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A dynamic model for liquid fossil fuel production based on gross product/EROI coupling

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Abstract

Since 1940, many attempts to model world oil production have been proposed. These approaches, using increasing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its Energy Return On energy Invested (EROI). The model is based on mass and energy conservation and can be applied to all extracted liquid fossil fuels. After comparison with oil extraction and EROI dynamics, it highlights the existence of an external, controlling parameter: the investment rate, which account for the re-investment in newly operated liquid fuel sources. The dynamic of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the fossil liquid fuel production in order to sustain a globally linear production, starting around 1943: at short time scale (shorter than 30-36 years), the investment rate evolved linearly. However, in order to keep a linearly growing production at long time scale, the investment rate had to evolve exponentially: this was achieved through a piecewize linear control, where the investment rate and its derivative doubled every 30-36 years.

The model also allows to highlight a major issue in liquid fossil fuel production: even if the gross product can be controlled and keeps growing linearly, the net product, which account for the energy delivered by the oil industry to the consumer, can decrease before the gross product peaks, due to the decay of EROI. At this point, the energy benefit of the oil industry will

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inevitably decrease and fossil liquid fuel production will slow down. Based on the present model and a sensibility study on its parameters, this tipping point will happen between 2032 and 2039. Net product of fossil liquid fuels could therefore keep growing linearly until this point, where a steep decay is expected. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model.

Keywords: Oil production, EROI, dynamic model, investment rate

1 Introduction

Models that account for oil production have been published from 1962 [1], 2 with increasing complexity ([2, 3, 4], citing only very few of them). These 3 models rely on a production dynamic with constant parameters. The aim of this study will be to evaluate how the parameters could evolve in time, 5 based on a coupling between oil production (all extracted liquid fuels) and 6 its Energy Return On energy Invested (EROI) at the wellhead, as defined 7 in [5] or [6]. Through this dependency, it is expected to explain why the prediction of peak is always delayed. For simplicity, "all liquid fuels" in the 9 following refers to "all extracted liquid fuels" or "all liquid fossil fuels". 10

The model suggested is based on mass and energy conservation for liquid fuel gross production. It is worth noticing that the structure of the obtained ODE ressembles a predator type equation of a Lotka-Volterra set. In this model, production of liquid fuels Q appears to be the EROI predator as the production "feeds" on EROI to grow. It is in line with former use of Lotka-Volterra equations to model dynamic systems in ecology [7] or in economy [8].

The article is organised as follow: A first part is dedicated to a presenta-18 tion of an assumption on a mean global EROI for liquid fuels, suggesting why 19 the model applies to the production of all liquid fossil fuels. It also presents 20 a discussion on a production averaged EROI and its consequence in term of 21 production modelling. The gross product equation is then derived. A fitting 22 of the model parameters based on historical dynamics of oil production and 23 mean EROI is performed, using an inverse method for the investment rate. 24 An analysis of the latter, a forcing parameter, is done, suggesting a control 25 of the investment to keep a linearly growing production. Some possible links 26 between this control and the oil shocks are also presented, suggesting the 27 next oil shock will occur around 2039. 28

A second part is dedicated to studying net liquid fuel production and energy benefit. It higlights the existence of a tipping point between 2032 and 2039, where net product will begin to decrease. It suggests two potential limit scenarios for liquid fossil fuel production after this point, which are investigated. This analysis suggests that net energy from fossil liquid fuel to the consumer around 2060-2072, will be that of 1943, just a few years before going down to zero.

³⁶ 1. Modelling the interaction between oil production and EROI

This section is dedicated to the description of the interactions between production of all liquid fuels Q (in Gbbl) and EROI at the wellhead, as defined in [5] or [6].

