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Disclaimer

I would like to apologise to the reader for the crudeness of this version. References are sparse and results lack discussion. Nevertheless I had the feeling the model should be published rapidly in order to discuss it with readers. In forthcoming versions, this paper will be updated based on these comments.

Forcing a dynamic model for oil production and EROI evolution: The Oil Game

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Abstract

Since 1940, many attempts to model world oil production have been proposed. Those approach, using growing complexity, consider the growing and decay of production independently of external, time-varying, causes. It is here proposed to extend the production equation by modelling a dynamic dependency between oil production and its EROI, using Lotka-Volterra equations. The model obtained, after comparison with oil extraction and EROI evolution on the period 1960-2010, illustrates the production dynamic and the existence of an external, controlling parameter: the production effort which account for the re-investment in the production process. The evolution of this parameter provides some possible explanations about the progress of the oil shocks and also some possible explanations about the peak prediction issues of the classical Hubbert model. Studying this evolution also suggests an attempt to control the oil production in order to obtain a linear time evolution on the period 1960-2010: the oil game. Since the end of the oil shocks, this control has been slightly inflected for the first time around 2000-2005, what could explain the evolution in fossil fuel investment from that time. Unfortunately, in order to keep playing the oil game, this control has now to be strongly inflected, in a manner that our economy never faced before. Finally, it is suggested that even if this new dynamic is kept for twenty years, the production will be at the peak between 2040 and 2048 for the liquid fossil fuels, rushing the EROI decay down to 3.2-4.5 at the peak.

Keywords: Oil production, EROI, dynamic model, production effort

1 Introduction

Models that account for oil production have been published from 1962 (Hubbert (1962)), with increasing complexity (Bartlett (2000); Cavallo (2002); Duncan (2003), citing only very few of them). These models rely on a production dynamic with constant parameters. The aim of this study will be to evaluate how the parameters could evolve in time, based on a coupling between oil production and its EROI. Through this dependency, it is expected to explain why the prediction of peak is always delayed. The model suggested is based on a Lotka-Volterra set of equations, linking an oil production Q with its mean EROI. In this approach, Q is the EROI predator.

The article is organised as follow: A first part is dedicated to a presentation of an assumption on the oil distribution as a function of EROI, which should allow to extend this study to any liquid fossil fuel. It also presents a discussion on a “physical” meaning of a mean EROI, as presented in studies like Hill (2015) where EROI is derived based on thermodynamics assumptions. The model itself and the set of equation are then presented, along with the meaning of the parameters that appear in the model. Some sub-models are here suggested for these parameters. A fitting of the model parameters based on historical evolution of oil production and mean EROI is then performed. Finally, an analysis of the production effort, a forcing parameter, is done, suggesting the existence of a benefit/production optimization: the oil game.

A second part is dedicated to the projection of the production effort that followed the same trend from the mid eighty’s to 2000. An extension to the year 2020 is also discussed, showing an evolution of the production dynamic and a need to update the dynamic in 2020 in order to keep a linearly growing production.

Finally, a few conclusions are suggested about the potential peak and an opening is presented, proposing a strategy to adapt the oil extraction dynamic to an acceptable ecological impact and acceptable economy dynamic.

1. Modelling the interaction between oil production and EROI

1.1. *An assumption about the oil production distribution as a function of EROI*

This section is dedicated to the description of the interactions between oil production Q and EROI. This description is based on the assumption that

36 the EROI considered is representative of the mean EROI at a given time.
37 Therefore, it is suggested that this value can only decrease with time, even
38 if at a given time some wells with higher EROI than the mean value can
39 exist. Also, the following assumption is suggested: if there is no oil (in any
40 form) available at a given EROI, there will always be some existing oil at
41 a lower EROI. This assumption suppose the existence of an oil distribution
42 as a function of EROI. It should allow extending this model from conven-
43 tional oil to non-conventional oil and liquid fossil fuels. This definition does
44 not exactly fit the reality of “measured” EROI, nevertheless it has some
45 “physical” meaning which presents some interest for modelling purpose. The
46 value of this mean EROI should be close to actual wells EROI for a given
47 year. A good candidate to represent this quantity could be the thermody-
48 namic model suggested by Hill (2015), which behave as described above and
49 presents values at different times which are consistent with actual, measured
50 values.

