle flow over day $\frac{T_i}{h_i}$, the per vehicle-kilometer cruising distance $t_{m,r,i,c}^v(x) \cdot v_{c,i}$, and the per vehicle-kilometer cost θ_c . And the cost related is expressed as:

$$28 \quad C_{f,m}^c = \theta_c \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L \frac{T_i}{h_i} \cdot t_{m,r,i,c}^v(x) \cdot v_{c,i} dx, \quad (27)$$

$$29 \quad m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

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Where, θ_c is the vehicle cost per unit distance covered at cruising speed (dollar per vehicle per kilometer).

Similarly, the cost associated with distance coasted $C_{f,m}^{co}$, the cost associated with acceleration and deceleration $C_{f,m}^{ad}$ are expressed as follows, respectively:

$$38 \quad C_{f,m}^{co} = \theta_{co} \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L \frac{T_i}{h_i} \cdot \frac{v_{c,i}^2 - v_{co,m,r,i}^2(x)}{2c_v} \cdot \delta_{m,r}(x) dx \quad (28)$$

$$C_{f,m}^{ad} = \theta_{ad} \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L \frac{T_i}{h_i} \cdot \left(\frac{v_{c,i}^2}{2a_v} + \frac{v_{co,m,r,i}^2(x)}{2d_v} \right) \cdot \delta_{m,r}(x) dx, \quad (29)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\};$$

Where

θ_{co} : the unit cost per kilometer traveled by coasting (dollar per vehicle per kilometer);

θ_{ad} : the unit cost per kilometer covered at accelerating from bus station or braking to bus station (dollar per vehicle per kilometer);

Pollutant Cost

In this model, we primarily take HC, CO, and NO_x into account. The n^{th} pollutant volume can be determined as the product of emission rate for pollutant n ($n=1$: HC; $n=2$: CO; $n=3$: NO_x) and the per-kilometer travel time of different driving regimes, as expressed below:

$$P_{m,r,n} = e_n^a \cdot t_{a,m,r,i}^v(x) + e_n^v \cdot t_{c,m,r,i}^v(x) + e_n^{co} \cdot t_{co,m,r,i}^v(x) + e_n^b \cdot t_{b,m,r,i}^v(x) + e_n^d \cdot t_{m,r,i}^d(x), \quad (30)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}, n \in \{1,2,3\};$$

where e_n^a , e_n^v , e_n^{co} , e_n^b , and e_n^d represent the emission rates for pollutant n , under acceleration, cruising, coasting, deceleration, and standing regimes, respectively, in unit of (ton/hour).

Therefore, the pollutants cost generated while buses are operating on the corridor is:

$$C_{P,m} = \sum_{n=1}^3 \sum_{r=eb,wb} \sum_{i=pk,op} \int_0^L P_{m,r,n} \cdot \theta_{p,n} \cdot \frac{T_i}{h_i} dx, \quad (31)$$

$$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}, n \in \{1,2,3\};$$

where, $\theta_{p,n}$ is the unit vehicle-related damage cost of pollutant (\$/ton).

Model Optimization

In this model, the objective function is the generalized cost, which is the sum of user cost, operator cost, and pollutant cost. The expression of minimization problem is:

$$\min_{\delta_{m,r}(x), h_i, v_{co,m,r,i}(x)} \{C_{u,m} + C_{o,m} + C_{P,m}\} \quad (32)$$

1
2 Subject to:

$$3 \quad \begin{cases} Q_i \leq \frac{cap_{bus}}{h_i} \\ (al_{r,i}(x) + bo_{r,i}(x)) \cdot h_i \leq cap_{stop} \cdot \delta_{m,r}(x) \\ 0 \leq h_i, 0 \leq \delta_{m,r}(x), 0 \leq v_{co,m,r,i}(x) \leq v_{c,i} \end{cases} \quad (33)$$

4 $m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}$

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6

7 The first constraint indicates that the bus's capacity must feed the total passenger demand
8 ($i \in \{pk, op\}$). The second constraint is the stop capacity should satisfy the total demand of
9 boarding and alighting. In addition, the optimal results of headway, stop density, and coasting
10 speed should be positive.

11 The objective function has three variables/functions: $\delta_{m,r}(x)$, h_i , $v_{co,m,r,i}(x)$, and the
12 constrains are nonlinear. Those factors increase the complexity of the optimization. To obtain the
13 analytical expression of each variable/function, two alternative procedures are proposed. Firstly,
14 we introduced an initial value of headway to reduce the dimension. We can obtain the expression
15 of coasting speed and stop density by solving first order condition $v_{co,m,r,i}(x)^* = f(x, h_i)$ and
16 $\delta_{m,r}^*(x) = f(x, h_i, v_{co,m,r,i}(x)^*)$. Through this approach, the model is transformed into a problem
17 that has h_i variables.

18 The second procedure contains two steps: in the first place, $\delta_{m,r}(x)$ and $v_{co,m,r,i}(x)$ are
19 replaced by the optimal expression $\delta_{m,r}^*(x)$ and $v_{co,m,r,i}(x)^*$. As the optimal function of headway
20 also contains stop density and coasting speed, which is $h_i^* = f(x, \delta_{m,r}(x)^*, v_{co,m,r,i}(x)^*)$, the next
21 step is to iterate the analytical expression of headway, stop density, and coasting speed until the
22 convergence is reached under constraints.

