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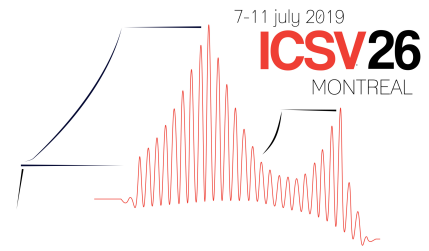
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# CHARACTERIZATION OF A NONLINEAR SOUND ABSORBER AT LOW FREQUENCIES AND HIGH SOUND LEVELS

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The objective of this work is to develop identification techniques to characterize nonlinear acoustic loads. These nonlinear acoustic loads are here considered as a one-port acoustic system, analyzed at low frequencies and for increasing excitation levels. A specific setup of impedance tube named "Short Kundt's Tube" was built to reach high sound levels at low frequencies. Two approaches, developed in the frequency domain, are discussed in this work. The two approaches require a single acoustic pressure measurement inside the Short Kundt's Tube, and a calibration of the Short Kundt's Tube acoustic source from a set of known (linear) acoustic loads. The first approach is defined as a linearization method and gives access to the acoustic impedance which is excitation level dependent. The second one is based on a nonlinear model able to characterize energy transfer to higher harmonics. It is defined as a scattering matrix linking the harmonic components of the input and output pressure wave amplitudes. These techniques are applied for the study of two different nonlinear vibroacoustic absorbers based on the concept of targeted energy transfer.

Keywords: Noise reduction, non linear vibrations, non linear impedance

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## 1. Introduction

Numerous sound absorbers dedicated to noise reduction at low frequencies are based on nonlinear properties. This is the case for example considering micro-perforated plates or nonlinear Helmholtz resonators. Recently, nonlinear vibroacoustic absorbers also known as Nonlinear Energy Sink (NES) have been developed. Such devices consist in a thin structure submitted to large deformations, which exhibits non linear resonances used to absorb sound energy (see for example [1, 2]).

Characterization of such acoustic elements is necessary for many applications. Classically (*i.e.* under linear assumption), the impedance is sufficient to describe the behaviour of a sound absorber considered

as a one-port acoustic system and measurement methods are available. Note that in tubes, when considering only the plane wave mode, the impedance is defined for any cross-section of the tube as the ratio of the mean acoustic pressure by the volume velocity and can be equivalently replaced by the reflection coefficient.

The objective of this work is to develop system identification techniques to characterize or analyze nonlinear elements at low frequencies and for increasing excitation levels. This dynamic and frequency range requires a suitable no standard equipment which was developed. It consists in an impedance tube named "Short Kundt's Tube" able to reach very high levels ( $> 300$  Pa *i.e.* 143.5 dB) at low frequencies.

Two approaches, developed in the frequency domain, are discussed in this work. One is defined as a linearization method and the other is based on a nonlinear model able to characterize energy transfer to higher harmonics. The linearization method gives access to the acoustic impedance and/or the reflection coefficient which are excitation level dependent. The nonlinear model is defined as a scattering matrix[3] relating the harmonic components of the input pressure wave amplitudes with the harmonic components of the output pressure wave amplitudes. In the first approach, we use a swept-sine signal (also called chirp) exhibiting an exponential instantaneous frequency as excitation. In the second approach, the source signal is a stepped sine. We measure the acoustic pressure inside the tube, at one point only, in front of the device under study, after a calibration of the source from measurements of known acoustic loads (*i.e.* characteristics can be obtained analytically).

The use of these techniques is illustrated for the analysis of two different nonlinear vibroacoustic absorbers.

