



HAL
open science

Forcing a dynamic model for oil production and EROI evolution: The Oil Game

Aymeric Lamorlette

► **To cite this version:**

Aymeric Lamorlette. Forcing a dynamic model for oil production and EROI evolution: The Oil Game. 2020. hal-02383025v3

HAL Id: hal-02383025

<https://hal.science/hal-02383025v3>

Preprint submitted on 11 Aug 2020 (v3), last revised 4 Jul 2023 (v7)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Forcing a dynamic model for oil production and EROEI evolution: The Oil Game

A. Lamorlette

Aix Marseille Univ, Marseille, France

Correspondence to aymeric.lamorlette@univ-amu.fr

Tel (33) 491 113 811 ; Fax (33) 491 118 502

Abstract

Since 1940, many attempts to model world oil production have been proposed. These approaches, using increasing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its Energy Return on Energy Invested (ERoEI), based on mass and energy conservation. The EROEI equation is derived according to the second principle. It leads to a Lotka-Volterra set of equations, which can be applied to all extracted liquid fossil fuels. The model obtained, after comparison with oil extraction and EROEI evolution on the period 1960-2010, illustrates the production dynamic and the existence of an external, controlling parameter: the investment rate, which account for the re-investment in newly operating liquid fuel sources. The evolution of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the liquid fuel production in order to obtain a linear time evolution on the period 1960-2010 through an apparently linearly growing investment rate: the oil game. Unfortunately, in order to keep a linearly growing production at long time scale, the investment rate has actually to evolve exponentially: the linear growth is in fact a short time scale approximation of the control required to play the oil game. The model also allows to highlight a major issue in liquid fuel production: even if the gross product can be controlled and keeps growing linearly, the net product, which account for the energy delivered by the oil industry to the world, is falling down faster and faster, due to the decrease of EROEI. At some point, the

net energy benefit will be equal to zero and liquid fuel production will stop, except if energy is given to the oil industry to keep extracting oil. In anyway, liquid fuels would become an energy sink instead of an energy source. Based on the model presented in this study, this will happen between 2027 an 2033. Production of liquid fuels could therefore keep growing linearly until this point, where a quick collapse is expected. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model.

Keywords: Oil production, EROEI, dynamic model, investment rate

1 Introduction

2 Models that account for oil production have been published from 1962 [1],
3 with increasing complexity ([2, 3, 4], citing only very few of them). These
4 models rely on a production dynamic with constant parameters. The aim
5 of this study will be to evaluate how the parameters could evolve in time,
6 based on a coupling between oil production (all extracted liquid fuels) and
7 its Energy Return on Energy Invested (EROEI) at the wellhead, as defined
8 in [5] or [6]. Through this dependency, it is expected to explain why the
9 prediction of peak is always delayed. For simplicity, "all liquid fuels" in the
10 following refers to "all extracted liquid fuels" or "all liquid fossil fuels".

11 The model suggested is based on mass and energy conservation for the
12 production equation and derived in accordance with the second principle for
13 the EROEI equation. It is worth noticing that the structure of the obtained
14 ODE set is equivalent to a Lotka-Volterra set of equations, linking oil produc-
15 tion of all liquid fuels Q with its mean EROEI. In this model, Q appears to
16 be the EROEI predator as the production "feeds" on EROEI to grow. It is in
17 line with former use of Lotka-Volterra equations to model dynamic systems
18 in ecology [7] or in economy [8].

19 The article is organised as follow: A first part is dedicated to a pre-
20 sentation of an assumption on the oil distribution as a function of EROEI,
21 suggesting why the model applies to the production of all liquid fossil fu-
22 els. It also presents a discussion on a production averaged EROEI and its
23 consequence in term of production modelling. The set of equation is then
24 derived and the prey-predator analogy is presented. A fitting of the model
25 parameters based on historical evolution of oil production and mean EROEI
26 is then performed. An analysis of the investment rate, a forcing parameter,

27 is done, suggesting a control of the investment to keep a linearly growing
28 production: the oil game.

29 A second part is dedicated to the the study of net liquid fuel production,
30 relative energetic benefit and relative investment rate. This analysis sug-
31 gests some possible explanations for the evolution of world economy in the
32 beginning of the 80's, recent recessions and a future collapse of liquid fuel
33 production rather than a slow, progressive decline. It also gives an estima-
34 tion of the remaining "reachable" liquid fuels. Finally, a short discussion on
35 oil price evolution is presented.

36 1. Modelling the interaction between oil production and ERoEI

37 This section is dedicated to the description of the interactions between
38 production of all liquid fuels Q (in Gbbl) and ERoEI at the wellhead, as
39 defined in [5] or [6].

40 1.1. An assumption about a mean, production averaged, ERoEI at the well- 41 head

42 The global modelling of all liquid fuels requires the assumption that the
43 ERoEI considered is representative of the mean ERoEI of all liquid fossil fuels
44 at a given time. Considering N liquid fuel sources in the world, $ERoEI_i$ the
45 ERoEI of a given source and Q_i its production, it is here suggested to take
46 $ERoEI = \frac{1}{Q} \sum_{i=1}^N Q_i \cdot ERoEI_i$. This will allow to derive a single equation
47 for the production of all liquid fuels, instead of having a set of N equations.

