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Aymeric Lamorlette. A dynamic model for liquid fossil fuel production based on gross product/EROI coupling. 2023. hal-02383025v7

HAL Id: hal-02383025

<https://hal.science/hal-02383025v7>

Preprint submitted on 4 Jul 2023

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A dynamic model for liquid fossil fuel production based on gross product/ERoEI coupling

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Abstract

Since 1940, many attempts to model oil production have been proposed. These approaches, using increasing complexity, consider growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by a dynamic dependency between oil and Energy Return on Energy Invested (ERoEI). The model is based on mass and energy conservation and can be applied to all extracted liquid fossil fuels. After comparison with oil extraction and *ERoEI* dynamics, it highlights the existence of an external, controlling parameter: the investment rate, which account for the re-investment in newly operated liquid fuel sources. Its dynamic provides explanations about the oil shock and some explanations about the peak prediction issues of the Hubbert model. Studying this evolution suggests an attempt to control the oil production in order to sustain a globally linear production, starting around 1943: at short time scale (shorter than 28-36 years), the investment rate evolved linearly. However, in order to keep a linearly growing production at long time scale, the investment rate had to evolve exponentially: this was achieved through a piecewise linear control, where the investment rate and its derivative doubled every 28-36 years. The link between this control and the oil shocks suggests the next oil shock will occur around 2035-2040.

The model also allows to highlight a major issue in liquid fossil fuel production: even if gross product can be controlled and keeps growing linearly, net product, which account for the energy delivered by the oil industry to the consumer, can decrease before the gross product peaks, due to the decay of *ERoEI*. At this point, the energy benefit of the oil industry will inevitably

decrease and oil production will slow down. Based on the present model and a sensibility study on its parameters, this tipping point will happen between 2026 and 2039. Net product of fossil liquid fuels could therefore keep growing linearly until this point, where a steep decay is expected. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model. Production will finally be close to zero around 2063-2072.

Keywords: Oil production, EROEI, dynamic model, investment rate

Introduction

Models that account for oil production have been published from 1962 [1], with increasing complexity ([2, 3, 4], citing only very few of them). These models rely on a production dynamic with constant parameters. The main drawback of such model is that the maximum production (its main parameter), had to be reevaluated several times since [1] proposed his model, leading to several false estimation of the peak oil. Besides, these models do not apply to unconventional oil such as oil sands for instance. The aim of this study will be to develop a model for all liquid fossil fuel, including unconventional oil, and to evaluate how the model parameters could evolve in time, based on a coupling between oil production (all extracted liquid fuels) and its Energy Return on Energy Invested (EROEI) at the wellhead, as defined in [5] or [6]. Through this dependency, it is expected to explain why the prediction of peak is always delayed. For simplicity, "all liquid fuels" in the following refers to "all extracted liquid fuels" or "all liquid fossil fuels".

The model suggested is based on mass and energy conservation for liquid fuel gross production. It is worth noticing that the structure of the obtained ODE resembles a predator type equation of a Lotka-Volterra set. In this model, production of liquid fuels Q appears to be the *EROEI* predator as the production "feeds" on EROEI to grow. It is in line with former use of Lotka-Volterra equations to model dynamic systems in ecology [7] or in economy [8].

The article is organised as follow: A first part is dedicated to a presentation of an assumption on a mean, volume averaged, global EROEI for liquid fuels, suggesting why the model applies to the production of all liquid fossil fuels. It also presents a discussion on a production averaged EROEI and its consequence in term of production modelling. The gross product equation is

then derived. A fitting of the model parameters based on historical dynamics of oil production and mean EROEI is performed, using an inverse method for the investment rate. An analysis of the latter, a forcing parameter, is done, suggesting a control of the investment to keep a linearly growing production. Some possible links between this control and the oil shocks are also presented, suggesting the next oil shock will occur around 2035-2040.

A second part is dedicated to studying net liquid fuel production and energy benefit. It highlights the existence of a tipping point between 2026 and 2039, where net product will begin to decrease. It suggests two potential limit scenarios for liquid fossil fuel production after this point, which are investigated. This analysis suggests that net energy from fossil liquid fuel to the consumer around 2063-2072, will be that of 1943, just a few years before going down to zero.

1. Modelling the interaction between oil production and EROEI

This section is dedicated to the description of the interactions between yearly production of all liquid fuels Q and $ERoEI$ at the wellhead, as defined in [5] or [6]: $ERoEI$ is the ratio of the gross energy delivered at the wellhead on the energy required to extract it. $ERoEI$ is therefore dimensionless. The unit of Q must be discussed here. Since the model presented is based on energy conservation, Q should be expressed in energy unit. However, it seems more practical to express it in term of volume (or mass). These three expressions are correlated through energy content per unit mass (or volume) and density (or API degrees). As extraction of unconventional oil increases, the oil density changes since unconventional are heavier oil, but the energy contained per unit mass is not. Therefore, the three expressions of Q (as energy, volume or mass) used to be strictly equivalent over time, but it is not the case anymore. Nevertheless, the density difference between conventional oil and heavy oil is roughly 10%. Even if oil production becomes pure unconventional oil, density variation cannot exceed this value. For instance, the density deviated over time from less than 1% in 2006 and less than 2% in 2018, according to IEA [9] data. Based on this remark, the energy/volume/mass formulation equivalence seems acceptable and the yearly production Q is expressed in volumes (Giga-barrels or Gbbl).