## 2. Measurement setup

### 2.1 Short Kundt's Tube (SKT)

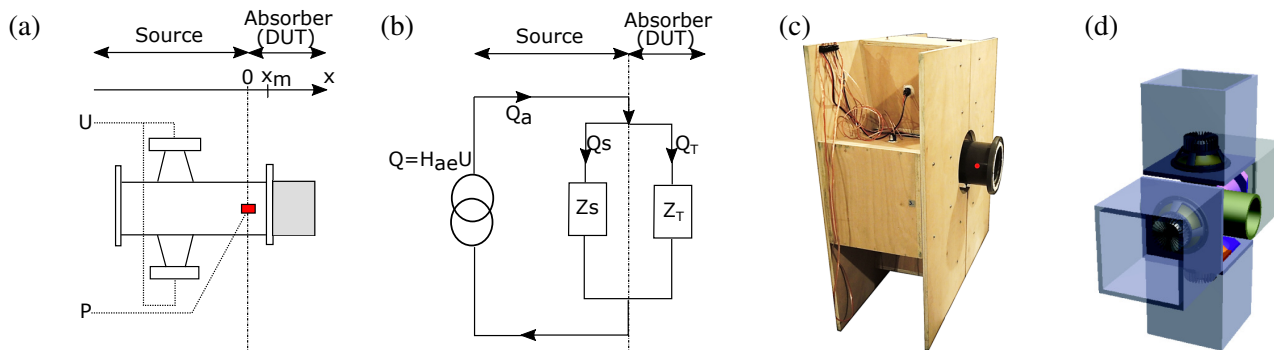


Figure 1: Scheme of (a) the experimental set-up and (b) the equivalent electroacoustic circuit of the one-microphone identification method. (c) Picture of the source. (d) Scheme showing the four loudspeakers.

Fig. 1(a) presents the acoustical scheme of the set-up. It is made of a sound source controlled by a voltage  $u(t)$  (or  $U(f)$  in the frequency domain) and a sample or any device under study (DUT), connected through a tubular section of minimal length (hence the name "Short" Kundt's Tube). A particular source (Fig. 1(c,d)) was designed for the low frequency range (typically 10 to 600 Hz), in order to expose the DUT to high pressure levels ( $> 300$  Pa). Attention was paid to the linearity and the smooth frequency response of the source. This source is composed of four loudspeakers (type Beyma 10LW30/N, diam 0.25 m) connected perpendicularly at the mid length of a tube with length 0.71 m and diameter  $D_t = 0.175$  m. The rear face of each loudspeaker is enclosed in a 37 ℓ air-tight box. The front side of each loudspeaker is

connected to the tube by smooth 3D-printed horns with initial diameter 0.25 m and final diameter 0.1 m. The four loudspeakers are connected at the same abscissa (see Fig. 1(d)), and contribute equally to the volume velocity of the source. The two ends of the source tube can be fitted by a lid or by any load. For the present study, the left side of the source is closed by an air-tight disc placed inside the tube next to the horns exits. Only the right part is changed and the microphone is localized 0.035 m away from the DUT (resulting in  $x_m = 0.035$  m). The air-tightness of the source is critical and has been checked.

## 2.2 Acquisition chain

The acquisition chain is controlled by a self-made program running on a computer fitted with an RME HDSPe MADi sound card. It uses a Ferrofish A16 MK-II AD/DA Converter. The source signal drives a power amplifier TIRA, BAA120, connected to the four loudspeakers of the source. Two pairs of loudspeakers connected in series are connected in parallel to the amplifier. This connection makes the four loudspeakers circuit get the same impedance as a single loudspeaker. The pressure measurements are made with a high pressure microphone (G.R.A.S 40BH 1/4), connected to a G.R.A.S 26AC-1 preamplifier and a G.R.A.S. 12AA amplifier. The computer sends logarithmic sweeps (chirps) or sines as driving signal, to the amplifier.

## 3. Linear impedance model

### 3.1 Principe of the method

The DUT is characterized by an equivalent impedance  $Z_T(f)$ , which is defined as the ratio of the fundamental components of the spectra of the pressure and the volume velocity, both considered over the tube section in the measurement plane. Plane waves are assumed and the local pressure  $p(t)$  measured by the microphone is considered as a good estimate of the pressure averaged over the measurement section. The local volume velocity is not measured, but is indirectly obtained through the setup calibration. When the DUT is nonlinear,  $Z_T(f)$  corresponds to a linear approximation of the relationship between the pressure and volume velocity. In this case,  $Z_T(f)$  is an indicator depending on the DUT, obviously, and depending also of the experimental excitation (*i.e.* amplitude and frequency content of the voltage  $u(t)$ ).

