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Aymeric Lamorlette. Forcing a dynamic model for oil production and EROEI evolution: The Oil Game. 2020. hal-02383025v5

HAL Id: hal-02383025

<https://hal.science/hal-02383025v5>

Preprint submitted on 20 Aug 2020 (v5), last revised 4 Jul 2023 (v7)

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Forcing a dynamic model for oil production and EROEI evolution: The Oil Game

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Abstract

Since 1940, many attempts to model world oil production have been proposed. These approaches, using increasing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its Energy Return on Energy Invested (ERoEI), based on mass and energy conservation. The EROEI equation is derived according to the second principle. It leads to a Lotka-Volterra set of equations, which can be applied to all extracted liquid fossil fuels. The model obtained, after comparison with oil extraction and EROEI evolution on the period 1960-2010, illustrates the production dynamic and the existence of an external, controlling parameter: the investment rate, which account for the re-investment in newly operating liquid fuel sources. The evolution of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the liquid fuel production in order to obtain a linear time evolution on the period 1960-2010 through an apparently linearly growing investment rate: the oil game. Unfortunately, in order to keep a linearly growing production at long time scale, the investment rate has actually to evolve exponentially: the linear growth is in fact a short time scale approximation of the control required to play the oil game. The model also allows to highlight a major issue in liquid fuel production: even if the gross product can be controlled and keeps growing linearly, the net product, which account for the energy delivered by the oil industry to the world, is falling down faster and faster, due to the decrease of EROEI. At some point, the

net energy benefit will be equal to zero and liquid fuel production will stop, except if energy is given to the oil industry to keep extracting oil. In anyway, liquid fuels would become an energy sink instead of an energy source. Based on the model presented in this study, this will happen between 2027 and 2033. Production of liquid fuels could therefore keep growing linearly until this point, where a quick collapse is expected. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model.

Keywords: Oil production, EROEI, dynamic model, investment rate

1 Introduction

2 Models that account for oil production have been published from 1962 [1],
3 with increasing complexity ([2, 3, 4], citing only very few of them). These
4 models rely on a production dynamic with constant parameters. The aim
5 of this study will be to evaluate how the parameters could evolve in time,
6 based on a coupling between oil production (all extracted liquid fuels) and
7 its Energy Return on Energy Invested (EROEI) at the wellhead, as defined
8 in [5] or [6]. Through this dependency, it is expected to explain why the
9 prediction of peak is always delayed. For simplicity, "all liquid fuels" in the
10 following refers to "all extracted liquid fuels" or "all liquid fossil fuels".

11 The model suggested is based on mass and energy conservation for the
12 production equation and derived in accordance with the second principle for
13 the EROEI equation. It is worth noticing that the structure of the obtained
14 ODE set is equivalent to a Lotka-Volterra set of equations, linking oil produc-
15 tion of all liquid fuels Q with its mean EROEI. In this model, Q appears to
16 be the EROEI predator as the production "feeds" on EROEI to grow. It is in
17 line with former use of Lotka-Volterra equations to model dynamic systems
18 in ecology [7] or in economy [8].

19 The article is organised as follow: A first part is dedicated to a pre-
20 sentation of an assumption on the oil distribution as a function of EROEI,
21 suggesting why the model applies to the production of all liquid fossil fu-
22 els. It also presents a discussion on a production averaged EROEI and its
23 consequence in term of production modelling. The set of equation is then
24 derived and the prey-predator analogy is presented. A fitting of the model
25 parameters based on historical evolution of oil production and mean EROEI
26 is then performed. An analysis of the investment rate, a forcing parameter,

27 is done, suggesting a control of the investment to keep a linearly growing
28 production: the oil game.

29 A second part is dedicated to the the study of net liquid fuel production,
30 relative energetic benefit and relative investment rate. This analysis sug-
31 gests some possible explanations for the evolution of world economy in the
32 beginning of the 80's, recent recessions and a future collapse of liquid fuel
33 production rather than a slow, progressive decline. It also gives an estima-
34 tion of the remaining "reachable" liquid fuels. Finally, a short discussion on
35 oil price evolution is presented.

36 1. Modelling the interaction between oil production and ERoEI

37 This section is dedicated to the description of the interactions between
38 production of all liquid fuels Q (in Gbbl) and ERoEI at the wellhead, as
39 defined in [5] or [6].

40 1.1. An assumption about a mean, production averaged, ERoEI at the well- 41 head

42 The global modelling of all liquid fuels requires the assumption that the
43 ERoEI considered is representative of the mean ERoEI of all liquid fossil fuels
44 at a given time. Considering N liquid fuel sources in the world, $ERoEI_i$ the
45 ERoEI of a given source and Q_i its production, it is here suggested to take
46 $ERoEI = \frac{1}{Q} \sum_{i=1}^N Q_i \cdot ERoEI_i$. This will allow to derive a single equation
47 for the production of all liquid fuels, instead of having a set of N equations.