48 1.2. A dynamic model for oil production and ERoEI evolution

49 1.2.1. Production equation

50 In order to derive the production equation, a mass balance is considered
51 over the whole set of liquid fuel sources, based on a one year time laps ($\Delta t = 1$
52 year). The ODE is then derived taking $\Delta t \rightarrow dt$. For simplicity, the balance
53 is based on gross product Q_g and the net product Q_n is deduced from Q_g
54 afterwards:

55 On a given year n , a gross product Q_g^n is extracted from the N liquid fuel
56 sources. A fraction of this production k_0 (an investment rate, in year^{-1}) is
57 used to extract liquid fuels from new sources. Let us consider Δh an energy
58 density contained in the liquid fuel (similar to a heat of combustion).

59 The work W_{ex}^n available for extraction in the new sources is then $W_{ex}^n =$
60 $k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$ with η being the efficiency of all the processes needed

61 to turn the extracted liquid fuel into work. This includes: transportation of
62 liquid fuel to refinery, refining, combustion and transformation of heat into
63 work, but also the exploration and structure development (such as wells and
64 platforms) required to get this amount of liquid fossil fuel. This efficiency
65 has been studied for crude oil and is equal to 0.2045 according to Hill [9]. For
66 all liquid fuels, due to the use of production averaged quantities, this value
67 should be about the same.

68 According to the definition of EROEI at wellhead, with $ERoEI^n$ being
69 the mean EROEI on year n , this work allows to get the following amount of
70 energy at the next time laps: $ERoEI^n \cdot k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$, corresponding to
71 an increase in production $\Delta^+ Q_g^{n+1}$ which follows $\Delta^+ Q_g^{n+1} \cdot \Delta h = ERoEI^n \cdot$
72 $k_0 \cdot Q_g^n \cdot \Delta h \cdot \eta \cdot \Delta t$.

73 From the initial gross product Q_g^n , it remains $(1 - k_0 \cdot \Delta t) \cdot Q_g^n$, there-
74 fore, considering only the increase in production due to the newly exploited
75 sources, one gets : $Q_g^{n+1} - Q_g^n = Q_g^n \cdot k_0 \cdot \Delta t \cdot (\eta \cdot ERoEI - 1)$.

76 During the same time laps, the producing fuel sources show a decline
77 which follows the model described in Sorrell [10]: Considering k_1 as the
78 mean oil source decline rate (in year⁻¹), the associate decrease in production
79 is equal to $Q_g^{n+1} - Q_g^n = -k_1 \cdot \Delta t$.

80 Both phenomena occur at the same time, during the same time laps. Since
81 they are linear, it is possible to use superimposition to get: $Q_g^{n+1} - Q_g^n =$
82 $Q_g^n \cdot [k_0 \cdot (\eta \cdot ERoEI - 1) - k_1] \cdot \Delta t$. Taking $\Delta t \rightarrow dt$ leads to:

$$\dot{Q}_g = k_0 \cdot Q_g \cdot (\eta \cdot ERoEI - 1) - k_1 \cdot Q_g. \quad (1)$$

83 Now, based on EROEI definition, it is possible to derive net product Q_n
84 from gross product Q_g : Since $ERoEI = \frac{Q_g}{W_{ext}}$, one can evaluate Q_{ext} , the
85 amount of liquid fuel used for extraction: $ERoEI = \frac{Q_g}{\eta \cdot Q_{ext}}$. Since $Q_{ext} =$
86 $Q_g - Q_n$, one gets: $Q_n = \frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI} Q_g$

87 1.2.2. EROEI equation

88 Based on Eq.(1) and using a prey-predator analogy, Q_g seems to “feed”
89 on EROEI to grow. More precisely, according to Eq.(1) structure, the “natu-
90 ral” prey of Q_g is $\eta \cdot ERoEI - 1$. Following this analogy, the prey should be
91 decreasing proportionally to $\eta \cdot ERoEI - 1$ and Q_g , and should be grow-
92 ing due to the renewal of fossil fuels. This is neglected since it can be
93 considered as happening at geological times. Considering a decline rate

94 k_2 (in (Gtbl.year)⁻¹) for ERoEI, this rational leads to the following equa-
 95 tion: $\eta \cdot ERoEI = -k_2 \cdot Q_g \cdot (\eta \cdot ERoEI - 1)$, equivalent to $ERoEI =$
 96 $-k_2 \cdot Q_g \cdot ERoEI + \frac{k_2}{\eta} \cdot Q_g$. It is interesting to notice that if $k_2 \cdot Q_g = A$ with
 97 A being a constant (as it is the case here, according to Fig.2), this equation
 98 reads: $ERoEI = -A \cdot ERoEI + \frac{A}{\eta}$. It leads to the inverse of the logistic
 99 function, which is the solution obtained for ERoEI evolution in the work of
 100 Hill [9]. The Lotka-Volterra approach allows to get the equation that lead
 101 to the solution obtained using the second principle, it can be considered as
 102 equivalent. This emphasizes the relevancy of the prey-predator analogy for
 103 liquid fuel product and its ERoEI.