The source is considered as linear and time-invariant, so it can be represented using the Norton equivalence by the electro-acoustic scheme shown in Fig. 1(b). It is characterized by its impedance  $Z_s(f)$  and its volume velocity  $Q_a(f)$ . Equivalently, it may be characterized by  $Z_s(f)$  and the transfer function  $H_{ae}(f)$  between the acoustic volume velocity  $Q_a(f)$  and the control voltage  $U(f)$ , defined as  $Q_a(f) = H_{ae}(f) U(f)$ . The equivalent impedance  $Z(f)$  loading the volume velocity source and the transfer function  $H_m(f)$  between the control voltage  $U(f)$  of the source and the acoustic pressure  $P(f)$  at the microphone are

$$Z(f) = \frac{Z_T(f)Z_s(f)}{Z_T(f) + Z_s(f)} \text{ and } H_m(f) = \left( \frac{1}{Z_s(f)} + \frac{1}{Z_T(f)} \right)^{-1} H_{ae}(f). \quad (1)$$

The impedance  $Z_T(f)$  and the reflection coefficient  $R_T(f)$  of the DUT can be expressed as

$$Z_T(f) = \frac{H_m(f)Z_s(f)}{H_{ae}(f)Z_s(f) - H_m(f)} \text{ and } R_T(f) = \frac{H_m(f)Z_s(f) - Z_c(f)(Z_s(f)H_{ae}(f) - H_m(f))}{H_m(f)Z_s(f) + Z_c(f)(Z_s(f)H_{ae}(f) - H_m(f))} \quad (2)$$

where  $Z_c = \rho c/S$  is the specific acoustic impedance of air,  $\rho$  is the density of air,  $c$  the sound wave velocity and  $S$  the surface of the waveguide section.

Thus, the determination of the impedance  $Z_T(f)$  and the reflection coefficient  $R_T(f)$  of the DUT only requires a measurement of the transfer function  $H_m(f)$  and the prior knowledge of the source characteristics  $H_{ae}(f)$  and  $Z_s(f)$ .

The setup calibration consists in determining the quantities  $H_{ae}(f)$  and  $Z_s(f)$ , from measurements of known acoustic loads. In view of Eq. (2), the measurements obtained from two different (but known) loads are theoretically sufficient to determine  $H_{ae}(f)$  and  $Z_s(f)$ . In order to avoid errors and improve the results, an over-determination of the calibration data is performed by combining more than two known loads [4]. A typical setup calibration is reported on Fig. 2 obtained from a logarithmic sweep excitation source. We observe a smooth behaviour of the source characteristics.

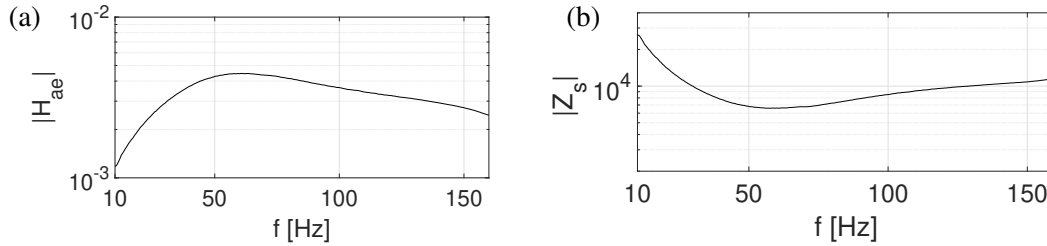


Figure 2: Estimated source parameters (a)  $H_{ae}$  and (b)  $Z_s$  from six known acoustic loads.

### 3.2 Application to a nonlinear vibroacoustic absorber

The procedure has been applied to characterize two nonlinear acoustic absorbers, a thin viscoelastic circular membrane and the same membrane with a plywood box clamped on its rear face which are described in [2]. Fig. 3 shows modulus of impedance  $Z_T$  and reflection coefficient  $R_T$  measured for five excitation levels using a logarithmic sweep excitation source over [10, 160] Hz. At low excitation level, both systems present a linear behavior (see Fig. 3(a,c)). We observe that, for the two absorbers, resonance frequency increase with the excitation level, characterizing a nonlinear behaviour of systems and their hardening nature. Moreover the reflection coefficient allows to compare the amount of energy removed out of the system by the absorber to the case of a total energy reflection  $|R_T|^2 = 1$ . Here, the energy is extracted by the absorber over a frequency range widening with the increase of the excitation level (see Fig. 3(b,d)).