1.1. *An assumption about a global, production averaged, EROEI at the well-head*

The dynamic modelling of all liquid fuels presented in this study requires an *ERoEI* function that would be representative of a mean global EROEI of all liquid fossil fuels. Considering N liquid fuel sources in the world, $ERoEI_i$ the *ERoEI* of a given source and Q_i its production, mass and energy conservation requires to take $ERoEI = \frac{1}{Q} \sum_{i=1}^N Q_i \cdot ERoEI_i$, a volume averaged *ERoEI*, as it allows to derive a single equation for the production of all liquid fuels, instead of having a set of N equations.

Such *ERoEI* data seem impossible to gather, since $ERoEI_i$ are unlikely to be available, therefore it seems adequate to look for some existing model of global, mean EROEI, instead of reconstructing *ERoEI* based on $ERoEI_i$ and Q_i . It could however be aggregated by oil categories, as suggested in [10], instead of oil sources, in considering i categories of liquid fuels (considering conventional crude oil, shale oil, off-shore, ultra-deep water oil, etc...) and then proceed to a categories averaging which will therefore be equivalent to the previously described volume averaged *ERoEI*. As a first approximation, it seems nevertheless acceptable to evaluate the mean *ERoEI* as the *EroEI* of conventional oil. Indeed, the ratio of unconventional on conventional oil *ERoEI* and production suggest so: According to [11], in 2014 for example, *ERoEI* of global oil production was roughly 17, while *ERoEI* of typical unconventional oil was roughly 11. Fig.1 shows the evolution of unconventional oil production over time and allows to evaluate the relationship between mean, conventional and unconventional *ERoEI*, using a weighted averaging. Based on these data, the ratio of mean *ERoEI* on conventional oil *ERoEI* was roughly 0.94 in 2014. Suggesting a constant ratio over time between conventional and unconventional oil *ERoEI* (an acceptable assumptions according to the different scenarios presented in [10]) and a linear approximation for the evolution of unconventional oil in the global production (fitted on Fig.1 data), the ratio of mean on conventional *ERoEI* will be roughly 0.88 in 2040, assessing the relevancy of the discussed assumption. It would obviously lead to a slight overestimation of the production, since non conventional fossil fuels have lower *ERoEI* than conventional.

In order to efficiently perform an inverse method from 1943 to 2021, this study also requires a continuous *ERoEI* function or at least yearly values on a large time laps rather than scattered values over time. Based on these remarks, three models are likely to be adequate: the mathematical model from [12], fitted using a price-based method by [13], the physical model of

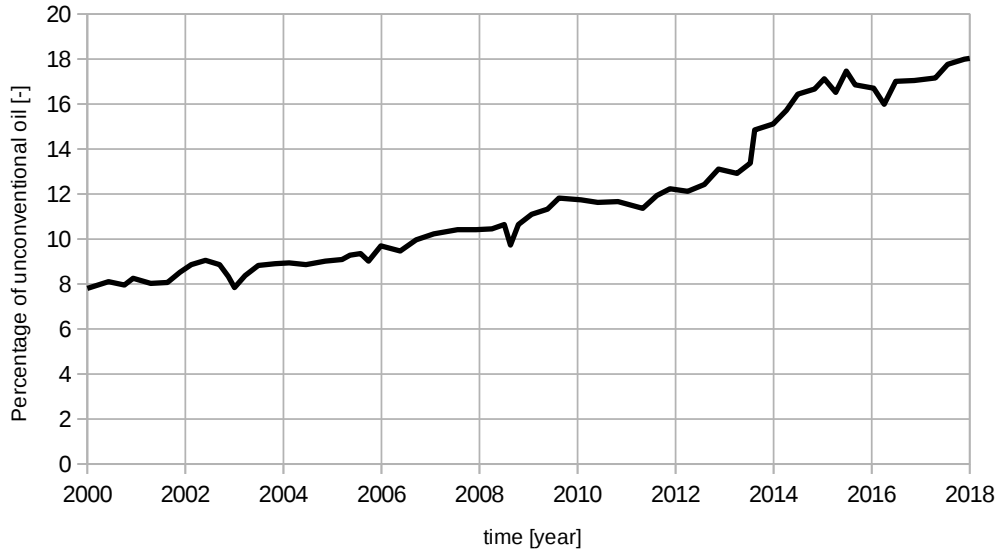


Figure 1: Percentage of unconventional oil in the global oil production, according to IEA [9]

