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# <sup>1</sup> Oscillating grid turbulence in shear-thinning polymer solutions

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Oscillating grid apparatus are well known and convenient tools for the fundamental study of turbulence and its interaction with other phenomena, since they allow to generate turbulence supposedly homogeneous, isotropic, and free of mean shear. They could in particular be used to study turbulence and mass transfer near the interface between non-Newtonian liquids and a gas, as already done in air-water situations. Although frequently used in water and Newtonian fluids, oscillating grid turbulence generation has yet been rarely applied and never characterized in non-Newtonian media. The present work consists in a first experimental characterization of the flow properties of shear-thinning polymer (Xanthan Gum, XG) solutions stirred by an oscillating grid. Various polymer concentrations are tested for a single grid stirring condition. The dilute and semi-dilute entanglement concentration regimes are considered. Liquid phase velocities are measured by Particle Image Velocimetry (PIV). The existing mean flow that establishes in the tank is described and characterized, as well as turbulence properties (intensity, decay rate, length-scales, isotropy etc.). Oscillating grid turbulence (OGT) in dilute polymer solutions induces an enhanced mean flow compared to water, a similar decay behaviour with yet different decay rates, and enhanced turbulence large scales and anisotropy. In the semi-dilute regime of XG, turbulence and mean flows are essentially damped by viscosity. The evolution of mean flow and turbulence indicators leads to the definition of several polymer concentration sub-regimes, within the dilute one. Critical concentrations around 20 ppm and 50 ppm are found, comparable to drag reduction characteristic concentrations.

### 6 I. INTRODUCTION

Oscillating grid stirred tanks have been used for many
purposes in research on turbulence, for example the study
of interactions between turbulence and solid impermeable
boundaries<sup>1,2</sup>, in stratified media<sup>3-6</sup>, or to study the behavior
of bubbles, cells, fibers and aggregates suspended in a turbulent liquid phase<sup>7-10</sup>.

Such devices are said to generate quasi homogeneous and 13 <sup>14</sup> isotropic turbulence in horizontal planes (parallel to the grid), 15 and to yield theoretically no mean flow, which is their major advantage compared to fixed grid setups. The absence of 16 strong mean shear avoids the destruction of complex flu-17 ids' components (fibers, polymer chains, cells) that is some-18 times observed in fixed grid turbulence<sup>11</sup>. The fact that tur-19 bulent structures are theoretically not advected by any mean 20 flow makes them more easily observable by advanced optical 21 techniques such as PIV and PLIF (Planar Laser Induced Fluorescence). It will yet be shown hereinafter that the validity 23 of this no-mean flow assumption is limited without any re-24 duction of turbulent properties. Note that when a mean flow does exist in oscillating grid systems, it takes the shape of a 26 set of stationary recirculation patterns, as will be extensively 27 discussed in this work. The term "mean flow" will hereinafter 28 refer to these recirculation, which can sometimes be called 29 secondary recirculation" in the literature. 30

For the previous reasons, this type of device has been extensively used for the study of turbulence and gas-liquid mass transfer at free surfaces<sup>12–15</sup>. The combination of numerical simulations<sup>16–18</sup> and experiments<sup>19–23</sup> has allowed to describe the behavior of turbulence close to a flat air-water interface,

36 and the influence of this near surface turbulence on the fun-37 damental and local mechanisms of gas dissolution into the 38 liquid phase. Yet these mechanisms are known to be mod-<sup>39</sup> ified when considering gas dissolution into a non-Newtonian 40 or surfactant-laden liquid phase. The influence of surface con-41 tamination by surface active agents has already been the sub- $_{42}$  ject of numerical simulations<sup>24,25</sup> and experimental works<sup>13</sup>, 43 but the effect of bulk fluid rheological properties remains to be 44 understood. This phenomenon is of great interest, since it is <sup>45</sup> frequently encountered in the chemical, pharmaceutical, and process industries<sup>26-28</sup>. In this context, experiments of non-46 47 Newtonian turbulence's interactions with flat free interfaces 48 are still needed. The convenience of oscillating grid setups, 49 often used in air-water situations, make them interesting can-<sup>50</sup> didates for such experiments.

Only a few studies of oscillating grid turbulence 51 52 (OGT) with non-Newtonian liquids can be found in the  $_{53}$  literature<sup>29-31</sup>, and none of them seems to tackle the effects <sup>54</sup> of variable viscosity on the possible mean flow and turbulence <sup>55</sup> properties mapping in the whole tank. Moreover, these studies <sup>56</sup> focus on turbulence below the grid, between the sweep region 57 and the bottom of the tank, for which the boundary condition 58 and thus mean flows are inherently different from the appli-<sup>59</sup> cation considered here. The aim of this paper is to study the 60 influence of a shear thinning behavior on the hydrodynamics <sup>61</sup> above the grid and below a free surface in an oscillating grid stirred tank, as a first necessary step for a further investigation of near surface turbulence and mass transfer in similar flu-63 ids. Such properties are given to an initially Newtonian fluid, 64 water, by addition of a minute amount of polymer (Xanthan 65 66 Gum, XG). Fluid velocity measurements are achieved using 67 Particle Image Velocimetry (PIV) in a region of the tank lo-68 cated between the upper position of the grid and the free surface. 69

<sup>70</sup> The objective is to describe precisely OGT in shear thin-<sup>71</sup> ning polymer solutions and see if its velocity field statistical

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72 properties can be compared to OGT in water. In other words, 73 can this device be used to generate controlled turbulence in such fluids, and does the hypothesis of a negligible mean flow 74 remain valid ? As an underlying question: can oscillating grid 75 apparatus be used to study turbulence near a free surface in shear thinning polymer solutions? To do so, indicators of tur-<sup>119</sup> Where,  $C_{TT} \simeq 0.1, C_{1_{HT}} \simeq 0.25, C_{2_{HT}} \simeq 1.2$ , and n = -1 bulence and mean flow topology are extracted. The effects of <sup>120</sup> in the original works. These relationships have been verified 77 shear thinning polymer solutions? To do so, indicators of tur- 119 <sup>79</sup> polymer concentration on the existing mean flow topology are <sup>121</sup> by a number of studies<sup>5,6,12,14,35–39</sup>. The values of the differ-78 so first studied. The evolution of turbulence properties with the 122 ent constants vary among the works in the following ranges: 81 83 regimes evidenced are compared to drag reduction character- <sup>126</sup> the S/M ratio<sup>4,40</sup>. 85 istics of XG.

### BACKGROUND 86 1.

### Oscillating grid turbulence in Newtonian liquids 87

The principle of an Oscillating Grid apparatus is to pro-88 <sup>89</sup> duce turbulence by making a grid oscillate at a frequency fand with an amplitude or stroke S. It is commonly said that 90 the jets and wakes behind the grid's holes and bars interact 91 to generate turbulence<sup>3,4,14,32</sup>, which then diffuses away from <sup>135</sup> 2. Mean flow the grid. Turbulence can be studied either above<sup>4</sup> or below  $_{94}$  the grid<sup>6</sup>, the bottom boundary condition being a rigid wall,  $_{136}$ 95 and the top boundary condition either a rigid wall<sup>3</sup> or a free 137 but only isotropic and homogeneous turbulence, was rather <sup>96</sup> surface<sup>20</sup>. The first oscillating grid apparatus were designed <sub>138</sub> well believed. With the development of PIV techniques al-97 by Rouse and Dodu<sup>33</sup> and Bouvard and Dumas<sup>34</sup>, but full 139 lowing further spatial investigations of the flow inside OGT 98 characterization of OGT in prismatic tanks only came with 140 tanks, it became clear that a mean flow always establishes, <sup>99</sup> the pioneer works of Thompson and Turner<sup>3</sup> and Hopfinger <sub>141</sub> even when matching the previous requirements. McKenna and Toly<sup>4</sup> (that are respectively referred to as TT and HT later 142 and McGillis<sup>41</sup> showed the existence of persistent mean flow <sup>101</sup> in the manuscript).

### Turbulence properties 102 **1**.

103 <sup>104</sup> and distance from the bottom of the tank defined by the previ-<sup>149</sup> OGT systems. Oscillating grid apparatus should thus be seen 105 107 geneity and isotropy are achieved in horizontal planes, far 152 free, homogeneous and isotropic turbulence. McCorquodale <sup>108</sup> enough from the walls and from the grid's extreme positions <sup>153</sup> and Munro<sup>42</sup> recently suggested a method for reducing mean 109 (2 or 3 times the mesh size M defined below, depending on the 154 flows in OGT using an inner box placed inside the stirred tank 110 study).

Expressions for the root mean square (rms) of horizontal 156 This will be discussed in section V B. 111 112 and vertical velocity fluctuations, respectively denoted  $u'_x$  and 157 113  $u'_{z}$ , as a function of the distance from the grid z have been 158 ous conclusions are yet only valid and established when con-<sup>114</sup> derived respectively by Hopfinger and Toly<sup>4</sup> as:

$$\begin{array}{ll} u'_x &= C_{1_{HT}}.f.S^{1.5}.M^{0.5}.z^n \\ u'_z &= C_{2_{HT}}.u'_x \end{array}$$
(1)

115 116 tions (or Stroke), and the mesh size M. An expression for the 165 briefly summarize the literature of turbulence in polymer solu-<sup>117</sup> evolution of the horizontal integral length scale of turbulence <sup>166</sup> tions. Studies involving grid turbulence in polymer solutions 118 L with z comes from the work of Thompson and Turner<sup>3</sup>:

$$L = C_{TT}.z \tag{2}$$

distance from the grid and polymer concentration is then ad- 123  $C_{TT} \in [0.1..0.4], C_{1_{HT}} \in [0.2..0.3], C_{2_{HT}} \in [1.1..1.4]$ . The dressed. Finally, possible causes for the apparent mean flow 124 proportionality coefficient between the integral length scale enhancement are discussed, and the different concentration 125 and the distance to the grid  $C_{TT}$  was also found to depend on

> It should also be noted that the previous laws are valid for 127 <sup>128</sup> turbulence both above and below the grid<sup>4,6</sup>, regardless of the 129 boundary condition, except at the vincinity of the either liquid/solid or liquid/gas interface where turbulence is affected 1 30 by boundary layer interactions<sup>1,2,12,23</sup>. However, mean flow patterns that develop in the tank should depend on boundary 133 conditions. In this work, we will more specifically focus on 134 the case of the flow above the grid and below a free surface.

