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# Impact of uncertainties of lead times and expiration dates on the stability of inventory levels in a distribution system

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**Abstract:** In this paper, we discuss the impact of uncertainties of lead times and expiration dates on the stability of the inventory regulation problem in productions systems using feedback control law structure, in the conception phase. The inventory control system is considered as an input-delay system with uncertainties on customer demands, and positive constraints due to the specifications of the agricultural supply chain. Also, the system is characterized by the presence of delay due to the process time and the distribution time, and the perishable products are modeled by a fixed preemption rate. We have first found the necessary and sufficient conditions that prove the existence and the admissibility of the control law. Secondly, a comparative analysis of impact of production delay and expiration date uncertainties on a robust design is given.  
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*Keywords:* Distribution systems, inventory control, production management, delayed systems, robustness analysis.

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## 1. INTRODUCTION

Delays are present everywhere in the supply chain and they are essentially linked to the flow movements. Since the delay is encountered in various production systems, the dynamic behavior of many physical processes inherently contains time delays and uncertainties, that are often the main cause of the instability of control systems. So for this reason we have a big interest in research into robust stabilization for uncertain time-delay systems. Several approaches in production and inventory control studied delayed systems through the years. The first was [Simon H. A. (1952)] who used Laplace Transform to analyze a supply line dynamics. Then, lots of authors modeled in their works [Forrester J. (1973)], [Kharitonov V. (1998)], [Moon Y.S. et al. (2001)], [Riddalls C. and Bennett S. (2002)], [Chiasson J. and Loiseau J.J. (2007)], [Tarbouriech S. et al. (2011)], [Wang X. et al. (2012)] the production system by using block diagrams and controlled feedback structure. In particular, [Ignaciuk P. and Bartoszewicz A. (2011)] studied the control of perishable inventory systems using smith predictor principle. After, different optimization frameworks were proposed using programming techniques, empirical experiences and control theory methods, in order to satisfy at each time the customer's demand [Nakasumi (2017), Tripoli and Schmidhuber (2018)]. Our concern focused on the use of the control theory methods which provide an analytic and formal framework, since such systems can be considered as time-delayed systems with uncertainties. Moreover, the invariance principle is one of the basic notions in control

theory. It answers as well to the issues of existence of feasible controllers for constrained systems. So we are interested in these questions of invariance and  $\mathcal{D}$ -invariance, in the context of solving constrained control problems for logistic systems.

In this section, we discuss the stability for the inventory regulation problem in agri-food productions systems using feedback control law structure, in the conception phase. The inventory control system is considered as an input-delay system with uncertainties on customer demands, and positive constraints due to the specifications of the agricultural supply chain. Also, the system is characterized by the presence of delay due to the process time and the distribution time, and the agri-food products are perishable with fixed preemption rate. Due to the lead time of the control law and loss factor, the objective is to define a control law which permits to satisfy the end-customer demand and for which the agricultural production system requirements will be completely met.

The remaining section of the study is organized as follows. The Section 2 deals with the problem statement and objective. We give the system description and model formulation, and describe the main problem. state of the art. In section 3, the different steps of the proposed control strategy is developed. In section 4 a comparative analysis of impact of transportation delay and expiration date uncertainties on a robust design is given through an illustrative example, followed by the robustness analysis. Results with numerical applications and discussions are carried out in Section 5. Finally, we conclude our study and suggest an area for future research in section 6.

## 2. PROBLEM STATEMENT AND OBJECTIVE

### 2.1 System description

we consider a production system composed of a transport unit due to the mobility of goods and flow services, and a storage unit that is characterized by the incoming flow of products, and the outgoing flow of products leaving the system due to the customers demand and sales made. Such a system makes it possible to describe the basic distribution processes, namely the routing and the delivery as well as the planning and the management of the purchases. The generic model for the output inventory level  $w(t)$  is described by the following first order delayed equation:

$$\dot{w}(t) = \begin{cases} -\sigma w(t) + o(t - T) - d(t) & , \text{for } t \geq T, \\ -\sigma w(t) + WIP(t) - d(t) & , \text{for } 0 \leq t < T. \end{cases} \quad (1)$$

This model was introduced by Simon in (1952). Then, it was used by Blanchini in (1990). He treated the communication networks control using the same model.

