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A dynamic model for liquid fossil fuel production based on gross product/EROI coupling

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Abstract

Since 1940, many attempts to model world oil production have been proposed. These approaches, using increasing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its Energy Return On energy Invested (EROI). The model is based on mass and energy conservation and can be applied to all extracted liquid fossil fuels. After comparison with oil extraction and EROI dynamics, it highlights the existence of an external, controlling parameter: the investment rate, which account for the re-investment in newly operated liquid fuel sources. The dynamic of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the fossil liquid fuel production in order to sustain a globally linear production, starting around 1943: at short time scale (shorter than 30-36 years), the investment rate evolved linearly. However, in order to keep a linearly growing production at long time scale, the investment rate had to evolve exponentially: this was achieved through a piecewize linear control, where the investment rate and its derivative doubled every 30-36 years.

The model also allows to highlight a major issue in liquid fossil fuel production: even if the gross product can be controlled and keeps growing linearly, the net product, which account for the energy delivered by the oil industry to the consumer, can decrease before the gross product peaks, due to the decay of EROI. At this point, the energy benefit of the oil industry will

inevitably decrease and fossil liquid fuel production will slow down. Based on the present model and a sensibility study on its parameters, this tipping point will happen between 2032 and 2039. Net product of fossil liquid fuels could therefore keep growing linearly until this point, where a steep decay is expected. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model.

Keywords: Oil production, EROI, dynamic model, investment rate

¹ Introduction

2 Models that account for oil production have been published from 1962 [1], ³ with increasing complexity $(2, 3, 4]$, citing only very few of them). These models rely on a production dynamic with constant parameters. The aim of this study will be to evaluate how the parameters could evolve in time, based on a coupling between oil production (all extracted liquid fuels) and its Energy Return On energy Invested (EROI) at the wellhead, as defined in $[5]$ or $[6]$. Through this dependency, it is expected to explain why the prediction of peak is always delayed. For simplicity, "all liquid fuels" in the following refers to "all extracted liquid fuels" or "all liquid fossil fuels".

 The model suggested is based on mass and energy conservation for liquid fuel gross production. It is worth noticing that the structure of the obtained ODE ressembles a predator type equation of a Lotka-Volterra set. In this model, production of liquid fuels Q appears to be the EROI predator as the production "feeds" on EROI to grow. It is in line with former use of Lotka- Volterra equations to model dynamic systems in ecology [7] or in economy $17 \quad [8]$.

 The article is organised as follow: A first part is dedicated to a presenta- tion of an assumption on a mean global EROI for liquid fuels, suggesting why the model applies to the production of all liquid fossil fuels. It also presents a discussion on a production averaged EROI and its consequence in term of production modelling. The gross product equation is then derived. A fitting of the model parameters based on historical dynamics of oil production and mean EROI is performed, using an inverse method for the investment rate. An analysis of the latter, a forcing parameter, is done, suggesting a control of the investment to keep a linearly growing production. Some possible links between this control and the oil shocks are also presented, suggesting the next oil shock will occur around 2039.

 A second part is dedicated to studying net liquid fuel production and energy benefit. It higlights the existence of a tipping point between 2032 and 2039, where net product will begin to decrease. It suggests two potential limit scenarios for liquid fossil fuel production after this point, which are investigated. This analysis suggests that net energy from fossil liquid fuel to the consumer around 2060-2072, will be that of 1943, just a few years before going down to zero.

1. Modelling the interaction between oil production and EROI

 This section is dedicated to the description of the interactions between production of all liquid fuels Q (in Gbbl) and EROI at the wellhead, as $_{39}$ defined in [5] or [6].

1.1. An assumption about a global, production averaged, EROI at the well-head

 The dynamic modelling of all liquid fuels presented in this study requires an EROI function that would be representative of a mean global EROI of all ⁴⁴ liquid fossil fuels. Considering N liquid fuel sources in the world, $EROI_i$ the ⁴⁵ EROI of a given source and Q_i its production, mass and energy conservation requires to take $EROI = \frac{1}{C}$ ⁴⁶ requires to take $EROI = \frac{1}{Q} \sum_{i=1}^{N} Q_i \cdot EROI_i$, as it allows to derive a single equation for the production of all liquid fuels, instead of having a set of N equations.