40 1.1. An assumption about a global, production averaged, EROI at the well 41 head

The dynamic modelling of all liquid fuels presented in this study requires an EROI function that would be representative of a mean global EROI of all liquid fossil fuels. Considering N liquid fuel sources in the world, $EROI_i$ the EROI of a given source and Q_i its production, mass and energy conservation requires to take $EROI = \frac{1}{Q} \sum_{i=1}^{N} Q_i \cdot EROI_i$, as it allows to derive a single equation for the production of all liquid fuels, instead of having a set of N equations.

Such EROI data seem impossible to gather, since $EROI_i$ are unlikely to 40 be available, therefore it seems adequate to look for some existing model of 50 global, mean EROI, instead of reconstructing EROI based on $EROI_i$ and 51 Q_i . It could however be aggregated by oil categories instead of oil sources, 52 in considering i categories of liquid fuels (considering conventional crude 53 oil, shale oil, off-shore, ultra-deep water oil, etc...). Since fossil liquid fuels 54 production is still nowadays mainly composed of conventional crude oil, using 55 EROI values of conventional oil for an aggregated, global model seems a fair 56 assumption. It would obviously lead to a slight overestimation, since non 57 conventional fossil fuels have lower EROI than conventional [9]. 58

The physical model suggested by [10], despite its basic development, fits the requirements presented above. It provides values of EROI that account for the entire crude oil production and presents values at different times which are consistent with actual, measured values of active oil sources. Hence, it will be used in this study, due to the lack of other available, consistent EROI data on a period which is long enough to fit the model parameters with an inverse method. Using the values of [10] to calibrate this model will certainly not allow to estimate precisely the investment rate time evolution, but it should allow to evaluate its global trend over the last decades to estimate its evolution in the forthcoming decade(s).

⁶⁹ 1.2. A dynamic model for oil production and ERoEI evolution

In order to derive the production equation, an energy balance is considred over the whole set of liquid fuel sources, based on a one year time laps $(\Delta t = 1 \text{ year})$. The ODE is then derived taking $\Delta t \rightarrow dt$. For simplicity, the balance is based on gross product Q_g and the net product Q_n is deduced from Q_g afterwards:

On a given year n, a gross product Q_g^n is extracted from the N liquid fuel sources. A fraction k_0 (the investment rate, in year⁻¹) of the associated net product Q_n is used to extract liquid fuels from new sources.

The increase in gross product due to this investment is $k_0^n Q_n^n EROI^n \Delta t$, 78 based on the definition of EROI at the wellhead. The net product can 79 be calculated based on the "losses" of the oil industry, as in [10]: From 80 [11, 12, 13, 14], it is possible to estimate the cost of refinery, transportation 81 infrastructures, transportation to consumer and the loss in non-fuel refinery 82 products. Let c_t be the relative cost for all the previously listed "losses". 83 One gets $c_t = 0.54$, according to [12], Table 3. Energy conservation implies 84 that the remaining part of the gross product is composed of the extrac-85 tion/production cost plus the net product (or net energy to the consumer), 86 hence: 87

$$Q_g = Q_n + Q_{ext} + Q_t, \tag{1}$$

with Q_g the gross product, Q_n the net product, Q_{ext} the extraction/production cost and Q_t the previously listed costs, with $Q_t = c_t Q_g$. According to EROI definition, $Q_{ext} = Q_g / EROI$, hence:

$$Q_n = \frac{EROI(1-c_t)-1}{EROI}Q_g.$$
(2)

⁹¹ Therefore the increase in gross product at the next time laps is $k_0^n Q_g^n (ERoEI^n(1-c_t)-1)\Delta t$.

