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1 Introduction

E-commerce has become a common practice since customers can shop during the whole day from wherever they are. Global e-commerce sales will reach approximately $2.8 trillion by the end of 2018 and will hit $4.5 trillion in 2021 [1]. This growing e-commerce poses a huge challenge for the last mile delivery services.

Currently, most of the deliveries are performed at customer's home and workplace where customers wait to get their online orders.

Recently companies like Amazon and FedEx developed locker delivery. When customers shop online, they can choose a nearby locker as their pickup location. In the past two years, a new concept called *trunk delivery* has been proposed. Here, customers' orders can be delivered to the trunks of their cars. Volvo launched its world-first in-car delivery service in Sweden in 2016. The courier has a one-time digital code to get access to the car. Trunk delivery is different from home delivery and locker delivery since the car moves and may be in different locations during different periods of time. As a consequence, synchronization between the car and the courier is required to perform the delivery.

This work studies a last-mile system that combines home/workplace, locker and car trunk delivery services. We call the resulting problem the generalized vehicle routing problem with time windows (GVRPTW). This work aims to develop an efficient solving method for the GVRPTW. This work follows a work from the authors where an efficient branch-and-cut algorithm for the one vehicle case has been developed [4].

2 Problem definition and related works

The GVRPTW can be formally defined as follows. Given a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, the set of vertices $\mathcal{V} = \{0, 1, ..., N\}$ is partitioned into $\mathcal{C}_0 = \{0\}, \mathcal{C}_1, ..., \mathcal{C}_K$ clusters. $\mathcal{K} = \{0, 1, ..., K\}$ denotes the cluster index set. Cluster $\mathcal{C}_0$ contains only the depot 0 where is located a fleet of M homogeneous vehicles, each with capacity Q. Cluster $\mathcal{C}_k, k \in \mathcal{K} \setminus \{0\}$ represents the set of alternative locations on which customer k can be delivered. Each customer has a demand Q_k. Each vertex is associated with a TW $[E_i, L_i], i \in \mathcal{V}$. $[E_0, L_0] = [0, T]$ represents the overall time horizon. A location can only be visited during its TW: an early arrival leads to the waiting time while a late arrival causes infeasibility. Arcs are only defined between vertices belonging to different clusters, that is, $\mathcal{A} = \{(i, j) | i \in \mathcal{C}_k, j \in \mathcal{C}_l, k \neq l, k, l \in \mathcal{K}\}$. Each arc $(i, j) \in \mathcal{A}$ is associated with a traveling cost C_{ij} and a traveling time T_{ij}. The GVRPTW consists in finding M vehicle routes such that the traveling cost is minimized and : (i) every route starts and ends at the depot; (ii) each cluster is visited exactly once; (iii) the total demand of customers served by one route do not exceed Q; (iv) the service time at vertex i is between its TW $[E_i, L_i]$, and every vehicle leaves the depot and returns to the depot during $[0, T]$.

To the best of our knowledge, very few works related to GVRPTW can be found in the literature. [3] have recently studied a special case where TWs within the same cluster do not overlap. They were inspired by the trunk delivery system and called this problem VRP with Roaming Delivery Locations (VRPRDL). They developed construction and improvement heuristics, and their results proved the advantage of using trunk delivery. Following this work, VRPRDL was formulated as a set-partitioning problem and a branch-and-price algorithm was proposed [2]. This algorithm is able to deal with a hybrid delivery strategy combining trunk delivery and home delivery, in which case the TWs within a cluster are no longer non-overlapping.

The GVRPTW can be formulated as a set covering problem. Let $\mathcal{R}$ denotes the set of all feasible routes, i.e., respecting capacity and time window constraints. Let C_r be the cost of route $r \in \mathcal{R}$ and A_{kr} indicate whether cluster $\mathcal{C}_k$ is visited on route r. We introduce a set of binary variables $z_r, r \in \mathcal{R}$, representing if route r is chosen in the solution. The set covering formulation of the GVRPTW is as follows:

$$\text{minimize} \quad \sum_{r\in\mathcal{R}} C_r z_r \tag{1}$$

$$\text{s.t.} \quad \sum_{r\in\mathcal{R}} A_{kr} z_r \geq 1 \qquad \forall k \in \mathcal{K} \setminus \{0\}, \tag{2}$$

$$\sum_{r\in\mathcal{R}} z_r \leq M, \tag{3}$$

$$z_r \in \{0,1\} \qquad \forall r \in \mathcal{R}. \tag{4}$$

The objective function (1) minimizes the total traveling cost. Constraints (2) ensure that each cluster is visited at least once. Constraints (3) ensure that the number of routes in the solution is lower than the size of the vehicle fleet.

3 Resolution method

The key idea of our approach is to first fill the route subset $\mathcal{R}' \subset \mathcal{R}$ with interesting routes and then obtain a solution solving the set-covering formulation proposed in Section 2 on set $\mathcal{R}'$ instead of set $\mathcal{R}$.

The heuristic to fill $\mathcal{R}'$ is a randomized greedy construction procedure with parallel insertions. The set of seed customers is chosen according to some problem-specific criteria. The selection of the next customer to be inserted is based on a generalized regret measure over all routes. Moreover interesting routes may be optimized with the branch-and-cut algorithm [4] we already developed for the one vehicle case.

In order to test the efficientcy of the proposed algorithm, results will be presented on the instances for VRPRDL and be compared to that obtained in [2]. Promising results are obtained and will be presented at the conference.

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