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# Forcing a dynamic model for oil production and EROI evolution: The Oil Game

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## Abstract

Since 1940, many attempts to model world oil production have been proposed. Those approach, using increasing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its Energy Return On (Energy) Invested (EROI), using Lotka-Volterra equations and to apply the model to all liquid fossil fuels. The model obtained, after comparison with oil extraction and EROI evolution on the period 1960-2010, illustrates the production dynamic and the existence of an external, controlling parameter: the production/exploration effort which account for the re-investment in the production and exploration processes. The evolution of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the oil production in order to obtain a linear time evolution on the period 1960-2010 through an apparently linearly growing production/exploration effort: the oil game. Since the end of the oil shocks, this control has been slightly inflected for the first time around 2000-2005, what could explain the evolution in fossil fuel investment from that time. Unfortunately, in order to keep a linearly growing production at long time scale, the production effort has actually to evolve exponentially: the linear growth is in fact a short time scale approximation of the control required to play the oil game. Therefore the production effort will require more frequent and stronger inflection in the future, what suggests more frequent and stronger recession period, since the economy seems strongly correlated to the production effort, as an oil price

chart comparison with the production effort suggests. Finally, playing the oil game until the end would lead to a peak of all liquid fossil fuels between 2080 and 2090, when EROI=1, followed by a quick collapse of oil production. Hence production will be strongly asymmetric regarding the peak, contrary to the prediction suggested by Hubbert's model.

*Keywords:* Oil production, EROI, dynamic model, production effort

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## 1 Introduction

2 Models that account for oil production have been published from 1962 [1],  
3 with increasing complexity ([2, 3, 4], citing only very few of them). These  
4 models rely on a production dynamic with constant parameters. The aim of  
5 this study will be to evaluate how the parameters could evolve in time, based  
6 on a coupling between oil production and its Energy Return On (Energy)  
7 Invested (EROI), as primarily defined in [5] or [6]. Through this dependency,  
8 it is expected to explain why the prediction of peak is always delayed. The  
9 model suggested is based on a Lotka-Volterra set of equations, linking an oil  
10 production  $Q$  with its mean EROI. In this approach,  $Q$  is the EROI predator  
11 as the production "feeds" on EROI to grow.

12 The article is organised as follow: A first part is dedicated to a pre-  
13 sentation of an assumption on the oil distribution as a function of EROI,  
14 suggesting why the model applies to the production of all liquid fossil fuel.  
15 It also presents a discussion on a "physical" meaning of a mean EROI, as  
16 presented in studies like [7] where EROI is derived based on thermodynamics  
17 assumptions. The model itself and the set of equation are then presented,  
18 along with the meaning of the parameters that appear in the model. Some  
19 sub-models are here suggested for these parameters. A fitting of the model  
20 parameters based on historical evolution of oil production and mean EROI  
21 is then performed. Finally, an analysis of the production effort, a forcing  
22 parameter, is done, suggesting the existence of a benefit/production opti-  
23 mization: the oil game.

24 A second part is dedicated to the projection of the production effort that  
25 followed the same trend from the mid eighty's to 2000. An extension to the  
26 year 2020 is also discussed, showing an evolution of the production dynamic  
27 and a need to update the dynamic in 2020 in order to keep a linearly growing  
28 production.

29 A business as usual scenario is also studied, to evaluate the ultimate peak  
30 (which is reached when  $EROI=1$ ) and the following collapse of oil production.  
31 Finally, a relation is suggested between long time scale oil price evolution and  
32 production effort.

## 33 1. Modelling the interaction between oil production and EROI

### 34 1.1. An assumption about the oil production distribution as a function of 35 EROI

36 This section is dedicated to the description of the interactions between  
37 production of all liquid fuel  $Q$  and EROI. This description is based on the  
38 assumption that the EROI considered is representative of the mean EROI of  
39 all liquid fossil fuel at a given time. Therefore, it is suggested that this value  
40 can only decrease with time, even if at a given time some oil sources with  
41 higher EROI than the mean value can exist. Also, the following assumption  
42 is suggested: if there is no oil (in any form) available at a given EROI,  
43 there will always be some existing oil at a lower EROI. This description  
44 does not exactly fit the reality of “measured” EROI, nevertheless it has some  
45 “physical” meaning which presents some interest for modelling purpose. The  
46 value of this mean EROI should be close to actual oil sources EROI as those  
47 presented in [5].