49 Such EROI data seem impossible to gather, since $EROI_i$ are unlikely to be available, therefore it seems adequate to look for some existing model of global, mean EROI, instead of reconstructing EROI based on $EROI_i$ and \mathcal{Q}_i . It could however be aggregated by oil categories instead of oil sources, $\frac{1}{53}$ in considering i categories of liquid fuels (considering conventional crude oil, shale oil, off-shore, ultra-deep water oil, etc...). Since fossil liquid fuels production is still nowadays mainly composed of conventional crude oil, using EROI values of conventional oil for an aggregated, global model seems a fair assumption. It would obviously lead to a slight overestimation, since non conventional fossil fuels have lower EROI than conventional [9].

 The physical model suggested by [10], despite its basic development, fits the requirements presented above. It provides values of EROI that account for the entire crude oil production and presents values at different times which are consistent with actual, measured values of active oil sources. Hence, it will be used in this study, due to the lack of other available, consistent EROI ⁶⁴ data on a period which is long enough to fit the model parameters with an ϵ ₆₅ inverse method. Using the values of [10] to calibrate this model will certainly ⁶⁶ not allow to estimate precisely the investment rate time evolution, but it $67 \text{ should allow to evaluate its global trend over the last decades to estimate its$ $\frac{68}{100}$ evolution in the forthcoming decade(s).

⁶⁹ *1.2. A dynamic model for oil production and ERoEI evolution*

⁷⁰ In order to derive the production equation, an energy balance is consid-⁷¹ ered over the whole set of liquid fuel sources, based on a one year time laps τ_2 ($\Delta t = 1$ year). The ODE is then derived taking $\Delta t \rightarrow dt$. For simplicity, ⁷³ the balance is based on gross product Q_q and the net product Q_n is deduced 74 from Q_g afterwards:

 σ ₇₅ On a given year *n*, a gross product Q_g^n is extracted from the N liquid fuel ⁷⁶ sources. A fraction k_0 (the investment rate, in year⁻¹) of the associated net π product Q_n is used to extract liquid fuels from new sources.

The increase in gross product due to this investment is $k_0^n Q_n^n EROI^n \Delta t$, ⁷⁹ based on the definition of EROI at the wellhead. The net product can ⁸⁰ be calculated based on the "losses" of the oil industry, as in [10]: From 81 [11, 12, 13, 14], it is possible to estimate the cost of refinery, transportation ⁸² infrastructures, transportation to consumer and the loss in non-fuel refinery \mathbf{B} s products. Let c_t be the relative cost for all the previously listed "losses". 84 One gets $c_t = 0.54$, according to [12], Table 3. Energy conservation implies ⁸⁵ that the remaining part of the gross product is composed of the extrac-⁸⁶ tion/production cost plus the net product (or net energy to the consumer), ⁸⁷ hence:

$$
Q_g = Q_n + Q_{ext} + Q_t,\tag{1}
$$

88 with Q_g the gross product, Q_n the net product, Q_{ext} the extraction/production 89 cost and Q_t the previously listed costs, with $Q_t = c_t Q_g$. According to EROI 90 definition, $Q_{ext} = Q_g/EROI$, hence:

$$
Q_n = \frac{EROI(1 - c_t) - 1}{EROI} Q_g.
$$
\n
$$
(2)
$$

91 Therefore the increase in gross product at the next time laps is $k_0^n Q_g^n (ERoEI^n(1-p))$ 92 c_t) – 1) Δt .

⁹³ During the same time laps, the producing fuel sources show a decline 94 which follows the model described in Sorrell [15]: Considering k_1 the mean 95 oil source decline rate (in year⁻¹), the associate decrease in production is ⁹⁶ equal to $Q_g^{n+1} - Q_g^n = -k_1 \cdot \Delta t$.