The system variables are  $o(t)$ ,  $w(t)$  and  $d(t)$  that are non-negative entities. The output  $w(t)$  presents the instantaneous warehouse or the storage level. The system disturbance  $d(t)$  corresponds to the flow of products leaving the storage at any moment  $t$  due to the customer demand, and the input of the system  $o(t)$  presents the order to produce and deliver the flow of products.

Our study deals with the perishable products which means that we have a deterioration on the goods. So these items are modeled by a loss factor  $\sigma$  with  $0 \leq \sigma < 1$ .

In reality, to obtain the products, a non-negligible lead time is necessary, and it is noted by  $T$  and supposed to be constant. It corresponds to the time needed since taking the decision to produce the goods until delivering the goods to the storage unit.

### 2.2 Constraints and objectives

This problem is used to describe the basic logistics processes, namely production, routing and delivery process. So the system is subject to two types of limitations. First, physical quantities like production order  $o(t)$  and inventory level  $w(t)$  can take only non-negative values. The second specificity of the system is that these entities are limited resources, so maximum storage capacities and maximum production orders are imposed and must be respected. So the controller design should take into account these positive and saturation constraints that are formulated as follows.

For the inventory level  $w(t)$ ,

$$0 < w_{min} \leq w(t) \leq w_{max}. \quad (2)$$

For  $o(t)$ , we suppose that

$$0 < o_{min} \leq o(t) \leq o_{max}. \quad (3)$$

Finally, the customer demand  $d(t)$  must always verify

$$0 < d_{min} \leq d(t) \leq d_{max}. \quad (4)$$

The system is studied in terms of a time-delayed system with a constant lead time  $T$ , where the specifications are

introduced in form of constraints imposed to the controller. So the problematic consists of finding a robust strategy for this system so that the storage level  $w(t)$  and the flow of goods  $o(t)$  verify their constraints already mentioned, and this for any disturbance that varies arbitrary in the range  $[d_{min}, d_{max}]$ . The necessary and sufficient conditions for the existence of admissible control laws are then interpreted in order to ensure the stability of the transport unit by forbidding any overrun on the delivery orders and on the storage level, and to absorb the uncertainties on the losses and the customer demands  $d(t)$ . So that we can finally determine the decision support system for transport and logistic supply chain.

## 3. CONTROL STRATEGY AND MAIN RESULTS

The control approach is build as follows. First step is to find an equivalent delay-free system by applying a state feedback prediction to the system. After that the basic conceptual idea is to verify the system constraints in terms of invariance by determining the  $\mathcal{D}$ -invariance conditions to the prediction system. Following this approach we propose to apply two different types of the control to the dynamic system. Finally we end the study by verifying the admissibility of the control theory and by finding the attainable bounds for the system output and the necessary and sufficient conditions of the original time delay system.

As developed in [Bou Farraa B. et al. (2018)] and [Abbou R. et al. (2017)], the proposed approach to control systems with delayed inputs is based on a prediction state feedback principle. This structure permits both the stability the system in closed loop and the compensation of the delay effects present in the loop. We denote  $z(t)$  the prediction of the future state of the storage level  $w(t)$ . This prediction is expressed by

$$z(t) = e^{-\sigma T} w(t) + \int_{t-T}^t e^{-\sigma(t-\tau)} o(\tau) d\tau. \quad (5)$$

By time derivation of (5) and using (1), we obtain a feedback-predictor structure known as model reduction [Artstein Z. (1982)], and that is expressed as follows

$$\dot{z}(t) = -\sigma z(t) + o(t) - e^{-\sigma T} d(t). \quad (6)$$

The Artstein reduction can be expressed by the general form  $\dot{w}(t) = f(w(t), o(t), d(t))$ , with the interval of prediction  $W = [w_{min}, w_{max}]$  and the interval of the disturbance  $d(t)$ ,  $\mathcal{D} = [d_{min}, d_{max}]$ . Thus we can apply the invariance theorem developed in [Blanchini F. (1990)] to the non-delayed system (6).

