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Modelling of heat transfer in open cell foam described as graphs associated to the solid network using Port-Hamiltonian Systems.

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Highlights

- Modelling of heat transfer in metallic open-cell foam.
- New network approach based on Port-Hamiltonian systems.
- Graph representation are obtained using *X-ray CT* and *iMorph*.

1. Introduction

This paper deals with the modelling of the heat transfer in metallic open-cell foams using their geometric and topological properties. Several works have considered simplified geometries such as regular paving by Kelvin's cell [1] or random cellular morphologies based on the weighted-Voronoi (Laguerre) tessellations [2]. These approaches, however, fail to represent accurately the actual, often non-homogeneous and non-regular, structure of foams and may lead to poor estimation of their effective properties. Another approach is to use, X-ray CT (Computed Tomography) to construct 3D realistic structure which can be used in a CFD software and perform a numerical simulation of the heat transfer equations using finite elements or finite volume methods (FEM, FVM). The main two drawbacks of this approach are (1) the discretization of the computational domain, which leads to numerical models of extremely high complexity (2) the use of staircase boundary approximations, which may lead to inconsistent results.

In this paper, we follow an alternative approach, based on the derivation of two graphs representing the solid and fluid phases obtained using X-ray CT and image analysis [3]. From these two dual graphs, we derive the Port Hamiltonian formulation [4] of the heat diffusion. In a first instance, the work presented here is limited to modelling of the heat transfer in the solid phase.

2. Methodology

From 3D images and using *iMorph* software [3] (as shown in **Fig. 1**), two graphs are derived. For the graph of solid, one associates nodes to adjacent half-struts and edges to interfaces. For the graph of fluid, one associates nodes with finite volumes and edges to surfaces at the interface of two volumes. Furthermore, some geometrical properties such as strut lengths and diameters, are extracted and associated with the elements of the graph.

The heat diffusion in the solid phase may be formulated as the Port Hamiltonian System

$$\begin{pmatrix} \frac{\partial u}{\partial t} \\ F \end{pmatrix} = \begin{pmatrix} 0 & -div \\ -grad & 0 \end{pmatrix} \begin{pmatrix} \tau \\ \phi \end{pmatrix} \text{ and } \begin{pmatrix} f^b \\ e^b \end{pmatrix} = \text{tr} \begin{pmatrix} \tau \\ \phi \end{pmatrix}$$

where u is the internal energy density, τ is the reciprocal temperature, ϕ the heat flux and $\begin{pmatrix} f^b \\ e^b \end{pmatrix}$ denotes the port boundary variables where tr is the trace operator at the boundary of the

spatial domain. The Hamiltonian functional is the total entropy $H_0 = S$, the state variable is the internal energy density u and the co-state variable is the reciprocal temperature, $\tau = \frac{\delta S}{\delta u}$ corresponding to the Gibbs-Massieu equation $ds = \tau du$. The heat flux obeys Fourier's law hence, $\phi = -\frac{\lambda}{\tau^2} F$ with the heat conductivity λ .

Writing the set of discrete balance equations, $\frac{d}{dt} \int_{V_i} u_i dv = - \int_{\partial V_{ij}} \phi_{ij} ds$, for every control volume V_i associated with node n_i with boundary faces ∂V_{ij} associated with the edges e_{ij} , yields the finite-dimensional Port Hamiltonian Systems [5]:

$$\begin{pmatrix} \frac{du_i}{dt} \\ F_{ij} \end{pmatrix} = \begin{pmatrix} 0 & D \\ -D^T & 0 \end{pmatrix} \begin{pmatrix} \tau_i \\ \phi_{ij} \end{pmatrix} + \begin{pmatrix} D_b \\ 0 \end{pmatrix} \phi_b \text{ and } F_b = -D_b^T \tau_i$$

where D and D_b are the co-incidence matrices, which relate the control volumes to the interfaces (b for boundary). Simulations will be presented in comparison with experimental results [6].

3. Conclusions & perspectives

We have presented the Port Hamiltonian model of the heat conduction in the solid phase of a metallic foam, based on its graph structure obtained by image analysis and tomography. Thereby we take account of the real structure of the foam, the non-regularity of its geometry. Future work will consider the dual graph of the fluid phase and the coupling with the fluid phase and complete the heat transport with mass transport.

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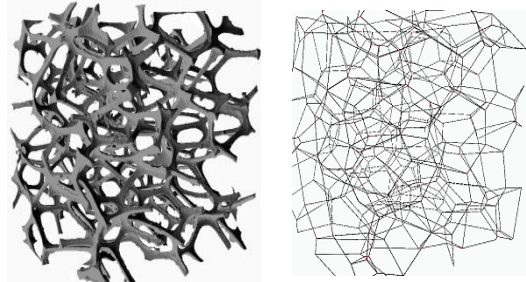
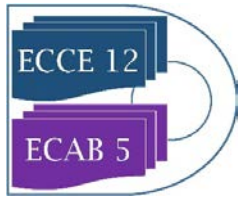


Figure 1. X-ray CT image reconstruction and the solid graph associated (623 nodes and 203 cells).



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