⁹⁷ Both phenomena occur at the same time, during the same time laps. Since ⁹⁸ they are linear, it is possible to use superimposition to get: $Q_g^{n+1} - Q_g^n =$ 99 $Q_g^n[k_0^n(EROI^n(1-c_t)-1)-k_1]\Delta t$. Taking $\Delta t \to dt$ leads to:

$$
\dot{Q}_g = k_0 Q_g (EROI(1 - c_t) - 1) - k_1 Q_g \,. \tag{3}
$$

¹⁰⁰ *1.3. Fitting the model parameters on the period 1943-2018*

 Since the study [10] provides an EROI dynamic from 1930, k_0 could be 102 investigated on the period 1930-2018, for which both EROI and Q_g dynamics are available. The analysis however focus on a period which begins in 1943. Indeed, the production curve shows a clear change of production rate in 1943. From that time, the gross product is mostly linear, with several overshots and undershots. Using Hubbert's logistic fit as in [16], this part of the curve is very close from being linear. Also, a linear fit of the gross product on the period 1943-2019 shows a determination coefficient higher than 0.95. These remarks suggest that the business as usual of the oil industry began around 1943. This is why the investment rate is investigated on the period 1943-2019, in order to evaluate how this business as usual is sustained.

 k_1 represents the oil sources mean decline rate. This parameter should be ¹¹³ extracted from experimental measurements, using inverse methods. Based $_{114}$ on the results of [15], the mean value lies in the range $4.1 - 6.7\%$ but is ¹¹⁵ increasing with the exploitation of new non-conventional sources. Therefore, $_{116}$ k₁ is set equal to 6% (a mean value based on previous remark) and k₀ can ¹¹⁷ be fitted.

118 Now, based on Eq.(3), setting a value for k_1 , it is possible to plot k_0 119 dynamic, based on historical data for Q_g and a model for EROI:

$$
k_0 = \frac{\dot{Q}_g}{Q_g(EROI(1-c_t) - 1)} + \frac{k_1}{EROI(1-c_t) - 1)}.
$$
 (4)

120 The oil production data is extracted from [16, 17]. As stated in section $121 \cdot 11$, the EROI dataset is taken from [10], considering both the optimistic and ¹²² the pessimistic fit. The analysis is performed on the period 1943-2019 using ¹²³ a three-year averaging on Q_g and a second order upwind method to calculate $_{124}$ Q_g derivative.

¹²⁵ *1.4. Studying the investment rate*

¹²⁶ *1.4.1. History of the investment rate*

127 The investment rate can be evaluated through the value of k_0 , which lies in the range [0; 1[. It is nevertheless suggested to study $k_{eff} = k_0 \frac{EROI(1-c_t)-1}{EROI}$

 $k_0 \frac{Q_n}{Q_n}$ ¹²⁹ $k_0 \frac{Q_n}{Q_g}$ instead of k_0 , for function fitting requirements. The parameter k_{eff} , which represents the forcing of the system is plot over time in Fig.1. Its analysis provides some possible characteristics of the oil extraction strategy, which are presented below.

133 On the period 1943-1968, k_{eff} is relatively linear, as Fig.1 shows. In 1969, k_{eff} begins to decrease, most likely due to the geopolitical event that led to the first oil shock. This shock is then responsible for an instantaneous ¹³⁶ drop of k_{eff} in 1973, but k_{eff} is kept constant just after the shock, hence no clear disruption appears in oil production. Due to the mathematical ¹³⁸ structure of the model, a constant k_{eff} is not sufficient to sustain a linearly growing production for more than a few years. This is likely to be the cause ¹⁴⁰ of the second oil shock, which corresponds to another, longer drop of k_{eff} . Contrary to the first shock, it creates a disruption both in investment rate and oil production from 1979 to 1985, where investment is way under its mean value, as observed in Fig.1. It is therefore suggested here that the first shock is mainly geopolitic, whereas the second one is mainly economic.

145 On the period 1943-1978, k_{eff} is mostly linear, despite the first shock. The solid line fits the data with a mean relative error equal to 13%.

 After the second shock, k_{eff} seems to evolve (globally) linearly, with raises and plateaus during the period 1986-2006. The dashed line fits the data with a mean relative error of less than 5%. Compared to the period 1943-1978, ¹⁵⁰ one can see that the slope has roughly doubled. This behaviour allows Q_q to $_{151}$ grow linearly in time again. The plateaus of k_{eff} are responsible every time $\frac{1}{152}$ for a slow damping of Q_q , which corresponds to past predictions of a nearby peak, using Hubbert's curves. This phenomenon leads every time to an economical recession and a raise in oil price (this can be shown in comparing Fig.1 with an oil price chart), at the moment where the oil industry needs ¹⁵⁶ to increase its investment to keep k_{eff} close to the behavior that ensure a linearly growing Q_q . The following assumption is suggested to explain the origin of this raise/plateaus dynamic: With time, the production of an oil source eventually decreases, meaning that exploration is firstly needed to extract more oil. It means that if exploration does not suggests new sources to exploit, the production stagnates because it is not possible to invest in new sources, therefore k_{eff} is constant or slightly decreasing. When new sources become available, the investment rate can quickly increase until the new sources become less available and then exploration has to start again.