*Proposition 1.* Given the system of form (1), there exists a control law so that the prediction interval  $[z_{min}, z_{max}]$  is  $\mathcal{D}$ -invariant for the closed-loop system (6), if and only if the following two conditions are verified.

$$\sigma z_{min} + e^{-\sigma T} d_{max} \leq o_{max} \quad (7)$$

$$o_{min} \leq \sigma z_{max} + e^{-\sigma T} d_{min} \quad (8)$$

These conditions, (7) and (8) are the necessary and sufficient conditions for the  $\mathcal{D}$ -invariance of the interval

$[z_{min}, z_{max}]$ , and they are deduced verifying the following inequality

$$z_{min} \leq z_{max}. \quad (9)$$

We can propose properly two different types of control law  $o(t)$  that verify (10), and that the interval  $[z_{min}, z_{max}]$  is  $\mathcal{D}$ -invariant for the closed-loop system (6). The big advantage is that the control can be easily used first for both continuous and hybrid type of production and transportation flows, and second for a large type of logistic and distribution systems. To do that we define first two values of the control law  $o(t)$ ,  $o_1$  and  $o_2$  verifying (3) and expressed by

$$o_1, o_2 \in [o_{min}, o_{max}]. \quad (10)$$

*Affine control law* This type of the control law  $o(t)$  takes the form of a linear feedback defined as

$$o(t) = \begin{cases} K(z_0 - z(t)) & , \text{for } o_1 \neq o_2, \\ o_1 = o_2 & , \text{for } o_1 = o_2. \end{cases} \quad (11)$$

$K = \frac{o_1 - o_2}{z_{max} - z_{min}}$  is a static gain, and  $z_0 = \frac{o_1 z_{max} - o_2 z_{min}}{o_1 - o_2}$  is a reference value.

*Bang-bang control law* This law is expressed for a hybrid system. It's given by the following expression.

$$o(t) = \begin{cases} o_1 & , \text{for } z(t) \leq z_{min}, \\ o_2 & , \text{for } z(t) \geq z_{max}. \end{cases} \quad (12)$$

Its behavior is described by the hybrid automaton of the figure below.

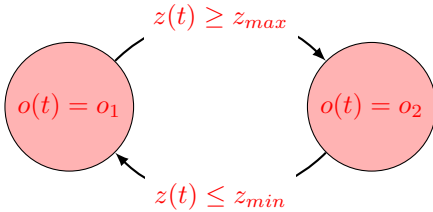


Fig. 1. Automaton of Bang-bang control law

Based on the above results, we will deduce the necessary and sufficient conditions of the control law admissibility for  $t \geq T$  for the reduced model (Bou Farraa B. et al. (2018)).

*Proposition 2.* Given the system of the form (1), there exists  $o_1$  and  $o_2$  such that the control law  $o(t)$  is admissible if and only if the parameters  $z_{min}$  and  $z_{max}$  satisfy (10), (7), (8), (9),

$$w_{min} \leq z_{min} - \frac{1 - e^{-\sigma T}}{\sigma} d_{max}, \quad (13)$$

$$z_{max} - \frac{1 - e^{-\sigma T}}{\sigma} d_{min} \leq w_{max}. \quad (14)$$

Or these conditions are written in form of inequalities that depend on the intrinsic parameters  $T$ ,  $\sigma$ , the system parameters  $w_{min}$  and  $w_{max}$ ,  $o_{min}$  and  $o_{max}$  and  $d_{min}$  and  $d_{max}$ , and the prediction parameters  $z_{min}$  and  $z_{max}$ . The conditions (7), (8), (9), (13) and (14) are reformulated

in the following theorem (Bou Farraa B. et al. (2018)), in order to eliminate  $z_{min}$  and  $z_{max}$  from the above inequalities.