[14], which shows values in line with [13] and the aggregated model of [10], based on the model of [12], which should be the most suited dataset since it takes into account the increasing extraction of unconventional oil. The data presented on Fig.(2) in reference [10] are however discarded, since they show an asymptotic $ERoEI$ value (much) greater than unity, what is contrary to the assumption suggested by [12] in its original $ERoEI$ model for non-renewable energy sources. The models of [13] (for global $ERoEI$ values of oil and gas) and [14] (both models presented) are therefore used in this study. Both references lead to very similar investment rate trend, as discussed later in this article. It is here added that, during the review process of this article, authors of [10] provided supplementary $ERoEI$ data (one dataset satisfying the original assumption of [12] and taking into account the increasing extraction of unconventional oil), which lead to investment rate trend similar to those obtained with the models of [13] and [14], with slightly more pessimistic results, in accordance with the previous remark. This dataset will allow to study the effect of increasing unconventional oil production.

1.2. A dynamic model for oil production and $ERoEI$ evolution

In order to derive the production equation, an energy balance is considered over the whole set of liquid fuel sources, based on a one year time laps

($\Delta t = 1$ year). The ODE is then derived taking $\Delta t \rightarrow 0$. For simplicity, the balance is based on gross yearly product Q_g (in Gbbl) and the net yearly product Q_n (in Gbbl) is deduced from Q_g afterwards.

On a given year n , a gross product Q_g^n is extracted from the N liquid fuel sources. A fraction k_0 (the investment rate, in year^{-1}) of the associated net product Q_n is used to extract liquid fuels from new sources. Let Q_{ext} be the volume of oil equivalent to the energy used for extraction (in Gbbl). For these new sources, on year n , it reads $Q_{ext}^n = k_0^n Q_n^n \Delta t$. Now using the energy/volume equivalence, $ERoEI$ can be expressed the following way: $ERoEI = Q_g/Q_{ext}$. Therefore the increase in gross product due to this investment is $Q_g^{n+1} - Q_g^n = Q_{ext}^n ERoEI^n = k_0^n Q_n^n ERoEI^n \Delta t$.

In order to evaluate the net product, it is here suggested to use the work of [15] who noticed that, besides the energy cost of oil extraction, subsidiary energy costs exist in the oil industry, before net energy is delivered to the consumer. These costs include refinery, transportation infrastructures, transportation to consumer and the loss in non-fuel refinery products (a more detailed description can be found in [15]). Let Q_s be the volume of oil equivalent to the subsidiary energy costs (in Gbbl), volume conservation implies:

$$Q_g = Q_n + Q_{ext} + Q_s, \quad (1)$$

Based on ERoEI definition, $Q_{ext} = Q_g/ERoEI$. According to [16, 15, 17, 18], Q_s is estimated as a fraction of Q_g (constant over time): $Q_s = c_s Q_g$, with $c_s = 0.54$, according to [15], Table 3. Since the quantity c_s is a relative energy cost, it is dimensionless. This leads to:

$$Q_n = \frac{ERoEI(1 - c_s) - 1}{ERoEI} Q_g. \quad (2)$$

The increase in gross product at the next time laps becomes $Q_g^{n+1} - Q_g^n = k_0^n Q_g^n (ERoEI^n (1 - c_s) - 1) \Delta t$.

During the same time laps, the producing fuel sources show a decline which follows the model described in Sorrell [19]: Considering k_1 the mean oil source decline rate (in year^{-1}), the associate decrease in production is equal to $Q_g^{n+1} - Q_g^n = -k_1 Q_g^n \Delta t$.

Both phenomena occur at the same time, during the same time laps. It is possible to use superimposition to get: $Q_g^{n+1} - Q_g^n = Q_g^n [k_0^n (ERoEI^n (1 - c_s) - 1) - k_1] \Delta t$. Taking $\Delta t \rightarrow 0$ leads to $(Q_g^{n+1} - Q_g^n)/\Delta t = \dot{Q}_g$ and

$$\dot{Q}_g = k_0 Q_g (ERoEI(1 - c_s) - 1) - k_1 Q_g. \quad (3)$$

As mentioned in the introduction, this equation has the same structure as the predator equation of the Lotka-Volterra model, where Q_g is the predator and $ERoEI$ (technically $ERoEI(1 - c_s) - 1$) is the prey; k_0 is the predator growth rate and k_1 its mortality.