23 The first-order-condition expressions of stop density in each direction is:

$$26 \quad \delta_{m,r}(x) = \sqrt{\frac{\sum_{i=pk,op} (al_{r,i}(x) + bo_{r,i}(x)) \cdot T_i \cdot \theta_a}{4v_a \left(f_{2,m,r,i}(x) + \sum_{i=pk,op} [f_{3,m,r,i}(x) + \theta_c \cdot f_{4,m,r,i}(x)] \frac{T_i}{h_i} \right) + \sum_{n=1,2,3} \sum_{i=pk,op} \frac{\theta_p \cdot n \cdot T_i}{h_i} \left(f_{5,m,r,i}(x) + \frac{e_n^v}{v_{c,i}} \cdot f_{4,m,r,i}(x) \right)}} \quad (34)$$

27 $m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}, n \in \{1,2,3\};$

28
29

30 Where

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32

$$33 \quad f_{1,m,r,i}(x) = \left(\frac{1}{c_v} - \frac{1}{d_v} \right) \cdot \frac{v_{co,m,r,i}^2(x)}{2v_{c,i}} - \left(\frac{1}{c_v} - \frac{1}{d_v} \right) \cdot v_{co,m,r,i}(x) + \frac{v_{c,i}}{2a_v} + \frac{v_{c,i}}{2c_v};$$

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35

$$f_{2,m,r,i}(x) = \sum_{i=pk,op} \left(\theta_v \cdot T_i \cdot P_{r,i}(x) + \frac{T_i}{h_i} \cdot \theta_h \right) \cdot \left(f_{1,m,r,i}(x) + t_d(x) \right) + \frac{\theta_b (f_{1,m,r,pk}(x) + t_{d,pk}(x))}{h_{pk}} + \theta_f + \theta_0 T ;$$

$$f_{3,m,r,i}(x) = \theta_d \cdot t_d(x) + \theta_{co} \cdot \frac{v_{c,i}^2 - v_{co,m,r,i}^2(x)}{2c_v} + \theta_{ad} \left(\frac{v_{c,i}^2}{2a_v} + \frac{v_{co,m,r,i}^2(x)}{2d_v} \right) ;$$

$$f_{4,m,r,i}(x) = - \left(\frac{v_{c,i}^2}{2a_v} + \frac{v_{c,i}^2 - v_{co,m,r,i}^2(x)}{2c_v} + \frac{v_{co,m,r,i}^2(x)}{2d_v} \right) ;$$

$$f_{5,m,r,i}(x) = e_n^a \cdot \frac{v_{c,i}}{a_v} + e_n^{co} \cdot \frac{v_{c,i} - v_{co,m,r,i}(x)}{c_v} + e_n^b \cdot \frac{v_{co,m,r,i}(x)}{d_v} + e_n^d \cdot t_d(x) ;$$

It should be mentioned that the stop density function is different on two sides of corridor to reflect the flexibility of locating curbside bus stops.

The expressions of optimal headway in peak hour and off-peak hour are as follows, respectively:

$$h_{pk} = \sqrt{\frac{\sum_{r=eb,wb} \int_0^L \left[(\theta_b + \theta_h \cdot T_{pk}) (t_{m,r,pk}^v(x) + t_{m,r,pk}^d(x)) + g_{1,m,r,pk}(x) + \sum_{n=1}^3 T_{pk} \cdot \theta_{p,n} \cdot P_{m,r,n}(x) \right] dx}{\sum_{r=eb,wb} \int_0^L b_{or,pk}(x) \cdot \frac{T_{pk}}{2} \cdot \theta_w + P_{r,pk}(x) \cdot T_{pk} \cdot g_{2,m,r,pk}(x) \cdot \theta_v dx}} \quad (35)$$

$$h_{op} = \sqrt{\frac{\sum_{r=eb,wb} \int_0^L \left[\theta_h \cdot T_{op} \cdot (t_{m,r,op}^v(x) + t_{m,r,op}^d(x)) + g_{1,m,r,op}(x) + \sum_{n=1}^3 T_{op} \cdot \theta_{p,n} \cdot P_{m,r,n}(x) \right] dx}{\sum_{r=eb,wb} \int_0^L b_{or,op}(x) \cdot \frac{T_{op}}{2} \cdot \theta_w + P_{r,op}(x) \cdot T_{op} \cdot g_{2,m,r,op}(x) \cdot \theta_v dx}} \quad (36)$$

$$m \in \{SC, CNG, Li, Di\}, r \in \{eb, wb\}, n \in \{1, 2, 3\} ;$$

Where

$$g_{1,m,r,i}(x) = T_i \cdot \theta_c \cdot t_{c,m,r,i}^v(x) \cdot v_{c,i} + \theta_{ad} \cdot \left(\frac{v_{c,i}^2}{2a_v} + \frac{v_{co,m,r,i}^2(x)}{2d_v} \right) \cdot \delta_{m,r}(x) \cdot T_i + T_i \cdot \theta_d \cdot t_{m,r,i}^d(x) + T_i \cdot \theta_{co} \cdot \frac{v_{c,i}^2 - v_{co,m,r,i}^2(x)}{2c_v} \cdot \delta_{m,r}(x) ;$$

$$g_{2,m,r,i}(x) = \max(b_{ori}(x) \cdot t_m^b, a_{li}(x) \cdot t_m^a).$$

1
2 Once the optimal value of headway is obtained, we can determine $v_{co,m,r,i}(x)$. The first
3 order condition is applied to obtain the unconstrained optimal $\tilde{v}_{co,m,r,i}(x)$ as follows:
4
5

$$6 \quad \tilde{v}_{co,m,r,i}(x) = \frac{\left(\frac{1}{c_v} - \frac{1}{d_v}\right) \cdot u_{r,i}(x) + \sum_{n=1}^3 \frac{T_i}{h_i} \cdot \theta_{p,n} \cdot \left(\frac{e_n^{co}}{c_v} - \frac{e_n^b}{d_v}\right)}{\left(\frac{1}{c_v} - \frac{1}{d_v}\right) \cdot \left[\frac{u_{r,i}(x)}{v_{c,i}} + \frac{T_i}{h_i} \cdot (\theta_c - \theta_{co} + \theta_{ad} + \sum_{n=1}^3 \frac{e_n^v}{v_{c,i}} \cdot \theta_{p,n})\right]} \quad (37)$$

7 $m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}, n \in \{1, 2, 3\};$
8
9

10 Where $u_{r,i}(x) = \theta_v \cdot T_i \cdot P_{r,i}(x) + \frac{\theta_b}{h_{pk}} + \theta_h \cdot \frac{T_i}{h_i};$

11 Considering the constraint ($0 \leq v_{co,m,r,i}(x) \leq v_{c,i}$), the optimal expression of
12 $v_{co,m,r,i}(x)$ is given by:
13
14

$$15 \quad v_{co,m,r,i}(x) = \text{mid}(0, \tilde{v}_{co,m,r,i}(x), v_{c,i}), \quad (38)$$

16 $m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}, n \in \{1, 2, 3\};$
17 where function $\text{mid}(x, y, z)$ return the middle value among $x, y,$ and z .
18