At first, the concept of OGT generating no mean flow, 143 structures, with relative high kinetic energy level as compared 144 to the turbulent kinetic energy. Moreover, this mean flow 145 seems to be poorly repeatable and strongly dependent on ini-<sup>146</sup> tial conditions<sup>14</sup>. It is therefore really hard to predict the mean 147 flow that could occur in the oscillating tank during a spe-When fulfilling a set of conditions on grid shape, solidity 148 cific measurement, and this is one of the main limitations of ous authors, OGT is supposed to yield a quasi-homogeneous 150 as ways of generating controlled turbulence with low mean and isotropic turbulence with negligible mean flow. Homo- 151 flow rather than as a tool to produce completely mean shear 155 to separate the wall-induced vortices from the rest of the flow.

Nevertheless, it is important to point out that all the previ-159 sidering OGT in water (or other Newtonian fluids). Character-160 ization of turbulence velocity fluctuations and integral length scales, and evidence of mean flows associated to OGT in shear 161 162 thinning polymer solutions are yet to be performed. This is the 163 scope of this study. In order to understand the changes that Where f is the grid frequency, S the amplitude of oscilla- 104 may arise upon polymer addition, the following paragraphs 167 are detailed in section II C.

### 168 B. Drag reduction and turbulence in polymer solutions

169 <sup>170</sup> concentration of polymer diluted in a solvent could drastically <sup>228</sup> mer chain increases: polymer chains store energy at small 171 it may decrease the pressure drop in a pipe by up to  $80 \%^{44}$ . 172 As it is of paramount interest in many applications such as 173 174 studies have already addressed this topic<sup>45</sup>. 175

176 and by boundary layer interactions at the grid's bar level, drag 177 reduction should be one of the mechanisms at the origin of 178 the modification of flow properties by the polymer. The ba-179 sic principle is that the polymer molecules introduced in the 180 solvent act as molecular springs, undergoing coil-stretch tran-181 sition thus storing part of the kinetic energy of the flow<sup>46,47</sup>. 182 This is especially efficient in high shear regions close to the 183 walls and leads to an apparent increase in the buffer layer 242 184 thickness of the boundary layer and a drastic reduction of the 185 186 187 <sup>190</sup> reduction<sup>47</sup>. The higher the polymer concentration, the lower <sup>248</sup> burst events generated at the bottom of a channel flow by in-192 193 194 195 196 199 200 201 202 relatively rigid, elongated and undeformable molecules. XG <sup>262</sup> chains tend to align with the turbulent front<sup>57</sup>. Apart from 205 stresses the importance of polymer conformation and rigidity 206 in its interactions with the flow. 207

The physical mechanisms of drag reduction relies on the in-208 teractions between polymer chains and the flow, and the scales 209 at which polymers can take or give energy to the fluid. Poly-210 mer chains' structure gives them the ability to deform elasti-211 cally and store energy at small scales<sup>46</sup>, which tends to trun-21.2 cate or modify the flow energy cascade before reaching the 268 213 214 215 216 218 219 220 duced at small scales in viscoelastic solutions. The same con- 275 in grid turbulence of dilute polymer solutions and evidence clusions can also be drawn using an experimental Lagrangian 276 high Reynolds number viscoelastic turbulence. 222 approach. Crawford et al. 61 for example performed time re- 277 experiments, they found that both the shape of the energy 223 solved measurement of Lagrangian acceleration of tracer par- 278 spectrum and the elastic ("Lumley") scale evolved with time, <sup>224</sup> ticles in a *washing-machine* turbulence and found that the <sup>279</sup> which they explained by the destruction of polymer chains by

<sup>225</sup> rms and isotropy of particle accelerations were substantially <sup>226</sup> decreased in dilute polymer solutions as compared to water. It has been known since the late 1940's<sup>43</sup> that a very small <sup>227</sup> Viscous dissipation is reduced as the dissipation by the polyreduce drag. This phenomenon is called drag reduction, since <sup>229</sup> scales and so the small scales of turbulence are damped. But 230 the effect of polymer also propagates to larger scales. This 231 can translate for example into an increase of integral length hydraulics systems or oil and gas industry, a huge number of <sup>232</sup> scales and of large scale fluctuations of velocity<sup>56,59,60</sup>. This <sup>233</sup> is also the case in inelastic shear-thinning solutions<sup>62</sup>. Nguyen Since OGT is driven both by large scale motion of the grid <sup>234</sup> *et al.*<sup>55</sup> recently studied the flow-polymer interactions in the <sup>235</sup> dissipative range and showed that the polymer pumps energy 236 at small scale when it is deformed by flow structures, and 237 can give back a part of this energy to the flow when relax-238 ing, thus explaining the up-scales propagation observed ex-239 perimentally. As for OGT, we should thus expect an influence 240 of the polymer on the large scale flow patterns. This is indeed 241 described in section IV A.

In channels or pipes, most of the strain field to be re-243 duced can be found near the walls, and the forcing is done friction. For a given concentration of polymer, the friction <sup>244</sup> at small scales (roughness of the wall), hence the polymer coefficient variations as a function of the Reynolds number <sup>245</sup> effects are felt at low concentrations, typical of drag reducdeparts from the classical Prandtl-Karman law when reaching<sup>246</sup> tion. For example, Cai *et al.*<sup>63</sup> observed that viscoelastican onset Reynolds number. This is called the onset of drag <sup>247</sup> ity strongly decreased both the frequency and the intensity of the Reynolds number at the onset. When further increasing Re<sup>249</sup> hibiting the small scale coherent structures in the sheared subafter the onset, the friction factor decreases extremely rapidly<sup>250</sup> layer. However, when forcing turbulence by energy injection until it reaches an asymptotic trend. Depending on the nature <sup>251</sup> at scales larger than the cutoff scale, polymers do not play of the polymer, two types of drag reduction leading to two <sup>252</sup> a role at the turbulence production step, and non-Newtonian asymptotic slopes are possible. In type A drag reduction, the <sup>253</sup> effects are found to appear for concentrations larger than typfriction coefficient tends to what is called the maximum drag<sup>254</sup> ical drag reduction concentrations<sup>59</sup>. Finally, an interesting reduction asymptote empirically determined by Virk<sup>48</sup> (see <sup>255</sup> feature observed in flows with strong mean shears such as Sreenivasan and White<sup>47</sup>, figure 1). In type B drag reduction, <sup>256</sup> channel flows<sup>63,64</sup> is an increase of turbulence anisotropy. It it ultimately follows a Prandtl-Karman slope shifted towards<sup>257</sup> comes from the tendency of polymer chains to align with the lower friction coefficients<sup>49</sup>. The drag reduction level of this <sup>258</sup> main shear direction, leading to an increase in tangential velast curve is fixed by the polymer concentration. Type A is <sup>259</sup> locity fluctuations and a decrease in the normal ones. This characteristic of flexible random coiled and highly deformable <sup>260</sup> effect was lately observed to happen during the propagation polymer chains (eg. PolyEthylene Oxide, PEO), and type B of <sup>261</sup> of a turbulent/non-turbulent interface as well, where polymer studied here is of this second type<sup>50</sup>. This last observation <sup>263</sup> elasticity, shear-thinning induced by the presence of polymer 264 chains is also known to affect many features of flows, turbu-<sup>265</sup> lent or not, from large to small scales, as described recently in 266 several works<sup>62,65,66</sup>.

## 267 C. Grid turbulence in polymer solution

The influence of non-Newtonian behavior on grid Kolmogorov scale<sup>51–55</sup>. Such interactions should also impact <sup>269</sup> turbulence has been mainly studied for fixed grid turbulence in the homogeneous isotropic case, without high 270 configuration<sup>44,67–71</sup>. It was found that grid turbulence shear and boundary layers effects. It has been shown from 271 for such fluids was much more anisotropic, as expected from direct numerical simulations<sup>56,57</sup> and experiments<sup>53,58-60</sup> that 272 flows with a preferential shear direction, but also that it dethe Eulerian quantities of the bulk turbulence (strain, enstro- 273 cayed more slowly. Recently, Vonlanthen and Monkewitz<sup>11</sup> phy, Reynolds stress, velocity gradients fields) are strongly re- 274 used PIV measurements to look at turbulent spectra and scales In their

280 the strong shears in the vicinity of the grid. This shows one of the limits of fixed grid devices for the study of turbulence in 281 polymer solutions: reaching high levels of turbulence requires 282 high flow rates which may cause important degradation of the 283 polymer throughout the measurements. One may thus prefer 284 using OGT rather than fixed grid. 285

Citing Vonlanthen and Monkewitz<sup>11</sup>, "The difference be-286 287 tween the effect of polymers on turbulence without and with 288 mean shear is that in the former case the polymers can only 289 provoke additional local energy dissipation, while in the lat-290 ter case they can in addition modify the mean shear and with it long-range energy exchange by instabilities". The non-291 <sup>292</sup> Newtonian properties of the fluid are in such a way supposed to affect the OGT turbulence decay law, since they act on lo-293 <sup>294</sup> cal turbulence dissipation. The previous sentence also implies 295 that the secondary flows inside the grid stirred tank are very likely to be different in dilute polymer solution than for the 296 Newtonian case. 297