1.3. Fitting the model parameters on the period 1943-2019

Since the study [14] provides an $ERoEI$ dynamic from 1930, k_0 could be investigated on the period 1930-2018, for which both $ERoEI$ and Q_g dynamics are available. The analysis however focus on a period which begins in 1943, where the production shows a clear change of rate. From that time, the gross product is mostly linear, with several overshoots and undershoots due to political, technical and economic factors. Using Hubbert's logistic fit as in [20], this part of the curve is very close from being linear. Also, a linear fit of the gross product on the period 1943-2019 shows a determination coefficient higher than 0.95. These remarks suggest that the business as usual of the oil industry, a constant production derivative, began around 1943. This is why the investment rate is investigated on the period 1943-2019, in order to evaluate how it is sustained.

k_1 represents the oil sources mean decline rate. This parameter should be extracted from experimental measurements, using inverse methods. Based on the results of [19] on the period 1975-2009, the mean value lies in the range 4.1 – 6.7% but is increasing with the exploitation of new non-conventional sources. Therefore, k_1 is set equal to 6% (a mean value based on previous remark) and k_0 can be fitted. This is an optimistic assumption since k_1 is expected to slightly increase in the future with the increase of non conventional extraction. Nevertheless, a sensitivity study has been conducted on k_1 : In the range [6-9]%, it accounts for less than a one year variation on the tipping point prediction and a two year variation on the exponential forcing characteristic time of the investment rate (see section 1.4.2), what is small in comparison to the variation due to $ERoEI$ uncertainties. Noticing that 9% is a pessimistic value, even with increasing production of unconventional oil (according to [19]), using a constant value of 6% seems an acceptable assumption for the purpose of this study.

Now, based on Eq.(3) and using an inverse method, setting a value for k_1 , it is possible to plot k_0 dynamic, based on historical data for Q_g and values for $ERoEI$:

$$k_0 = \frac{\dot{Q}_g}{Q_g(ERoEI(1 - c_s) - 1)} + \frac{k_1}{ERoEI(1 - c_s) - 1}. \quad (4)$$

\dot{Q}_g is calculated from Q_g using a second order upwind finite difference scheme, then using Q_g^n , $ERoEI^n$, c_s and k_1 values, k_0^n can be computed with Eq.(4), for each year in the interval 1943-2019.

The oil production data is extracted from [20, 21]. As stated in section 1.1, the $ERoEI$ dataset are taken from [13] (data of global $ERoEI$ for oil and gas), [14] (considering both the optimistic and the pessimistic fit) and from supplementary data of [10] (one dataset). The analysis is performed on the period 1943-2019 using a moving three-year averaging on Q_g (i.e. using a three-year windowing for the averaging process).

1.4. Studying the investment rate

1.4.1. History of the investment rate

The investment rate can be evaluated through the value of k_0 , which lies in the range $[0; 1[$. It is nevertheless suggested to study $k_{eff} = k_0 \frac{ERoEI(1-c_s)-1}{ERoEI} = k_0 \frac{Q_g^n}{Q_g}$ instead of k_0 , for function fitting requirements. The parameter k_{eff} , which represents the forcing of the system is plot over time in Fig.2. Its analysis provides some possible characteristics of the oil extraction strategy, which are presented below.

On the period 1943-1968, k_{eff} is relatively linear, as Fig.2 shows. In 1969, k_{eff} begins to decrease, most likely due to the geopolitical event that led to the first oil shock. This shock is then responsible for an instantaneous drop of k_{eff} in 1973, but k_{eff} is kept constant just after the shock, hence no clear disruption appears in oil production. Due to the mathematical structure of the model, a constant k_{eff} is not sufficient to sustain a linearly growing production for more than a few years. This is likely to be the cause of the second oil shock, which corresponds to another, longer drop of k_{eff} . Contrary to the first shock, it creates a disruption both in investment rate and oil production from 1979 to 1985, where investment is way under its mean value, as observed in Fig.2. This analysis seems rather in line with the historical developments of oil shocks presented in [22].

On the period 1943-1978, k_{eff} is mostly linear, despite the first shock. The solid line fits the data with a mean relative error equal to 13%.

After the second shock (which is responsible for a clear disruption in the data), k_{eff} seems to evolve (globally) linearly, with raises and plateaus during the period 1986-2006. The dashed line fits the data with a mean relative error of less than 5%. Compared to the period 1943-1978, one can see that the slope has roughly doubled. This behaviour allows Q_g to grow