48 A good candidate to represent this quantity could be the thermodynamic  
49 model suggested by [7], which behave as described above and presents values  
50 at different times which are consistent with actual, measured values. Using  
51 the values of [7] to calibrate this model will certainly not allow to estimate  
52 precisely the time evolution of oil production effort, but it should allow to  
53 evaluate its global trend over the last decades to estimate its evolution in the  
54 forthcoming decades.

### 55 1.2. A dynamic model for oil production and EROI evolution

56 The approach suggested in this study considers oil production  $Q$  (in billion  
57 barrels) as a predator of its mean EROI. Indeed, in a prey/predator analogy,  
58 oil production is feeding on EROI to grow, as suggested by the following set  
59 of equations.

60 The oil production  $Q$  is suggested to be growing proportionally to EROI  
61 and  $Q$ , and has a natural decay due to the mortality rate of oil sources,  
62 therefore proportional to  $Q$  only, as the mortality rate (also called decay or  
63 depletion rate) defined in [8] suggests. The production time derivative is

64 composed of two terms: the growing term and the decaying term. The grow-  
 65 ing term can be justified the following way: At a given time  $t$ , a production  
 66  $Q$  is available, so that the largest quantity that can be extract on time  $t + dt$   
 67 is equal to  $Q \cdot EROI$ , according to the EROI definition of [6]. The maximum  
 68 growth  $dQ$  during a time  $dt$  is equal to  $(Q \cdot EROI - Q) \cdot dt = Q \cdot (EROI - 1) \cdot dt$ .  
 69 Let's consider a parameter  $k_0$  (in year<sup>-1</sup>), called production effort. Hence,  
 70 the effective growth  $dQ$  during a time  $dt$  is equal to  $k_0 \cdot Q \cdot (EROI - 1) \cdot dt$ ,  
 71 with  $k_0$  in  $[0; 1]$ .

72 The decaying term of  $Q$  is simply equal to  $k_1 \cdot Q \cdot dt$ , where  $k_1$  is the mean  
 73 oil source mortality (in year<sup>-1</sup>), as defined in [8].

74 The prey is decreasing proportionally to EROI and  $Q$ , and is supposedly  
 75 growing due to the renewal of fossil fuels, which can be considered as happen-  
 76 ing at geological times. This effect is therefore neglected. The EROI decay  
 77 can be explained the following way: For each unit of  $Q$  that is extracted,  
 78 the relative EROI variation is proportional to a parameter  $k_2$  (in (billion  
 79 barrels.year)<sup>-1</sup>), the EROI decaying term is then equal to  $k_2 \cdot Q \cdot EROI \cdot dt$ .  
 80 Parameter  $k_2$  is expected to decrease in time, as our ability to extract oil prop-  
 81 erly increase, and according to the natural distribution of oil as a function of  
 82 its availability on earth. In order to model  $k_2$ , the following dependency is  
 83 proposed:  $k_2 = C/(t - t_0)$  where  $C$  is a constant (in (billion barrels)<sup>-1</sup>) and  
 84  $t_0$  (in year) is a time offset.  $C^{-1}$  can be interpreted as a technology evolution  
 85 rate.

86 The following equations are obtained for production and EROI dynamic:

$$\dot{Q} = k_0 \cdot Q \cdot (EROI - 1) - k_1 \cdot Q, \quad (1)$$

$$\dot{EROI} = -k_2 \cdot Q \cdot EROI. \quad (2)$$

88 It is noticed that, amongst these parameters,  $k_1$  and  $k_2$  seem to represent  
 89 some effective “physical” properties of the system, whereas  $k_0$  represents a  
 90 forcing parameter on which oil producers can play to adapt the oil extraction  
 91 to their need.