In 2007-2008, another oil shock happened, which again caused a sudden

¹⁶⁶ drop of k_{eff} . From this point, k_{eff} seems to be linear again, but again the ¹⁶⁷ slope has doubled in comparison to the previous period. It can be seen in ¹⁶⁸ Fig.1, where the dotted line fits the data with a mean relative error equal to 169 7% .

Figure 1: k_{eff} dynamic on the period 1943-2018. Symbols are measured values: diamonds on the period 1943-1978, crosses on the period 1979-1985, triangles on the period 1986- 2006 and stars on the period 2007-2019. Lines are linear fits: solid line on the period 1943-1978, dashed line on the period 1986-2006 and dotted line on the period 2007-2019.

¹⁷⁰ *1.4.2. The limit of a linear dynamic*

 171 Studying k_{eff} at long time scale shows the real dependency of Q_g on ¹⁷² k_{eff} : in order to keep a linearly growing Q_g , k_{eff} should follow a behavior ¹⁷³ close to an exponential. This corresponds to the description of the previous ¹⁷⁴ section: A piecewize linear behavior where the slope doubles at fixed time 175 interval. It means that in order to sustain a linear production dynamic, k_{eff} 176 should follow $k_{eff} \sim \exp(t/\tau)$ with $\tau = 48 \pm 4$ year, depending on the EROI 177 dynamic (optimistic or pessimistic) that is used to fit τ . Fig.2 shows k_{eff}

 on a period 1943-2040 along with a projection based on a linearly increasing production and the exponential fit. It means that the investment rate and 180 its derivative has to double every $\Delta t_{shock} = \tau \ln(2) = 33 \pm 3$ year, which fit both the time between the beginning of the business as usual in 1943 and the major disruption in oil production in 1979, and the time between this disruption and the oil shock of 2007-2008. It suggests this is the main reason why oil shocks have to happen. It would mean that the next oil shock will happen around 2040.

Figure 2: k_{eff} dynamic on the period 1943-2040. Diamonds are measured values on the period 1943-2019 and triangles are projected values on the period 2020-2040. The solid line is the exponential fit.

185

¹⁸⁶ 2. Analysis of net production and energy benefit

¹⁸⁷ In order to evaluate the future dynamic of oil extraction based on this ¹⁸⁸ scenario, numerical simulations are done with a 0.5 year time step and a Runge-Kutta 4 method, starting in 2020, after a numerical validation on the period 1930-2019. The results presented here are based on historical data for the period 1943-2019 and based on numerical simulations for the period 2020-2080.

2.1. The tipping point

 In order to evaluate how long the business as usual of the oil industry can be sustained, it is suggested to study the absolute energy benefit of the oil 196 industry, which reads $\epsilon_A = (1 - k_0)Q_n$. As observed in Fig.3, where ϵ_A has been normalized by its maximum value, it reaches this maximum between $198 \quad 2032$ and 2039, at EROI=9-10. This is in line with [12, 9] who suggest the energy cliff happens at EROI=10. It is the point at which industrial societies 200 cannot keep growing the way it used to. When ϵ_A begins to decrease, the oil industry cannot grow anymore, even with a gross product that keeps growing linearly. It suggests the oil business will not be viable anymore as it will get a decreasing benefit while requiring an increasing investment. Besides, k_{eff} fits an exponential function, but the investment rate k_0 is also 205 exponential only as long as Q_n/Q_q is constant. From that point, the effect of Q_n/Q_g cannot be neglected anymore (i.e. the energy cliff has been reached), ₂₀₇ therefore Δt_{shock} is not constant anymore and k_0 would have to double in about 16 years, then 12,6 and so on. As the oil industry can take a few years to recover from a shock, it might be impossible to recover from a shock when the next one happens. Such scenario is therefore physically possible, yet it seems economically impossible.