104 However, the obtained equation cannot fit the purpose here, since its
 105 structure is decoupling Q_g and ERoEI. In order to keep this coupling, the
 106 following form is retained:

$$ERoEI = -k_2 \cdot Q_g \cdot ERoEI. \quad (2)$$

107 Parameter k_2 is expected to decrease in time, according to the natural
 108 distribution of oil as a function of its availability on earth. In order to model
 109 k_2 , the following dependency is proposed: $k_2 = C/(t - t_0)$ where C is a
 110 constant (in (Gtbl)⁻¹) and t_0 (in year) is a time offset. C can be interpreted
 111 as the effect of oil distribution as a function of ERoEI, regarding the rate at
 112 which oil is extracted. It suggests that the largest amount of oil on earth is
 113 available at the lowest ERoEI.

114 The work of Hill [9] nevertheless suggests that the “natural” physical cou-
 115 pling between ERoEI and Q_g is established through a distribution $ERoEI =$
 116 $f(Q_g)$, which is the expression of the Etp equation, derived by Hill [9] based
 117 on the second principle. Therefore, this distribution is now studied to see
 118 how it could be a surrogate to Eq.(2).

119 In [9], the ERoEI(t) function is derived based on a production which fol-
 120 lows a Hubbert’s curve for crude oil only (cumulative product $Q_p = 2357, 15$
 121 Gtbl, peak at $t_m = 2001$, $Q_m = Q(t = t_m) = b \cdot Q_p/4$). Based on the previ-
 122 ous remarks, $Q_g = f^{-1}(ERoEI)$ can be explicitly derived for a Hubbert like
 123 extracting scenario:

$$Q_g(ERoEI) = b \cdot Q_p \frac{\left(\frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI_m - 1}\right)^{b/a}}{\left[1 + \left(\frac{\eta \cdot ERoEI - 1}{\eta \cdot ERoEI_m - 1}\right)^{b/a}\right]^2}. \quad (3)$$

124 With $ERoEI_m = 13.3$ being the value of EROEI at time t_m and $a = 0.0537$
125 the parameter of the EROEI solution (the inverse of a logistic function) cal-
126 culated in [9]. Eq.(3) is general, but the values of Q_p and b are representative
127 of a production which follow the Hubbert's scenario presented earlier. For
128 all liquid fuels, this scenario is not realistic, at least for the values of Q_p and
129 b previously suggested. Therefore, in order to estimate function f^{-1} for all
130 liquid fuels, an adapted scenario has to be establish. This will be discuss
131 later in this article, and distribution $ERoEI = f(Q_g)$ will be calculated for
132 all liquid fuels.

133 1.3. Fitting the model parameters on the period 1960-2010

134 k_1 represents the oil sources mean decline rate. This parameter should be
135 extracted from experimental measurements, using inverse methods. Based
136 on the results of [10], the mean value lies in the range 4.1 – 6.7% but is
137 increasing with the exploitation of new non-conventional sources. Therefore,
138 k_1 is set equal to 6% (a mean value based on previous remark) and k_0 , k_2
139 and $k_2 \cdot Q_g$ can be fitted. Now, based on Eq.(1) and (2), setting a value for
140 k_1 , it is possible to plot k_0 and k_2 time evolution, based on historical data of
141 Q_g and EROEI:

$$k_0 = \frac{\dot{Q}_g}{Q_g \cdot (\eta \cdot ERoEI - 1)} + \frac{k_1}{(\eta \cdot ERoEI - 1)}, \quad (4)$$

142

$$k_2 = -\frac{ERoEI}{Q_g \cdot ERoEI}. \quad (5)$$

143 The oil production data is extracted from [11]. Unfortunately, there is
144 no available data for the mean EROEI defined in section 1.1. The thermo-
145 dynamic model suggested by [9], which fits the requirements described in
146 section 1.1 and presents values at different times which are consistent with
147 actual, measured values of active oil sources is used. Using the values of [9]
148 to calibrate this model will certainly not allow to estimate precisely the in-
149 vestment rate time evolution, but it should allow to evaluate its global trend
150 over the last decades to estimate its evolution in the forthcoming decade(s).

151 The analysis is performed on the period 1960-2010 using a three-year av-
152 eraging on Q_g and a second order upwind method to calculate Q_g and EROEI
153 derivatives. The evolution obtained for k_2 is presented in Fig.1. The contin-
154 uous line represents the model $k_2 = C/(t - t_0)$, with $C = 0.06036 \text{ Gbbl}^{-1}$
155 and $t_0 = 1950.5$ year. The model seems to fit adequately the data, with a

156 mean relative error of 1.05%. The evolution shows two periods: The first one,
 157 before the first oil shock, corresponds to a rapid and smooth evolution of k_2 .
 158 The second one, after the second oil shock, shows some jumps which could
 159 correspond to the exploitation of fossil fuels that were not exploited before
 160 due to their low EROEI, in comparison with the mean EROEI of the moment.
 161 When these sources become of interest and start being exploited, the value
 162 of k_2 suddenly drops because exploiting these sources does not affect much
 163 the mean EROEI.