During the same time laps, the producing fuel sources show a decline which follows the model described in Sorrell [15]: Considering k_1 the mean oil source decline rate (in year⁻¹), the associate decrease in production is equal to $Q_g^{n+1} - Q_g^n = -k_1 \cdot \Delta t$. ⁹⁷ Both phenomena occur at the same time, during the same time laps. Since ⁹⁸ they are linear, it is possible to use superimposition to get: $Q_g^{n+1} - Q_g^n =$ ⁹⁹ $Q_q^n [k_0^n (EROI^n(1-c_t)-1) - k_1] \Delta t$. Taking $\Delta t \to dt$ leads to:

$$\dot{Q}_g = k_0 Q_g (EROI(1-c_t)-1) - k_1 Q_g .$$
 (3)

100 1.3. Fitting the model parameters on the period 1943-2018

Since the study [10] provides an EROI dynamic from 1930, k_0 could be 101 investigated on the period 1930-2018, for which both EROI and Q_q dynamics 102 are available. The analysis however focus on a period which begins in 1943. 103 Indeed, the production curve shows a clear change of production rate in 1943. 104 From that time, the gross product is mostly linear, with several overshots 105 and undershots. Using Hubbert's logistic fit as in [16], this part of the curve 106 is very close from being linear. Also, a linear fit of the gross product on the 107 period 1943-2019 shows a determination coefficient higher than 0.95. These 108 remarks suggest that the business as usual of the oil industry began around 109 1943. This is why the investment rate is investigated on the period 1943-2019, 110 in order to evaluate how this business as usual is sustained. 111

 k_1 represents the oil sources mean decline rate. This parameter should be extracted from experimental measurements, using inverse methods. Based on the results of [15], the mean value lies in the range 4.1 - 6.7% but is increasing with the exploitation of new non-conventional sources. Therefore, k_1 is set equal to 6% (a mean value based on previous remark) and k_0 can be fitted.

Now, based on Eq.(3), setting a value for k_1 , it is possible to plot k_0 dynamic, based on historical data for Q_g and a model for EROI:

$$k_0 = \frac{\dot{Q}_g}{Q_g(EROI(1-c_t)-1)} + \frac{k_1}{EROI(1-c_t)-1)} \,. \tag{4}$$

The oil production data is extracted from [16, 17]. As stated in section 121 1.1, the EROI dataset is taken from [10], considering both the optimistic and 122 the pessimistic fit. The analysis is performed on the period 1943-2019 using 123 a three-year averaging on Q_g and a second order upwind method to calculate 124 Q_g derivative.

125 1.4. Studying the investment rate

126 1.4.1. History of the investment rate

¹²⁷ The investment rate can be evaluated through the value of k_0 , which lies in ¹²⁸ the range [0; 1[. It is nevertheless suggested to study $k_{eff} = k_0 \frac{EROI(1-c_t)-1}{EROI} =$ $k_0 \frac{Q_n}{Q_g}$ instead of k_0 , for function fitting requirements. The parameter k_{eff} , which represents the forcing of the system is plot over time in Fig.1. Its analysis provides some possible characteristics of the oil extraction strategy, which are presented below.

On the period 1943-1968, k_{eff} is relatively linear, as Fig.1 shows. In 133 1969, k_{eff} begins to decrease, most likely due to the geopolitical event that 134 led to the first oil shock. This shock is then responsible for an instantaneous 135 drop of k_{eff} in 1973, but k_{eff} is kept constant just after the shock, hence 136 no clear disruption appears in oil production. Due to the mathematical 137 structure of the model, a constant k_{eff} is not sufficient to sustain a linearly 138 growing production for more than a few years. This is likely to be the cause 139 of the second oil shock, which corresponds to another, longer drop of k_{eff} . 140 Contrary to the first shock, it creates a disruption both in investment rate 141 and oil production from 1979 to 1985, where investment is way under its 142 mean value, as observed in Fig.1. It is therefore suggested here that the first 143 shock is mainly geopolitic, whereas the second one is mainly economic. 144

¹⁴⁵ On the period 1943-1978, k_{eff} is mostly linear, despite the first shock. ¹⁴⁶ The solid line fits the data with a mean relative error equal to 13%.