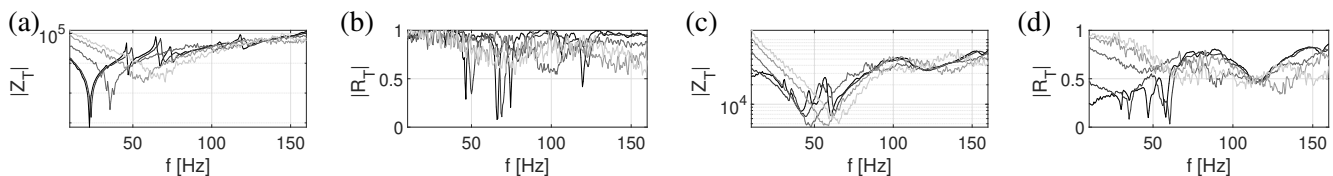


Figure 3: Estimation of (a,c) impedance  $Z_T$  and (b,d) reflection coefficient  $R_T$  for (a,b) viscoelastic membrane and (c,d) membrane with a plywood box clamped on its rear face, for excitation level  $U_{rms} = [0.05, 0.19, 1.87, 5.61, 9.36]$  V (black to grey)

## 4. Nonlinear impedance model

To increase the understanding of the acoustic load when nonlinearities are present, the linear impedance model has to be extended to take nonlinear energy transfer to higher harmonics into account.

### 4.1 Principe of the method

#### 4.1.1 The nonlinear model

We start with the multi-port models proposed in [3]. We assume that the acoustic source generates an excitation at only the excitation frequency  $f$  (the fundamental frequency). The impedance formulation of multi-port model characterizes the relationship between the harmonic terms ( $P_n(f)$ ) of the acoustic pressure at the microphone position and the harmonic terms ( $Q_k(f)$ ) of the corresponding acoustic volume velocity as

$$P_n(f) = \sum_{k=1}^{\infty} Z_{nk}(f, |P_1(f)|) Q_k(f) \text{ for } n = 1, 2, \dots \quad (3)$$

The impedance term,  $Z_{nk}(f, |P_1(f)|)$ , represents the opposition at the frequency  $nf$  that the acoustic load presents to the acoustic flow at the frequency  $kf$  from the fundamental excitation frequency  $f$ . This term depends on the excitation frequency  $f$  and on the amplitude level of the acoustic pressure represented here by  $|P_1(f)|$  (amplitude of the first harmonics). To simplify the notation, we have omitted the subscript "T" in the impedance terms.

An equivalent formulation can be obtained using a scattering-matrix approach as

$$P_n^-(f) = \sum_{k=1}^{\infty} S_{nk}(f, |P_1(f)|) P_k^+(f) \text{ for } n = 1, 2, \dots \quad (4)$$

or in matrix expression

$$\begin{pmatrix} P_1^-(f) \\ P_2^-(f) \\ P_3^-(f) \\ \vdots \end{pmatrix} = \begin{pmatrix} S_{11}(f, |P_1(f)|) & S_{12}(f, |P_1(f)|) & S_{13}(f, |P_1(f)|) & \cdots \\ S_{21}(f, |P_1(f)|) & S_{22}(f, |P_1(f)|) & S_{23}(f, |P_1(f)|) & \cdots \\ S_{31}(f, |P_1(f)|) & S_{32}(f, |P_1(f)|) & S_{33}(f, |P_1(f)|) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} P_1^+(f) \\ P_2^+(f) \\ P_3^+(f) \\ \vdots \end{pmatrix} \quad (5)$$

where  $P_n^-(f)$  (respectively  $P_k^+(f)$ ) denotes the  $n^{\text{th}}$  harmonic of the reflected  $p^-(t)$  (respectively incident  $p^+(t)$ ) acoustic wave. The term  $S_{nk}(f, |P_1(f)|)$  represents the transfer of energy from the harmonic term ( $P_k^+(f)$ ) of the acoustic pressure at the microphone position to the harmonic term ( $P_n^-(f)$ ). As shown in [3], the matrix impedance can be written in terms of the scattering matrix (5).