19 As for supercharge bus and Lithium-ion bus, the pollutant emitted can be neglected, so
20 the optimal expressions of stop density, headway, and coasting speed are simplified as follows:
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22

$$23 \quad \delta_{m,r}(x) = \sqrt{\frac{\sum_{i=pk,op} (a_{r,i}(x) + b_{o,r,i}(x)) \cdot T_i \cdot \theta_a}{4v_a (f_{2,m,r,i}(x) + \sum_{i=pk,op} [f_{3,m,r,i}(x) + \theta_c \cdot f_{4,m,r,i}(x)]) \cdot \frac{T_i}{h_i}}} \quad (39)$$

$$24 \quad h_{pk} = \sqrt{\frac{\sum_{r=eb,wb} \int_0^L [(\theta_b + \theta_h \cdot T_{pk}) (t_{m,r,op}^v(x) + t_{m,r,op}^d(x)) + g_{1,m,r,pk}(x)] dx}{\sum_{r=eb,wb} \int_0^L b_{o,r,pk}(x) \cdot \frac{T_{pk}}{2} \cdot \theta_w + P_{r,pk}(x) \cdot T_{pk} \cdot g_{2,m,r,pk}(x) \cdot \theta_v dx}}, \quad (40)$$

$$27 \quad h_{op} = \sqrt{\frac{\sum_{r=eb,wb} \int_0^L [\theta_h \cdot T_{op} \cdot (t_{m,r,op}^v(x) + t_{m,r,op}^d(x)) + g_{1,m,r,op}(x)] dx}{\sum_{r=eb,wb} \int_0^L b_{o,r,op}(x) \cdot \frac{T_{op}}{2} \cdot \theta_w + P_{r,op}(x) \cdot T_{op} \cdot g_{2,m,r,op}(x) \cdot \theta_v dx}}, \quad (41)$$

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$$\tilde{v}_{co,m,r,i}(x) = \frac{u_{r,i}(x)}{\frac{u_{r,i}(x)}{v_{c,i}} + \frac{T_i}{h_i}(\theta_c - \theta_{co} + \theta_{ad})} \quad (42)$$

$$v_{co,m,r,i}(x) = \text{mid}(0, \tilde{v}_{co,m,r,i}(x), v_{c,i}), \quad (43)$$

$m \in \{SC, CNG, Li, Di\}, i \in \{pk, op\}, r \in \{eb, wb\}, n \in \{1, 2, 3\};$

NUMERICAL APPLICATION

System Inputs

In this section, we apply continuum model to the 7th bus route, Yaan (City), China, where the first supercharge bus route is in operation. The studied corridor is approximately 11 km in length, 21 stops in the east direction and 22 stops in the west, as shown in Figure 1. The bus's peak-hour cruising speed and off-peak hour cruising speed are given by Shu Tong Transportation Agency. All the operator cost items are shown in Table 1 as follows:



FIGURE 1 The No.7 bus route, Yaan (China)

TABLE 1 Cost Parameters of Four Transit Modes

Supercharge Cost Parameters		
Parameters	Value	Source
\$I-S infrastructure stop cost (\$/stop/h)	\$0.47	Derived from Gu et al (2016), with additional construction cost 2,590,000 yuan for charging facilities.

Operating Costs (Distance)		
Maintenance cost per veh-km	\$0.012	Shu Tong Transpotation Agency
Energy cost per km (\$/km)	\$0.088	Electricity price 2018
\$v, Cost per veh-km (\$/veh-km)	\$0.1	
Operating Costs (Time)		
Employees per vehicle	1.5	
Average wage (\$/h)	\$6.15	Yaan City average wage standard
Labour cost per hour	\$9.225	
Vehicle cost (\$)	\$257,353	
Vehicle lifespan (years)	8	
Depreciation cost per hr (\$/hr)	\$6.3	Assumed straight-line depreciation, work 14 hr per day
CNG Cost Parameters		
Parameters	Value	Source
\$I-S infrastructure stop cost (\$/stop/h)	\$0.35	Derived from Gu et al (2016)
Operating Costs (Distance)		
Maintenance cost per veh-km	\$0.02	Shu Tong Transpotation Agency
Energy cost per km (\$/km)	\$0.238	CNG price 2018
\$v, Cost per veh-km (\$/veh-km)	\$0.258	
Operating Costs (Time)		
Employees per vehicle	2	
Average wage (\$/hr)	\$6.15	Yaan City average wage standard
Labor cost per hour	\$12.3	
Vehicle cost (\$)	\$73,529	
Vehicle lifespan (years)	8	
Depreciation cost per hr (\$/hr)	\$1.8	Assumed straight-line depreciation, work 14 hr per day
Lithium-ion battery Bus Cost Parameters		
Parameters	Value	Source
\$I-S infrastructure stop cost (\$/stop/h)	\$0.58	Derived from Gu et al (2016), with an additional cost 5,000,000yuan for supplement equipment
Operating Costs (Distance)		
Maintenance cost per veh-km	\$0.242	Shu Tong Transpotation Agency
Energy cost per km (\$/km)	\$0.088	Electricity price 2018
\$v, Cost per veh-km (\$/veh-km)	\$0.33	
Operating Costs (Time)		
Employees per vehicle	1.7	
Average wage (\$/hr)	\$6.15	Yaan City average wage standard
Labour cost per hour	\$10.455	
Vehicle cost (\$)	\$235,294	
Vehicle lifespan (years)	8	