### 92 1.3. Fitting the model parameters on the period 1960-2010

93 Since  $k_1$  represents the oil sources mean mortality, it is assumed it should  
 94 fit measured data. For instance, according to [8], the mean value lies in the  
 95 range 4.1 – 6.7%. Therefore,  $k_1$  is taken as a constant and  $k_0$  and  $k_2$  are  
 96 fitted for the different values of  $k_1$ , within the range 4 – 7%. Now, based on

97 Eq.(1) and (2), setting a value for  $k_1$ , it is possible to plot  $k_0$  and  $k_2$  time  
 98 evolution, based on historical data of  $Q$  and EROI:

$$k_0 = \frac{\dot{Q}}{Q \cdot (EROI - 1)} + \frac{k_1}{(EROI - 1)}, \quad (3)$$

99

$$k_2 = -\frac{E\dot{R}OI}{Q \cdot EROI}. \quad (4)$$

100 The oil production data is extracted from [9] and the EROI data is ex-  
 101 tracted from [7]. The analysis is performed on the period 1960-2010 using a  
 102 three-year averaging on  $Q$  and a second order upwind method to calculate  $Q$   
 103 and EROI derivatives. The evolution obtained for  $k_2$  is presented in Fig.1.  
 104 The continuous line represents the model  $k_2 = C/(t - t_0)$ , with  $C = 0.06036$   
 105 (billion barrels) $^{-1}$  and  $t_0 = 1950.5$  year.

106 The model seems to fit adequately the data, with a mean relative error  
 107 of 1.05%. The evolution shows two periods: The first one, before the first  
 108 oil shock, corresponds to a rapid and smooth evolution of  $k_2$ . The second  
 109 one, after the second oil shock, shows some jumps which could correspond to  
 110 technological jumps, or simply the exploitation of fossil fuels that were not  
 111 exploited before due to their low EROI, in comparison with the mean EROI  
 112 of the moment. When these sources become of interest and start being  
 113 exploited, the value of  $k_2$  suddenly drops because exploiting these sources  
 114 does not affect much the mean EROI.

#### 115 1.4. Studying the production effort

116 This section aims at studying the evolution of the investment in pro-  
 117 duction/exploration. This investment can be evaluated through the value  
 118 of  $k_0$ , which lies in the range  $[0; 1]$ . It is nevertheless suggested to study  
 119  $k_{eff} = k_0(EROI - 1)/EROI$  rather than  $k_0$ , as  $k_{eff}$  lies in the range  $[0; +\infty[$ ,  
 120 in order to have a broader choice of function that could fit the data. The  
 121 parameter  $k_{eff}$ , which represents the forcing of the system is plot over time  
 122 in Fig.2 for  $k_1 = 5\%$ . Its analysis provides some possible characteristics of  
 123 the oil extraction strategy, which are presented below.

124 On the period 1960-1968,  $k_{eff}$  shows a relatively linear behaviour. This  
 125 period corresponds to an evolution of oil extraction that begin to behave as  
 126 exponential around 1965. Due to the laws of market, and the oil price at  
 127 this period which is rather low, keeping an exponential growth for  $Q$  could  
 128 have been responsible for an important decrease in oil price. In order to keep