When nonlinearities are absent, only the terms  $Z_{11}(f, |P_1(f)|)$  and  $S_{11}(f, |P_1(f)|)$  are not equal to zero and they correspond respectively to the impedance and reflection coefficient of the acoustic load as introduced in Section 3.

The scattering-matrix model (5) can be simplified assuming that (i) energy exchange can occur only from low to high frequency ( $S_{nk}(f, |P_1(f)|) = 0$  for  $n < k$ ); (ii) the DUT satisfies the harmonic superposition principle [5] ( $S_{nk}(f, |P_1(f)|) = 0$  for  $1 < k < n$ ). It takes the resulting form

$$\begin{pmatrix} P_1^-(f) \\ P_2^-(f) \\ P_3^-(f) \\ \vdots \end{pmatrix} = \begin{pmatrix} S_{11}(f, |P_1(f)|) & 0 & 0 & \cdots \\ S_{21}(f, |P_1(f)|) & S_{22}(f, |P_1(f)|) & 0 & \cdots \\ S_{31}(f, |P_1(f)|) & 0 & S_{33}(f, |P_1(f)|) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} P_1^+(f) \\ P_2^+(f) \\ P_3^+(f) \\ \vdots \end{pmatrix} \quad (6)$$

Our objective is to develop a method to measure the non zero terms of the scattering matrix (6).

#### 4.1.2 Measurement procedure

The measurement method is based on the use of the SKT after calibration (known source impedance  $Z_s(f)$  and transfer function  $H_{ac}(f)$ ).

A measurement run consists in recording the pressure while the source is excited by the input voltage

$$u(t) = A \sin(2\pi ft + \phi) \quad (7)$$

where  $f$  is the excitation frequency and  $A$  is the associated amplitude. The phase  $\phi$  is introduced arbitrarily by the signal generator. Each experiment is done with constant values for  $A$  and  $f$ . The duration of an experiment must be limited for practical reasons, but must be long enough to capture the physics of the response. During a measurement run, the loudspeaker voltage response  $u(t)$  and the acoustic pressure  $p(t)$  are recorded simultaneously. Finally, the harmonics components of  $u(t)$  and  $p(t)$  defined as

$$u(t) = \text{Re}\left(\sum_{k=1}^{n_H} U_k(f, A) e^{-j2\pi kft}\right) \text{ and } p(t) = \text{Re}\left(\sum_{k=1}^{n_H} P_k(f, A) e^{-j2\pi kft}\right) \quad (8)$$

are extracted by signal processing, with  $n_H$  the number of harmonics. In all the cases, the terms  $U_k(f, A)$  for  $k > 1$  are insignificant and have been neglected.

For a given couple  $(f^e, A^e)$ , the scattering matrix (6) is obtained row by row. Each row needs a measurement run. Basically, we measure  $S_{11}$  from a first run excited at the fundamental frequency, which gives also responses at harmonic frequencies. With a second run we measure  $S_{22}$  at a proper pressure level, which permits to get  $S_{21}$  from the first measurement.

Row 1: A run is carried out choosing  $f = f^e$  and  $A = A^e$  for the voltage signal. Using the first harmonic terms  $U_1(f^e, A^e)$  and  $P_1(f^e, A^e)$ ,  $S_{11}(f^e, |P_1(f^e, A^e)|)$  can be obtained from (2) as

$$S_{11}(f^e, |P_1(f^e, A^e)|) = \frac{H_m(f^e)Z_s(f^e) - Z_c(f^e)(Z_s(f^e)H_{ac}(f^e) - H_m(f^e))}{H_m(f^e)Z_s(f^e) + Z_c(f^e)(Z_s(f^e)H_{ac}(f^e) - H_m(f^e))} \quad (9)$$

where

$$H_m(f^e) = \frac{P_1(f^e, A^e)}{U_1(f^e, A^e)}. \quad (10)$$

Moreover, combining the following two relations

$$P_1(f^e, A^e) = P_1^-(f^e, A^e) + P_1^+(f^e, A^e) \text{ and } S_{11}(f^e, |P_1(f^e, A^e)|) = \frac{P_1^-(f^e, A^e)}{P_1^+(f^e, A^e)} \quad (11)$$

we compute the reflected,  $P_1^-(f^e, A^e)$ , and incident,  $P_1^+(f^e, A^e)$ , contributions which are used in the next steps.