The first study of OGT in viscoelastic dilute polymer so-298 <sup>299</sup> lutions (PEO) was made by Liberzon *et al.*<sup>29</sup>, who observed the velocity propagation of the boundary between turbulent 300 and non-turbulent regions in the tank, at the first instants after 301 the onset of the grid's oscillations. They found that the tur-302 bulent/non turbulent interface moved globally faster in dilute 303 polymer solution than in water. However, the characteristics 304 of turbulence and mean flow in a steady state are not men-305 306 tioned, and the author later admitted that results could have 307 been contaminated by the presence of mean shear at the walls of the tank (*i.e.* by the unexpected mean flow)<sup>57</sup>. 308

Wang et al. 30,31 later used a two oscillating grid device to 351 tion : 309 310 study the viscoelastic effects of surfactants and dilute polymer on coherent structures. They confirmed that the addition 311 of polymer tends to decrease the small scale effects of tur-31 2 bulence, and that this decrease can not only be attributed to 31 3 314 the overall viscosity increase, since it is not associated with a 352 31 5 <sup>316</sup> property of the flow seemed to strongly modify the spectrum <sup>354</sup> and exponents a and p depend on the polymer concentration  $_{317}$  of turbulent structures at large wave numbers. They also  $_{355}$   $C_{XG}$ . The power law decay exponent of viscosity with the 318 found that the turbulent small scale suppression effect arose 356 increasing shear rate is (p-1). a is a parameter for the transition 319 only when reaching a critical polymer concentration (unfor-<sup>320</sup> tunately not quantified), which may not be the same in OGT <sup>358</sup> are reported in table I for  $C_{XG}$  variations over 3 decades. 321 than in channel flows or fixed grid experiments, and seems 359 322 to be higher than the critical concentration for drag reduction 360 with polymer concentration (figure 1), one can clearly evi-323 effects.

To our best knowledge, no existing study mentions the pos-324 325 sible mean flow that could have developed in grid stirred tanks, and even less its probable polymer concentration de- 364 326 pendency. Moreover, the previous studies focused on the case 365 327 of a viscoelastic polymer solution while the fluid studied here 366 328 is shear thinning and inelastic. The evolution of stationary 367 329 turbulence properties with the distance from the grid, such as 330 368 turbulent kinetic energy and velocity fluctuations rms also re-331 369 main unknown in single oscillating grid systems with dilute 332 370 polymer solutions (e.g., can profiles analog to HT's law 1 be 333 334 exhibited ?). The aim of this work is thus to characterize both 371 335 these turbulence properties and the mean flow that possibly 372 336 establishes in the tank. 373

Table I. Carreau-Yasuda fitting parameters for the shear thinning behavior of XG solutions at various concentrations.

### 337 III. MATERIALS AND METHODS

### 338 A. Polymer solutions

Shear-thinning properties are conferred to the liquid by ad-339 340 dition of Xanthan Gum (XG), into distilled water. Here XG 341 produced by Kelco under the commercial name Keltrol CG-T is used. Its average molar mass is  $M_w = 3.4 \times 10^6$  g mol<sup>-1</sup> and <sup>343</sup> its polydispersity equal to  $1.12^{72}$ . XG is chosen for its high 344 resistance to strong shear and extreme temperature and pH <sup>345</sup> conditions<sup>73</sup>. Such features are useful when using it nearby a <sup>346</sup> rigid oscillating grid (which can locally crate high shears), and <sup>347</sup> for future studies of scalar transfer at gas-liquid interfaces<sup>74</sup>. 348 The rheological properties of such solutions have been mea-349 sured using a MCR 302 Anton Paar Rheometer. Their shear 350 thinning behavior is modeled by a Carreau-Yasuda (CY) equa-

$$\frac{\mu - \mu_{\infty}}{\mu_0 - \mu_{\infty}} = \left(1 + \left(t_{CY} \dot{\gamma}\right)^a\right)^{\frac{p-1}{a}} \tag{3}$$

Where the zero shear rate and infinite shear rate Newtonian decrease of the turbulent kinetic energy. The non-Newtonian 353 viscosities (resp.  $\mu_0$  and  $\mu_{\infty}$ ), characteristic time scale  $t_{CY}$ , 357 between power law and Newtonian behaviours. Their values

> By plotting the evolution of the characteristic timescale <sup>361</sup> dence the three main entanglement concentration domains for 362 XG in aqueous salt-free solutions defined by Cuvelier and <sup>363</sup> Launay <sup>75</sup> and Wyatt and Liberatore <sup>76</sup>.

- The dilute regime (D) in which the interaction between isolated polymer chains and the flow are dominant, and interactions of polymer chains between each other are negligible.
- The semi-dilute (SD) regime, in which the electrostatic and mechanical interactions between molecule becomes significant
- The concentrated (C) regime, for which chain entanglement is the dominant mechanism of the liquid phase rheology.

The transition concentration between the three regimes are 424 time scale and the grid forcing time scale, the grid based Deb-374

<sup>375</sup> identical to the ones observed by Wyatt and Liberatore <sup>76</sup>, and <sup>425</sup> orah number De is defined as: so are the zero shear rate viscosity magnitudes. Time scales 376 measured for the present study are typically one order of mag-377 378 nitude lower. This is explainable by the fact that the proper-379 ties of XG are very sensitive to the molar mass  $M_w$  of the 380 polymer, which depends itself on production and dissolution conditions<sup>73</sup>. Moreover, in order to avoid the formation of 382 disordered chains during dissolution, and ensure the repro-383 ductibility of measurements, special care has to be taken in 428 C. PIV Measurements 384 the pocess of dissolving XG. Only distilled water is used wi-385 hout adding any salt, and moderate stirring and heating con-<sup>386</sup> ditions are applied as specified by Garcia-Ochoa *et al.*<sup>73</sup>. In <sup>429</sup> this rheological characterization, the uncertainty on  $C_{XG}$  is of 388 the order of 1%. The procedure used for the fabrication of XG solutions for PIV experiments yields uncertainty up to 10% on  $^{432}$  (ROI) is a vertical rectangle, in plane  $(\vec{x}, \vec{z})$ . Its width is close <sup>390</sup>  $C_{XG}$  (see Lacassagne <sup>74</sup> appendix A). The associated horizon-<sup>391</sup> tal error bars are not shown on following figures in the interest 392 of clarity, but this uncertainty has to be kept in mind, espe-<sup>393</sup> cially when discussing critical polymer concentrations in sec-<sup>394</sup> tion V A. In this work, the concentration range between 0 and <sup>437</sup> Quantel Nd: YAG laser emitting at  $\lambda = 532$  nm is used to il-<sup>395</sup> 500 ppm is explored. The focus is made on the dilute regime. 396 The onset of the semi-dilute regime is also considered, but <sup>397</sup> no measurement is performed in the inner semi-dilute regime <sup>398</sup> evidenced by Wyatt and Liberatore <sup>76</sup> between 100 ppm and 399 200 ppm (see figure 1 a), slope equal to -0.5).

### Oscillating grid setup 400 B.

401 402 by 277 mm inner cross section. The fluid height is set at 449 is  $\Delta t = 18$  ms. In the second case,  $f_{acq} = 10$  Hz and consequently  $_{403}$  H<sub>f</sub> = 450 mm and the distance between the surface and the  $_{450}$   $\Delta t$ =100 ms. Vector fields are computed with DaVis 8 software 404 average grid position is 250 mm. The vertical axis, oriented 451 using a multipass processing: a first pass with 64 by 64 pixels 405 upwards, is noted z, and x and y are the axis defined by the 452 and 2 following passes with 32 by 32 pixels round Gaussian 406 grid bars. The origin of the reference frame is placed at the 453 weighted interrogation windows, at a maximum 50% overlap. 407 410 has square section bars of width equal to 7 mm, and the mesh 457 <sup>411</sup> parameter (distance between two grid bars) is M = 35 mm. <sup>458</sup> est Kolmogorov length scale and Taylor micro scales of tur-412 This yields a solidity of 0.36, below the maximum value of 459 bulence are supposed to be found in the water case, for which 413 0.4 recommended by Thompson and Turner<sup>3</sup>. The frequency 400 the viscosity is always the lowest. Based on the velocity and  $_{414}$  is fixed at f = 1 Hz and the Stroke at S = 45 mm. It allows to  $_{401}$  length scales order of magnitudes shown in figures 7 and 9 415 define a grid-based Reynolds number using the definition of 462 (scales arbitrarily taken at z=20 mm), Kolmogorov and Tay-<sup>416</sup> Janzen *et al.*<sup>20</sup>, using the zero shear rate viscosity:

$$Re_g = \frac{\rho f S^2}{\mu_0} \tag{4}$$

417 418 ter, because of the very small mass of polymer added. For 470 Reynolds number viscoelastic turbulence. It is yet sufficient 419 this range of grid Reynolds number, it has been checked that 471 to discuss the and large scale effects. <sup>420</sup> polymer chains are indeed not destroyed during the experi- <sup>472</sup> <sup>421</sup> ments (maximum 2 hours of duration) by comparing viscosity <sup>473</sup> ing to a measurement time of 250 s at  $f_{acg} = 4$  Hz and 100 s <sup>422</sup> curves of the solution before and after the experiment. Finally, <sup>474</sup> at  $f_{acq} = 10$  Hz. Assuming the integral time-scale of turbu-423 in order to quantify the ratio between the polymer relaxation 475 lence to be of the order of magnitude of the grid period (this

$$De = \frac{t_{CY}}{T} = t_{CY}f \tag{5}$$

426 with T the period of oscillations. Values of  $Re_g$  and De are 427 reported in table II for each concentration studied.

Liquid phase velocity measurements in the tank are 430 achieved by Particle Image Velocimetry (PIV). The experi-431 mental setup is sketched in figure 2. The region of interest 433 to that of the tank, and it includes fluid heights between the 434 grid's top position and the free surface. The cameras used 435 are double frame LaVision sCmos sensors of 2560 by 2160 436 pixels, equipped with a 50 mm focal Macro lens. A pulsed 438 luminate 50  $\mu$ m diameter polyamide particles. The colimated 439 laser sheet thickness achieved is 200  $\mu$ m. It is estimated by 440 marking a photosensitive paper band placed on the wall of the tank closer to the laser head with a single laser pulse.