164 The evolution obtained for $k_2 \cdot Q_n$ is presented in Fig.2, showing the rel-
 165 evancy of the previous remark on the EROEI ODE and on the prey-predator
 166 analogy.

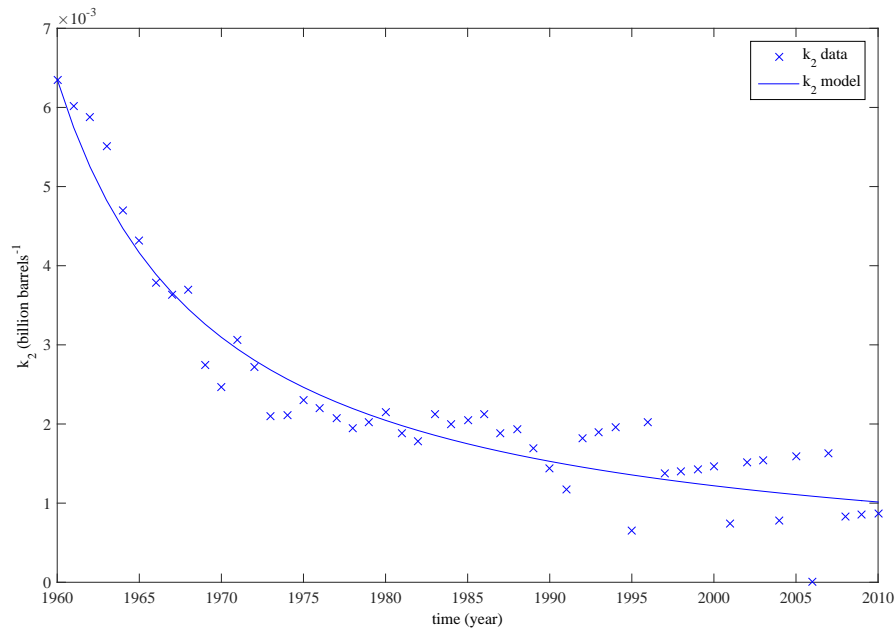


Figure 1: k_2 time evolution

167 *1.4. Studying the investment rate*

168 *1.4.1. History of the investment rate*

169 The investment rate can be evaluated through the value of k_0 , which lies
 170 in the range $[0; 1]$. It is nevertheless suggested to study $k_{eff} = k_0 \cdot \frac{\eta \cdot EROEI - 1}{\eta \cdot EROEI}$

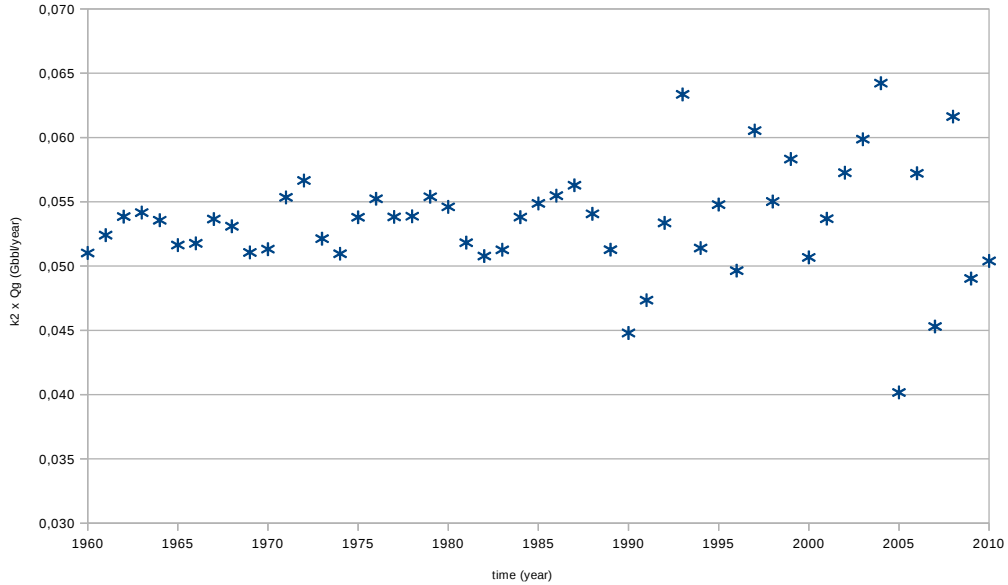


Figure 2: $k_2 \cdot Q_g$ time evolution

171 instead of k_0 , for function fitting requirements. The parameter k_{eff} , which
 172 represents the forcing of the system is plot over time in Fig.3. Its analysis
 173 provides some possible characteristics of the oil extraction strategy, which
 174 are presented below.

175 On the period 1960-1968, k_{eff} shows a relatively linear behaviour. This
 176 period corresponds to an evolution of oil extraction that begin to behave as
 177 exponential around 1965. Due to the laws of market, and the oil price at
 178 this period which is rather low, keeping an exponential growth for Q_g could
 179 have been responsible for an important decrease in oil price. In order to keep
 180 a decent benefit without using too much of their resources, producers have
 181 to reduce Q_g , by reducing k_{eff} . This strategy begins in 1969, according to
 182 Fig.3. However, due to the behaviour of k_2 at this time, the system shows a
 183 great inertia and damping k_{eff} is not sufficient to control instantaneously Q_g .
 184 Any reasons could have been sufficient to suddenly reduce k_{eff} and adapt
 185 Q_g . Three years after the first inflection of k_{eff} , the first oil shock happen
 186 and k_{eff} is adapted.