48 Such ERoEI data seem impossible to gather, since $ERoEI_i$ are unlikely
49 to be available, therefore it seems adequate to look for some existing model of
50 global, mean ERoEI, instead of reconstructing ERoEI based on $ERoEI_i$ and
51 Q_i . The thermodynamic model suggested by [9], despite many theoretical
52 flaws in its development, is close to the definition above. It provides values
53 of ERoEI that account for the entire oil production and presents values at
54 different times which are consistent with actual, measured values of active oil
55 sources. Besides, it's mathematical form (the inverse of a logistic curve) fits
56 the behaviour of oil extraction, that is an infinite ERoEI before oil extraction
57 began and an ERoEI that tends toward zero when the production tends
58 toward zero. Hence, it will be used as a comparison in this study, due to
59 the lack of other available, consistent ERoEI data on the period 1960-2020.
60 Using the values of [9] to calibrate this model will certainly not allow to
61 estimate precisely the investment rate time evolution, but it should allow to

62 evaluate its global trend over the last decades to estimate its evolution in the
 63 forthcoming decade(s).

64 1.2. A dynamic model for oil production and EROEI evolution

65 1.2.1. Production equation

66 In order to derive the production equation, a mass balance is considered
 67 over the whole set of liquid fuel sources, based on a one year time laps ($\Delta t = 1$
 68 year). The ODE is then derived taking $\Delta t \rightarrow dt$. For simplicity, the balance
 69 is based on gross product Q_g and the net product Q_n is deduced from Q_g
 70 afterwards:

71 On a given year n , a gross product Q_g^n is extracted from the N liquid fuel
 72 sources. A fraction of this production k_0 (an investment rate, in year⁻¹) is
 73 used to extract liquid fuels from new sources. Let us consider Δh an energy
 74 density contained in the liquid fuel (similar to a heat of combustion).

75 The work W_{ex}^n available for extraction in the new sources is then $W_{ex}^n =$
 76 $k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$ with η being the efficiency of all the processes needed
 77 to turn the extracted liquid fuel into work. This includes: transportation of
 78 liquid fuel to refinery, refining, combustion and transformation of heat into
 79 work, but also the exploration and structure development (such as wells and
 80 platforms) required to get this amount of liquid fossil fuel. This efficiency
 81 has been studied for crude oil and is equal to 0.2045 according to Hill [9]. For
 82 all liquid fuels, due to the use of production averaged quantities, this value
 83 should be about the same.

84 According to the definition of EROEI at wellhead, with $ERoEI^n$ being
 85 the mean EROEI on year n , this work allows to get the following amount of
 86 energy at the next time laps: $ERoEI^n \cdot k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$, corresponding to
 87 an increase in production $\Delta^+ Q_g^{n+1}$ which follows $\Delta^+ Q_g^{n+1} \cdot \Delta h = ERoEI^n \cdot$
 88 $k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$.

89 From the initial gross product Q_g^n , it remains $(1 - k_0 \cdot \Delta t) \cdot Q_g^n$, there-
 90 fore, considering only the increase in production due to the newly exploited
 91 sources, one gets : $Q_g^{n+1} - Q_g^n = Q_g^n \cdot k_0 \cdot \Delta t \cdot (\eta \cdot ERoEI - 1)$.

92 During the same time laps, the producing fuel sources show a decline
 93 which follows the model described in Sorrell [10]: Considering k_1 as the
 94 mean oil source decline rate (in year⁻¹), the associate decrease in production
 95 is equal to $Q_g^{n+1} - Q_g^n = -k_1 \cdot \Delta t$.

96 Both phenomena occur at the same time, during the same time laps. Since
 97 they are linear, it is possible to use superimposition to get: $Q_g^{n+1} - Q_g^n =$

98 $Q_g^n \cdot [k_0 \cdot (\eta \cdot ERoEI - 1) - k_1] \cdot \Delta t$. Taking $\Delta t \rightarrow dt$ leads to:

$$\dot{Q}_g = k_0 \cdot Q_g \cdot (\eta \cdot ERoEI - 1) - k_1 \cdot Q_g. \quad (1)$$

99 Now, based on ERoEI definition, it is possible to derive net product Q_n
 100 from gross product Q_g : Since $ERoEI = \frac{Q_g}{W_{ext}}$, one can evaluate Q_{ext} , the
 101 amount of liquid fuel used for extraction: $ERoEI = \frac{Q_g}{\eta \cdot Q_{ext}}$. Since $Q_{ext} =$
 102 $Q_g - Q_n$, one gets: $Q_n = \frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI} Q_g$

103 1.2.2. ERoEI equation

104 Based on Eq.(1) and using a prey-predator analogy, Q_g seems to “feed”
 105 on ERoEI to grow. More precisely, according to Eq.(1) structure, the “natu-
 106 ral” prey of Q_g is $\eta \cdot ERoEI - 1$. Following this analogy, the prey should be
 107 decreasing proportionally to $\eta \cdot ERoEI - 1$ and Q_g , and should be grow-
 108 ing due to the renewal of fossil fuels. This is neglected since it can be
 109 considered as happening at geological times. Considering a decline rate
 110 k_2 (in (Gbblyear)⁻¹) for ERoEI, this rational leads to the following equa-
 111 tion: $\eta \cdot ERoEI = -k_2 \cdot Q_g \cdot (\eta \cdot ERoEI - 1)$, equivalent to $ERoEI =$
 112 $-k_2 \cdot Q_g \cdot ERoEI + \frac{k_2}{\eta} \cdot Q_g$. It is interesting to notice that if $k_2 \cdot Q_g = A$ with
 113 A being a constant (as it is the case here, according to Fig.2), this equation
 114 reads: $ERoEI = -A \cdot ERoEI + \frac{A}{\eta}$. It leads to the inverse of the logistic
 115 function, which is the solution obtained for ERoEI evolution in the work of
 116 Hill [9].