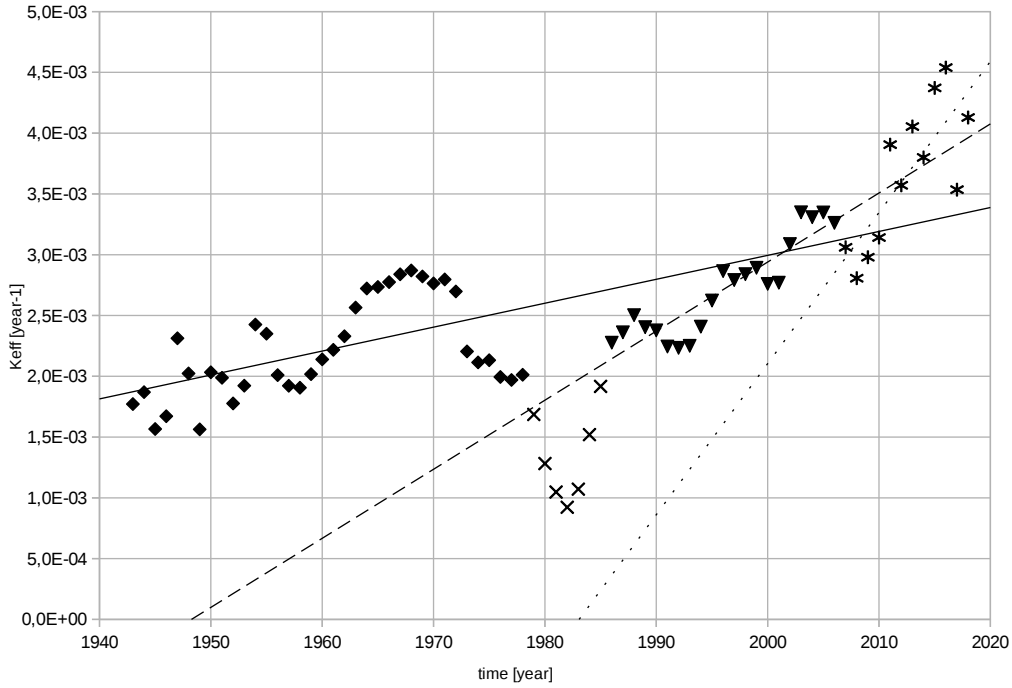


Figure 2: k_{eff} dynamic on the period 1943-2018. Symbols are measured values: diamonds on the period 1943-1978, crosses on the period 1979-1985, triangles on the period 1986-2006 and stars on the period 2007-2019. Lines are linear fits: solid line on the period 1943-1978, dashed line on the period 1986-2006 and dotted line on the period 2007-2019.

linearly in time again. The plateaus of k_{eff} are responsible every time for a slow damping of Q_g , which corresponds to past predictions of a nearby peak, using Hubbert's curves. This phenomenon leads every time to an economical recession and a raise in oil price (this can be shown in comparing Fig.2 with an oil price chart), at the moment where the oil industry needs to increase its investment to keep k_{eff} close to the behavior that ensure a linearly growing Q_g . The following assumption is suggested to explain the origin of this raise/plateaus dynamic: With time, the production of an oil source eventually decreases, meaning that exploration is firstly needed to extract more oil. It means that if exploration does not suggests new sources to exploit, the production stagnates because it is not possible to invest in new sources, therefore k_{eff} is constant or slightly decreasing. When new sources become available, the investment rate can quickly increase until the

new sources become less available and then exploration has to start again.

In 2007-2008, another oil shock happened, which again caused a sudden drop of k_{eff} and therefore a massive disruption in the data. From this point, k_{eff} seems to be linear again, but again the slope has doubled in comparison to the previous period. It can be seen in Fig.2, where the dotted line fits the data with a mean relative error equal to 7%.

1.4.2. The limit of a linear dynamic

Studying k_{eff} at long time scale shows the real dependency of Q_g on k_{eff} : in order to keep a linearly growing Q_g , k_{eff} should follow a behavior close to an exponential. This corresponds to the description of the previous section: A piecewise linear behavior where the slope doubles at fixed time interval, with no clear disruption on roughly 32 years periods and major disruptions between these periods, where the oil industry had to adapt to keep the oil flowing. It means that in order to sustain a linear production dynamic, k_{eff} should follow $k_{eff} \sim \exp(t/\tau)$ with $\tau = 46 \pm 6$ year: Optimistic and pessimistic values from [14] lead respectively to $\tau = 52$ year and $\tau = 43$ year, values from [13] lead to $\tau = 46$ year and supplementary data of [10] lead to $\tau = 40$ year. The comparison between these different results suggests that τ value is not too sensitive to the increase in unconventional oil production. Fig.3 shows k_{eff} on a period 1943-2040 along with a projection based on a linearly increasing production and the exponential fit.

It means that the investment rate and its derivative has to double every $\Delta t_{shock} = \tau \ln(2) = 32 \pm 4$ year, which fit both the time between the beginning of the business as usual in 1943 and the major disruption in oil production in 1979, and the time between this disruption and the oil shock of 2007-2008. It suggests this is the main reason why oil shocks have to happen. It would mean that the next oil shock will happen around 2035-2040.

In other words, oil industry began its business as usual (i.e. a constant gross production derivative) in 1943 with a linear k_1 , which could sustain a steadily increasing production at short time scale, smaller than 46 years (the characteristic time of the exponential forcing). Therefore, after roughly 32 years, this linear forcing was too far from what the investment should have been to keep a constant gross production derivative and the production eventually decreased, creating an oil shock. The price raise due to the shock then allowed the oil industry to set k_1 dynamic on the updated short time scale linear approximation and the process repeated itself until the 2007-2008 oil shock. Therefore this process is most likely going to repeat itself, causing