*Theorem 3.* Given the dynamic system (1), there exists control laws in affine type (11) or bang-bang type (12) that stabilize the system, and for which the constraints (2) and (3) are fulfilled for any disturbance verifying (4), if and only if the following conditions remain always true.

$$\sigma w_{min} + d_{max} \leq o_{max} \quad (15)$$

$$o_{min} \leq \sigma w_{max} + d_{min} \quad (16)$$

$$w_{min} + \frac{1 - e^{-\sigma T}}{\sigma} d_{max} \leq w_{max} + \frac{1 - e^{-\sigma T}}{\sigma} d_{min} \quad (17)$$

At the end of this approach, we have found the necessary and sufficient conditions (15), (16) and (17) for the existence and admissibility of the control laws for the original delayed system, in order to stabilize the perishable dynamic inventory system.

In the following, a comparative analysis of the impact of production delay and expiration date uncertainties on the system co-design has been given and followed by a robustness analysis.

## 4. ROBUSTNESS ANALYSIS AND DISCUSSIONS

### 4.1 Admissible area analysis

We first start to determine the admissible area of the decision-making system by expressing the necessary and sufficient conditions (15), (16) and (17) in terms of the intervals  $\Delta o$ ,  $\Delta w$  and  $\Delta d$ . We note that  $\Delta o = o_{max} - o_{min}$ ,  $\Delta w = w_{max} - w_{min}$  and  $\Delta d = d_{max} - d_{min}$ . In the following, the specification values for the production order are  $o(t) \in [20, 45]$  with  $T = 6$ , and  $w(t) \in [0, 85]$  with  $\sigma = 0.2$  for the inventory level. In addition, the customer demand bounds are  $d(t) \in [25, 35]$ .

First, we can notice the equivalence between the condition (17) and the following condition

$$\Delta w \geq \frac{1 - e^{-\sigma T}}{\sigma} \Delta d. \quad (18)$$

Then the conditions (15) and (16) can be expressed in terms of intervals by

$$\Delta o \geq \Delta d - \sigma \Delta w. \quad (19)$$

As a result, based on (18) and (19) we obtain the following two curves that correspond to the limits of the admissible area of the system.

$$\Delta w = \frac{1 - e^{-\sigma T}}{\sigma} \Delta d, \quad (20)$$

and

$$\Delta w = -\frac{1}{\sigma} \Delta o + \frac{1}{\sigma} \Delta d. \quad (21)$$

As a conclusion, based on the necessary and sufficient conditions of the existence of admissible control laws and respecting the system constraints, we are able to define the admissible area that is bounded by (20), (21),  $o_{max}$  and  $w_{max}$  as shown in the figure (2). Consequently the admissible area is useful to define the decision-making support system that size properly the system parameters.