117 However, the obtained equation cannot fit the purpose here, since its
 118 structure is decoupling Q_g and ERoEI. In order to keep this coupling, the
 119 following form is retained:

$$ERoEI = -k_2 \cdot Q_g \cdot ERoEI. \quad (2)$$

120 Parameter k_2 is expected to decrease in time, according to the natural
 121 distribution of oil as a function of its availability on earth. In order to model
 122 k_2 , the following dependency is proposed: $k_2 = C/(t - t_0)$ where C is a
 123 constant (in (Gbblyear)⁻¹) and t_0 (in year) is a time offset. C can be interpreted
 124 as the effect of oil distribution as a function of ERoEI, regarding the rate at
 125 which oil is extracted. It suggests that the largest amount of oil on earth is
 126 available at the lowest ERoEI.

127 The work of Hill [9] nevertheless suggests that the “natural” physical cou-
 128 pling between ERoEI and Q_g is established through a distribution $ERoEI =$

129 $f(Q_g)$, which is the expression of the Etp equation, derived by Hill [9] based
 130 on the second principle. Therefore, this distribution is now studied to see
 131 how it could be a surrogate to Eq.(2).

132 In [9], the EROEI(t) function is derived based on a production which fol-
 133 lows a Hubbert's curve for crude oil only (cumulative product $Q_p = 2357, 15$
 134 Gbbl, peak at $t_m = 2001$, $Q_m = Q(t = t_m) = b \cdot Q_p/4$). Based on the previ-
 135 ous remarks, $Q_g = f^{-1}(ERoEI)$ can be explicitly derived for a Hubbert like
 136 extracting scenario:

$$Q_g(ERoEI) = b \cdot Q_p \frac{\left(\frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI_m - 1}\right)^{b/a}}{\left[1 + \left(\frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI_m - 1}\right)^{b/a}\right]^2}. \quad (3)$$

137 With $ERoEI_m = 13.3$ being the value of EROEI at time t_m and $a = 0.0537$
 138 the parameter of the EROEI solution (the inverse of a logistic function) cal-
 139 culated in [9]. Eq.(3) is general, but the values of Q_p and b are representative
 140 of a production which follow the Hubbert's scenario presented earlier. For
 141 all liquid fuels, this scenario is not realistic, at least for the values of Q_p and
 142 b previously suggested. Therefore, in order to estimate function f^{-1} for all
 143 liquid fuels, an adapted scenario has to be establish. This will be discuss
 144 later in this article, and distribution $ERoEI = f(Q_g)$ will be calculated for
 145 all liquid fuels.

146 1.3. Fitting the model parameters on the period 1960-2010

147 k_1 represents the oil sources mean decline rate. This parameter should be
 148 extracted from experimental measurements, using inverse methods. Based
 149 on the results of [10], the mean value lies in the range 4.1 – 6.7% but is
 150 increasing with the exploitation of new non-conventional sources. Therefore,
 151 k_1 is set equal to 6% (a mean value based on previous remark) and k_0 , k_2
 152 and $k_2 \cdot Q_g$ can be fitted. Now, based on Eq.(1) and (2), setting a value for
 153 k_1 , it is possible to plot k_0 and k_2 time evolution, based on historical data of
 154 Q_g and EROEI:

$$k_0 = \frac{\dot{Q}_g}{Q_g \cdot (\eta \cdot ERoEI - 1)} + \frac{k_1}{(\eta \cdot ERoEI - 1)}, \quad (4)$$

155

$$k_2 = -\frac{ERoEI \cdot \dot{ERoEI}}{Q_g \cdot ERoEI}. \quad (5)$$

156 The oil production data is extracted from [11]. As stated in section 1.1,
 157 the EROEI dataset is taken from [9]. The analysis is performed on the period
 158 1960-2010 using a three-year averaging on Q_g and a second order upwind
 159 method to calculate Q_g and EROEI derivatives. The evolution obtained for
 160 k_2 is presented in Fig.1. The continuous line represents the model $k_2 =$
 161 $C/(t - t_0)$, with $C = 0.06036 \text{ Gbbl}^{-1}$ and $t_0 = 1950.5 \text{ year}$. The model seems
 162 to fit adequately the data, with a mean relative error of 1.05%. The evolution
 163 shows two periods: The first one, before the first oil shock, corresponds to a
 164 rapid and smooth evolution of k_2 . The second one, after the second oil shock,
 165 shows some jumps which could correspond to the exploitation of fossil fuels
 166 that were not exploited before due to their low EROEI, in comparison with
 167 the mean EROEI of the moment. When these sources become of interest and
 168 start being exploited, the value of k_2 suddenly drops because exploiting these
 169 sources does not affect much the mean EROEI.