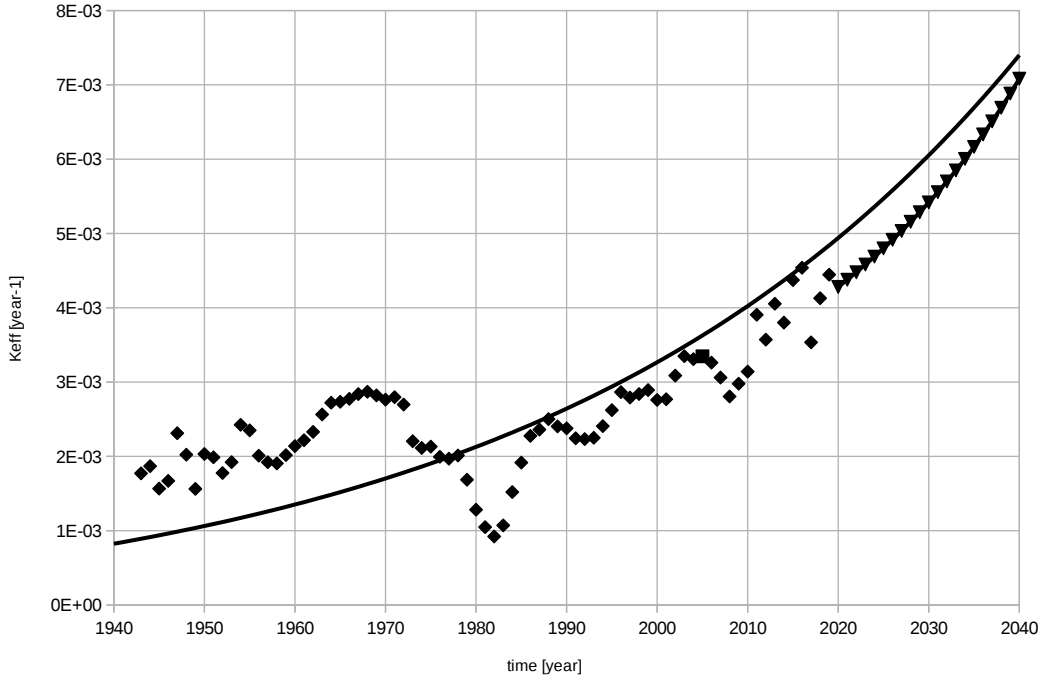


Figure 3: k_{eff} dynamic on the period 1943-2040. Diamonds are measured values on the period 1943-2019 and triangles are projected values on the period 2020-2040. The solid line is the exponential fit.

the next oil shock around 2035-2040.

2. Analysis of net production and energy benefit

In order to evaluate the future dynamic of oil extraction based on this scenario, numerical simulations are done with a 0.5 year time step and a Runge-Kutta 4 method, starting in 2020 (using the exponential fit for k_{eff}), after a numerical validation on the period 1930-2019 (which used the history of k_{eff}). The mean relative error between the computed production and production history is less than 4% on the validation period. The mean relative error between the computed production and the business as usual is less than 2% on the projected period until the tipping point, as explained in the next section. The results presented here are based on historical data for the period 1943-2019 and based on numerical simulations for the period

2020-2080.

2.1. The tipping point

In order to evaluate how long the business as usual of the oil industry can be sustained, it is suggested to study the absolute energy benefit of the oil industry, which reads $\epsilon_A^n = (1 - k_0^n)Q_n^n\Delta t$ (in Gbbl). As observed in Fig.4, where ϵ_A has been normalized by its maximum value, it reaches this maximum between 2032 and 2039, at $ERoEI = 9 - 10$, for $ERoEI$ dynamics from [14]. The results obtained with the data of [13] lies between the two extreme results, with a maximum value for ϵ_A in 2036, at $ERoEI = 9.7$. Additional data provided by [10] give a maximum value for ϵ_A in 2026 (way sooner than the prediction of the other $ERoEI$ dataset), at $ERoEI = 12$. It suggests that, contrary to the oil shock prediction, an accurate prediction of the tipping point requires to properly take into account the increase in unconventional oil extraction into the $ERoEI$ dataset. Based on this remark, one could suggest the tipping point is most likely to happen between 2026 and 2032, rather than between 2032 and 2039.

This is in line with [11, 15] who suggest the energy cliff happens at $ERoEI = 10$. It is the point at which industrial societies cannot keep growing the way it used to. When ϵ_A begins to decrease, the oil industry cannot grow anymore, even with a gross product that keeps growing linearly. It suggests the oil business will not be viable anymore as it will get a decreasing benefit while requiring an increasing investment. Besides, k_{eff} fits an exponential function, but the investment rate k_0 is also exponential only as long as Q_n/Q_g is constant. From that point, the effect of Q_n/Q_g cannot be neglected anymore (i.e. the energy cliff has been reached), therefore Δt_{shock} is not constant anymore and k_0 would have to double in about 16 years, then 12,6 and so on. Anyway, from that point, the business as usual cannot be sustained anymore, even with an exponential k_{eff} . As the oil industry can take a few years to recover from a shock, it might be impossible to recover from a shock when the next one happens. Such scenario is therefore physically possible, yet it seems economically impossible.