51 *1.2. A dynamic model for oil production and EROI evolution*

52 The approach suggested in this study considers oil production Q (in billion
53 barrels) as a predator of a mean EROI.

54 The oil production Q is then growing as a function of EROI and Q , and
55 has a natural decay due to the mortality rate of wells, therefore proportional
56 to Q . The production time derivative is composed of two terms: the growing
57 term and the decaying term. The growing term can be justified the following
58 way: At a given time, a production Q is available, so that an amount of
59 oil $Q \cdot EROI$ could be extracted the next year. Lets consider a parameter
60 k_0 (in year⁻¹), called production effort. The growth of Q is then equal to
61 $k_0 \cdot Q \cdot EROI$. The decaying term of Q is simply equal to $k_1 \cdot Q$, where k_1 is
62 the mean well mortality (in year⁻¹).

63 The prey is decreasing proportionally to EROI and Q , and is supposed-
64 edly growing due to the renewal of fossil fuels, which can be considered as
65 happening at geological times. This effect is therefore neglected. The decay
66 can be explained the following way: For each unit of Q that is extracted,
67 the relative EROI variation is proportional to a parameter k_2 (in (billion
68 barrels.year)⁻¹) that is expected to decrease in time, as our ability to extract
69 oil properly increase, and according to the distribution of oil as a function
70 of its availability on earth. In order to model k_2 , the following dependency
71 is proposed: $k_2 = C/(t - t_0)$ where C is a constant (in (billion barrels)⁻¹)

72 and t_0 (in year) is a time offset. The EROI decaying term is then equal to
 73 $k_2 \cdot Q \cdot EROI$.

74 The following equations are then suggested for production and EROI dy-
 75 namic:

$$\dot{Q} = k_0 \cdot Q \cdot EROI - k_1 \cdot Q, \quad (1)$$

76

$$E\dot{ROI} = -k_2 \cdot Q \cdot EROI. \quad (2)$$

77 It is noticed that, amongst these parameters, it seems that k_1 and k_2
 78 represent some effective “physical” properties of the system, whereas k_0 rep-
 79 represents a forcing parameter on which oil producers can play to adapt the oil
 80 extraction to their need.

81 1.3. Fitting the model parameters on the period 1960-2010

82 Since k_1 represents the wells mean mortality, it is assumed it should fit
 83 measured data. For instance, according to Sorrell et al. (2012), this value lies
 84 in the range 4.1 – 6.7%. Now, based on the previous model and suggesting a
 85 value for k_1 , it is possible to plot k_0 and k_2 time evolution, based on historical
 86 data of Q and EROI:

$$k_0 = \frac{\dot{Q}}{Q \cdot EROI} + \frac{k_1}{EROI}, \quad (3)$$

87

$$k_2 = -\frac{E\dot{ROI}}{Q \cdot EROI}. \quad (4)$$

88 The oil production data is extracted from Rodrigue et al. (2016) and the
 89 EROI data is extracted from Hill (2015). The analysis is performed on the
 90 period 1960-2010. The evolution obtained for k_2 is presented in Fig.1. The
 91 continuous line represents the model $k_2 = C/(t - t_0)$, with $C = 0.06036$
 92 (billion barrels) $^{-1}$ and $t_0 = 1950.5$ year.

93 The model seems to fit adequately the data, with a mean relative error
 94 of 1.05%. The evolution shows two periods: The first one, before the first
 95 oil shock, corresponds to a rapid and smooth evolution of k_2 . The second
 96 one, after the second oil shock, shows some jumps which could correspond
 97 to technological jumps, or simply the exploitation of fossil fuels that were
 98 not exploited before due to their EROI. When these wells become of interest
 99 and start being exploited, the value of k_2 suddenly drops because exploiting
 100 these wells does not affect much the mean EROI.

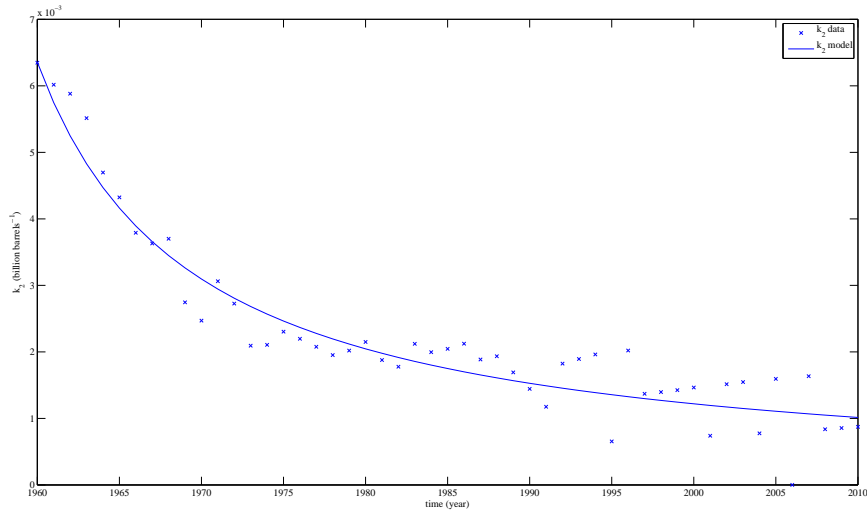


Figure 1: k_2 time evolution

101 *1.4. Studying the production effort*

102 The parameter k_0 , which represents the forcing of the system is plot over
 103 time in Fig.2 for $k_1 = 5\%$. Its analysis provides some possible characteristics
 104 of the oil extraction strategy, which are presented below.