170 The evolution obtained for $k_2 \cdot Q_n$ is presented in Fig.2 to check that
 171 EROEI behaviour is similar to the inverse of a logistic curve.

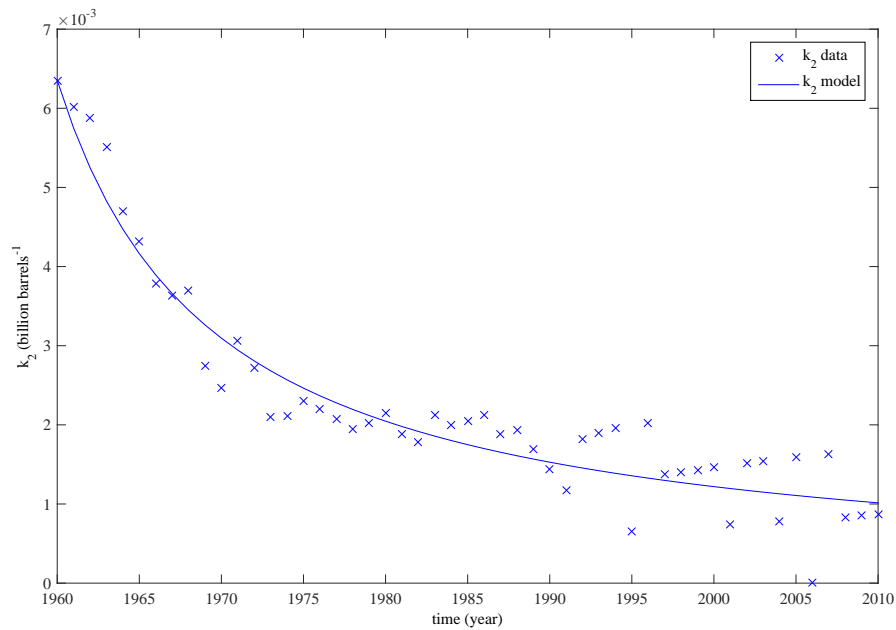


Figure 1: k_2 time evolution

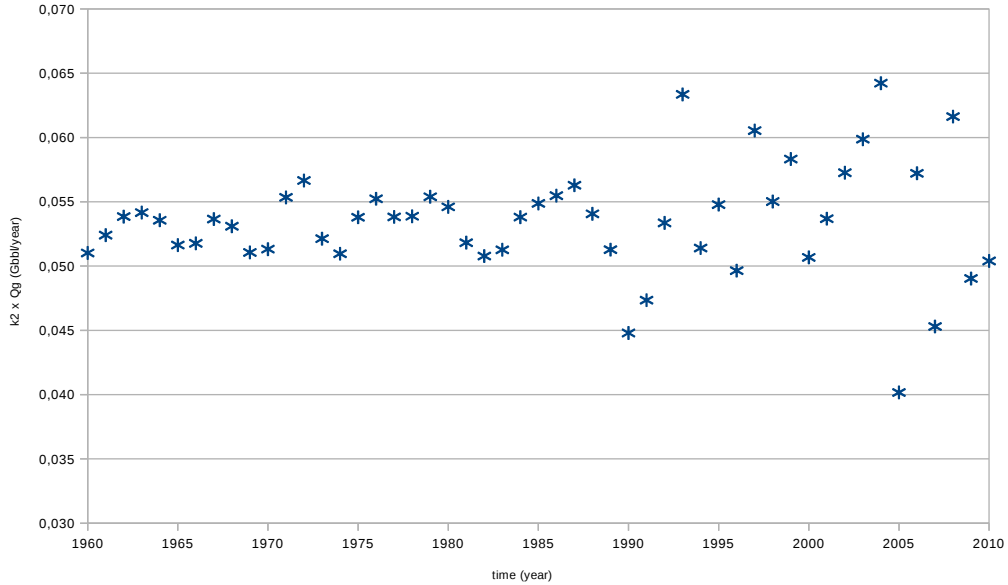


Figure 2: $k_2 \cdot Q_g$ time evolution

172 1.4. Studying the investment rate

173 1.4.1. History of the investment rate

174 The investment rate can be evaluated through the value of k_0 , which lies
 175 in the range $[0; 1[$. It is nevertheless suggested to study $k_{eff} = k_0 \cdot \frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI}$
 176 instead of k_0 , for function fitting requirements. The parameter k_{eff} , which
 177 represents the forcing of the system is plot over time in Fig.3. Its analysis
 178 provides some possible characteristics of the oil extraction strategy, which
 179 are presented below.

180 On the period 1960-1968, k_{eff} shows a relatively linear behaviour. This
 181 period corresponds to an evolution of oil extraction that begin to behave as
 182 exponential around 1965. Due to the laws of market, and the oil price at
 183 this period which is rather low, keeping an exponential growth for Q_g could
 184 have been responsible for an important decrease in oil price. In order to keep
 185 a decent benefit without using too much of their resources, producers have
 186 to reduce Q_g , by reducing k_{eff} . This strategy begins in 1969, according to
 187 Fig.3. However, due to the behaviour of k_2 at this time, the system shows a

188 great inertia and damping k_{eff} is not sufficient to control instantaneously Q_g .
189 Any reasons could have been sufficient to suddenly reduce k_{eff} and adapt
190 Q_g . Three years after the first inflection of k_{eff} , the first oil shock happen
191 and k_{eff} is adapted.