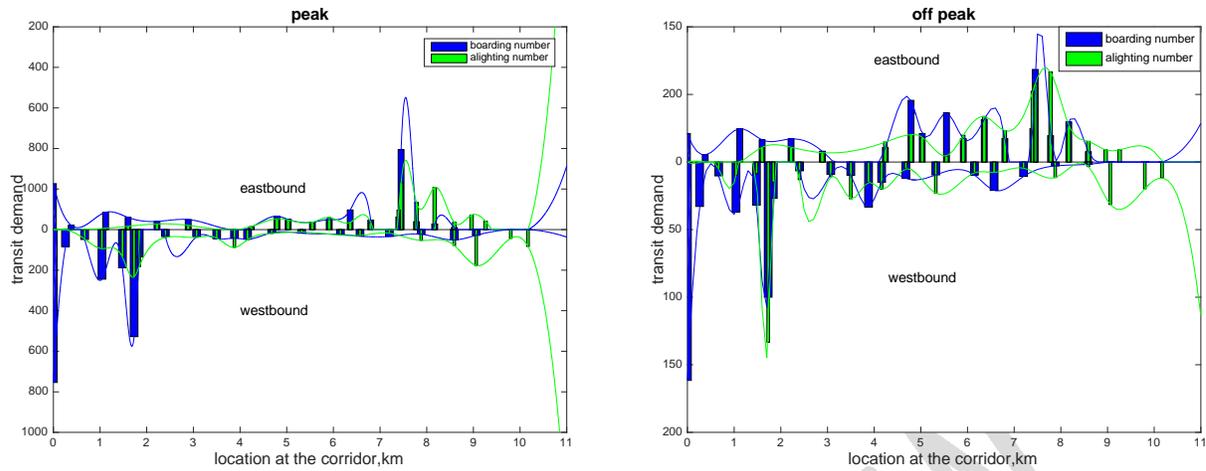
Depreciation cost per hr (\$/hr)	\$5.76	Assumed straight-line depreciation, work 14 hr per day
Diesel Bus Cost Parameters		
Parameters	Value	Source
\$I-S infrastructure stop cost (\$/stop/h)	\$0.35	Derived from Gu et al (2016)
Operating Costs (Distance)		
Maintenance cost per veh-km	\$0.025	Shu Tong Transportation Agency
Energy cost per km (\$/km)	\$0.512	Diesel price 2018
\$v, Cost per veh-km (\$/veh-km)	\$0.537	
Operating Costs (Time)		
Employees per vehicle	2	
Average wage (\$/hr)	\$6.15	Yaan City average wage standard
Labor cost per hour	\$12.3	
Vehicle cost (\$)	\$73,529	
Vehicle lifespan (years)	8	
Depreciation cost per hr (\$/hr)	\$1.8	Assumed straight-line depreciation, work 14 hr per day

In this analysis, four different vehicle types are considered, among which two of them are clean-energy bus (i.e., supercharge bus and Lithium-ion bus), the others are conventional buses (i.e., CNG buses and diesel buses). It should be mentioned that all the vehicles are 12m in length. The emission standard for CNG bus and Diesel bus is both of China National IV standard. The emission rates of conventional buses at different driving regimes are given in Table 2, the data is adopted from previous studies (24-25) and is summarized below.

TABLE 2 Emission Rates of Pollutants at Different Driving Cycles

12m China National IV CNG bus				
Pollutant	Idling	Acceleration	Deceleration	Constant velocity
NO_x (g/s)	0.0036	0.0152	0.0071	0.0115
HC(g/s)	0.0012	0.0034	0.0021	0.0027
CO (g/s)	0.0211	0.0473	0.0303	0.0363
12m China National IV Diesel Bus				
NO_x (g/s)	0.0226	0.0973	0.0503	0.0766
HC (g/s)	0.0013	0.0022	0.0014	0.0018
CO (g/s)	0.0070	0.0212	0.0069	0.0137

The boarding and alighting density function is obtained by using the on and off data which is collected on 6th April, 2017 by Shu Tong Transportation Agency. All the programming process is performed on Matlab platform. Figure 2 shows the boarding and alighting densities along the corridor in terms of different time periods (i.e., peak hour and off-peak hour).

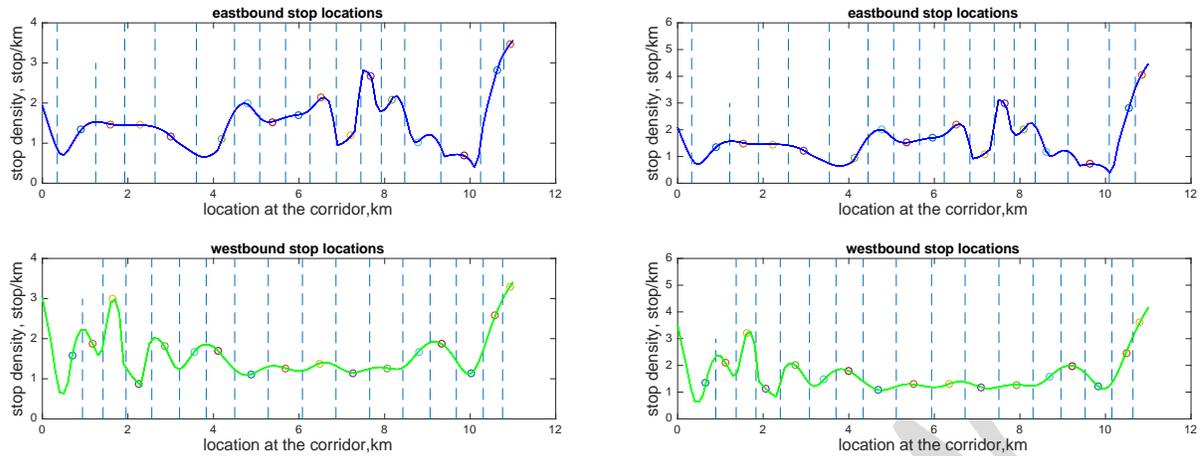


1
2 **FIGURE 2 Boarding and alighting densities along the corridor in different time periods**

3
4 The values of walking time, riding time, and waiting time are derived from (16), and are set
5 at 4.09\$/hour, 1.64\$/hour, 2.73\$/hour, respectively. Passengers access/egress bus stop at 3.6km/h.
6 The time for opening and closing doors is 2s at each stop. According to the schedule provided by
7 Shu Tong Transportation Agency, in the day time, from 7:00 to 17:00, the headway in current
8 system is 7 min; and during the evening peak, which is 17:00-19:00, the service headway is 12min;
9 for 19:00-20:00, the service headway is 15 min; 20:00-21:00, the service headway prolonged to
10 20min.