187 After the first shock, k_{eff} is surprisingly constant, with a linear time
188 evolution for Q_g . The second shock corresponds to another, longer drop of
189 k_{eff} .

190 After the second shock, k_{eff} seems to evolve (globally) linearly, with raises
191 and plateaus during the period 1985-2000. The solid line fits the data with
192 a mean relative error of less than 1%. This behaviour allows Q_g to grow
193 linearly in time. Also, since the system inertia has evolved in time with
194 k_2 , the plateaus are responsible every time for a slow damping of Q_g , which
195 corresponds to past predictions of a nearby peak, using Hubbert's curves.
196 This phenomenon leads every time to an economical recession and a raise in
197 oil price (this can be shown in comparing Fig.3 with an oil price chart), at
198 the moment where producers need to increase their investment to keep k_{eff}
199 close to the solid line that ensure a linearly growing Q_g . The origin of this
200 raise/plateaus dynamic can be explained the following way: With time, the
201 production of an oil source eventually decreases, meaning that exploration
202 is firstly needed to extract more oil. It means that if exploration does not
203 suggests new sources to exploit, the production stagnates because it is not
204 possible to invest in new sources, therefore k_{eff} is constant or slightly decreas-
205 ing. When new sources become available, the investment rate can quickly
206 increase until the new sources become less available and then exploration has
207 to start again.

208 1.4.2. Projections based on the constant 1985-2000 dynamic

209 Following this line using raises and plateaus, allow to optimize the oil
210 benefit and production: It could be compared to a game where k_{eff} should
211 be kept on this line to optimize benefits. This strategy can then be extended
212 to forthcoming years. One can observe that k_{eff} begins to deviate on the
213 period 2000-2010. It seems that, in order to keep a constant derivative for
214 Q_g , k_{eff} should not follow the same trend any more. The data of [12] is an
215 extension of [11] data. It is used to evaluate the evolution of this slope. On
216 the period 2000-2020, the slope seems to be different from the one observed
217 on the period 1985-2000. Instead of plateaus, between 2005 and 2010, drops
218 are required on k_{eff} to fasten the effect on Q_g , and the mean slope has to be
219 higher than before.

220 1.4.3. The limit of the 2000-2020 dynamic

221 The extension of that game actually shows the real rule: in order to
222 keep a linearly growing Q_g , k_{eff} has to evolve exponentially in time. To

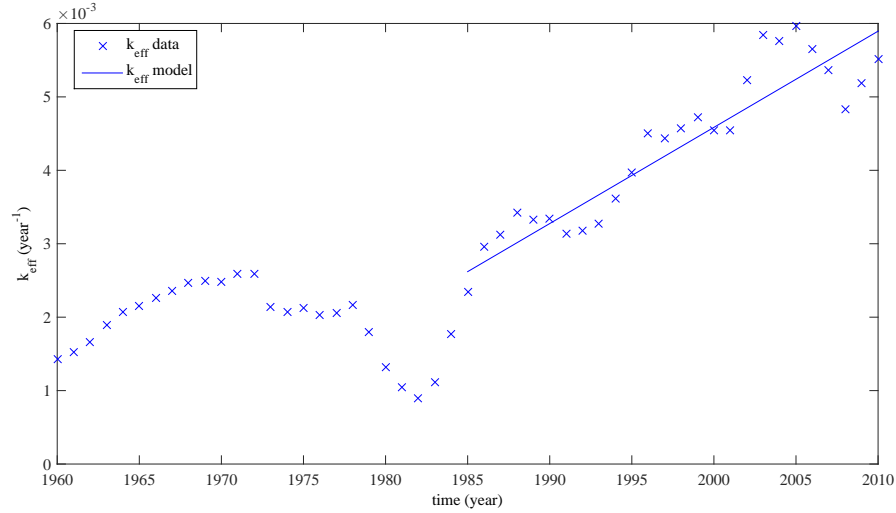


Figure 3: k_{eff} time evolution for $k_1 = 6\%$

223 keep playing that game the way it started, k_{eff} should follow the equation
 224 $k_{eff} = 1.2650 \cdot 10^{-25} \exp(t/\tau)$ with $\tau = 37.32$ years. Fig.4 shows the extension
 225 of k_{eff} on the period 2005-2020 along with a projection using the exponential
 226 fit. This projection suggests that the slope has again to be inflected, by
 227 strongly inflecting the shoots between the plateaus/drops. It also requires
 228 exploration to be more and more efficient, to ensure that the plateaus won't
 229 last for too long. It is worth noticing that according to [13] for instance, the
 230 exact opposite is expected.