192 After the first shock, k_{eff} is surprisingly constant, with a linear time
193 evolution for Q_g . The second shock corresponds to another, longer drop of
194 k_{eff} .

195 After the second shock, k_{eff} seems to evolve (globally) linearly, with raises
196 and plateaus during the period 1985-2000. The solid line fits the data with
197 a mean relative error of less than 1%. This behaviour allows Q_g to grow
198 linearly in time. Also, since the system inertia has evolved in time with
199 k_2 , the plateaus are responsible every time for a slow damping of Q_g , which
200 corresponds to past predictions of a nearby peak, using Hubbert's curves.
201 This phenomenon leads every time to an economical recession and a raise in
202 oil price (this can be shown in comparing Fig.3 with an oil price chart), at
203 the moment where producers need to increase their investment to keep k_{eff}
204 close to the solid line that ensure a linearly growing Q_g . The origin of this
205 raise/plateaus dynamic can be explained the following way: With time, the
206 production of an oil source eventually decreases, meaning that exploration
207 is firstly needed to extract more oil. It means that if exploration does not
208 suggests new sources to exploit, the production stagnates because it is not
209 possible to invest in new sources, therefore k_{eff} is constant or slightly decreas-
210 ing. When new sources become available, the investment rate can quickly
211 increase until the new sources become less available and then exploration has
212 to start again.

213 1.4.2. Projections based on the constant 1985-2000 dynamic

214 Following this line using raises and plateaus, allow to optimize the oil
215 benefit and production: It could be compared to a game where k_{eff} should
216 be kept on this line to optimize benefits. This strategy can then be extended
217 to forthcoming years. One can observe that k_{eff} begins to deviate on the
218 period 2000-2010. It seems that, in order to keep a constant derivative for
219 Q_g , k_{eff} should not follow the same trend any more. The data of [12] is an
220 extension of [11] data. It is used to evaluate the evolution of this slope. On
221 the period 2000-2020, the slope seems to be different from the one observed
222 on the period 1985-2000. Instead of plateaus, between 2005 and 2010, drops
223 are required on k_{eff} to fasten the effect on Q_g , and the mean slope has to be
224 higher than before.

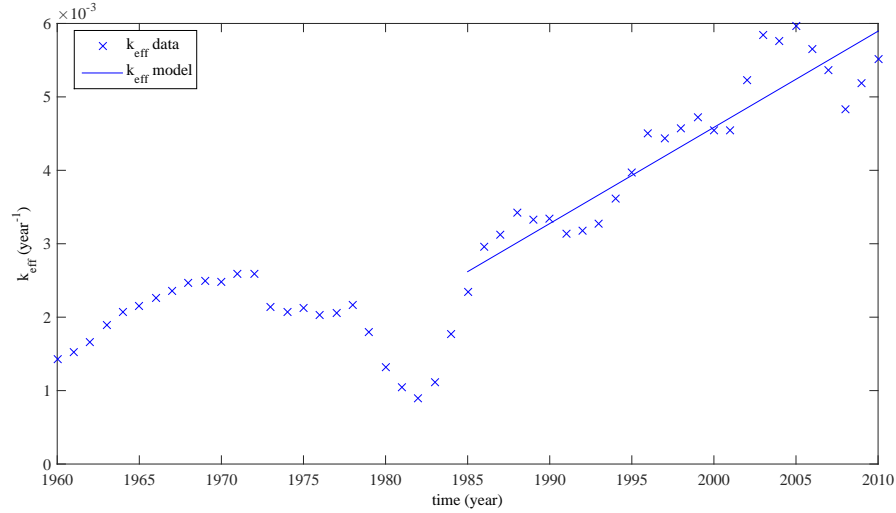


Figure 3: k_{eff} time evolution for $k_1 = 6\%$

225 *1.4.3. The limit of the 2000-2020 dynamic*

226 The extension of that game actually shows the real rule: in order to
 227 keep a linearly growing Q_g , k_{eff} has to evolve exponentially in time. To
 228 keep playing that game the way it started, k_{eff} should follow the equation
 229 $k_{eff} = 1.2650 \cdot 10^{-25} \exp(t/\tau)$ with $\tau = 37.32$ years. Fig.4 shows the extension
 230 of k_{eff} on the period 2005-2020 along with a projection using the exponential
 231 fit. This projection suggests that the slope has again to be inflected, by
 232 strongly inflecting the shoots between the plateaus/drops. It also requires
 233 exploration to be more and more efficient, to ensure that the plateaus won't
 234 last for too long. It is worth noticing that according to [13] for instance, the
 235 exact opposite is expected.