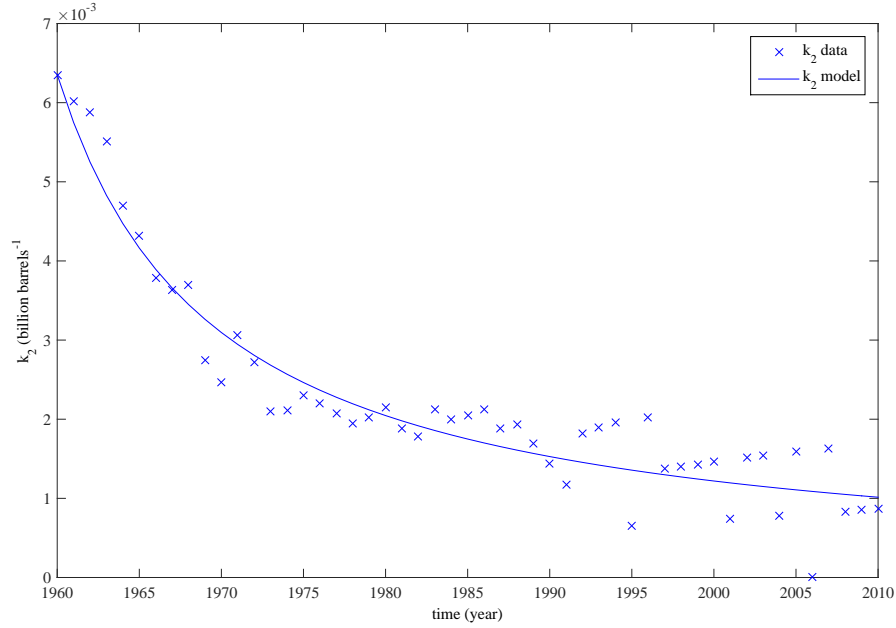


Figure 1:  $k_2$  time evolution

129 a decent benefit without using too much of their resources, producers have  
 130 to reduce  $Q$ , by reducing  $k_{eff}$ . This strategy begins in 1969, according to  
 131 Fig.2. However, due to the behaviour of  $k_2$  at this time, the system shows a  
 132 great inertia and damping  $k_{eff}$  is not sufficient to control instantaneously  $Q$ .  
 133 Any reasons could have been sufficient to suddenly reduce  $k_{eff}$  and adapt  $Q$ .  
 134 Three years after the first inflection of  $k_{eff}$ , the first oil shock happen and  
 135  $k_{eff}$  is adapted.

136 After the first shock,  $k_{eff}$  is surprisingly constant, with a linear time  
 137 evolution for  $Q$ . The second shock corresponds to another, longer drop of  
 138  $k_{eff}$ .

139 After the second shock,  $k_{eff}$  seems to evolve (globally) linearly, with raises  
 140 and plateaus during the period 1985-2000. The solid line corresponds to the  
 141 equation  $k_{eff} = 0.00349 + 0.000173 \cdot (t - 1985)$ , which fits the data with a mean  
 142 relative error of less than 1%. This behaviour allows  $Q$  to grow linearly in  
 143 time. Also, since the system inertia has evolved in time with  $k_2$ , the plateaus  
 144 are responsible every time for a slow damping of  $Q$ , which corresponds to

145 past predictions of a nearby peak, using Hubbert's curves. This phenomenon  
 146 leads every time to an economical recession and a raise in oil price (this can  
 147 be shown in comparing Fig.2 with an oil price chart), at the moment where  
 148 producers need to increase their investment to keep  $k_{eff}$  close to the solid line  
 149 that ensure a linearly growing  $Q$ . The origin of this raise/plateaus dynamic  
 150 can be explained the following way: With time, the production of an oil  
 151 source eventually decreases, meaning that exploration is firstly needed to  
 152 extract more oil. It means that if exploration does not suggests new sources  
 153 to exploit, the production stagnates because it is not possible to invest in new  
 154 sources, therefore  $k_{eff}$  is constant or slightly decreasing. When new sources  
 155 become available, the production effort can quickly increase until the new  
 156 sources become less available and then exploration has to start again.