105 On the period 1960-1968, k_0 shows a relatively linear behaviour. This
 106 period corresponds to an evolution of oil extraction that begin to behave as
 107 exponential around 1965. Due to the laws of market, and the oil price at
 108 this period which is rather low, keeping an exponential growth for Q could
 109 have been responsible for an important decrease in oil price. In order to
 110 keep a decent benefit without using too much of their resources, producers
 111 have to reduce Q , by reducing k_1 . This strategy begins in 1969, according to
 112 Fig.2. However, due to the behaviour of k_2 at this time, the system shows a
 113 great inertia and damping k_0 is not sufficient to control instantaneously Q .
 114 Any reasons could have been sufficient to suddenly reduce k_0 and adapt Q .
 115 Three years after the first inflection of k_0 , the first oil shock happen and k_0
 116 is adapted.

117 After the first shock, k_0 is surprisingly constant, with a linear time evo-
 118 lution for Q . This strategy seems to optimize the production of a limited
 119 resource (as oil). The second shock corresponds to another, longer drop of

120 k_0 .

121 After the second shock, k_0 seems to evolve (globally) linearly, with rises
122 and plateaus during the period 1985-2000. The solid line corresponds to the
123 equation $k_0 = 0.00349 + 0.000173 \cdot (t - 1985)$, which fits the data with a
124 mean relative error less than 1%. This behaviour allow Q to grow linearly in
125 time. Also, since the system inertia has evolved in time with k_2 , the plateaus
126 are responsible every time for a slow damping of Q , which corresponds to
127 past predictions of a nearby peak, using Hubbert's curves. This phenomenon
128 allows every time a rise in oil price (this can be shown in comparing Fig.2
129 with an oil price chart), precisely at the moment where producers need to
130 increase their investment to keep k_1 close to the solid line that ensure a
131 linearly growing Q .

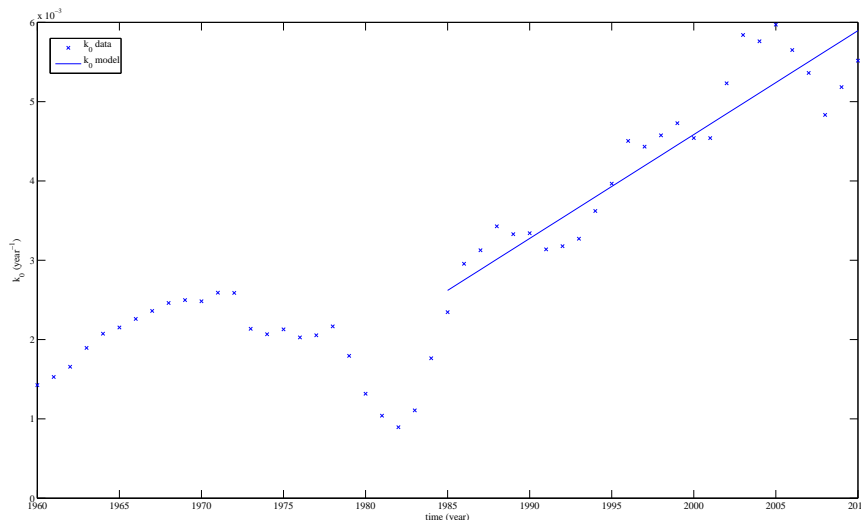


Figure 2: k_0 time evolution for $k_1 = 5\%$

132 It is here noticed that k_0 depends on the choice of k_1 according to Eq.(1),
133 hence the parameter k_0 has been fitted for different values of k_1 in the range
134 4 – 7%. A sensitivity study is conducted on k_1 , which lead to various results
135 in test simulations, always in a 5% range of previsions for production and
136 EROI evolution, for k_1 in the range 4 – 6%. $k_1 = 7\%$ is discarded in this
137 study since it always lead to underestimation of production and peak. Since

138 the value of k_1 cannot be set with a greater accuracy, all the results presented
139 in this study will be calculated using a range on k_1 which lead to a range on
140 the $\{Q, \text{EROI}\}$ results.