 It is most likely that investment will go to other energy sources of higher EROI. Nevertheless, renewables with EROI higher than 10 need their whole lifespan to generate the energy after the energy investment is done. It mean that the "classical" return on investment is faster with oil than with renew- able, what might keep investing in oil to sustain a minimum level of energy if the development of renewable is not sufficient when this point is reached.

 Two extreme scenarios are investigated, based on this analysis. The first 219 one corresponds to $k_0 = 0$ after the tipping point (a transition scenario) and 220 the second corresponds to a sustained linear gross product until $Q_n/Q_g = 0$ (a business as usual scenario).

2.2. Remaining net energy from liquid fossil fuel

 In order to evaluate the net amount of energy that can be delivered by liquid fossil fuel, the net product is investigated. The future net product

Figure 3: Normalized absolute energy benefit, for both optimistic (black line) and pessimistic (grey line) EROI dynamics

 trend is firstly compared to past and nowadays trends. The total amount (i.e. integral or cumulated) of remaining net energy to the consumer is also compared to the net energy already used by our industrial societies and finally a point to point comparison is done. In every case, a range is suggested, based on the two proposed scenarios and the two EROI dynamics used in this study. Fig.4 shows the net energy from oil on the period 1943-2080. The solid lines represent the business as usual scenario and the dotted lines represent the transition scenario. One can see that the trend will remain almost the same from 2021 to the tipping point as it was on the period 1985-2020, suggesting the available energy evolution after the tipping point might be very abrupt (i.e. "we can't see it coming based on the evolution of net available energy"). If the transition scenario happens, our industrial societies might struggle to adapt to such a steep drop of available energy. If the business as usual can be sustained for a few years, a plateau of net energy will occur before the drop. Either way, the slope of the drop is roughly twice the inverse of that of the rise.

 Depending on the scenario and the EROI dynamic, the remaining net en- ergy from 2021 to the end of oil extraction is in the range [320-680]Gbbl, with 540Gbbl and 680Gbbl for the business as usual scenario and with 320Gbbl and 470Gbbl for the transition scenario. It corresponds to the net energy from 1930 or 1995 to 2021 (1930 or 1980 to 2021 for business as usual and 1970 or 1995 to 2021 for transition). Either way, it means that the mid-point of net energy from oil has already been past. Most likely, the remaining net energy from oil is less than what we already used from the first oil shock until 2020.

 Fig.4 also highlights that the net available energy from oil in 2040-2050 will be that of 1970 with the transition scenario (in 2060-2068 with the busi- ness as usual scenario) and in 2055-2063 it will be that of 1943 with the transition scenario (in 2066-2072 with the business as usual scenario). It fi- nally suggests the end of oil as an energy source around 2068-2072, whatever the scenario.

Conclusion

 This study proposes a production averaged model which allow to inves- tigate the extraction dynamic of fossil liquid fuels. This dynamic follows a typical Lotka-Volterra equation. An investigation of its parameter dynamic, based on an inverse method, along with numerical simulations to forecast production, allows to highlight the following:

- \bullet A parameter controlled by the oil industry, the investment rate, drives the production dynamic.
- ²⁶⁴ This rate has to evolve exponentially to keep the production growing linearly: the investment rate and its derivative roughly doubles every $_{266}$ 33 ± 3 year.
- This behavior seems to be the explanation for oil shocks, hence the next one will occur around 2039.
- It shows the existence of a tipping point between 2032 and 2039, at EROI=9-10, before the peak of gross product, where the oil industry cannot grow anymore. It corresponds to the energy cliff. From this point, investment might switch to other energy sources with higher

Figure 4: Net product: Energy delivered to the consumer, for both optimistic (black line) and pessimistic (grey line) EROI dynamics. The solid line is the business as usual scenario and the dotted line is the transition scenario.

 EROI. If this change of investment happens immediately after the tip- ping point, an extremely fast decay of the net product is expected. Otherwise a small plateau is expected before a fast decay.

 Either way, the net product of liquid fossil fuel will be zero around 2068-2072.

 Finally, the physical bases and the mathematical form of the model sug- gests this method applies to study the dynamics of other energy source pro-duction, such as gas, coal, nuclear or renewable.

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