236 **2. Results and analysis**

237 *2.1. A business as usual scenario*

238 Based on Eq.(1), the threshold value of $ERoEI$ for which oil production
 239 cannot grow any more is $\eta^{-1} \sim 4.89$. Another threshold could be the relative
 240 investment rate which cannot exceed 1 (i.e. the net investment cannot exceed
 241 the net product). This relative investment k_r reads: $k_r = \frac{Q_g}{Q_n} \cdot k_0 = \frac{\eta \cdot ERoEI}{\eta \cdot ERoEI - 1} \cdot$
 242 k_0 . Supposing the exponential investment dynamic could be sustained until

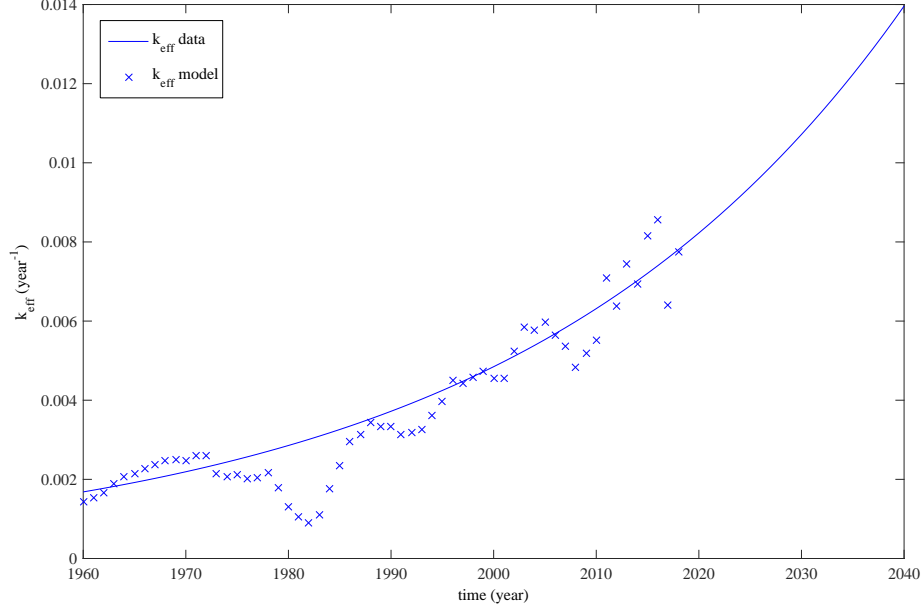


Figure 4: k_{eff} time evolution for $k_1 = 6\%$

243 the mean liquid fossil fuel $ERoEI$ reaches 4.89 or until the relative investment
 244 rate reaches 1 represents a business as usual scenario where oil is extracted
 245 until the energetic benefit drops to zero or until development of new sources
 246 becomes impossible. It could allow a linear growth of Q_g until 2040. This
 247 linear growth would be followed by a quick collapse of Q_g .

248 This business as usual scenario would be equivalent, for $t < 2040$ years to
 249 a Hubbert's curve with parameters : $Q_p = 5200$ Gbbl, a peak at $t_m = 2040$
 250 years and $b = 0.028$. Please note that $Q_p = 5200$ Gbbl has nothing to do with
 251 an amount of recoverable liquid fuels. This aspect will be discussed later.
 252 Using these parameters allows, based on the ERoEI model of Hill, to derive
 253 the associated distribution $Q_g = f(ERoEI)$. Now, in order to estimate the
 254 evolution of Q_g and ERoEI, one can use either Eq.(1) and Eq.(2), either
 255 Eq.(1) and Eq.(3) using the above values for Q_p and b .

256 In order to evaluate the end of oil extraction based on this scenario, three
 257 variables are studied: the net product Q_n , the relative energetic benefit $\epsilon_B =$
 258 $\frac{Q_n}{Q_g} = \frac{\eta ERoEI - 1}{\eta ERoEI}$ and the relative investment rate $k_r = \frac{k_0}{\epsilon_B}$. These variables are

259 studied with Eq.(1) and Eq.(2), then with Eq.(1) and Eq.(3). A 10% margin
260 on η is also considered, as this value has been fitted for crude oil and is now
261 applied to all liquid fuels.

262 2.2. Results

263 The numerical simulations are done with a 0.1 year time step and a
264 Runge-Kutta 4 method, starting in 2010, after a numerical validation on
265 the period 1960-2010. The first remark is that the results are within 3% re-
266 gardless the set of equations which is used. It suggests that the prey-predator
267 analogy makes sense, with Q_g being the predator of $ERoEI$. It also suggests
268 that this kind of Lotka-Volterra set of equations could apply to other energy
269 sources such as coal, gas and nuclear energy, if data such as $Q_g = f(ERoEI)$
270 are available, or can be derived based on thermodynamical considerations.

271 Plots are presented for the most pessimistic calculation (in term of re-
272 remaining liquid fuel) and all results are presented with a dispersion based on
273 the extreme calculations. Fig.5 shows the net product as a function of time.
274 It highlights that the maximum net energy delivered to the world by the oil
275 industry peaked in 1979. It also shows the real difference in energy delivered
276 by the oil industry to the world before and after the second oil shock, evol-
277 ving from a continuous increase to a steady state until 2000. The net product
278 begins to fall around 2000, but this is damped by the investment done in oil
279 production at this time. These investments seem to have maintain the net
280 product almost constant until 2005, at which point the net product start to
281 decline linearly, until 2017-2018 where the fall begins to accelerate. All these
282 remarks seem in line with the recessions that occurred since the year 2000.