It is most likely that investment will go to other energy sources of higher $ERoEI$. Nevertheless, renewables with $ERoEI$ higher than 10 need their whole lifespan to generate the energy after the energy investment is done. It mean that the “classical” return on investment is faster with oil than with renewable, what might keep investing in oil to sustain a minimum level of

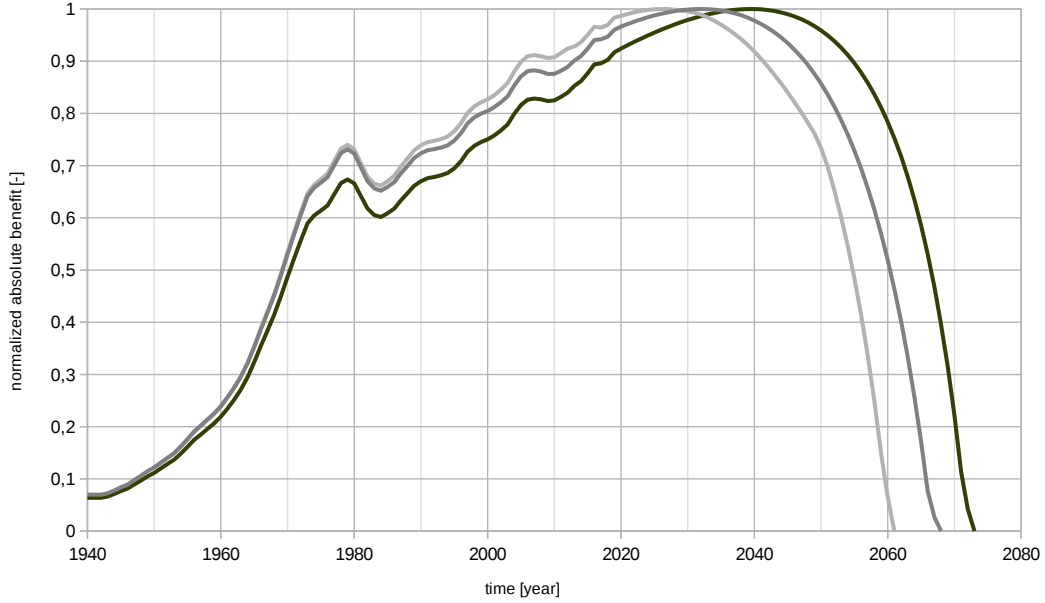


Figure 4: Normalized absolute energy benefit, for optimistic (black line) and pessimistic (dark grey line) $ERoEI$ dynamics from [14] and for supplementary data from [10] (light grey line)

energy if the development of renewable is not sufficient when this point is reached.

Two extreme scenarios are investigated, based on this analysis. The first one corresponds to $k_0 = 0$ after the tipping point (a transition scenario) and the second corresponds to a sustained linear gross product until $Q_n/Q_g = 0$ (a business as usual scenario).

2.2. Remaining net energy from liquid fossil fuel

In order to evaluate the net amount of energy that can be delivered by liquid fossil fuel, the net product is investigated. The future net product trend is firstly compared to past and nowadays trends. The total amount (i.e. integral or cumulated) of remaining net energy to the consumer is also compared to the net energy already used by our industrial societies. In every case, a range is suggested, based on the two proposed scenarios and the two extreme $ERoEI$ dynamics of [14]. The results obtained with the supplemen-

tary data of [10] are also presented in this section, allowing to evaluate how the increase in unconventional oil extraction affects this prediction.

Fig.5 shows the net energy from oil on the period 1943-2080. The solid lines represent the business as usual scenario and the dotted lines represent the transition scenario. One can see that the trend will remain almost the same from 2021 to the tipping point as it was on the period 1985-2020, suggesting the available energy evolution after the tipping point might be very abrupt (i.e. “we can’t see it coming based on the evolution of net available energy”). If the transition scenario happens, our industrial societies might struggle to adapt to such a steep drop of available energy. If the business as usual can be sustained for a few years, a plateau of net energy will occur before the drop. Either way, the slope of the drop is roughly twice the inverse of that of the rise.

Depending on the scenario and the *ERoEI* dynamic, the remaining net energy from 2021 to the end of oil extraction is in the range [220-680]Gbbbl: 540Gbbbl or 680Gbbbl (with [14] data), and 480Gbbbl (with the supplementary data of [10]) for the business as usual scenario. 320Gbbbl or 470Gbbbl (with [14] data), and 220Gbbbl (with the supplementary data of [10]) for the transition scenario. It corresponds to the net energy from 1930 or 2004 to 2021: From 1930 or 1980 to 2021 (with [14] data) and from 1980 to 2021 (with the supplementary data of [10]) for business as usual; From 1970 or 1995 to 2021 (with [14] data) and from 2004 to 2021 (with the supplementary data of [10]) for transition. Anyway, it means that the mid-point of net energy from oil has already been past. Most likely, the remaining net energy from oil is less than what we already used from the second oil shock until 2020.