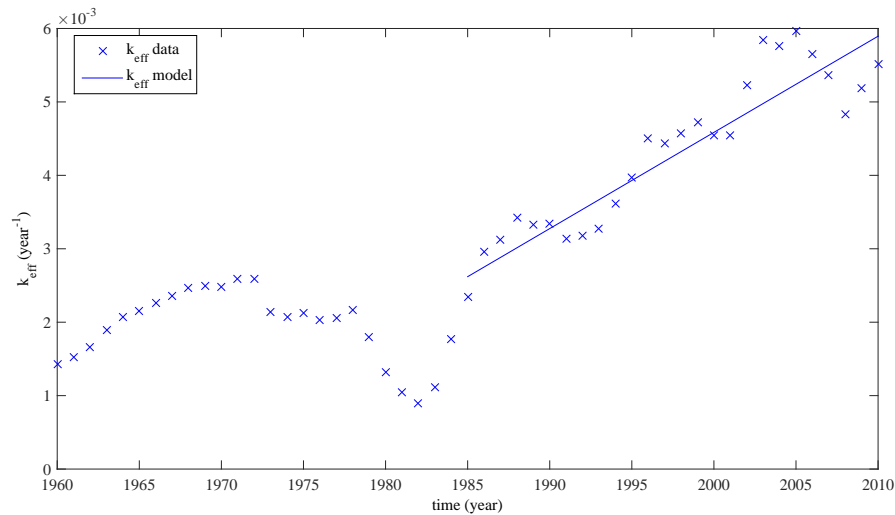


Figure 2:  $k_{eff}$  time evolution for  $k_1 = 5\%$

157 It is here noticed that  $k_{eff}$  depends on the choice of  $k_1$  according to  
 158 Eq.(1), hence the parameter  $k_{eff}$  has been fitted for different values of  $k_1$  in  
 159 the range 4 – 7%. A numerical sensitivity study is conducted on  $k_1$ , using  
 160 a Runge-Kutta 4 method with a 0.5 year time step to solve Eq.(1) and (2)  
 161 from 1960 to 2010. It leads to various results in test simulations, always in  
 162 a 5% range of previsions for production and EROI evolution, for  $k_1$  in the  
 163 range 4 – 6%.  $k_1 = 7\%$  is discarded in this study since it leads to slight



164 underestimation of production and peak, suggesting 7% is an overestimation  
165 of the actual value of  $k_1$ . Since the value of  $k_1$  cannot be set with a greater  
166 accuracy, all the results presented in this study will be calculated with  $k_1$  in  
167 the range 4 – 6%.

## 168 **2. Projections of the production effort**

### 169 *2.1. Projections based on the constant 1985-2000 dynamic*

170 Following this line using raises and plateaus, allow to optimize the oil  
171 benefit and production: It could be compared to a game where  $k_{eff}$  should be  
172 kept on this line to optimize benefits. This strategy can then be extended to  
173 forthcoming years. One can observe nevertheless that  $k_{eff}$  begins to deviate  
174 on the period 2000-2010. It seems that, in order to keep a constant derivative  
175 for  $Q$ ,  $k_{eff}$  should not follow the same trend any more. Extending the data  
176 using Eq.(1) and Eq.(2) to fit  $Q$  values allows to evaluate the evolution of  
177 this slope. On the period 2000-2020, the slope seems to be different from  
178 the one observed on the period 1985-2000. Instead of plateaus, between 2005  
179 and 2010, drops are required on  $k_{eff}$  to fasten the effect on  $Q$ , and the mean  
180 slope has to be higher than before.

### 181 *2.2. The limit of the 2000-2020 dynamic*

182 The extension of that game actually shows the real rule: in order to  
183 keep a linearly growing  $Q$ ,  $k_{eff}$  has to evolve exponentially in time. To  
184 keep playing that game the way it started,  $k_{eff}$  should follow the equation  
185  $k_{eff} = 1.3839 \cdot 10^{-27} \exp(t/\tau)$  with  $\tau = 34.57$  years  $\pm 0.8$  year. Fig.3 shows  
186 the extension of  $k_{eff}$  on the period 2005-2020 along with a projection using  
187 the exponential fit. This projection suggests that the slope has again to be  
188 inflected, by strongly inflecting the shoots between the plateaus/drops. It  
189 also requires exploration to be more and more efficient, to ensure that the  
190 plateaus won't last for too long. According to [10] for instance, the exact  
191 opposite is expected for exploration, but such studies do not take into account  
192 that  $k_{eff}$  represents for the moment less than 1% of the available investment,  
193 which suggests that oil producers could invest way more in exploration. It is  
194 nevertheless in accordance with [7] and [10] on one point: This is the end of  
195 cheap oil, in the sense “it is the end of cheap oil extraction”.