283 Fig.6 shows the evolution of the relative investment rate. It highlights
284 the rate at which investment have to increase from now on to keep playing
285 the oil game. It also shows that the study of the net product hides the
286 potential investment issues: at some point it becomes impossible to invest
287 enough energy to keep the gross production increasing even if the net product
288 (or the relative benefit) is still above zero. The relative investment rate is
289 therefore an appropriate metric to evaluate the end of the oil age. Once its
290 value reaches one, it becomes impossible to put new sources in operation; the
291 production relies only on sources which are already operating, hence gross
292 product will decrease by 6% (according to k_1 value) every year from this
293 point. Considering the extreme calculations, this “dead-point” of the oil
294 industry is reached between 2027 and 2033.

295 These calculations are done for a given scenario, it cannot be done differ-
296 ently due to formulation of the equation set which is solved. The reality of
297 extraction dynamic could be different. However, the amount of “reachable”
298 liquid fuels is fixed by these equations. This study hence suggests that the
299 remaining reachable amount of gross liquid fuels lies between 305 and 560
300 Gbbl. In term of net product, it represents only 76 to 172 Gbbl. As a com-
301 parison, this is equal to the net energy delivered by the oil industry to the
302 world during the last 5 to 10 years.

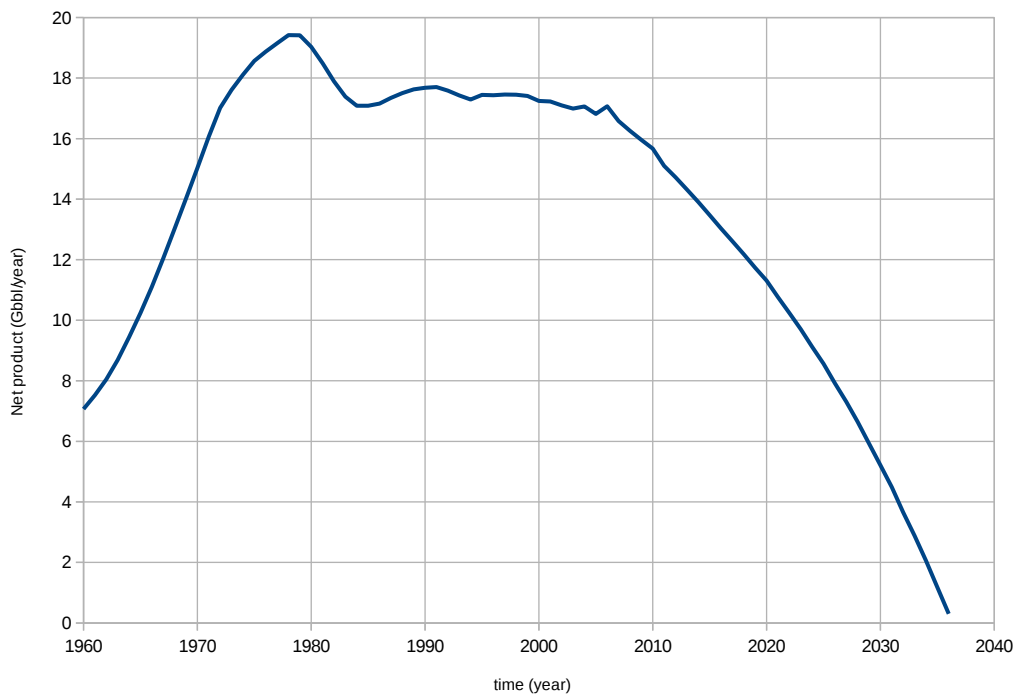


Figure 5: Net product: energy delivered to the world

303 *2.3. A relation between production and oil price*

304 The origin of oil price variation is a highly discussed topic. According to
305 some authors as [14], it might not follow the law of market any more. It is
306 also discussed as being a consequence of geopolitical events, as discussed in