After the second shock, k_{eff} seems to evolve (globally) linearly, with raises 147 and plateaus during the period 1986-2006. The dashed line fits the data with 148 a mean relative error of less than 5%. Compared to the period 1943-1978, 149 one can see that the slope has roughly doubled. This behaviour allows Q_q to 150 grow linearly in time again. The plateaus of k_{eff} are responsible every time 151 for a slow damping of Q_q , which corresponds to past predictions of a nearby 152 peak, using Hubbert's curves. This phenomenon leads every time to an 153 economical recession and a raise in oil price (this can be shown in comparing 154 Fig.1 with an oil price chart), at the moment where the oil industry needs 155 to increase its investment to keep k_{eff} close to the behavior that ensure a 156 linearly growing Q_q . The following assumption is suggested to explain the 157 origin of this raise/plateaus dynamic: With time, the production of an oil 158 source eventually decreases, meaning that exploration is firstly needed to 159 extract more oil. It means that if exploration does not suggests new sources 160 to exploit, the production stagnates because it is not possible to invest in 161 new sources, therefore k_{eff} is constant or slightly decreasing. When new 162 sources become available, the investment rate can quickly increase until the 163 new sources become less available and then exploration has to start again. 164

In 2007-2008, another oil shock happened, which again caused a sudden

drop of k_{eff} . From this point, k_{eff} seems to be linear again, but again the slope has doubled in comparison to the previous period. It can be seen in Fig.1, where the dotted line fits the data with a mean relative error equal to 7%.

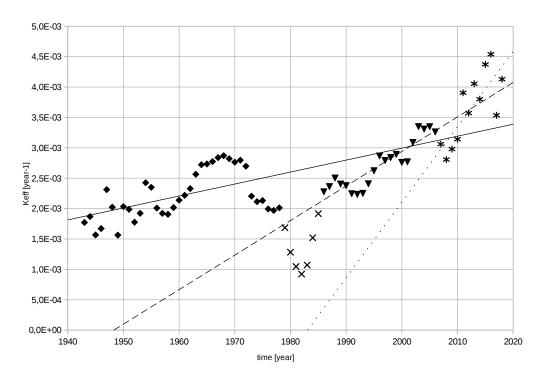


Figure 1: k_{eff} dynamic on the period 1943-2018. Symbols are measured values: diamonds on the period 1943-1978, crosses on the period 1979-1985, triangles on the period 1986-2006 and stars on the period 2007-2019. Lines are linear fits: solid line on the period 1943-1978, dashed line on the period 1986-2006 and dotted line on the period 2007-2019.

170 1.4.2. The limit of a linear dynamic

Studying k_{eff} at long time scale shows the real dependency of Q_g on k_{eff} : in order to keep a linearly growing Q_g , k_{eff} should follow a behavior close to an exponential. This corresponds to the description of the previous section: A piecewize linear behavior where the slope doubles at fixed time interval. It means that in order to sustain a linear production dynamic, k_{eff} should follow $k_{eff} \sim \exp(t/\tau)$ with $\tau = 48 \pm 4$ year, depending on the EROI dynamic (optimistic or pessimistic) that is used to fit τ . Fig.2 shows k_{eff} on a period 1943-2040 along with a projection based on a linearly increasing production and the exponential fit. It means that the investment rate and its derivative has to double every $\Delta t_{shock} = \tau \ln(2) = 33 \pm 3$ year, which fit both the time between the beginning of the business as usual in 1943 and the major disruption in oil production in 1979, and the time between this disruption and the oil shock of 2007-2008. It suggests this is the main reason why oil shocks have to happen. It would mean that the next oil shock will happen around 2040.

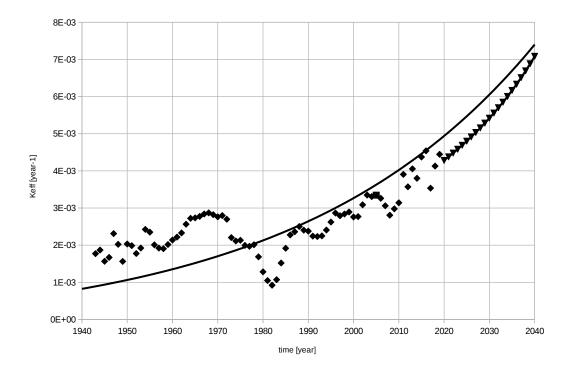


Figure 2: k_{eff} dynamic on the period 1943-2040. Diamonds are measured values on the period 1943-2019 and triangles are projected values on the period 2020-2040. The solid line is the exponential fit.