231 2. Results and analysis

232 2.1. A business as usual scenario

233 Based on Eq.(1), the threshold value of $ERoEI$ for which oil production
 234 cannot grow any more is $\eta^{-1} \sim 4.89$. Another threshold could be the relative
 235 investment rate which cannot exceed 1 (i.e. the net investment cannot exceed
 236 the net product). This relative investment k_r reads: $k_r = \frac{Q_g}{Q_n} \cdot k_0 = \frac{\eta \cdot ERoEI}{\eta \cdot ERoEI - 1} \cdot$
 237 k_0 . Supposing the exponential investment dynamic could be sustained until
 238 the mean liquid fossil fuel $ERoEI$ reaches 4.89 or until the relative investment
 239 rate reaches 1 represents a business as usual scenario where oil is extracted
 240 until the energetic benefit drops to zero or until development of new sources

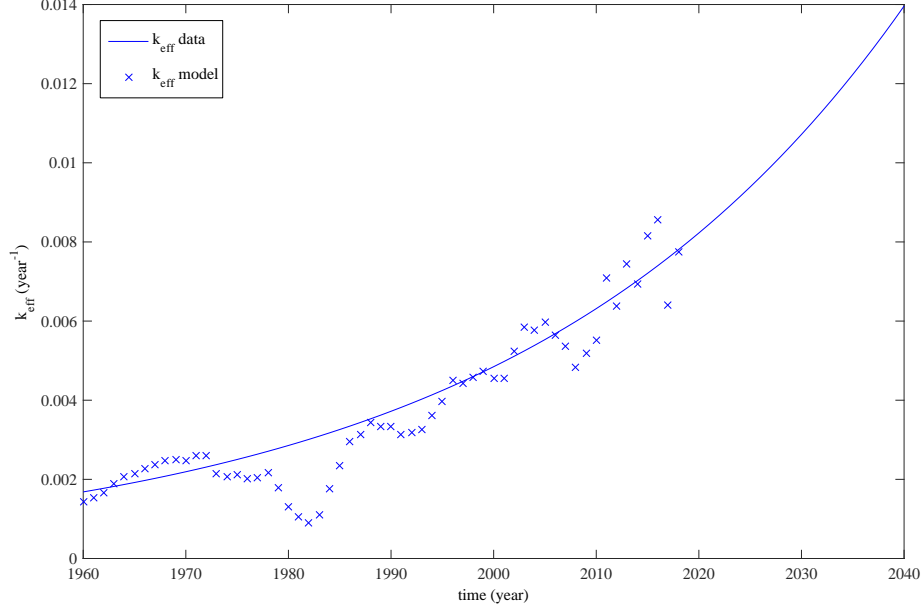


Figure 4: k_{eff} time evolution for $k_1 = 6\%$

241 becomes impossible. It could allow a linear growth of Q_g until 2040. This
 242 linear growth would be followed by a quick collapse of Q_g .

243 This business as usual scenario would be equivalent, for $t < 2040$ years to
 244 a Hubbert's curve with parameters : $Q_p = 5200$ Gbbl, a peak at $t_m = 2040$
 245 years and $b = 0.028$. Please note that $Q_p = 5200$ Gbbl has nothing to do with
 246 an amount of recoverable liquid fuels. This aspect will be discussed later.
 247 Using these parameters allows, based on the ERoEI model of Hill, to derive
 248 the associated distribution $Q_g = f(ERoEI)$. Now, in order to estimate the
 249 evolution of Q_g and ERoEI, one can use either Eq.(1) and Eq.(2), either
 250 Eq.(1) and Eq.(3) using the above values for Q_p and b .

251 In order to evaluate the end of oil extraction based on this scenario, three
 252 variables are studied: the net product Q_n , the relative energetic benefit $\epsilon_B =$
 253 $\frac{Q_n}{Q_g} = \frac{\eta E_{roEI} - 1}{\eta E_{RoEI}}$ and the relative investment rate $k_r = \frac{k_0}{\epsilon_B}$. These variables are
 254 studied with Eq.(1) and Eq.(2), then with Eq.(1) and Eq.(3). A 10% margin
 255 on η is also considered, as this value has been fitted for crude oil and is now
 256 applied to all liquid fuels.

257 *2.2. Results*

258 The numerical simulations are done with a 0.1 year time step and a
259 Runge-Kutta 4 method, starting in 2010, after a numerical validation on
260 the period 1960-2010. The first remark is that the results are within 3% re-
261 gardless the set of equations which is used. It suggests that the prey-predator
262 analogy makes sense, with Q_g being the predator of $ERoEI$. It also suggests
263 that this kind of Lotka-Volterra set of equations could apply to other energy
264 sources such as coal, gas and nuclear energy, if data such as $Q_g = f(ERoEI)$
265 are available, or can be derived based on thermodynamical considerations.

266 Plots are presented for the most pessimistic calculation (in term of re-
267 maining liquid fuel) and all results are presented with a dispersion based on
268 the extreme calculations. Fig.5 shows the net product as a function of time.
269 It highlights that the maximum net energy delivery to the world by the oil
270 industry peaked in 1979. It also shows the real difference in energy delivered
271 by the oil industry to the world before and after the second oil shock, evol-
272 ving from a continuous increase to a steady state until 2000. The net product
273 begins to fall around 2000, but this is damped by the investment done in oil
274 production at this time. These investments seem to have maintain the net
275 product almost constant until 2005, at which point the net product start to
276 decline linearly, until 2017-2018 where the fall begins to accelerate. All these
277 remarks seem in line with the recessions that occurred since the year 2000.