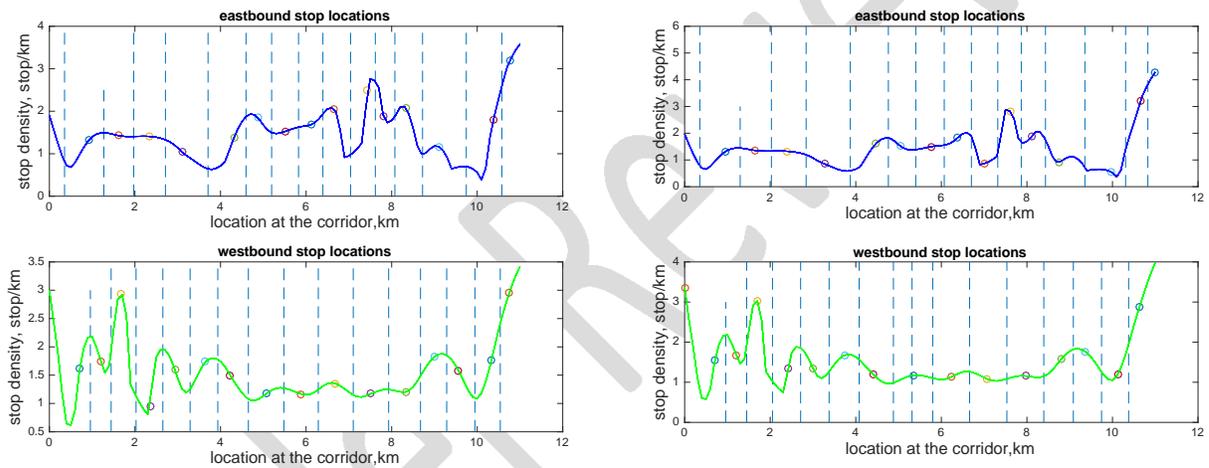
11 **Optimal Design Solution Analysis**

12 Figure 3 presents the discretization of the bi-directional stop density functions for four transit
13 modes. We discretize the bi-directional stop density functions by locating stops when the integral
14 of its left boundary and right boundary is 1. Here, we take the center line of each stop spacing as
15 the left/right boundary. The detailed description of the discretization method is derived from (16).
16 In the Figure, the circles on the stop density curves represent the optimal location of stops and the
17 dash lines represent the boundaries of each stop coverage market. The system characteristics of
18 current and optimized 7th route in Yaan (City) are summarized in Table 3.
19
20



(a) Supercharge bus stop locations

(b) CNG bus stop locations



(c) Li-thium-ion bus stop locations

(d) Diesel bus stop locations

FIGURE 3 Optimal stop locations

In Table 3, we compare the optimal system costs of four vehicle types. The optimal results indicate that when CNG buses are operating on the route, the system cost is the lowest. Note that the cost gap between CNG buses system and supercharge buses system is slight (8080.9\$/day vs. 8127.4\$/day). Then, we discretize the optimal results based on CA approach and we obtain the real results when CA models are applied practically, which are shown in the third row. Comparing with the optimal results with discretized results, the outcomes of discrete models are in neighborhood with that of CA models, with error less than 3%. In other words, if we employ the discretized results on a real bus route, the system cost will be closed to optimal value, which verified the applicability and accuracy of CA models. Finally, we compare the current system costs (detailed in user cost, operator cost, and pollutant cost for conventional buses), with optimal results and discretized results. Remarkably, the optimal results have saved more than 50% of the system cost, especially in user cost.

1 In terms of system design, comparing with the current route configuration, the average stop
 2 spacing is increased by up to 31.3%. As for the service headway, the optimal headway in peak hour
 3 is ranging from 6.55 min to 8.30 min with respect to vehicle type. Among them, CNG is the most
 4 frequently emitted bus type. According to optimal design, the optimal headway in peak hour has
 5 been decreased by up to 31%, while in off-peak hour, the optimal headways are approximately
 6 equal to the observed values in current system.

7
 8 **TABLE 3 System Characteristics of Current and Optimized 7th route in Yaan**
 9

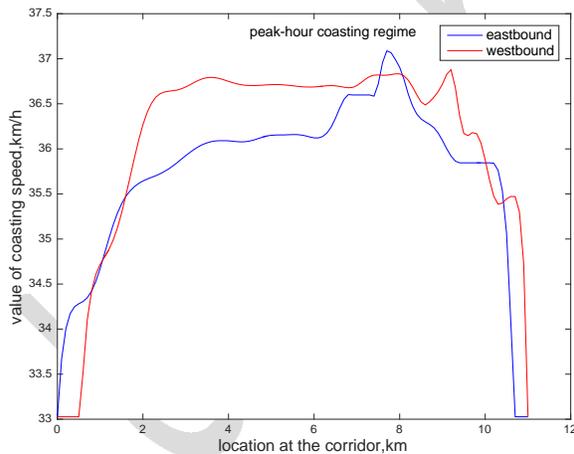
Supercharge Bus Route Design			
System metrics	Current corridor	Optimal results	Real results
h_{peak} , min	9.50 (weighted)	7.76	7.76
$h_{off-peak}$, min	9.64 (weighted)	8.12	8.12
$Spacing_{eb}$, km	0.52	0.65 (17 stops)	0.65
$Spacing_{wb}$, km	0.55	0.61 (18 stops)	0.61
C_u , \$/day	14,081.0	5,567.4	5,540.5
C_o , \$/day	2,624.2	2,559.9	2,359.4
TC , \$/day	16,706.2	8,127.3	7,899.9
Cost Saving, %	na	51.3%	52.7%
Difference% (CA and discrete models)	na	2.8%	
CNG Bus Route Design			
System metrics	Current corridor	Optimal results	Real results
h_{peak} , min	9.50 (weighted)	6.55	6.55
$h_{off-peak}$, min	9.64 (weighted)	10.68	10.68
$Spacing_{eb}$, km	0.52	0.65 (17 stops)	0.65 (17 stops)
$Spacing_{wb}$, km	0.55	0.61 (18 stops)	0.61 (18 stops)
C_u , \$/day	15,365.0	5,594.7	5,796.2
C_o , \$/day	2,165	2,398.2	2,412.7
C_p , \$/day	252.1	88.0	91.3
TC , \$/day	17,782.1	8,080.9	8,300.2
Cost Saving, %	na	54.6%	53.3%
Difference% (CA and discrete models)	na	2.7%	
Lithium-ion Battery Bus Route Design			
System metrics	Current corridor	Optimal results	Real results
h_{peak} , min	9.50 (weighted)	8.3	8.3
$h_{off-peak}$, min	9.64 (weighted)	10.3	10.3
$Spacing_{eb}$, km	0.52	0.69 (16 stops)	0.69 (16 stops)
$Spacing_{wb}$, km	0.55	0.65 (17 stops)	0.65 (17 stops)
C_u , \$/day	14,320.0	5,911.1	6,054.6
C_o , \$/day	3,223.1	2,818.0	2,880.2
TC , \$/day	17,543.1	8,729.1	8,934.8
Cost Saving, %	na	50.2%	47.8%