Depending on polymer concentration, the order of magni-442 443 tude of measured velocities may differ considerably. In order to keep the measured particle displacement of the order of 445 one third of the interrogation window, two types of PIV are used: double frame PIV for  $C_{XG} < 250$  ppm, and single frame <sup>447</sup> PIV for  $C_{XG} \ge 250$  ppm. In the first case, the acquisition fre-Turbulence is generated in a transparent tank of a 277 mm  $_{448}$  quency  $f_{acq}$  is 4 Hz, and the time interval between laser pulses grid average position (z=0) at the crossing between the two 454 Spurious vectors are removed from PIV fields by applying a central bars. In this study only polymer concentration is var- 455 threshold of 1.2 on the peak ratio, and replaced using median ied and all oscillations parameters are kept constant. The grid 456 filtering (always less than 10% of the total number of vectors).

The final spatial resolution achieved is 2.3 mm. The small-463 lor length scales are evaluated to be respectively of about 464 0.19 mm and 2.24 mm. For 100 ppm XG solutions, using a 465 constant viscosity equal to  $\mu_{\infty}$ , they increase to 0.33 mm and 466 3.25 mm, and up to 4.35 mm and 18.17 mm when using  $\mu_0$ 467 as the scale viscosity. The 2.3 mm spatial resolution is thus 468 quite coarse and would unfortunately not allow to evidence The density  $\rho$  of the fluid is assumed equal to that of wa- 400 energy variations at large wave numbers characteristic of high

1000 vector fields are recorded for each run, correspond-



Figure 1. Evolution of characteristic time scale  $t_{CY}$  (a) and zero shear rate viscosity  $\mu_0$  (b) with polymer concentration  $C_{XG}$ . Data and trends for the present study (resp. full squares and dashed lines) are compared to the work of Wyatt and Liberatore<sup>76</sup> (empty circles and dasheddotted lines, for which the slopes are indicated by italic numbers). Dilute (D), semi-dilute (SD) and concentrated (C) concentration regimes are separated by dotted lines at  $C_{XG} = 100$  ppm and  $C_{XG} = 2000$  ppm on both sub-figures.



Figure 2. Sketch of the oscillating grid and PIV setup.

476 hypothesis could be checked for water and dilute regime XG 491 IV. RESULTS

477 solutions), this ensures a statistical analysis over at least 100

Mean flows and re-circulations

478 uncorrelated events. Each experiment is moreover performed Α. 479 twice.

> Topology of the mean flow 493 **1**.

In order to check for statistical convergence and estimate 480 uncertainty on velocities, sliding statistics on 500 subsequent 494 481 482 483 484 485 486  $_{487}$  range. Typical velocity uncertainties thus evaluated are  $\pm 6\%$  scale motion and allowing mean flow patterns to persist once 488 and  $\pm 5\%$  for respectively mean and rms quantities (spatial av- 501 developed<sup>77</sup>). In the ROI, the mean flow is structured in two 489 erage on the whole ROI). Statistical convergence is ensured up 502 main recirculation vortices close to the grid and near the walls. 490 to the previous confidence intervals.

# The existence of mean flows and recirculations is most of snapshots (time interval equal to $1/f_{acq}$ ) are computed for 495 the time an unwanted feature of OGT, which was initially both experiments. For mean velocities, and rms of velocity 496 meant to study turbulence alone in absence of mean shear<sup>4</sup>, fluctuations, uncertainties are then estimated as the standard 497 but can unfortunately not be avoided. They are supposedly deviation of the variations of these samples' statistics around 498 due to the grid tendency to drift out of alignment, and to inithe "statistically converged" result obtained using the full data 499 tial minor fluid motion favouring the development of large <sup>503</sup> Figure 3 shows in three columns respectively the average ve-

<sup>504</sup> locity field magnitude  $\sqrt{\overline{U}_x^2 + \overline{U}_z^2}$  (with  $\overline{U}_x$  and  $\overline{U}_z$  respec-<sup>505</sup> tively the horizontal and vertical mean flow components), the <sup>558</sup> <sup>506</sup> associated streamlines, and the vorticity of the average veloc-<sup>507</sup> ity field  $\overline{\Omega} = \nabla \wedge \overline{\mathbf{U}}$ . These quantities are represented for wa-508 ter (first line) and three polymer concentrations: one in the 561 <sup>509</sup> dilute regime (10 ppm, second line), one in the semi-dilute <sup>510</sup> regime (500 ppm last line) and one in the transition region 563 511 (100 ppm, third line). The recirculation patterns observed on 5 64 <sup>512</sup> the streamline figures (central column) are evidenced by two 565 <sup>513</sup> opposite sign high magnitude vorticity patches on both sides 566 <sup>514</sup> of the tank, just above the grid sweep zone (right column). <sup>515</sup> Isovalue lines of vorticity are drawn at the arbitrary threshold 568 516  $\overline{\Omega} = \pm 0.02\overline{\Omega}_m$  whith  $\overline{\Omega}_m$  the maximum vorticity value at a 569 <sup>517</sup> given concentration defined section IVA2. They represent 518 an arbitrary estimation of the boundaries of the mean flow 570 vortices. Similar flow patterns have also been observed by 571 tration (indicator 1) is shown in figure 4 and table II. It in-McKenna and McGillis<sup>13</sup> for water. 520

521 522 523 524 526 527 528 529 530 531 532 ppm, as seen on figures 3, left and central columns. A further 585 100 and 500 ppm. 533 increase in concentration leads to the collapse of these recir-534 535 <sup>536</sup> columns), and the central up-flowing region progressively dis- <sup>588</sup> Since here T = 1/f = 1 s, scaling based on  $t_{CY}$  or De are <sup>537</sup> appears. Hypothesis will later be made on the probable rea- <sup>589</sup> equivalent. The dimensionless quantity  $\overline{\Omega}t_{CY}$  is represented <sup>538</sup> sons for this higher concentration behavior.

### Mean flow indicators 2 539

Simple indicators can be defined in order to measure the in- 595 54.0 541 the mean flow enhancement. 542

543 544 54 5 547 548 54 9 550 multiplying  $\Omega$  by the polymer time scale  $t_{CY}$ . 551

552 553 554 555 556 • The last indicator defined here is the position along z of the maximum vertical velocity on the x = 0 line (regardless of the value of this velocity). This is an estimation of the area of influence of the mean flow rather than of its intensity and comes as a complement of the two other indicators. It can be further normalized by the mesh parameter M.

The evolution of the peak vorticity magnitude with concen-<sup>572</sup> creases with polymer concentration in the [0-150] ppm range Based on the observation of figure 3 and additional inter- 573 and brutally decreases for the last two points. The critical mediate concentrations (18, 25, 35, 50, 75, 150 and 250 ppm, 574 concentration above which vortex intensity stops increasing, not shown in the figure), the effect of polymer addition on 575 here 150 ppm, can be related to the transition concentration the mean flow seems to be the following. When adding poly- 576 between the dilute and semi-dilute regime, 100 ppm. In the mer to water, the flow organizes into two main regions dom- 577 dilute regime, increasing polymer concentration and conseinated by the two counter rotative side vortices illustrated by 578 quently the solution typical response time leads to an increase the streamlines or the vorticity isovalues. Increasing polymer 579 in maximum vorticity. When reaching the semi-dilute regime, concentration first causes the mean flow vortices to grow in 580 further increase in concentration on the contrary reduces vorsize until they reach the top of the fluid volume, at a concen- 581 ticity associated to mean recirculations. These observation are tration between 10 and 25 ppm. The flow is then divided into 582 consistent with the streamline patterns observed in figure 3, wo main regions separated by a global up-going region at 583 central column: well defined structures build up upon polymer the center of the plane for concentrations between 25 and 100 584 addition and grow in the dilute regime, and collapse between

In the dilute regime, the characteristic time scale depends culating regions: their size reduces (figure 3 central and right  $_{587}$  on concentration with the scaling  $t_{CY} \sim C_{XG}^{1.5}$  (see figure 1). <sup>590</sup> as a function of De (figure 4 b). It follows a power law such <sup>591</sup> that  $\overline{\Omega}t_{CY} \sim De^{1.39}$ . Since here  $t_{CY} \sim De$  (equation 5), it im-<sup>592</sup> plies that  $\overline{\Omega} \sim De^{0.39}$ . It should be mentioned that because of <sup>593</sup> the limited range of time scales achievable in the dilute regime (difficulty of making polymer solutions at  $C_{XG} < 10$  ppm) and of the single grid frequency used, this scaling is derived from tensity and region of influence of the mean flow, and quantify 596 a limited range of De. Larger variations of the Deborah num-<sup>597</sup> ber could be achieved by varying the grid frequency. Yet in <sup>598</sup> the available range, the correlation is quite good. The inten-• The first criterion is the value of the curl of the average <sup>599</sup> sity of mean recirculations increases with the solution time velocity field at the center of the side vortices. This is 600 scale, but this increasing rate is reduced as the Deborah numa measurement of the maximum vorticity in the mean 601 ber increases (exponent below 1). Mean flow enhancement flow, and thus of the intensity of the side eddies. Here 602 by polymer addition thus seems to be a low-concentration efit is defined as  $\Omega_m = max(\overline{\Omega}(O_A), \overline{\Omega}(O_B))$  where  $O_A$  is 603 fect that loses efficiency approaching the semi-dilute regime. the central point of the left vortex and  $O_B$  the central 604 This may be explained by the fact that as the relaxation time point of the right vortex (see figure 3, right column). A 605 increases with polymer concentration, viscosity also does. In non dimensional vorticity indicator can be constructed,  $^{606}$  the dilute regime, the zero shear rate viscosity scales as  $C_{XG}^{1.6}$ 607 (from figure 1 b)), hence it increases more rapidly than the re-<sup>608</sup> laxation time scale. In the semi-dilute regime, viscosity keeps The second indicator is the peak-to-peak amplitude A  $_{600}$  increasing with the same trend but  $t_{CY}$  stays constant (note (difference between maximum and minimum value) of 610 that Wyatt and Liberatore<sup>76</sup> observed a change of slope for the horizontal profile (along x) for the vertical compo-  $_{011}$  the semi-dilute regime that is not observed here since no  $C_{XG}$ nent,  $\overline{U}_z(x,z=Z_p)$ , taken at a given probing altitude 612 value is used in this domain). In that sense, it may be assumed  $Z_p$  on the fields illustrated figure 3, left column. It is  $_{613}$  that viscosity and time scale increase would have two differ-