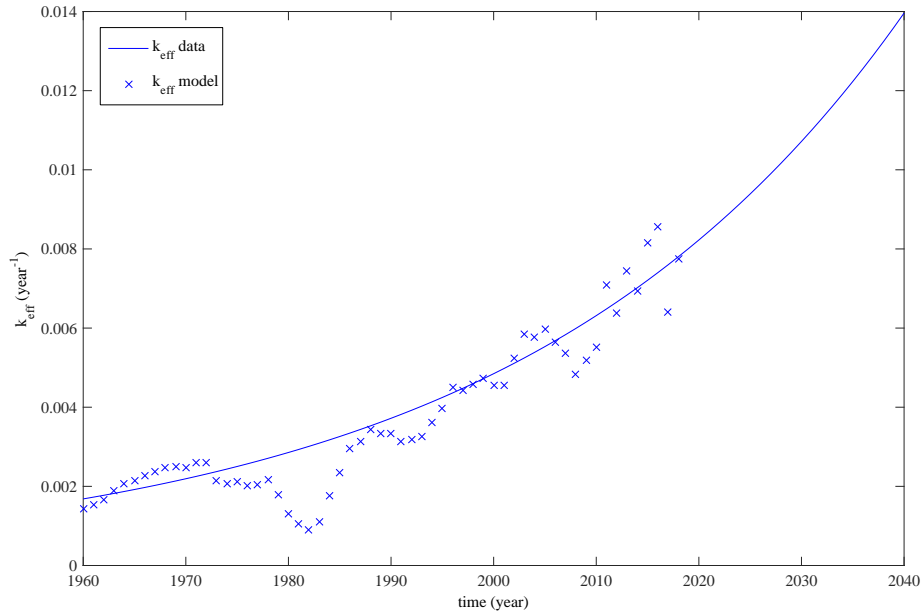


Figure 3:  $k_{eff}$  time evolution for  $k_1 = 5\%$

196 **3. A business as usual scenario**

197       Supposing the exponential investment dynamic could be sustained until  
198 the mean liquid fossil fuel *EROI* drops to 1 represents a business as usual  
199 scenario where oil is extracted until the energetic benefit drops to zero. It  
200 would allow a linear growth of  $Q$  until 2088-2090, depending on the actual  
201 value of mortality rate  $k_1$ : 2088 is the limit for the highest realistic value of  
202 mortality rate ( $k_1 = 6\%$ ) and 2090 is the limit for the lowest realistic value  
203 ( $k_1 = 4\%$ ). This linear growth is followed by a quick collapse of  $Q$  down to  
204 zero in 12 to 18 years, again depending on the value of  $k_1$ . This scenario  
205 supposes that oil can be extracted until  $k_0 = 1$ , i.e. when oil extraction  
206 cannot generate benefits any more. It does not take into account all the  
207 marginal costs of the oil business, hence collapse of oil production will occur  
208 before  $k_0$  reaches 1. In order to let the reader evaluates by herself/himself  
209 the oil peak based on a threshold value of  $k_0$ , a plot of  $k_0$  as a function of  
210 time is presented in Fig.4 for the extreme values of  $k_1$ .

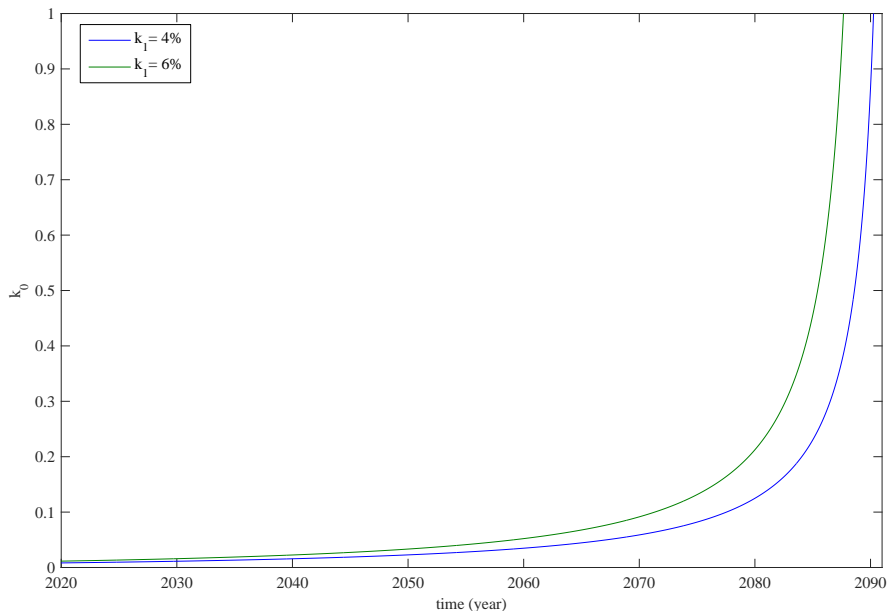


Figure 4:  $k_0$  time evolution for  $k_1$  in the range 4-6%

#### 211 4. A relation between production effort and oil price

212 The origin of oil price variation is a highly discussed topic. According  
 213 to some authors as [11], it might not follow the law of market any more. It  
 214 is also discussed as being a consequence of geopolitical events, as discussed  
 215 in [12] or [13] for instance. In this article, it is suggested, as mentioned in  
 216 section 1.4, that rapid increase in oil price are due to drops or plateaus in  
 217 investment, what is likely linked to geopolitical events. It is now suggested  
 218 that long time scale oil price variations should follow the same global trend  
 219 as the production effort, in order to keep a constant relative benefit for oil  
 220 producers.