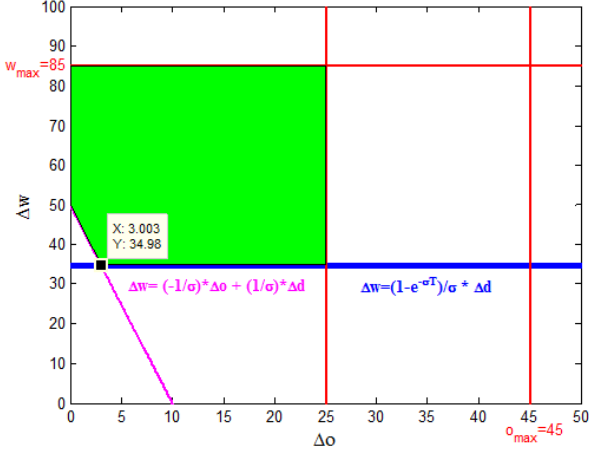


Fig. 2. Illustration of the admissible area

### Discussions

- In the considered example, the values of different parameters of the studied system are such that  $w_{min} = 0$ ,  $w_{max} = 85$ ,  $o_{min} = 20$  and  $o_{max} = 45$ . In addition, if we suppose that the minimum value of  $o_{min} = 0$  which is an accepted value, we obtain  $\Delta o = o_{max}$ . In this case, the admissible area is not limited by the green area, but extended until the value of  $o_{max} = 45$ .
- From the equation (18) we notice that

$$\Delta d \leq \frac{\sigma}{1 - e^{-\sigma T}} \Delta w \leq 25 \quad (22)$$

We can conclude that to be able to size  $\Delta o$  and  $\Delta w$  according to the variations of the customer demand  $\Delta d$ , it is necessary that (22) is always checked. Otherwise we can not determine property the admissible zone due to the presence of the Bullwhip effect all over the logistic-transportation line.

### 4.2 Impact of $(\sigma, T)$ variations on the robust system, $\sigma \neq 0$

We first start to see the impact of the delay on the admissible area of the system. It is clear that the curve (21) does not depend on  $T$ , while (20) as shown in the following figure, can vary from  $\Delta w = 0$  for  $T = 0$ , up to  $\Delta w = 50$  for large values of  $T$ . It is well explained by the fact that we don't need a security storage when we have direct access to the products without taking any lead time to produce and transport the products. And inversely we have an interest in having an important warehouse when the process time to deliver the products become larger.

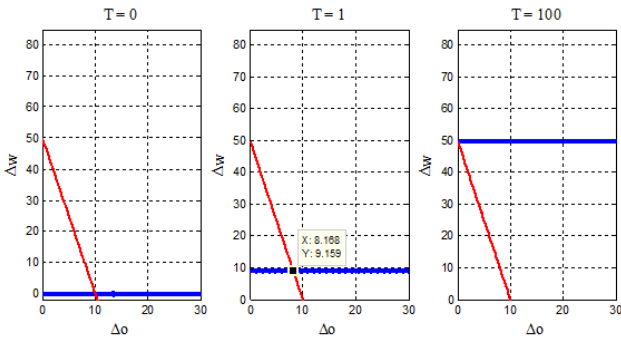


Fig. 3. The impact of  $T$  on the curves (20) and (21)

Then, referring to the figure (4) and table 4.2, we can study the variations on the loss factor and show its impact on the admissible zone for the distribution robust system.

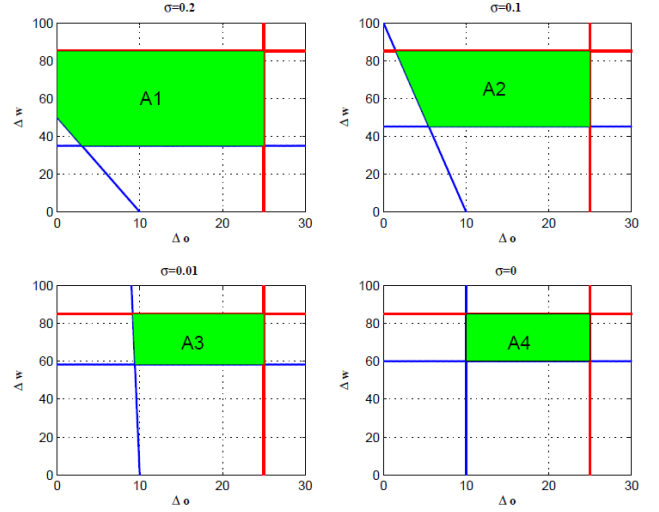


Fig. 4. The admissible areas  $A_i$  for different values of  $\sigma$

Table 1.  $\Delta o$  and  $\Delta w$  of the polytopes vertices  $A_i$  for different values of  $\sigma$