In the next steps, we also need the reflected and incident contributions of the harmonic terms  $P_n(f^e, A^e)$  for  $n > 2$ . For a given  $n$ , they can be obtained by solving with respect to  $(P_n^-(f^e, A^e), P_n^+(f^e, A^e))$  the two equations

$$P_n(f^e, A^e) = P_n^-(f^e, A^e) + P_n^+(f^e, A^e) \text{ and } -\frac{P_n(f^e, A^e)}{Z_s(nf^e)} = \frac{1}{\rho c}(P_n^-(f^e, A^e) - P_n^+(f^e, A^e)). \quad (12)$$

The second equation expresses the  $n^{\text{th}}$  harmonic term  $Q_{T_n}(f^e, A^e)$  of the acoustic volume velocity with respect to  $(P_n^-(f^e, A^e)$  and  $P_n^+(f^e, A^e))$ . Referring to the equivalent electroacoustic circuit (Fig. 1) and assuming  $Q_{a_n}(f^e, A^e) = 0$ , we obtain  $Q_{T_n}(f^e, A^e) = -Q_{s_n}(f^e, A^e) = P_n(f^e, A^e)/Z_s(nf^e)$ .

Row 2: A run is carried out choosing  $f = 2f^e$  and selecting  $A = A_2^e$  such that  $|P_1(2f^2, A_2^e)|$  is close to  $|P_2(f^e, A^e)|$ . Using the first harmonic terms,  $U_1(2f^e, A_2^e)$  and  $P_1(2f^e, A_2^e)$ ,  $S_{22}(f^e, |P_1(f^e, A^e)|)$  can be obtained from (2) as

$$S_{22}(f^e, |P_1(f^e, A^e)|) = \frac{H_m(2f^e)Z_s(2f^e) - Z_c(2f^e)(Z_s(2f^e)H_{ae}(2f^e) - H_m(2f^e))}{H_m(2f^e)Z_s(2f^e) + Z_c(2f^e)(Z_s(2f^e)H_{ae}(2f^e) - H_m(2f^e))} \quad (13)$$

where

$$H_m(2f^e) = \frac{P_1(2f^e, A_2^e)}{U_1(2f^e, A_2^e)}. \quad (14)$$

Finally the second term  $S_{21}(f^e, |P_1(f^e, A^e)|)$  is obtained from the second equation of (6) as

$$S_{21}(f^e, |P_1(f^e, A^e)|) = \frac{P_2^-(f^e, A^e) - S_{22}(f^e, |P_1(f^e, A^e)|)P_2^-(f^e, A^e)}{P_1^-(f^e, A^e)}. \quad (15)$$

The same procedure can be extended to the other rows.

## 4.2 Application to a passive nonlinear absorber

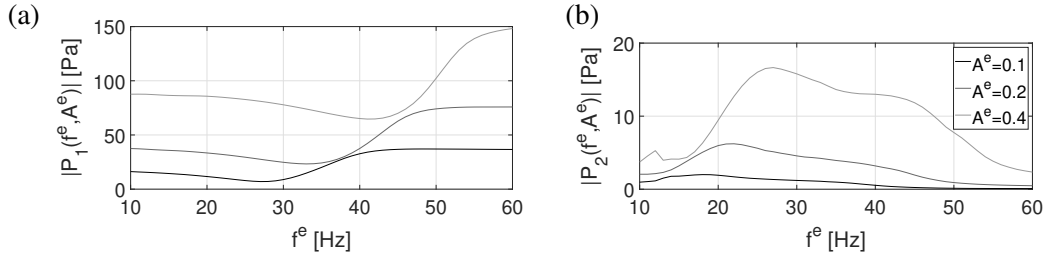


Figure 4: Measured acoustic pressure for the three excitation levels: (a) fundamental and (b) second harmonic terms.