It finally suggests the end of oil as an energy source around 2064-2072, whatever the scenario. It is worth noticing that taking into account the increase in unconventional oil extraction in the *ERoEI* dataset leads to lower prediction of the absolute remaining energy, especially in the case of the transition scenario for which the prediction is almost halved. However the prediction of relative net remaining energy (i.e. the ratio of remaining energy on used energy) and the end of oil are way less sensitive.

Conclusion

This study proposes a production averaged model which allow to investigate the extraction dynamic of fossil liquid fuels. This dynamic follows a Lotka-Volterra predator equation with one extra parameter, hence the model

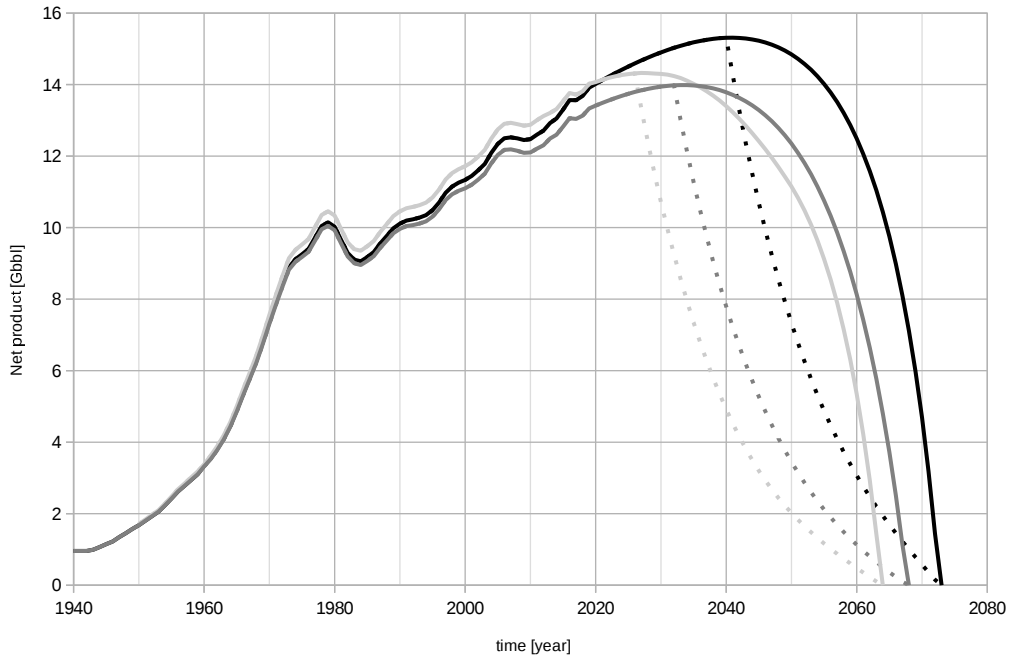


Figure 5: Net product: Energy delivered to the consumer, for optimistic (black line) and pessimistic (dark grey line) $ERoEI$ dynamics from [14] and for supplementary data from [10] (light grey line). The solid line is the business as usual scenario and the dotted line is the transition scenario.

has three parameters. The first one is strictly constant and the second one shows variations over time that are small enough to be neglected for the purpose of this study. An investigation of the remaining parameter dynamic, based on an inverse method, along with numerical simulations to forecast production, allows to highlight the following:

- A parameter controlled by the oil industry, the investment rate, drives the production dynamic.
- This rate has to evolve exponentially to keep the production growing linearly: the investment rate and its derivative roughly doubles every 32 ± 4 year.
- This behavior seems to be the explanation for oil shocks, hence the next one will occur around 2035-2040.

- It shows the existence of a tipping point between 2026 and 2039, at $ERoEI \sim 10$, before the peak of gross product, where the oil industry cannot grow anymore. It corresponds to the energy cliff; the $ERoEI$ value at the cliff is in accordance with predictions of prior work. From this point, investment might switch to other energy sources with higher $ERoEI$. If this change of investment happens immediately after the tipping point, an extremely fast decay of the net product is expected. Otherwise a small plateau is expected before a fast decay. Based on the model sensitivity to the increase in unconventional oil extraction, this point is most likely to happen between 2026 and 2032, rather than between 2032 and 2039.
- The net product of liquid fossil fuel will be close to zero around 2064-2072.
- Neglecting the increase in unconventional oil extraction in the $ERoEI$ dataset is an acceptable assumption to investigate the global trend of net production. However, an efficient prediction of the tipping point requires to take it into account.

Finally, the physical bases and the mathematical form of the model suggests this method applies to study the dynamics of other energy source production, such as gas, coal, nuclear or renewable.

Acknowledgments

Author would like to thanks Louis Delannoy for providing supplementary data from his work on oil aggregated EROEI modeling, and also the reviewers who helped enhancing this manuscript.

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