278 Fig.6 shows the evolution of the relative investment rate. It highlights
279 the rate at which investment have to increase from now on to keep playing
280 the oil game. It also shows that the study of the net product hides the
281 potential investment issues: at some point it becomes impossible to invest
282 enough energy to keep the gross production increasing even if the net product
283 (or the relative benefit) is still above zero. The relative investment rate is
284 therefore an appropriate metric to evaluate the end of the oil age. Once its
285 value reaches one, it becomes impossible to put new sources in operation; the
286 production relies only on sources which are already operating, hence gross
287 product will decrease by 6% (according to k_1 value) every year from this
288 point. Considering the extreme calculations, this “dead-point” of the oil
289 industry is reached between 2027 and 2033.

290 These calculations are done for a given scenario, it cannot be done differ-
291 ently due to formulation of the equation set which is solved. The reality of
292 extraction dynamic could be different. However, the amount of “reachable”
293 liquid fuels is fixed by these equations. This study hence suggests that the
294 remaining reachable amount of gross liquid fuels lies between 305 and 560

295 Gbbl. In term of net product, it represents only 76 to 172 Gbbl. As a com-
296 parison, this is equal to the net energy delivered by the oil industry to the
297 world during the last 5 to 10 years.

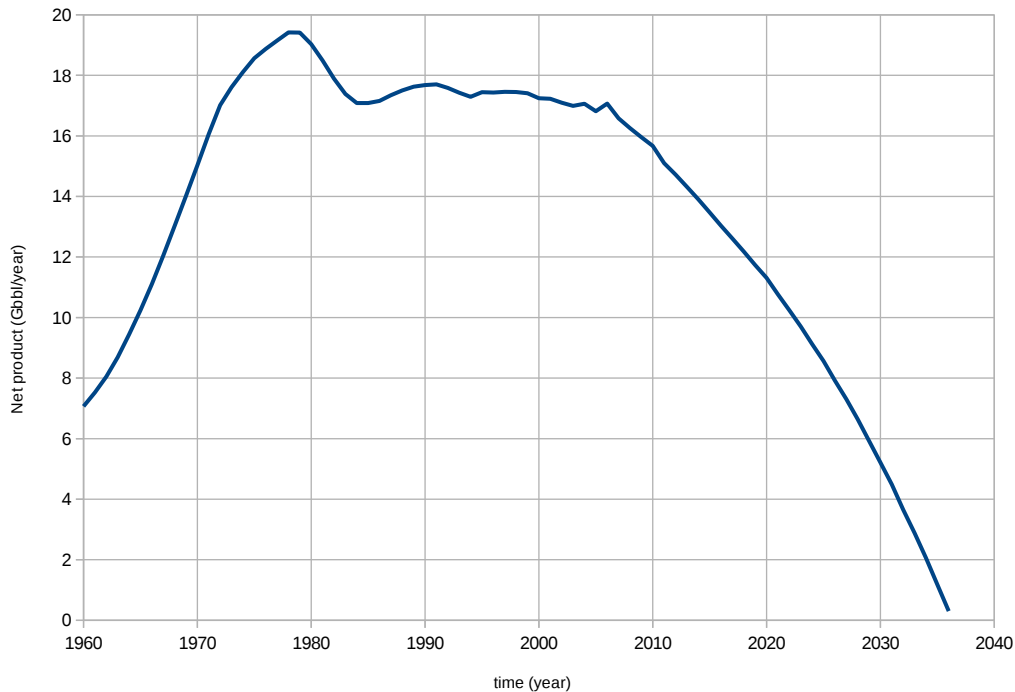


Figure 5: Net product: energy delivered to the world

298 *2.3. A relation between production and oil price*

299 The origin of oil price variation is a highly discussed topic. According to
300 some authors as [14], it might not follow the law of market any more. It is
301 also discussed as being a consequence of geopolitical events, as discussed in
302 [15] or [16] for instance. In this article, it is suggested that oil price should
303 be able to pay employees and share-holder of the oil industry only, since the
304 energetic cost has already been taken into account through η . This would
305 suggest that benefit (in money) is proportional to the gross product, while
306 the actual benefit is the oil price multiplied by the net product. The price