This method has been applied to an adjustable nonlinear acoustic absorber, made of a loudspeaker membrane described in [1]. We have chosen a duration of 13 s for each measurement run and we limited our study to two harmonics. Three excitation levels were considered combined with 51 excitation frequencies starting from  $f^e = 10$  Hz to  $f^e = 60$  Hz with a frequency step equal to 1 Hz. The first two harmonic terms of the acoustic pressure are shown Fig.4. The first term is predominant (from 50 to 300 Pa) and both terms increase with excitation level. Coefficients of the scattering and impedance matrices are reported Fig.5(a,b,c) and (e,f,g).

The term  $|S_{11}|$  presents a characteristic curve of reflection coefficient (see Fig.5(a)), with  $|S_{22}|$  its continuation at  $2f^e$  the lowest level (see Fig.5(b)). The impedance  $|Z_{11}|$  is characteristic of a nonlinear absorber, and we can observe the frequency shift of the resonance when the excitation level increase (see Fig.5(d)). Its behavior is similar to the first absorbers described in Section 3.2. As for scattering coefficient,  $|Z_{22}|$  is the continuation of  $|Z_{11}|$  at  $2f^e$  and the lowest level (see Fig.5(e)). Finally,  $|S_{21}|$  and  $|Z_{21}|$  show the presence of energy transfer between the fundamental and the second harmonics, which increases with the excitation level (see Fig.5(c,f)).

## 5. Conclusion

Acoustic properties of nonlinear vibroacoustic absorbers have been determined at low frequency and high sound level using a short impedance tube named "Short Kundt's Tube". Two approaches have been



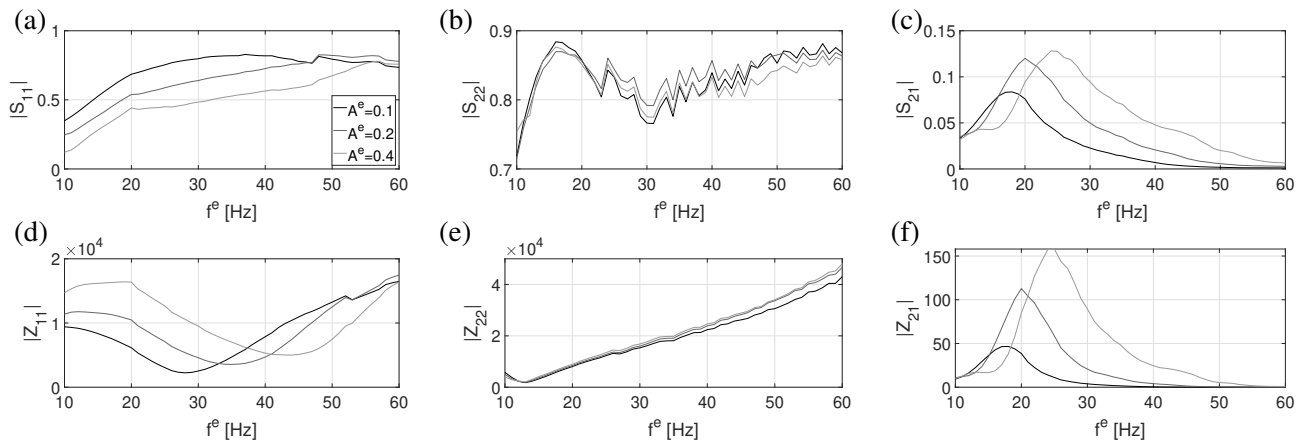


Figure 5: Estimation of (a,b,c) scattering matrix and (d,e,f) impedance coefficients for three excitation levels.

developed to estimate them. The first approach is based on the measure of the impedance. From the characterization of the source of the Short Kundt's Tube and a pressure measurement in the tube, impedance and reflection coefficients can be obtained for absorbers. A second approach considers the nonlinear behavior of absorbers and gives information about nonlinear energy transfer to higher harmonics. Representation of nonlinear acoustic properties is done in two different but equivalent ways: a scattering matrix couples the harmonic terms of the incident and reflected acoustic pressures, and an impedance matrix couples the harmonic terms of acoustic pressure and acoustic volume velocity. Results confirm a nonlinear behavior of the different absorbers and the nonlinear representation of the system shows an energy transfer to higher harmonics.

In future works the quantification of this energy transfer will be done with a larger number of harmonics, in order to quantify all the wave conversions present in the tube. Numerical simulations will be performed to validate these experimental results.

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