| $\sigma$ | $\Delta o$                            | $\Delta w$                                  |
|----------|---------------------------------------|---|
| 0.2      | $\Delta o = [3.01 \ 25 \ 25 \ 0 \ 0]$ | $\Delta w = [34.94 \ 34.94 \ 85 \ 85 \ 50]$ |
| 0.1      | $\Delta o = [5.49 \ 25 \ 25 \ 1.50]$  | $\Delta w = [45.12 \ 45.12 \ 85 \ 85]$      |
| 0.01     | $\Delta o = [9.42 \ 25 \ 25 \ 9.15]$  | $\Delta w = [58.24 \ 58.24 \ 85 \ 85]$      |
| 0        | $\Delta o = [10 \ 25 \ 25 \ 10]$      | $\Delta w = [60 \ 60 \ 85 \ 85]$            |

### Discussions

We can notice the following remarks.

- The admissible areas  $A_1$ ,  $A_2$ ,  $A_3$  and  $A_4$  are defined as polytopes in the plan  $(\Delta o, \Delta w)$  that are limited by vertices as mentioned in table 4.2. When  $\sigma$  increases the areas  $A_i$  become larger, so we have  $A_4 \subset A_3 \subset A_2 \subset A_1$ .
- When  $\sigma$  decreases, the losses decrease, and the admissible area  $A_i$  of the system become smaller. So we have more limitations on the storage variations  $\Delta w$  and on the production variations  $\Delta o$ .
- When  $\sigma$  increases, the losses increase. So we have a larger gap in the transport  $\Delta o$  in order to deliver more quantities of products and compensate losses, and a storage gap  $\Delta w$  more important to store all the products that have been produced and distributed. On the other hand, a simple drop on the outgoing flow of products  $d(t)$  can cause a big loss on the warehouse level. That's why we notice this big gap on  $\Delta w$  and  $\Delta o$  in order to act against the bullwhip effect.
- In addition to inequalities (18) and (19) which define the intervals of variations on the storage level and the production order, the average levels of production and storage, (15) and (16) are also involved to limit the sizing area of the system.

- We can optimize the cost of storage by choosing  $w_{min} = 0$ . We obtain the following conditions.

$$\begin{aligned} d_{max} &\leq o_{max} \\ \frac{1 - e^{-\sigma T}}{\sigma} \Delta d &\leq w_{max} \\ o_{min} &\leq \sigma w_{max} + d_{min} \end{aligned}$$

We have the possibility to increase  $o_{min}$  by increasing  $w_{max}$ , but it may cause an unfair and expensive increase in general.

#### 4.3 Impact of $(\sigma, T)$ variations on the robust system, $\sigma = 0$

In the distribution system, we suppose in this case that we are dealing with non perishable products that have an unlimited lifetime, which correspond to  $\sigma = 0$ . In this case we don't have losses during the production and distribution processes. The conditions (15), (16) and (17) become

$$d_{max} \leq o_{max}, \quad (23)$$

$$o_{min} \leq d_{min}, \quad (24)$$

$$w_{min} + T d_{max} \leq w_{max} + T d_{min}. \quad (25)$$

Therefore, the intervals of variation are reduced to  $\Delta w \geq T \Delta d$ , and  $\Delta o \geq \Delta d$ , and the admissible area of the system, limited by both straight curves and constraints on  $o$  and  $w$ , is presented by A4 in the figure (4).