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Figure 3. Mean flow inside the grid stirred tank at different XG concentrations. Left column: norm of the 2D mean velocity field  $\sqrt{U_x^2} + \overline{U_z^2}$ in m/s. Central colums: streamlines of the mean flow. Right column: Vorticity of the mean velocity field  $\overline{\Omega} = \nabla \wedge \overline{U}$ , with contour lines drawn at 2% of the maximum vorticity value.

ent and maybe opposite effects on mean flows: the increase 622 5 a) shows that polymer addition tends to promote a central  $_{615}$  of the solution's time scale seems to enhance re-circulations,  $_{623}$  up-going effect: the  $\overline{U}_z(x, 2M)$  black dotted curve for water  $_{616}$  whereas increasing viscosity tends to dissipate the flow struc-  $_{624}$  does not have its maximum value close to x = 0 mm, while it 617 tures and ultimately make them collapse.

61 8 619 the second criterion with polymer concentration: the peak-to 628 defined as the curves' "peak-to-peak amplitude", that is to say peak amplitude A of the horizontal profile for the vertical ve- 629 the difference between the maximum and the minimum values locity component, probed at z = Zp = 2M,  $\overline{U}_z(x, 2M)$ . Figure

625 is the case for every polymer solutions at any concentrations. 626 This peak effect is increased with polymer concentration until A similar trend is observed when looking at the evolution of  $_{627}$  it reaches its maximum for  $C_{XG} = 100$  ppm. The indicator A is



Figure 4. Evolution of vorticity "strength" associated to mean flow structures as a function of polymer concentration and Deborah number. Plot a) is the maximum vorticity  $\overline{\Omega}$  between the two counter-rotatives vortex plotted for all concentrations. The relative uncertainty is estimated to be less than  $\pm 12\%$ , coming from the uncertainties on velocity measurement and from the spatial uncertainty on the location of  $O_A$  and  $O_B$ . Plot b) shows the non-dimensional quantity  $\overline{\Omega}_{CY}$  in the extended dilute regime ( $C_{XG} \leq 100$  ppm left of dashed dotted line in a)). The dotted line in a) denotes the transition between dilute (D) and semi-dilute (SD) concentration regimes, and the dashed dotted marks line in a) the change in trend at  $C_{XG} = 150$  ppm. The gray dashed line in b) is the power law fitting of  $\overline{\Omega}_{tCY}$  versus De, the scaling of which is reported in b) on top of the curve.

631 5 a). It is plotted as a function of the polymer concentration 665 very limited range. Figure 5f) seems to confirm the existence 632 633 634 even though the error bars on the data are also compatible with 669 quite small in the studied *De* range. 635 a constant A trend. It then collapses in the semi-dilute regime. 670 636 637 638 related to the polymer entanglement concentration regime. 639

The third indicator is illustrated by figures 5 b) and d). The  $^{674}$ 64 0 vertical profiles of vertical velocity at x = 0 mm plotted in 641 figure 5 b) all present a maximum value at small z (close to 642 the grid). Yet, the  $z = Z_m$  location of this maximum varies 64 3 with polymer concentration. This can also be observed on the 644 average velocity fields in figure 3, left column, and is a con-64 5 sequence of the flow organization into two side vortices. The 646 impact region of both vortex meet in the plane of symmetry 681 647 (X = 0 mm). Since the vortex are ellipsoidal and that their <sub>682</sub> 648 main axis is neither vertical nor horizontal, the maximum ver-649 tical velocity is found at a given altitude which depends on the 684 650 inclination of the vortices and their topology. This third cri-651 terion thus evaluates the vortex pair's spatial influence more 652 686 than their intensity. The altitude at which maximal vertical 653 velocity is found  $Z_m$  is plotted versus concentration in figure 5 687 654 d). It shows an increasing trend in the early part of the dilute 688 655 656 regime. However unlike the up-going intensity A, it begins 689 657 to decrease at a lower typical concentration, around 35 or 50 690 691 658 ppm.

692 Scaled second and third indicators can be plotted versus 659 693 the Deborah number in the dilute regime. The evolution of 660 694 A scaled by a reference grid velocity fS is shown in figure 5 661 695  $_{662}$  e), and that of  $Z_m$  scaled by the mesh parameter M in figure 663 5 f), both in log-log scales. The first plot shows that as for 696

630 of  $\overline{U}_z(x, 2M)$  along x, sketched by the dashed arrow on figure 664  $\overline{\Omega}t_{CY}$ , A/(fS) increases with the Deborah number, but on a in figure 5 c). An increase compared to the water case is ob- 666 of a critical concentration within the dilute regime: a peak of served for all polymer solutions in the dilute regime. Indicator  $_{667} Z_m/M$  is observed precisely at De = 0.55 corresponding to A seems to slightly increase with De within this dilute regime,  $_{608}$   $C_{XG} = 50$  ppm, even if the relative variations of  $Z_m/M$  are

One can quickly comment on the uncertainties of the plots This confirms that the intensity of the up-going central motion 671 in figure 5. In a) and b), the relative uncertainty on velocand that of the side vortices are coupled, and that they are both 672 ity value along these profiles is typically equivalent to that on <sup>673</sup> mean velocity, *i.e.* about  $\pm 6\%$ . Hence the relative uncertainty on A and A/(fS) is twice that value, about  $\pm 12\%$ . The ab-675 solute uncertainty on  $Z_m$  points in d) would be evaluated as  $_{676} \pm 1.15$  mm (from the PIV spatial resolution), which gives a <sup>677</sup> relative uncertainty on  $Z_m$  and  $Z_m/M$  less than 3%.

> The effects of polymer addition and polymer concentration 678 679 on mean flows inside the tank can thus be summarized as fol-680 lows:

- Polymer addition has an organizing effect on the mean flow. Existing vortices are enhanced by addition of even a small concentration of polymer, and the flow organizes into two distinct vorticity regions ruled by the pair of counter rotative eddies. A global up-going motion appears at the center of the tank.
- Increasing polymer concentration while staying in the dilute regime tends to increase the vortex and upgoing motion intensity. Both indicators for vortex and up-going motion intensity increase until concentration reaches the critical concentration  $C_{XG} = 100$  ppm for which solution switches from dilute to semi-dilute regimes. In the concentration range corresponding to the dilute regime, the maximum vorticity of the mean flow scales as a power law of the Deborah number.
- The region of influence of the mean flow is observed to



Figure 5. Effect of polymer concentration on vertical velocity. a) Horizontal profiles of the vertical velocity at z = 2M = 70 mm. Horizontal dashed lines and the vertical dashed arrow illustrate the peak-to-peak amplitude *A* indicator. b) Vertical profiles of the vertical velocity at x = 0 mm. Gray horizontal dashed lines are examples of  $z = Z_m$  depths for profile maxima. Inserts in a) and b) sketch the line along which profiles are plotted by a white dashed line, taking as an example the  $C_{XG} = 100$  ppm average velocity field magnitude. Markers in a) and b) are represented for only one in ten data points, for the sake of readability. c) Peak-to-peak amplitude *A* of the profiles of a). d) location  $Z_m$  along *z* of the maxima of profiles b) (marker size is reduced so that the error bar can be seen). e) and f) show the evolution with De in the dilute regime of the two previous indicators, respectively *A* and  $Z_m$ , scaled by respectively *fS* and *M* (in log scale). The dotted lines in c) and d) denote the dilute to semi-dilute entanglement transition ( $C_{XG} = 100$  ppm). The black dashed line in d) and f) corresponds to the sub-dilute critical concentration  $C_{XG} = 50$  ppm.

increase at the onset of polymer addition, but the maxi mum effect of the up-going motion is attained at a con centration lower than the dilute to semi-dilute transi tion. This suggests that the dilute regime is itself com posed of two sub-regimes: a very dilute one in which
 both vortex intensity and size increase, and a transition

to semi-dilute one in which the vortex intensity keeps increasing but its region of influence reduces.

• In the semi-dilute domain, vortex intensity and size both decrease. The hypothesis is that in this regime, viscosity keeps increasing upon polymer addition while elas-

708 ing polymer-polymer interactions. 709

### **Turbulence** properties B. 71 0

Turbulence properties are evaluated in the central region of 764 2. Mean flow to turbulence ratio 71 1 <sup>712</sup> the ROI, where they are the most homogeneous<sup>42</sup> by cropping out bands for x > 2.5M and x < -2.5M on the sides of the 765 713 714 vertical profiles (along z). This central region is illustrated in 71 5 <sup>716</sup> figure 6 where two examples (water and 25 ppm XG) of tur-<sup>767</sup> lute value of mean velocity component  $|\overline{U}_i|$  and the local rms <sup>717</sup> bulent kinetic energy fields k, computed as  $k = u'_z^2 + 2u'_x^2$ , are <sup>768</sup> of turbulent velocity fluctuations component  $u'_j$ , with i=x,y or <sup>718</sup> shown in log scale. Note that the kinetic energy is computed <sup>769</sup> Z.  $\Gamma_{ij}$  is a 9 component tensor, which reduces to a 4 compo-<sup>719</sup> under an assumption of horizontal isotropy. In the following <sup>770</sup> nent tensor with i, j = x or z for 2D PIV measurements. Most <sup>720</sup> analysis, all profiles are plotted versus the unscaled dimension <sup>771</sup> of the time, the values used to estimate the efficiency of a de-721 z in order to compare with dimensional HT and TT relation-722 ships for OGT (equations 1, 2).