185

¹⁸⁶ 2. Analysis of net production and energy benefit

¹⁸⁷ In order to evaluate the future dynamic of oil extraction based on this ¹⁸⁸ scenario, numerical simulations are done with a 0.5 year time step and a Runge-Kutta 4 method, starting in 2020, after a numerical validation on the
period 1930-2019. The results presented here are based on historical data
for the period 1943-2019 and based on numerical simulations for the period
2020-2080.

193 2.1. The tipping point

In order to evaluate how long the business as usual of the oil industry can 194 be sustained, it is suggested to study the absolute energy benefit of the oil 195 industry, which reads $\epsilon_A = (1 - k_0)Q_n$. As observed in Fig.3, where ϵ_A has 196 been normalized by its maximum value, it reaches this maximum between 197 2032 and 2039, at EROI=9-10. This is in line with [12, 9] who suggest the 198 energy cliff happens at EROI=10. It is the point at which industrial societies 199 cannot keep growing the way it used to. When ϵ_A begins to decrease, the 200 oil industry cannot grow anymore, even with a gross product that keeps 201 growing linearly. It suggests the oil business will not be viable anymore 202 as it will get a decreasing benefit while requiring an increasing investment. 203 Besides, k_{eff} fits an exponential function, but the investment rate k_0 is also 204 exponential only as long as Q_n/Q_g is constant. From that point, the effect of 205 Q_n/Q_g cannot be neglected anymore (i.e. the energy cliff has been reached), 206 therefore Δt_{shock} is not constant anymore and k_0 would have to double in 207 about 16 years, then 12,6 and so on. As the oil industry can take a few years 208 to recover from a shock, it might be impossible to recover from a shock when 209 the next one happens. Such scenario is therefore physically possible, yet it 210 seems economically impossible. 211

It is most likely that investment will go to other energy sources of higher EROI. Nevertheless, renewables with EROI higher than 10 need their whole lifespan to generate the energy after the energy investment is done. It mean that the "classical" return on investment is faster with oil than with renewable, what might keep investing in oil to sustain a minimum level of energy if the development of renewable is not sufficient when this point is reached.

Two extreme scenarios are investigated, based on this analysis. The first one corresponds to $k_0 = 0$ after the tipping point (a transition scenario) and the second corresponds to a sustained linear gross product until $Q_n/Q_g = 0$ (a business as usual scenario).

222 2.2. Remaining net energy from liquid fossil fuel

In order to evaluate the net amount of energy that can be delivered by liquid fossil fuel, the net product is investigated. The future net product

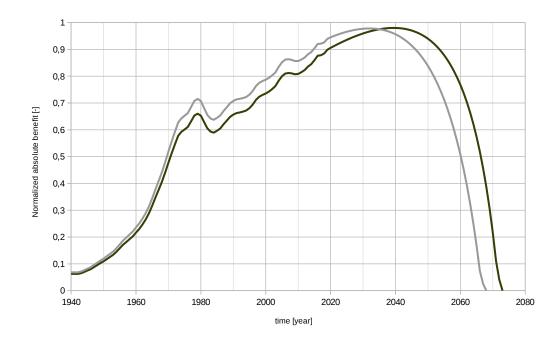


Figure 3: Normalized absolute energy benefit, for both optimistic (black line) and pessimistic (grey line) EROI dynamics