Difference% (CA and discrete models)	na	2.3%	
Diesel Bus Route Design			
System metrics	Current corridor	Optimal results	Real results
h_{peak} , min	9.50 (weighted)	7.60	7.60
$h_{off-peak}$, min	9.64 (weighted)	12.89	12.89
$Spacing_{eb}$, km	0.52	0.69 (16 stops)	0.69 (16 stops)
$Spacing_{wb}$, km	0.55	0.65 (17 stops)	0.65 (17 stops)
C_u , \$/day	16,130.0	6,305.8	6,146.9
C_o , \$/day	2,542.2	2,688.9	2,534.7
C_p , \$/day	248.9	62.5	106.5
TC , \$/day	18,921.1	9,057.2	8,788.1
Cost Saving, %	na	52.1%	53.6%
Difference% (CA and discrete models)	na	3%	

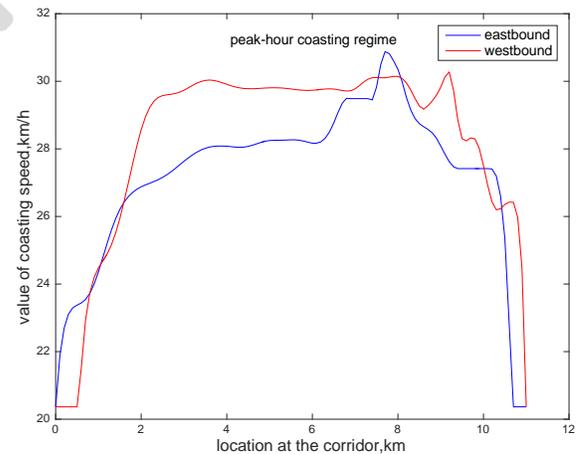
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The Effect of Coasting Regime

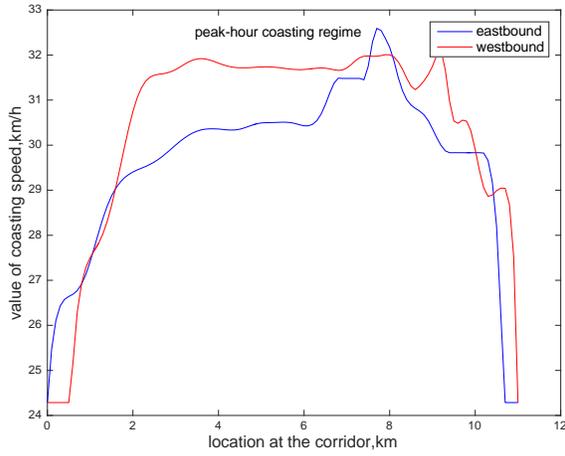
Previously, we have discussed the optimal values of stop density functions, service headways (peak and off-peak hour), and system costs for different bus types. This part will focus on analyzing the optimal value of coasting speed in different periods (peak hour and off-peak hour) for four vehicle types and investigating the interrelationship of coasting regime on system cost. The optimized profiles of coasting speed with respect to different vehicle types in peak/off-peak hours are presented as follows in Figure 4,5, respectively.