### Decay of turbulence intensity 1. 723

724 <sup>725</sup> For  $C_{XG} < 250$  ppm, it appears that all the profiles, water <sup>779</sup> that mean flow over turbulence ratio can locally reach higher 726 and polymer solutions, follow a power law of constant expo- 780 values of up to 10 in the central region where the mean flow is  $_{727}$  nent as predicted by the relationship of Hopfinger and Toly<sup>4</sup>  $_{781}$  very strong, and down to 0.1 in high turbulence intensity / low 729 decay of turbulence intensity of OGT in the dilute regime of 783 tration, which is a consequence of both the enhancement of 730 polymer solutions can be described by "HT-like" profiles. The 784 the mean flow and the decrease of turbulence intensity. Three <sup>731</sup> influence of polymer concentration on this decay is quantified <sup>785</sup> distinct regions can be seen on the plot of  $\Gamma$  versus z. The  $_{732}$  by the power law exponent. In the picture of Hopfinger and  $_{786}$  first one (1), for z < M and in which  $\Gamma < 2$ , lays just below Toly<sup>4</sup>,  $u'_x$  and  $u'_z$  should be proportional, resulting in a single <sup>787</sup> the main mean flow structure detailed previously. The second <sup>734</sup> value of *n*. Following this hypothesis, this value is estimated <sup>788</sup> one (2) between z = M and z = 4M corresponds to the region <sup>735</sup> for each concentration by fitting both the  $u'_x$  and  $u'_z$  (not shown <sup>789</sup> of the up-going motion and side recirculations, and yields the  $_{736}$  here) profiles with power laws, and averaging the two result-  $_{790}$  highest  $\Gamma$  values going along with the strongest mean flows. 737 738 yields the dispersion bars represented on figure 7 (correspond- 792 third region (3) corresponds to the distances not reached by ing to the uncertainties in table II). 739

740 real centration in figure 7 b) (for the whole  $C_{XG} < 250$  ppm range) res decaying. The values of  $\Gamma$  in the far grid region (3) dependence  $_{742}$  and c) (zoomed on  $C_{XG} < 100$  ppm). It appears that the  $_{796}$  on the existence or not of secondary re-circulations under the 743 n value for water is slightly above the n=-1 expected from 797 free surface. They are comparable to those found for water <sup>744</sup> HT. When adding polymer *n* starts increasing up to -0.716 at <sup>798</sup> and can even be lower than in water in some dilute polymer 745 18 ppm (figure 7 c)) before decreasing again, quasi linearly 799 solutions. In region (3), mean flow intensity is at worst twice <sup>746</sup> with polymer concentration, until  $C_{XG} = 150 \ ppm$ , where it <sup>800</sup> that of turbulence, no matter the working fluid. <sup>747</sup> is equal n = -1.479. The decay rate of oscillating grid tur-748 bulence thus varies with polymer concentration, staying quite 749 similar to the water case as long as one stays in the dilute 801 3. Integral length scales 750 regime.

751 752 from the HT behavior: the last two concentrations exhibit two sos the integral of correlation coefficients of velocity fluctuation res asymptotic slopes separated by a transition area, a first one n', see components i and j along dimension k, with i, jor k = x or z.  $_{754}$  at small z, which keeps decreasing with increasing  $C_{XG}$  but  $_{805}$  They are averaged over sampling regions at different depths.  $_{755}$  not following the linear trend of n, and a second one  $n^{2}=0$  so Length scales along x are computed on sampling regions wide 756 at large z. In this second region, turbulent velocity fluctua- 807 as the ROI and including 3 vectors in the z direction. Length 757 tions are extremely small and the mean flow is low as well sos scales along z are computed on sampling regions of width 758 (see figure 3). Viscous dissipation reduces the velocity mag- 809 equal to that of the ROI's and 10 mm height. The study of 759 nitudes, hence the local shear rate, which further increases \$10 integral scales is limited to the dilute regime. As for mean

763

A key quantity of oscillating grid turbulence is the local ROI before width averaging quantities (along x) and plotting resonant to turbulence ratio  $\Gamma_{ij} = \frac{|\overline{U}_i|}{u'_i}$ , between the local abso-<sup>773</sup> diagonal values  $\Gamma_{ii}$ . Here we compute the indicator  $\Gamma$  as the <sup>774</sup> width average of the quantity  $(\sum_i \Gamma_{ii}^2)^{0.5}$ . For OGT in water, a 775 review by Variano, Bodenschatz, and Cowen<sup>77</sup> reports local <sup>776</sup> typical best case values of  $\Gamma_{ii} = 0.25$  in a single coordinate <sup>777</sup> direction. Here, figure 8 shows that width averaged  $\Gamma$  lies be-Profiles of  $u'_x$  as a function of z are shown in figure 7 a). 778 tween 0.9 and 3.5 for all fluids. It yet has to be mentioned (equations 1). The first important observation is thus that the  $_{782}$  mean flow regions.  $\Gamma$  globally increases with polymer concening exponents. The difference between these two exponents  $_{791}$  In regions (1) and (2),  $\Gamma$  is always the lowest for water. The 793 the principal mean flow structure in polymer solutions, or in Values of n are reported in table II and plotted versus con-<sup>794</sup> which their intensity is reduced, but where turbulence keeps

In the semi-dilute regime, the decay of turbulence departs  $_{802}$  The integral length scales of turbulence  $L_{ii}^k$  are defined as



Figure 6. Fields of turbulent kinetic energy  $k = u_z'^2 + 2u_x'^2$  in log scale for water (a) and 25 ppm XG solution. Vertical dashed lines at  $x/M = \pm 2.5$  denote the boundaries of the domain for horizontally averaged turbulence properties.



Figure 7. "HT-like" plots of  $u'_x$  as a function of z (a), and estimation of the power law exponent evolution as a function of polymer concentration (b). c) is a zoom of b) in the  $C_{XG} < 100$  ppm range. Markers in a) are represented for only one in ten data points, for the sake of readability. The typical uncertainty on  $u'_x$  is  $\pm 5\%$ . Vertical bars in b) and c) show the disparity of *n* measurement from  $u'_x$  and  $u'_z$  profiles.

and rms velocities, uncertainties are estimated by computing at a scales  $L_{xx}^x$  and of  $\pm 12\%$  on vertical scales  $L_{zz}^z$ . s12 sliding statistics on 500 images samples and evaluating the 813 variations of the integral length-scales computed from these samples around the converged value derived from the full data  $_{\mathtt{s15}}$  range. This yields a typical uncertainty of  $\pm 10\%$  on horizon-

817 In the definition of Thompson and Turner<sup>3</sup>, L is equal to the <sup>818</sup> horizontal scale of horizontal velocity fluctuations  $L_{xx}^{x}$ . Figure <sup>819</sup> 9 shows the evolution of  $L = L_{xx}^x$  (a) and of the vertical scale <sup>820</sup> of vertical velocity fluctuations  $L_{zz}^z$  (b) as a function of the



Figure 8. Plots of the mean flow to turbulence indicator  $\Gamma$ , in log scale, as a function of the distance from the grid *z*. Dashed lines are plotted at z = M and z = 4M. Markers are represented for only one in ten data points, for the sake of readability. The typical uncertainty on  $\Gamma$  is  $\pm 11\%$ , sum of the uncertainties on  $\overline{U}$  ans  $u'_x$ .



Figure 9. Integral length scales of turbulence in the dilute regime. a) Horizontal scale of horizontal velocity fluctuations  $L_{xx}^{z}$  as a function of the distance to the grid z. b) Vertical scale of vertical velocity fluctuations  $L_{zz}^{z}$  as a function of the distance to the grid z. Markers in a) and b) are represented for only one in ten data points, for the sake of readability. c) Slopes  $\alpha$  and  $\beta$  of linear trends of respectively  $L_{xx}^{x}$  and  $L_{zz}^{z}$  a small z, as a function of polymer concentration. d) Ratio of these two slopes  $r = \beta/\alpha$ .

823 polymer solutions<sup>56,59,60</sup> 824

It appears that up to a given proximity of the free surface, sso for water. 825 which is located at z = 250 mm, both  $L_{xx}^x$  and  $L_{zz}^z$  scales follow <sub>881</sub> 826 alinear trend with z, as observed by Thompson and Turner<sup>3</sup> as figure 10 and values reported in table II), first moderately, and 830 831 832 833 (to be detailed in Lacassagne et al. 79). 834

835 sub spectively  $\alpha$  for  $L_{xx}^{x}$  and  $\beta$  for  $L_{zz}^{z}$ , they are reported in the sub- sub at concentrations above 100 ppm,  $C_{2HT}$  increases much faster <sup>837</sup> figure c) of figure 9 and compared with the value predicted by <sup>892</sup> with concentration. <sup>838</sup> Thompson and Turner<sup>3</sup> for water,  $C_{TT} = 0.1$ . The value of  $\alpha$ for  $C_{XG} = 0$  is quite close to the expected  $C_{TT}$  value. Both  $\alpha_{sy4}$  that for polymer solutions, a slight increase of  $I_s$  is observed and  $\beta$  increase with polymer concentration up to 50 ppm and system with increasing z. Barely visible at low concentrations, this seem to initiate a decrease for  $C_{XG} > 50$  ppm, meaning that seems all the more important that polymer concenthe growth of flow structure moving away from the grid is en-set tration is high, especially for  $C_{XG} \ge 250$  ppm. For the two hanced by the presence of polymer at the onset of the dilute  $_{898}$  higher concentrations studied,  $I_s$  is clearly no longer constant regime, and that this enhancement may no longer happen in with z. The estimation of  $C_{2_{HT}}$  by z-averaging of  $I_s$  should 845 the semi-dilute one.