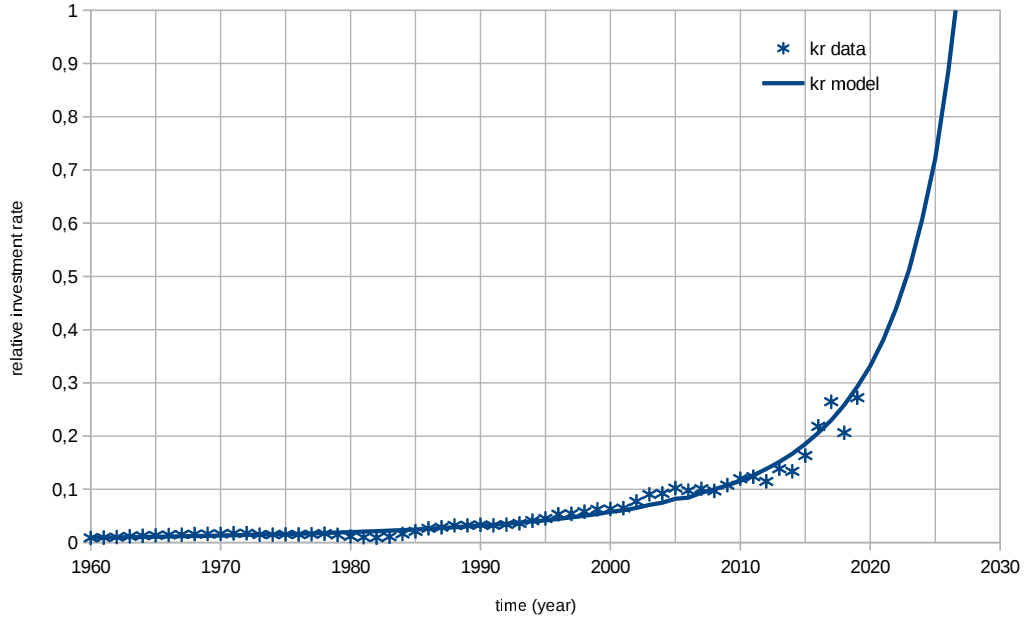


Figure 6: Relative investment rate

307 [15] or [16] for instance. In this article, it is suggested that oil price should
 308 be able to pay employees and share-holder of the oil industry only, since the
 309 energetic cost has already been taken into account through η . This would
 310 suggest that benefit (in money) is proportional to the gross product, while
 311 the actual benefit is the oil price multiplied by the net product. The price
 312 P should then be proportional to $1/\epsilon_B$. That would be the price at which
 313 oil should be sold to maintain the industry, hence oil price used to follow
 314 such behaviour. However, according to [17] for instance, the world cannot
 315 pay this price any more and the price should fall, along with the production
 316 due to some bankrupts in the oil industry. However, debt could prevent
 317 these bankrupts, to keep the system based on oil running as long as possible.
 318 Therefore discussing oil price seems out of the scope of this study; a rough
 319 approximate of the remaining reachable liquid fuels seems a more reliable
 320 metric of liquid fuel depletion.

321 **Conclusion**

322 This study proposes a production averaged model which allow to study
323 the mean extraction dynamic of all liquid fuels. This dynamic seems to follow
324 a typical prey-predator behaviour.

325 It shows that, whatever the possible investment of the oil industry, the
326 development will cease between 2027 and 2033 and the net energetic benefit
327 will follow. According to Garrett's theory on GDP and the contribution of
328 liquid fuels in the world energetic mix, it suggests that world GDP will be
329 reduced by roughly 35% in approximatively 10 years.

330 Finally, the prey-predator analogy suggests this method could also be
331 applied to study the dynamics of other energy source extraction, such as gas,
332 coal or nuclear energy.

333 **References**

- 334 [1] M. Hubbert, Energy resources: A report to the Committee on Natu-
335 ral Resources of the National Academy of Sciences–National Research
336 Council (1962).
- 337 [2] A. Bartlett, An analysis of U.S. and world oil production patterns using
338 Hubbert-style curves, *Mathematical Geology* 1 (2000).
- 339 [3] A. Cavallo, Predicting the peak in world oil production, *Natural*
340 *Ressources Research* 3 (2002).
- 341 [4] R. Duncan, Three world oil forecasts predict peak oil production, *Oil*
342 *and gas journal* 14 (2003).
- 343 [5] C. Cleveland, Net energy from extraction of oil and gas in the United
344 States, *Energy* (2004) 1–14.
- 345 [6] D. Murphy, A. Hall, Year in review-EROI or energy return on (energy)
346 invested, *Annals of the new york academy of sciences* (2010) 102–118.
- 347 [7] H. Freedman, Deterministic Mathematical models in population Ecol-
348 ogy.
- 349 [8] R. Goodwin, A Growth Cycle, paper presented at the First World
350 Congress of the Econometric Society (1965).

- 351 [9] B. Hill, Deplation - A determination of the world's petroleum reserve,
352 Hill's group (2015).
- 353 [10] S. Sorrell, J. Speirs, R. Bentley, R. Miller, E. Thompson, Shaping the
354 global oil peak: A review of the evidence on field sizes, reserve growth,
355 decline rates and depletion rates, *Energy* 37 (2012) 709 – 724. 7th
356 Biennial International Workshop Advances in Energy Studies.
- 357 [11] J. Rodrigue, C. Comtois, B. Slack, *The geography of transport systems*,
358 2016.
- 359 [12] 2020. [Https://transportgeography.org](https://transportgeography.org).
- 360 [13] M. Auzanneau, The European Union can expect to suffer oil depletion
361 by 2030, *The shift project* (2020).
- 362 [14] M. Heun, M. DeWit, Energy return on (energy) invested (EROI), oil
363 prices, and energy transitions, *Energy Policy* 40 (2012) 147–158.
- 364 [15] J. Hamilton, *Historical Oil Shocks*, University of California, San Diego
365 (2011).
- 366 [16] OPEC and Oil Opportunities, *Top Of Mind* 52 (2012) 16.
- 367 [17] B. Warm, Why is the average crude oil price decreasing since 2008 ? or:
368 Thermodynamics of oil production, *ASPO Deutschland* (2020).