221 Looking at a long time scale history of primary resource prices (such as  
 222 metal to begin with) suggests the existence of an average value over time, as  
 223 observed in [14] for iron, with variations around this value due to geopolitical  
 224 events. For oil, history begins the same way, but after the oil shocks of the  
 225 70's happen, the price never went down to its former, "usual", average value,

226 as it can be observed in data such as [15] or [16]. It is here suggested that oil  
227 shocks happen also because the production effort finally reached a value that  
228 could not be neglected any more. Said otherwise, the cost of oil production  
229 itself was negligible in the global cost of the oil business, hence there was no  
230 link between price and production effort until the late 60's.

231 The evolution of a “mean” oil price is now discussed, meaning that only  
232 the global trend is discussed, without considering short time scale variations  
233 due to geopolitical events. After the shocks of the 70's, it seems necessary for  
234 this mean price to follow the same trend as the production effort, to keep a  
235 constant benefit per barrel. This suggests oil price should rise exponentially,  
236 starting in the 70's, with a characteristic time of roughly 35 years.

237 This remark seems in accordance with observation, since a sudden raise  
238 in oil price, similar to the oil shocks, occurred around 2000-2005. If the  
239 link suggested in this section actually exists, the time gap between the two  
240 major raises of oil price is somehow a measurement of the production effort  
241 characteristic time  $\tau$ . This suggests the model led to a slight overestimation  
242 of  $\tau$ , most likely due to the difference between actual EROI history and the  
243 data used in this study to calibrate  $\tau$ . It means  $\tau$  is rather in the range 30-35  
244 years and that the higher estimation of  $k_0$  on Fig.4 should reach 1 around  
245 2080 rather than around 2088.

246 As a consequence, considering a business as usual assumption, oil price  
247 is expected to stabilize around 75 \$2014 per barrel in the forthcoming years,  
248 until 2030-2040 where a peak around 200 \$2014 should occur, followed by a  
249 stabilization around 150 \$2014. A rapid raise should happen again between  
250 2060 and 2075 around 400 \$2014 followed by a continuous raise until the end  
251 of oil production.

## 252 Conclusion

253 This study proposes a model which allow to study the oil extraction  
254 dynamic. This dynamic, which was constant in the period 1985-2000, seemed  
255 to deviate for a new dynamic from 2000 to 2020, what could be considered  
256 in agreement with the evolution of the energy economy since 2000.

257 Nevertheless, this inflection in investment is not sufficient to ensure enough  
258 oil production, as production/exploration investment in liquid fossil fuel ac-  
259 tually requires an exponential growth (with a characteristic time of roughly  
260 30-35 years) to keep a constant growth of  $Q$ . As business usually grows using  
261 a linear increase in investment, this required exponential behaviour of  $k_{eff}$

262 suggests major crisis in oil extraction every 30-35 years. Indeed a piecewise  
263 linear approximation of this curve shows a slope roughly 3 times steeper ev-  
264 ery 30-35 years, meaning that investment rate has to triple every 30-35 years  
265 instead of being kept constant.

266 It is also worth noticing that previous previsions of oil peak were due to  
267 local, momentary damping of the production effort. Every time, the pre-  
268 dicted peak has been postponed through a control of the production effort.  
269 It means that oil peak has been postponed several times in history and can  
270 be postponed again, possibly until 2090, based on the will of oil producers to  
271 increase the production effort up to a maximum value that would still allow  
272 to generate some benefits.

273 Finally, the comparison between oil price chart and production effort at  
274 long time scale suggests a need for the oil price to follow the same trend  
275 as the production effort. As a consequence, oil price is expected to rise  
276 exponentially, also with a characteristic time of 30-35 years.

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