#### Discussions

- From the obtained results, we can also optimize the level of the storage, the production and the sales by considering the minimum positive values. In our case study, we obtain the following values:  $w_{min} = o_{min} = d_{min} = 0$ . So the constraints (2), (3) and (4) become

$$0 \leq w(t) \leq w_{max},$$

$$0 \leq o(t) \leq o_{max},$$

$$0 \leq d(t) \leq d_{max}.$$

The conditions (23), (24) and (25) are reduced to

$$d_{max} \leq o_{max}, \quad (26)$$

$$T d_{max} \leq w_{max}. \quad (27)$$

- The following figure shows that when  $o_{max}$  increases,  $w_{max}$  needs to be larger consequently in order to store all the delivered products.

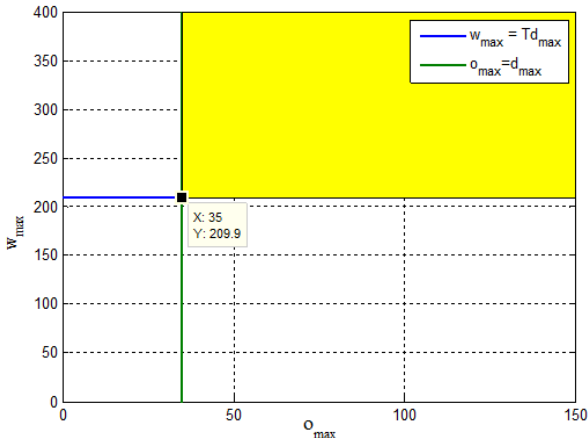


Fig. 5. The admissible area in the optimal case

## 5. NUMERICAL APPLICATION

In this numerical application we illustrate the effect of the proposed control strategy following the theoretical approach. We consider the elementary logistic system of the form (1), and we apply the proposed affine or bang-bang control laws.

For this system, we follow a co-design methodology in order to calculate the system parameters, so that the necessary and sufficient conditions given before are all satisfied. So supposing the customer demand as unknown but varying in  $[25, 35]$ , and a global lead time  $T = 6$ , with a loss factor  $\sigma = 0.2$  due to perishable products, we have obtained the values of the system parameters as seen in table 3.

Table 2. System parameters

|                     |                            |                      |
|---------------------|----------------------------|----------------------|
| Control law         | $o_{min} = 20$             | $o_{max} = 45$       |
|                     | Delay $T = 6$              |                      |
| Storage level       | $w_{min} = 0$              | $w_{max} = 85$       |
|                     | Loss factor $\sigma = 0.2$ |                      |
| Customer demand     | $d_{min} = 25$             | $d_{max} = 35$       |
| Prediction interval | $z_{min} = 123$            | $z_{max} = 148$      |
| Control parameters  | $o_2 = o_{min} = 20$       | $o_1 = o_{max} = 45$ |
|                     | $K = 1$                    | $z_0 = 168$          |
|                     | Initial conditions         |                      |
| $w(0) = 50$         | $WIP(t) = 33$              | $z(T) = 130$         |

*Case of affine control law* In this simulation, we consider an affine control law of the form (11). This law corresponds to the continuous flow of goods during production. Moreover the leaving flow of goods that is the customer demand is represented by a random signal varying between  $d_{min} = 25$  and  $d_{max} = 35$  units for instants between  $t = 0$  and  $t = 20$  time units. The results of this simulation are given by figure (6).

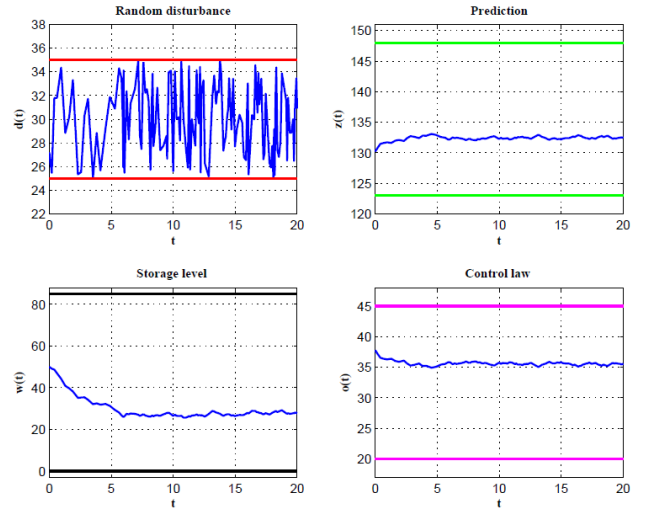


Fig. 6. Temporal evolution of  $o(t)$ ,  $z(t)$ ,  $w(t)$  and  $d(t)$

*Case of bang-bang control law* In this simulation, the leaving flow of products is represented by a rectangular