846 strain tified by computing the ratio  $r = \beta/\alpha$  at each concentration.  $_{902}$  the hypothesis of HT holds and anisotropy can be fully decreases faster that the horizontal one, and r < 1 the opposite.  $u_x$  it means that  $u'_x$  and  $u'_z$  are no longer proportional, and that Figure 9 d) shows that r is slightly below 1 for water, but  $u_z$  increases faster than  $u'_x$ . Hence the HT exponent for  $u'_z$ <sup>251</sup> increases up to a value of almost 2 for  $C_{XG} = 50$  ppm be-<sup>306</sup> is likely higher than the one for  $u'_x$ . Anisotropy is then exso to that is that the presence of polymer in the concentration  $u_{x}$  ponents for  $u'_{x}$  and  $u'_{z}$ . Increased anisotropy of turbulence in range  $C_{XG} \leq 50$  ppm not only enhances the growth of tur- 900 polymer solutions was already observed by Gupta, Sureshkusss bulence structures moving away from the grid, but also the sine mar, and Khomami<sup>64</sup> or Cai et al. <sup>63</sup> in channel flows of visanisotropy of the growth: the vertical elongation of eddies is and coelastic polymers, or in turbulent front propagation experi-<sup>857</sup> promoted. Both behaviour yet seem to reduce when approach-<sup>912</sup> ments by Cocconi et al.<sup>57</sup>. In the first case, it is explained 858 ing the semi-dilute regime.

### Isotropy Δ 859

860 861 863 865 horizontal one:  $I_s = u'_z / u'_x$ . 866

868 firming that the vertical velocity fluctuations rms is always 926 tank. larger than the horizontal one, for water and all polymer con- 927 869 870 872 portional to the rms of horizontal ones with a constant propor- 930 crease. This suggests that the free surface damps the vertical  $_{373}$  tionality coefficient  $C_{2_{HT}}$ .  $I_s$  profiles should thus be constant  $_{331}$  fluctuations of turbulence more efficiently than the horizontal ara along z, which is here only observed as a first approximation 932 ones (see Lacassagne *et al.*<sup>79</sup>). Not accounted for by the laws 875 in the dilute regime as will be discussed hereinafter. In this 933 of<sup>4</sup>, this could translate by a strong differentiation of the two

s21 distance to the grid z. Integral scales are larger in polymer s76 first approximation, the anisotropy coefficient, corresponding solutions compared to water. This was previously observed in  $\alpha_{77}$  to the second HT constant  $C_{2_{HT}}$ , can then be obtained for each several studies of turbulence in shear-thinning or viscoelastic  $\alpha_{78}$  working fluid as the average of  $I_s$  over z (figure 10). In the 879 literature (see section II A 1), its value is between 1.1 and 1.4

 $C_{2HT}$  is found to increase with polymer concentration (see in water. We notice that the change in trend trend happens at \*\*\* then faster above 100 ppm concentrations. The first concluhigher depths (smaller z) for the vertical scale  $L_{zz}^z$  than for the set sion is thus that the presence of polymer tends to promote horizontal one, and that this depths increases with polymer 885 turbulence anisotropy, and that this increased anisotropy is all concentration. This is consistent with the fact that vertical 886 the more important than polymer concentration is high. Yet, relocity structures are kinematically constrained by the hori- 1887 for concentrations lower than 100 ppm, the value of HT's seczontal interface, and larger for large polymer concentrations \*\*\* ond constant stay inside the range found in the literature for <sup>889</sup> water up to the error bar amplitude, despite the slight increase As for the slope of the different linear trends, denoted re- 800 observed. When entering the semi-dilute regime, that is to say

Nevertheless, looking at the evolution of  $I_s$  with z, we notice <sup>900</sup> thus be interpreted carefully: as long as  $I_s$  can be considered Finally, the relative evolution of  $L_{zz}^z$  and  $L_{xx}^x$  can be quan-  $_{901}$  constant with z, here to a first approach for  $C_{XG} \leq 100$  ppm, > 1 indicates that the vertical dimension of structures in-  $_{903}$  scribed by the evolution of  $C_{2_{HT}}$ . When  $I_s$  increases with z, fore decreasing again for  $C_{XG} = 100$  ppm. The conclusion  $_{907}$  pressed not only by  $C_{2_{HT}}$  but also by the ratio of decay ex-<sup>913</sup> by the ability of polymer chains to align with the mean flow. 914 In the second one, it likely comes from a reorganization of 915 turbulence leading to an alignment of polymers with vortic-916 ity, which is, as the authors stress out, somehow similar to the 917 mean-flow alignment observed in channel flows. Here, the For OGT in water, it is known that the vertical component 918 fact that  $C_{2_{HT}}$  is above unity and increases with  $C_{XG}$  fits well of turbulence is stronger than the horizontal one, because this <sup>919</sup> in this picture: the grid motion being vertical, polymer chains is the orientation of the grid forcing. Hence OGT is by nature 920 preferentially settle along the periodic vertical shear induced not fully isotropic in a vertical plane. An indicator of the 2D 921 by the grid, and vertical turbulent fluctuations are promoted. isotropy of turbulence in the  $(\vec{x}, \vec{z})$  plane,  $I_s$ , can be defined <sup>922</sup> The specific behavior observed in the semi-dilute regime has by dividing the HT profiles of the vertical component by the 923 to be tempered by the fact that for such concentrations tur-<sup>924</sup> bulence is very weak, event in the region close to the grid, The  $I_s$  indicator is as expected generally above 1, thus con- 925 and transitions to laminar or even motionless in the rest of the

As a last remark, one notices that when approaching the centrations. According to the description of Hopfinger and 928 free surface (located here at Z close to 250 mm), the previous Toly<sup>4</sup>, the rms of vertical velocity fluctuations should be pro-  $_{929}$  increasing trend of  $I_s$  with z is reversed, the values of  $I_s$  de-



Figure 10. Vertical profiles of anisotropy  $I_s$  for water and polymer solutions at different concentrations. a)  $I_s$  Isotropy profiles. The typical uncertainty on  $I_s$  is twice that of  $u'_x$  or  $u'_z$  profiles, i.e.  $\pm 10\%$ . b) Evolution of  $C_{2_{HT}}$ , computed as the average of  $I_s$  over z, with polymer concentration. Dashed lines are boundaries of the usual range of values found for water  $1.1 < C_{2\mu\tau} < 1.4$ . Vertical bars represent standard deviation of  $C_{2\mu\tau}$  along z around its average value for each concentration. Markers in a) are represented for only one in ten data points, for the sake of readability.

<sup>934</sup> power law exponents for  $u'_x$  and  $u'_z$ .

### 5. Homogeneity 935

936  $_{937}$  fine the quantity  $H_i^x$  as the standard deviation over an horizon-  $_{957}$  the dilute and semi-dilute regime, two specific critical con- $_{938}$  tal line at altitude z of the 2D field of rms velocity fluctuations  $_{958}$  centrations can be evidenced within the dilute regime itself: <sup>939</sup> in dimension j (j being x or z), normalized by the reference <sup>959</sup>  $C_{XG} = C_{D1} \simeq 20$  ppm and  $C_{XG} = C_{D2} \simeq 50$  ppm. This leads (width averaged) rms  $u'_j$  at this depth. The smaller the  $H_x^x \rightarrow \infty$  to three dilute sub-regimes referred to as D0, D1 and D2.  $H_z^x$  or  $H_z^x$  the higher the homogeneity in dimension x at a given  $H_z^x$  The first critical concentration marks the limit of an ex-<sup>942</sup> z. Homogeneity indicators  $H_z^x$  or  $H_x^x$  do not show any trend <sup>962</sup> tremely dilute behaviour of the polymer corresponding D0 to <sup>943</sup> along z. Values of  $H_z^x$  averaged over z, denoted H, are re- <sup>963</sup> the very onset of polymer action on both mean flows and turported in table II. Uncertainty on H value is computed as the 964 bulence. It is worth noting that the existence of such turbulent standard deviation of  $H_7^x$  over z. H does not seem to depend on seem to depend o <sup>946</sup> polymer concentration. This implies that polymer has a lesser <sup>966</sup> molecules (PEO, with a 25 ppm critical concentration<sup>29,80</sup>). 947 effect on turbulence homogeneity than on its isotropy. This is 967 The second one corresponds to the maximum mean flow 948 obviously only valid in the central region of the ROI defined 968 enhancement concentration reached between D1 and D2 (all 949 previously, for -2.5M < x < 2.5M.

### DISCUSSION ν.

### Concentration regimes and drag reduction Α

Several characteristic polymer concentrations emerge from 953 the previous results and observations. The different concentra-<sup>954</sup> tion sub-ranges and their characteristics in terms of mean flow 955 and turbulence are summarized in figure 11. Apart from the Finally, in order to measure horizontal homogeneity, we de- 956 100 ppm concentration which marks the transition between

<sup>969</sup> mean flow indicators increase between 0 and 50 ppm), and

$C_{XG}$ (ppm)	$Re_g$	De	$\overline{\Omega}$ (1/s)	-n (* or -n')	$C_{2_{HT}}$	Н
0	$2.03 \times 10^{3}$	0	2.57	$1.0\pm0.3$	$1.1\pm0.4$	$0.10\pm0.03$
10	$1.56 \times 10^3$	0.08	2.01	$0.7\pm0.1$	$1.2\pm0.1$	$0.08\pm0.02$
18	$7.43  imes 10^2$	0.28	2.69	$0.72\pm0.08$	$1.1\pm0.2$	$0.08\pm0.02$
25	$5.10 \times 10^2$	0.45	2.42	$0.89 \pm 0.06$	$1.3\pm0.1$	$0.10\pm0.03$
35	$3.00  imes 10^2$	0.49	3.20	$0.88\pm0.09$	$1.2\pm0.1$	$0.08\pm0.02$
50	$1.85 \times 10^2$	0.55	3.10	$0.9\pm0.2$	$1.6\pm0.3$	$0.11\pm0.02$
75	$9.26 \times 10^{1}$	1.06	4.70	$1.1\pm0.2$	$1.5\pm0.3$	$0.13\pm0.01$
100	$6.18 \times 10^{1}$	1.60	5.02	$1.2\pm0.2$	$1.4\pm0.3$	$0.11\pm0.03$
150	$3.74  imes 10^1$	1.60	5.37	$1.5\pm0.2$	$1.6\pm0.4$	$0.13\pm0.03$
250*	$2.10 \times 10^1$	1.60	2.31	$2.1\pm0.2$	$3\pm1$	$0.14\pm0.04$
500*	$9.97\times10^{0}$	1.60	1.30	$2.2\pm0.5$	$3\pm1$	$0.10\pm0.06$

Table II. Evolution of several properties (De, n,  $C_{2\mu\tau}$ ) and indicators ( $\overline{\Omega}$ ,H) with polymer concentration. H is the average of  $H_z^x$  over z.