trend is firstly compared to past and nowadays trends. The total amount 225 (i.e. integral or cumulated) of remaining net energy to the consumer is also 226 compared to the net energy already used by our industrial societies and finally 227 a point to point comparison is done. In every case, a range is suggested, based 228 on the two proposed scenarios and the two EROI dynamics used in this study. 229 Fig.4 shows the net energy from oil on the period 1943-2080. The solid 230 lines represent the business as usual scenario and the dotted lines represent 231 the transition scenario. One can see that the trend will remain almost the 232 same from 2021 to the tipping point as it was on the period 1985-2020, 233 suggesting the available energy evolution after the tipping point might be 234 very abrupt (i.e. "we can't see it coming based on the evolution of net 235 available energy"). If the transition scenario happens, our industrial societies 236 might struggle to adapt to such a steep drop of available energy. If the 237 business as usual can be sustained for a few years, a plateau of net energy 238 will occur before the drop. Either way, the slope of the drop is roughly twice 239 the inverse of that of the rise. 240

Depending on the scenario and the EROI dynamic, the remaining net en-241 ergy from 2021 to the end of oil extraction is in the range [320-680]Gbbl, with 242 540Gbbl and 680Gbbl for the business as usual scenario and with 320Gbbl 243 and 470Gbbl for the transition scenario. It corresponds to the net energy 244 from 1930 or 1995 to 2021 (1930 or 1980 to 2021 for business as usual and 245 1970 or 1995 to 2021 for transition). Either way, it means that the mid-point 246 of net energy from oil has already been past. Most likely, the remaining net 247 energy from oil is less than what we already used from the first oil shock 248 until 2020. 240

Fig.4 also highlights that the net available energy from oil in 2040-2050 will be that of 1970 with the transition scenario (in 2060-2068 with the business as usual scenario) and in 2055-2063 it will be that of 1943 with the transition scenario (in 2066-2072 with the business as usual scenario). It finally suggests the end of oil as an energy source around 2068-2072, whatever the scenario.

256 Conclusion

This study proposes a production averaged model which allow to investigate the extraction dynamic of fossil liquid fuels. This dynamic follows a typical Lotka-Volterra equation. An investigation of its parameter dynamic, based on an inverse method, along with numerical simulations to forecast production, allows to highlight the following:

- A parameter controlled by the oil industry, the investment rate, drives the production dynamic.
- This rate has to evolve exponentially to keep the production growing linearly: the investment rate and its derivative roughly doubles every 33 ± 3 year.
- This behavior seems to be the explanation for oil shocks, hence the next one will occur around 2039.
- It shows the existence of a tipping point between 2032 and 2039, at EROI=9-10, before the peak of gross product, where the oil industry cannot grow anymore. It corresponds to the energy cliff. From this point, investment might switch to other energy sources with higher

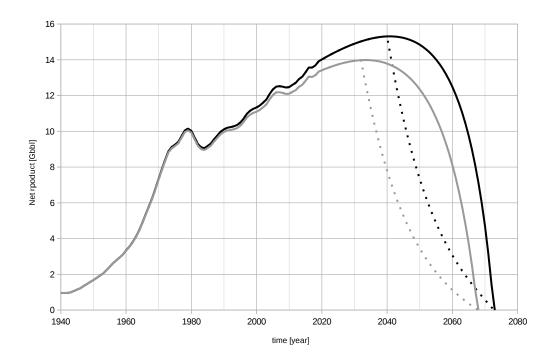


Figure 4: Net product: Energy delivered to the consumer, for both optimistic (black line) and pessimistic (grey line) EROI dynamics. The solid line is the business as usual scenario and the dotted line is the transition scenario.

EROI. If this change of investment happens immediately after the tipping point, an extremely fast decay of the net product is expected. Otherwise a small plateau is expected before a fast decay.

• Either way, the net product of liquid fossil fuel will be zero around 2068-2072.

Finally, the physical bases and the mathematical form of the model suggests this method applies to study the dynamics of other energy source production, such as gas, coal, nuclear or renewable.

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