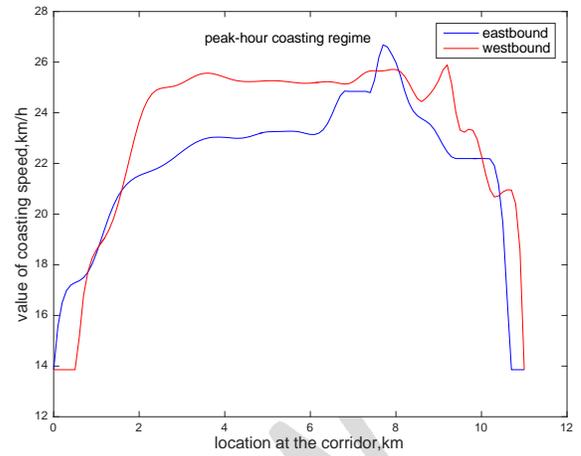
(a) Supercharge bus coasting profile



(b) CNG bus coasting profile

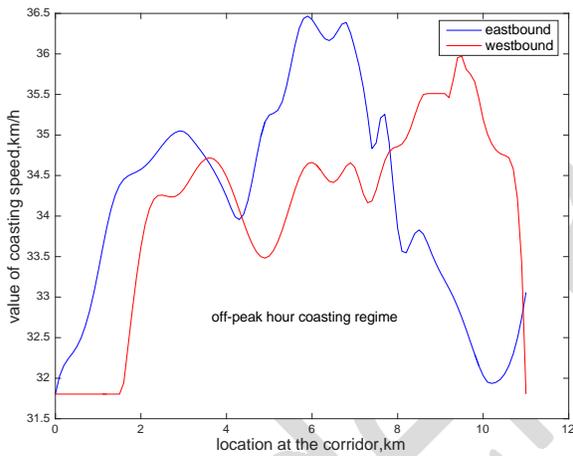


(c) Lithium-ion bus coasting profile

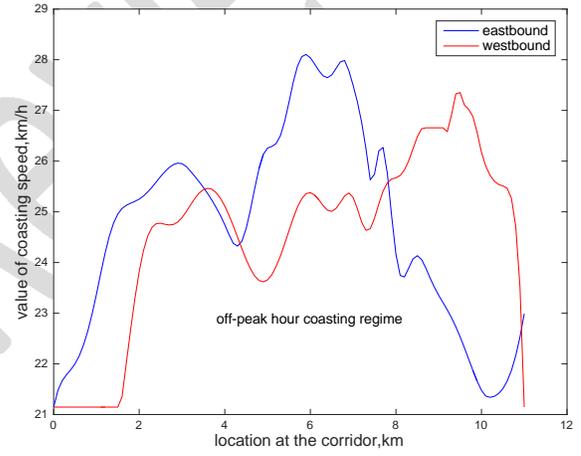


(d) Diesel bus coasting profile

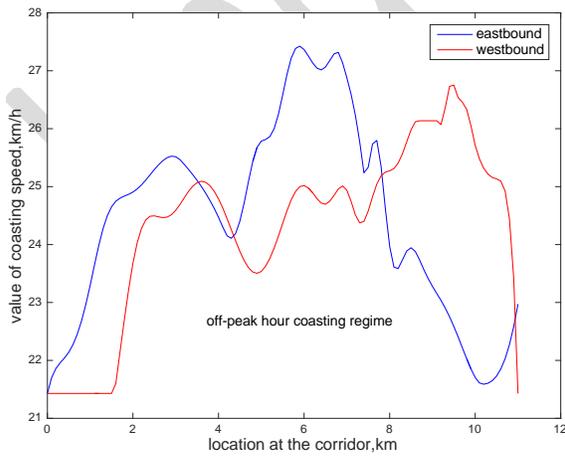
1 **FIGURE 4 Coasting speed profiles in peak hours for four vehicle types**
2



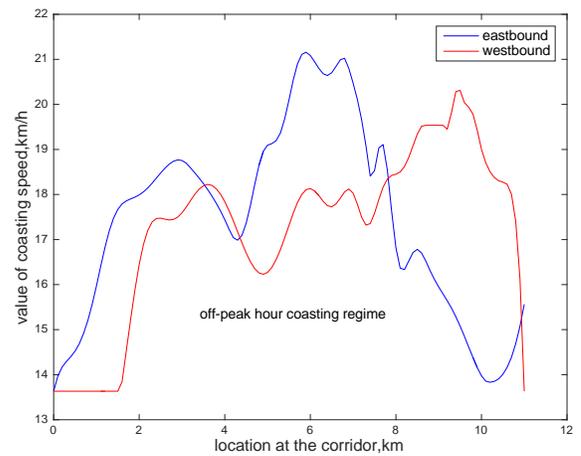
(a) Supercharge bus coasting profile



(b) CNG bus coasting profile



(c) Lithium-ion bus coasting profile



(d) Diesel bus coasting profile

3 **FIGURE 5 Coasting speed profiles in off-peak hour for four vehicle types**

From the two figures above, the optimal coasting speed profiles of four different vehicle types in the same period have the similar trend, but with distinguishing differences in average values. By discretizing the coasting speed functions, we can obtain the average values of coasting speeds in peak/off-peak hour on two directions. The results are summarized in Table 4, from which supercharge bus has the highest average coasting speed regardless of the time period. By contrast, diesel bus has the lowest average coasting speed. The results are reasonable because the diesel bus has the highest per-kilometer energy cost. In order to conserve energy, it's more beneficial to coast, as coasting regime will have less energy consumption than cruising regime. As for supercharge bus, whose per-kilometer energy cost is slight, it's more profitable to drive at cruising speed in order to save travel time.

TABLE 4 Optimal Values of Coasting Speed in Different Periods on Two Directions

Vehicle Types	$v_{co,eb}$ in peak (km/h)	$v_{co,eb}$ in off-peak (km/h)	$v_{co,wb}$ in peak (km/h)	$v_{co,wb}$ in off-peak (km/h)
Supercharge	35.23	36.12	35.42	35.00
CNG	25.00	23.07	25.57	22.80
Lithium-ion	28.79	24.95	29.23	24.71
Diesel	20.52	16.81	21.14	16.50

Driving regimes have direct impact on operation cost. In light of this, a system-cost comparison is conducted between optimized systems that introducing coasting regime and the one without considering coasting regime. The results are concluded in Table 5 and the operation costs in no-coasting system increase by up to 7%, while user costs decrease more than 12.9%, mainly in access/egress cost. The reason is that, introducing coasting regime will reduce the number of stops along corridor, leading to an increase in access/egress cost. However, the in-vehicle costs for two models are in neighborhood (with difference less than 5%), which means the travel time is closed for two systems, even when coasting regime is added. As a result, the distance traveled under cruising regime (with higher per-kilometer cost) will decrease. That's the reason why operation cost will decrease when introducing coasting regime in the system. Thus, coasting regime has a positive impact in view of operator, but passengers should accept a higher user cost. To summarize, for clean-energy buses (i.e., supercharge buses and Lithium-ion buses), it's more profitable to drive at cruising speed, while for conventional buses (i.e., CNG buses and diesel buses), introducing coasting regime is justifiable, considering the cost saving in operation and energy.

TABLE 5 System Cost with Coasting Regime vs. System Cost Without Coasting Regime

Supercharge Bus Route Design		
System metrics	No coasting regime	With coasting regime
h_{peak} , min	7.59	7.76
$h_{off-peak}$, min	8.94	8.12
$Spacing_{eb}$, km	0.50	0.65
$Spacing_{wb}$, km	0.48	0.61
C_u , \$/day	4,850.2	5,567.4
C_o , \$/day	2,762.2	2,559.9

TC , \$/day	7,612.4	8,127.3
CNG Bus Route Design		
h_{peak} , min	6.69	6.55
$h_{off-peak}$,min	11.92	10.68
$Spacing_{eb}$, km	0.50	0.65
$Spacing_{wb}$,km	0.46	0.61
C_u , \$/day	4,931.7	5,594.7
C_o , \$/day	2,596.6	2,398.2
C_p , \$/day	62.5	88.0
TC , \$/day	7,590.8	8,080.9
Lithium-ion Battery Bus Route Design		
h_{peak} , min	8.12	8.30
$h_{off-peak}$,min	11.95	10.30
$Spacing_{eb}$, km	0.52	0.69
$Spacing_{wb}$,km	0.50	0.65
C_u , \$/day	5,248.6	5,911.1
C_o , \$/day	2,999.9	2,818.0
TC , \$/day	8,248.5	8,729.1
Diesel Bus Route Design		
h_{peak} , min	7.94	7.60
$h_{off-peak}$,min	15.40	12.89
$Spacing_{eb}$, km	0.52	0.69
$Spacing_{wb}$,km	0.50	0.65
C_u , \$/day	5,456.4	6,305.8
C_o , \$/day	2,671.5	2,688.9
C_p , \$/day	270.6	62.5
TC , \$/day	8,398.5	9,057.2