141 **2. Projections of the production effort**

142 *2.1. Projections based on the constant 1985-2000 dynamic*

143 Following this line using rises and plateaus, allow to optimize the oil
144 benefit and production: It could be compared to a game where k_0 should be
145 kept on this line to optimize benefits. This strategy can then be extended to
146 forthcoming years. One can observe nevertheless that k_0 begins to deviate on
147 the period 2000-2010. It seems that, in order to keep a constant derivative
148 for Q , k_0 should not follow the same trend any more. Extending the data
149 using Eq.(1) and Eq.(2) to fit Q values allows to evaluate the evolution of
150 this slope. On the period 2000-2020, the slope seems to be different from the
151 one observed on the period 1985-2000. Instead of plateaus, between 2005 and
152 2010, drops are required on k_1 to fasten the effect on Q , and the mean slope
153 has to be higher than before. This behaviour could explain the evolution in
154 fossil fuel extraction that happened around 2000-2005.

155 *2.2. The limit of the 2000-2020 dynamic*

156 The extension of that game actually shows the real rule: in order to
157 keep a linearly growing Q , k_0 has to evolve exponentially in time. To
158 keep playing that game the way it started, k_0 should follow the equation
159 $k_0 = 1.3839 \cdot 10^{-27} \exp(t/34.57)$. Fig.3 shows the extension of k_1 on the
160 period 2005-2020 along with a projection using the exponential fit. This pro-
161 jection suggests that the slope has again to be inflected, either by strongly
162 inflecting the shoots between the plateaus/drops, either by shortening the
163 plateaus/drops. In either case, this dynamic of oil extraction has never oc-
164 curred. It means that in the forthcoming years, oil extraction strategies could
165 take a direction that economy never experienced before. Besides, this new
166 direction is highly unsustainable since it will have to be corrected very soon,
167 due to the exponential behaviour that is required to get a linearly growing
168 production. It suggests that, if these extractions strategies are kept in the
169 forthcoming years, the frequency of economic crises may also rise exponen-
170 tially.

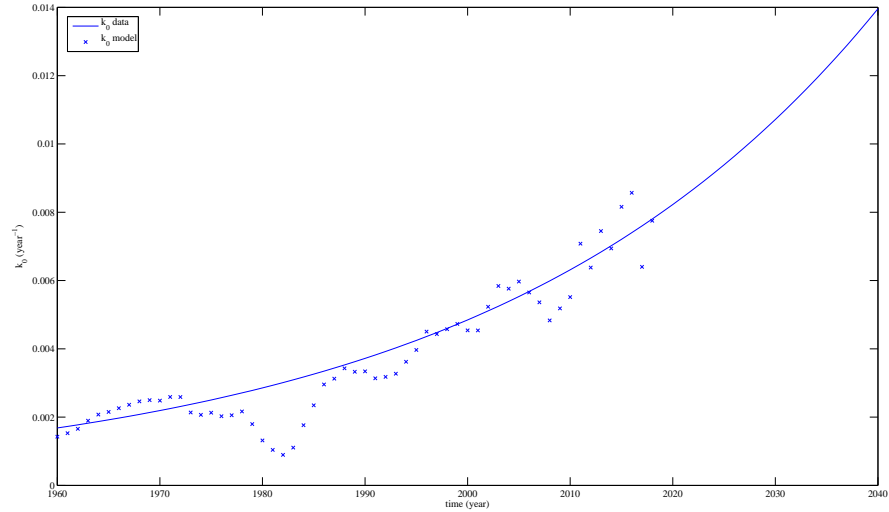


Figure 3: k_0 time evolution for $k_1 = 5\%$

171 **Conclusion and opening**

172 This study proposes a model which allow to study the oil extraction
 173 dynamic. This dynamic, which was constant in the period 1985-2000, seemed
 174 to deviate for a new dynamic from 2000 to 2020, what could be considered in
 175 agreement with the evolution of the energy economy since 2000. However, it
 176 suggests again an important evolution of the extraction dynamic, beginning
 177 in 2020. This new dynamic suggests an inflection in the economy that never
 178 occurs, which should be kept for twenty years in order to push back the peak
 179 between 2040 and 2048, rushing the EROI decay down to 3.2-4.5 at the peak.

180 The following methodology can nevertheless be suggested based on this
 181 model: considering an acceptable ecological impact and an acceptable econ-
 182 omy dynamic (a scenario for the production effort that could be sustained),
 183 it should be possible to evaluate an amount of oil that could be extracted,
 184 and therefore adapt an energy transition based on that amount of oil.

185 **Acknowledgement**

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