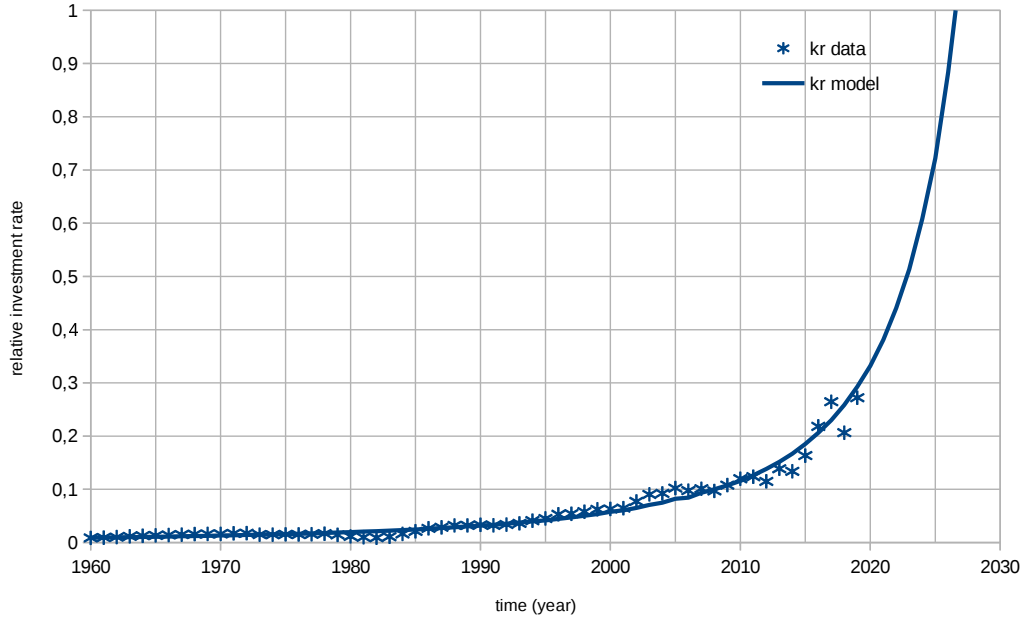


Figure 6: Relative investment rate

307 P should then be proportional to $1/\epsilon_B$. That would be the price at which
 308 oil should be sold to maintain the industry, hence oil price used to follow
 309 such behaviour. However, according to [17] for instance, the world cannot
 310 pay this price any more and the price should fall, along with the production
 311 due to some bankrupts in the oil industry. However, debt could prevent
 312 these bankrupts, to keep the system based on oil running as long as possible.
 313 Therefore discussing oil price seems out of the scope of this study; a rough
 314 approximate of the remaining reachable liquid fuels seems a more reliable
 315 metric of liquid fuel depletion.

316 Conclusion

317 This study proposes a production averaged model which allow to study
 318 the mean extraction dynamic of all liquid fuels. This dynamic seems to follow
 319 a typical prey-predator behaviour.

320 It is in accordance with the results obtained by Hill, with a few years
321 delay. This is due to the fact that Hill's study is focused on crude oil while
322 this study is extended to all extracted liquid fuels.

323 It shows that, whatever the possible investment of the oil industry, the
324 development will cease between 2027 and 2033 and the net energetic benefit
325 will follow. According to Garrett's theory on GDP and the contribution of
326 liquid fuels in the world energetic mix, it suggests that world GDP will be
327 reduced by roughly 35% in approximatively 10 years.

328 Finally, the prey-predator analogy suggests this method could also be
329 applied to study the dynamics of other energy source extraction, such as gas,
330 coal or nuclear energy.

331 **References**

- 332 [1] M. Hubbert, Energy resources: A report to the Committee on Natu-
333 ral Resources of the National Academy of Sciences–National Research
334 Council (1962).
- 335 [2] A. Bartlett, An analysis of U.S. and world oil production patterns using
336 Hubbert-style curves, *Mathematical Geology* 1 (2000).
- 337 [3] A. Cavallo, Predicting the peak in world oil production, *Natural*
338 *Ressources Research* 3 (2002).
- 339 [4] R. Duncan, Three world oil forecasts predict peak oil production, *Oil*
340 *and gas journal* 14 (2003).
- 341 [5] C. Cleveland, Net energy from extraction of oil and gas in the United
342 States, *Energy* (2004) 1–14.
- 343 [6] D. Murphy, A. Hall, Year in review-EROI or energy return on (energy)
344 invested, *Annals of the new york academy of sciences* (2010) 102–118.
- 345 [7] H. Freedman, Deterministic Mathematical models in population Ecol-
346 ogy.
- 347 [8] R. Goodwin, A Growth Cycle, paper presented at the First World
348 Congress of the Econometric Society (1965).
- 349 [9] B. Hill, Deplation - A determination of the world's petroleum reserve,
350 Hill's group (2015).

- 351 [10] S. Sorrell, J. Speirs, R. Bentley, R. Miller, E. Thompson, Shaping the
352 global oil peak: A review of the evidence on field sizes, reserve growth,
353 decline rates and depletion rates, *Energy* 37 (2012) 709 – 724. 7th
354 Biennial International Workshop Advances in Energy Studies.
- 355 [11] J. Rodrigue, C. Comtois, B. Slack, *The geography of transport systems*,
356 2016.
- 357 [12] 2020. <https://transportgeography.org>.
- 358 [13] M. Auzanneau, The European Union can expect to suffer oil depletion
359 by 2030, *The shift project* (2020).
- 360 [14] M. Heun, M. DeWit, Energy return on (energy) invested (EROI), oil
361 prices, and energy transitions, *Energy Policy* 40 (2012) 147–158.
- 362 [15] J. Hamilton, *Historical Oil Shocks*, University of California, San Diego
363 (2011).
- 364 [16] OPEC and Oil Opportunities, *Top Of Mind* 52 (2012) 16.
- 365 [17] B. Warm, Why is the average crude oil price decreasing since 2008 ? or:
366 Thermodynamics of oil production, *ASPO Deutschland* (2020).