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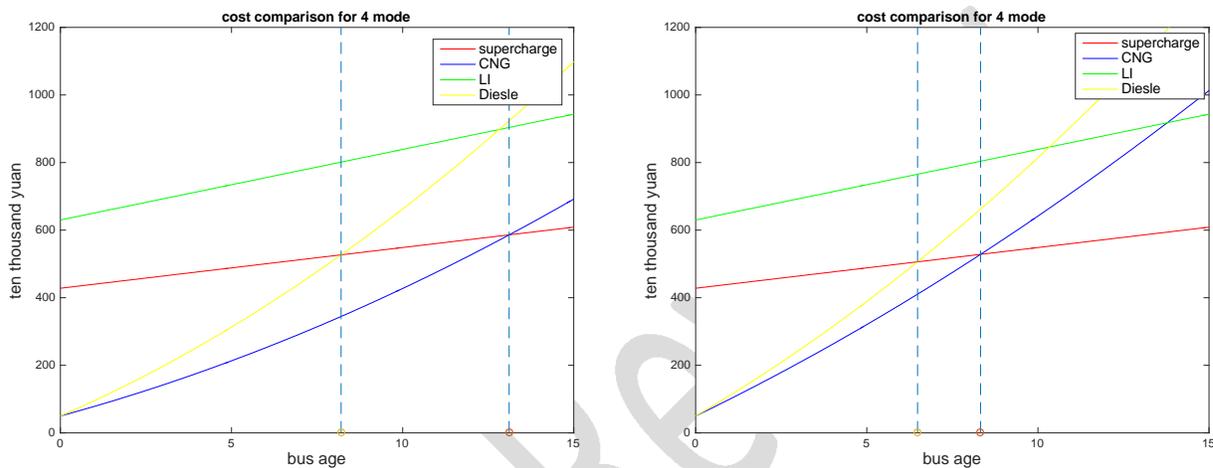
Cost Effectiveness Comparison

This part will analyze the life cycle cost for four vehicle types, which is from a macroscopic view. From Table 3, we can find the most economical vehicle type is CNG buses. Historical data shows that the maintenance cost for CNG and diesel bus will increase by year, even though their initial capital cost is lower. As a result, the cumulative costs of conventional buses in a life span will also be expensive. According to the maintenance cost data that Shu Tong Transportation Agency provided, clean-energy buses have a constant maintenance cost but a high initial capital cost. Thus, there exists a trade-off: which type is the most economical in a life span (8 years). Inspired by the previous studies (2, 26), which explored the varying maintenance cost by years in terms of diesel buses and CNG buses by using monthly maintenance data, we can roughly analyze the life cycle cost of our buses. The relationships of maintenance cost changing by year for diesel bus and CNG bus are shown respectively:

$y = 0.025 + 0.029 * z$, which y is presented the cost of maintenance per kilometer (\$/km) for diesel bus, z is the bus age (years).

1 $y = 0.02 + 0.017 * z$, which y is the maintenance cost per kilometer for CNG bus (\$/km), z is the
 2 bus age (years).

3
 4 We investigated a cost comparison for four bus types as shown in Figure 6. The results present that
 5 without including the pollutant cost into calculation, the cumulative costs of CNG bus will surpass
 6 those of supercharge bus after 13th year. On the contrary, when pollutant costs for conventional
 7 buses (i.e., CNG buses and diesel buses) are taken into consideration, supercharge bus will
 8 outperform other bus types after 8 years, which is still out of a life span. Thus, clean-energy buses
 9 are less competitive than conventional buses unless the government provides subsidies.
 10



(a) Life-cycle analysis without pollutant costs

(b) Life-cycle analysis with pollutant costs

11 **FIGURE 6 Life-span cost comparison**

12 **CONCLUSION AND FUTURE EXTENSION**

13 In this research, we developed a multi-period continuum approximation to optimize total cost
 14 including user cost, operator cost, and pollutant cost, where environmental impact and various
 15 driving regimes are explicitly considered. The proposed model is applied to four different vehicle
 16 types to optimize stop locations and service headway. The optimization of a real-world bus route
 17 results in significant reduction of total cost (over 50%). Comparing with current transit design, the
 18 average stop spacing is increased by 31.3%, and the peak-hour headway is decreased by up to 31%.
 19 The accuracy of proposed model is verified by a discrete model with error less than 3%. The huge
 20 gap of total system cost between current design and optimized design indicates that current design
 21 might not well fit the demand, adjustment should be done in terms of stop locations and headway.
 22

23 To investigate the effect of coasting regime, the optimal coasting profiles and the average
 24 values on two directions in multiple periods is compared in terms of different bus types. The
 25 results indicate that clean-energy buses have a shorter coasting distance while conventional buses
 26 need to coast longer in order to conserve energy. The analysis of the effect of coasting regime on
 27 operation cost suggest that introducing coasting regime will reduce operation cost (by up to 7%)
 28 but increase user cost (by up to 13%).

29 In the life-span cost effectiveness comparison amongst four different vehicle types, the
 30 economic feasibility of supercharge bus has been discussed. On a life-span scale (normally 8

1 years), the total cost of supercharge will be the lowest among four vehicle type after 8th years,
2 when pollutant costs are considered. It provides an insight that from the prespective of operator,
3 shifting conventional buses to new energy buses is not profitable unless subsidies are provided by
4 government.

5 For future extension, it is interesting to compare the stop-skip service with current all-stop
6 design. Additionally, the effect of coasting for different transit modes, such as rail and Bus Rapid
7 Transit (BRT) system can be further studied.

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12
13 Author contribution statement:

14 “The authors confirm contribution to the paper as follows: study conception and design:
15 Yue Su, Guo Lu, Wenbo Fan; data collection: Yue Su; analysis and interpretation of results: Yue
16 Su, Wenbo Fan; draft manuscript preparation: Yue Su, Wenbo Fan, Xiaobo Liu. All authors
17 reviewed the results and approved the final version of the manuscript”

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