signal that evolve during 20 units of time. In addition the bang-bang control law of the form (12), with  $o_1 = 45$  and  $o_2 = 20$ , describe very well the production and transport by batches or discrete flows of products moving to satisfy the customers demands. The simulation results are figured in (7).

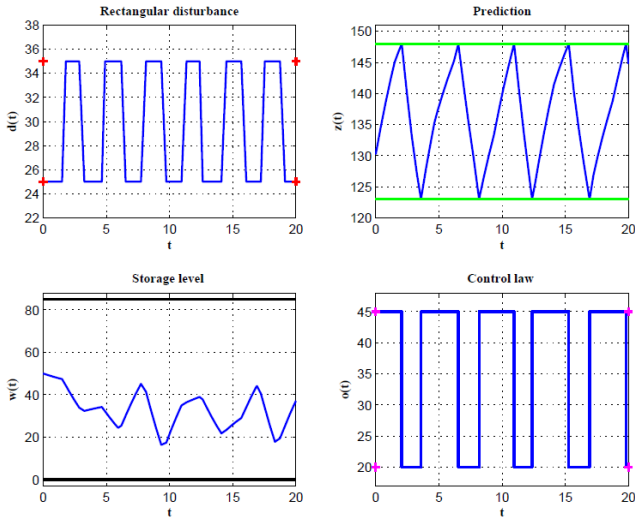


Fig. 7. Temporal evolution of  $o(t)$ ,  $z(t)$ ,  $w(t)$  and  $d(t)$

In the two case studies, we can notice that:

- the warehouse level  $w(t)$  has no overruns of  $w_{max}$ , and is always positive. The same for the control law  $o(t)$  that always remains between  $o_{min}$  and  $o_{max}$  for the affine control law, and switch between  $o_1$  and  $o_2$  with  $z(t)$  moving between  $z_{min}$  and  $z_{max}$ , and this is well verified in the temporal evolution of  $o(t)$  and of  $z(t)$ . So the constraints (2) and (3) are well verified.
- The system responses for different signals of  $d(t)$  show small variations. On the one hand, the affine control law  $o(t)$  varies very little (roughly between 35 and 38 units). In terms of regulation, it's perfect. In terms of production management, it makes possible the optimization of the production resources. On the other hand, the warehouse is in decreasing mode but remains quite high relative to  $w_{min} = 0$  and does not go below 19 units. In terms of production management, a large storage of products is expensive, so we have the possibility to reduce the warehouse level by applying algorithms and methods for storage costs optimization. This degree of freedom is very important in practice.

## 6. CONCLUSION

The paper deals with the problem of perishable inventory control of distribution system, subject to a constant loss factor and a constant lead time, using an approach based on control theory studied in terms of invariance. The inventory control system is considered as an input-delay system with uncertainties on customer demands, and positive constraints due to the specifications of the agricultural supply chain. We discuss the impact of uncertainties of the lead time and the expiration date on the stability of the inventory regulation problem within distribution system

using feedback control law structure.

As further work, it is interesting to reduce the uncertainty on the external demands by using customers demands estimation. By the way, our control approach is developed right now in the continuous-time domain, so we have to move on to the discrete analysis. Finally, we have already started to improve the performance of the proposed approach by considering variable lead time. Moreover, we have extended the study in the case of distributed systems that present real applications on logistic networks.

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