<sup>970</sup> also to the maximum large turbulent scale enhancement, in 1012 ports our observations that several hydrodynamic sub-regimes an anisotropic fashion (see figure 9). It can be inferred that 1013 of concentration can exist for XG and long chained polymers. 971 this state of maximize mean flow, found at 50 ppm, corre-1014 As it is the case for drag reducing properties, the values of 972 sponds to a given state of polymer-flow interactions (align-1015 these critical concentrations should depend on the conforma-973 ment, ratios between polymer relaxation time-scales and tur-1016 tion of the polymer chains, on the presence of salt that may 974 ulent time scales...). A maximized mean flow would imply 1017 modify this conformation<sup>84,85</sup>, on the entanglement state, and 975 that interactions between polymer and small turbulent struc-1018 on the molecular weight of the polymer chains,<sup>76,83</sup>. The first 976 tures are reduced, and energy transfer towards large scales 1019 critical concentration  $C_{D1}$ , comparing to low critical drag re-977 of the flow are favoured. The question is then the flowing: 1020 duction concentrations, is thus likely related to conformation since the polymer concentration lies inside the dilute entan-1021 effects and physical properties of polymer chains. The sec-979 glement regime (lower than 100 ppm) for which mechani-1022 ond, higher, critical concentration  $C_{D2}$  could be explainable 980 cal interactions between polymer chains is assumed negli-1023 by mean shear alignment effects. The critical concentration, 981 gible, what then causes polymer chains to exhibit variable 1024  $C_{XG} = 100$  ppm (and the 100 ppm to 150 ppm transition) is 982 response to the flow ? A possible explanation is the exis-1025 by definition connected to polymer-polymer interactions. 983 tence of polymer-polymer electrical interactions. It is indeed 984 known that when transitioning from the dilute to the semi di-985

lute regime, long chained XG molecules first see each other 1026 B. Possible mean flow production and feeding mechanisms 986 hrough repulsive and attractive electrical forces caused by the 987

presence of electrically charged complex on the polymer car-988 bonate backbone<sup>76</sup>. Would this mean that the critical 50 ppm  $_{1028}$  the origin of the mean flow in OGT is proposed. The authors 989 concentration marks the onset of polymer electrical interac-1029 argue that mean flow arises when there is "a significant dif-990 tions, "smoothing" the dilute to semi-dilute transition? This 1030 ference in the relative strengths of the jets produced by the 991 open question needs to be answered by a coupling between the 1031 oscillating grid in different regions of the tank", and relate it 992 knowledge of typical flow time-scales, and theoretical models 1032 to a Coanda effect applying on the jets closer to the walls. 993 of polymer conformation state and electrical interactions. 994

995 tration for drag reduction of XG solutions is found quite close, 1035 tensity is significantly reduced inside that inner box. What 996 at  $C_{XG} = 70$  ppm by Wyatt, Gunther, and Liberatore<sup>81</sup>. These 1036 can be added to this picture when the working fluid is a shear 997 previous concentrations can indeed be related to the drag re-1037 thinning polymer solution, is the fact that strong shearing of 998 duction properties (type B) of XG. Wyatt, Gunther, and Lib-1038 the fluid in the region between the grid and the wall tends to 999 eratore<sup>81</sup> evidenced drag reduction for XG flowing in pipes at 1039 locally reduce the average viscosity on the sides of the tank. 1000 XG concentrations down to 20 ppm. Sohn et al.<sup>82</sup> achieved 1040 Jets induced by the grid motion are thus by nature of even 1 0 0 1 drag reduction by XG in a rotating disk apparatus at concen-1041 more variable strength depending on their distance from the 1002 trations down to 10 ppm, but found a critical concentration for 1042 wall: near wall jets on each side of the tank see a statistically 1003 maximum drag reduction at 200 ppm. Pereira, Andrade, and 1043 lower viscosity than jets close to x = 0. The fact that mean 1 0 0 4 Soares<sup>83</sup> observed drag reduction for XG concentration as 1044</sup> flow is enhanced in presence of polymer in the dilute regime 1005 low as 2 ppm, and a critical concentration at 37.5 ppm below 1045 is thus consistent with the mean flow origin proposed by Mcwhich drag reduction efficiency falls with increasing  $C_{XG}$  and 1046 Corquodale and Munro<sup>42</sup>. 1007 above which it increases. They stressed that this last two-trend 1047 It has also been shown in several studies that the effect of 1008 behavior is significantly different from that of type A drag re-1048 polymer can propagate to larger scales (see section II B). This 1010 ducing polymers, for which drag reduction monotonously in-1049 can translate for example into an increase of integral length <sup>1011</sup> creases with polymer concentration. These last remarks sup-<sup>1050</sup> scales and of large scale fluctuations of velocity<sup>56,59,60</sup>. This

In McCorquodale and Munro<sup>42</sup>, a physical explanation of 1033 The authors check that when artificially separating the side It is in that sense interesting to notice that a critical concen-1034 jets from the central ones, using an inner box, mean flow in-

Turbulence									
Anisotropy	$\rm C_{2HT}$	Constant or slight increase			Strong anisotropy increase				
Integral scales	$a,\beta,r$	Increase		Decrease	?				
Decay rate	n	Increase	D	ecrease, single slope		2 slopes			
Mean flow									
Vorticity	$\overline{\Omega}, \ \overline{\Omega}t_{\scriptscriptstyle CY}$	Increase, correlated to De				Decrease			
Central velocity	A, A/(fS)	Increase			Decrease				
Altitude reached	$Z_m, Z_m/M$	Increase Dec		crease					
Concentration regimes									
Entanglement			D		SD				
Hydrodynamics		D0	D1	D2	SD1	SD2			
	l	) 20	0 5	0 10	00 1.	$50  C_{XG}$			

Figure 11. Summary of indicators and flow properties evolution with concentration, and sketch of concentration regimes and sub-regimes.

up-scale energy transfer<sup>55</sup> could be another feeding mecha-1088 centrations for drag reduction. A more extensive characterinisms for the mean flow, explaining its enhancement with in-1089 zation of turbulence properties would come from a parametric 1052 1053 creasing polymer concentration in the dilute regime. To con-1090 study of the influence of f, S or M on the velocity and integral firm such an hypothesis, it would be interesting to study turbu-1091 scales of turbulence. This could for example allow to dis-1054 lence inside the grid sweep region and try to evidence energy 1092 cuss the value of  $C_{1_{HT}}$  in polymer solutions, and its possible 1055 1056 transfer terms that could fuel the mean flow.

### CONCLUSION AND PERSPECTIVES 1057

In this work, the hydrodynamics in an oscillating grid 1058 1099 1059 stirred tank filled with either water or shear-thinning XG solutions have been investigated. It is found that the presence of <sup>1100</sup> region, and by the study of energy fluxes transferred between polymer tends to enhance the mean flow in the dilute regime, 1061 by structuring and increasing the two side recirculations al-1062 ready existing in water. This mean flow increase can be related 1063 to the shear-thinning Deborah number in the dilute regime. In 1 0 6 4 the semi-dilute regime, the further increase of viscosity not 1065 being followed by an increase in De leads to a collapse of the mean flow. As for turbulence, its properties also evolve with  $\frac{1}{1105}$ 1067 polymer concentration, especially its decay rate, its isotropy, 1106 1068 and the size of its largest turbulent structures. In the dilute 1107 1069 egime, the HT and TT descriptions of OGT in water can be<sup>1108</sup> 1070 1109 adapted to shear thinning XG solutions. 1071 1110

Oscillating grid apparatus can thus be used as tools to gen-1072 erate controlled turbulence in dilute regime shear thinning 1112 1073 polymer solutions. In this concentration range, laws for OGT 1113 1074 in dilute polymer solutions can be compared to those in water. 1114 1075 A non-negligible mean flow has to be accounted for, but it is  $\frac{1115}{1116}$ 1076 mostly acting in the bulk flow of OGT in the z < 4M region. 1117 Mean flow over turbulent intensity levels remain comparable 1118 1078 closer to the free surface for z > 4M, making it possible to<sup>1119</sup> 1079 study turbulence, mixing and mass transfer boundary layers<sup>1120</sup> 1080 in such fluids, as done in water. 1081 1122

The overall evolution of hydrodynamic properties with 1123 1083 polymer concentration can be described by several critical<sup>1124</sup> 1083 concentrations, splitting the dilute and semi-dilute entangle-1125 1 0 8 4 ment regimes into several hydrodynamic sub-regimes. In par- $\frac{1126}{1127}$ 1086 ticular, it is worth noting that transition exist at concentrations 1128 1087 as low as about  $C_{XG} \simeq 20$  ppm, comparable to critical con-1129 <sup>10</sup>M. T. Mahamod, W. H. M. W. Mohtar, and S. F. M. Yusoff, "Spatial and

1093 dependency on  $Re_g$ . Three-dimensional effects in turbulence and mean flow could be investigated by either additional PIV 1094 measurements in other planes of the tank, or directly by using three-dimensional particle tracking methods. Finally the 1096 1097 mechanisms of turbulence and mean flow production at the grid level remain to be fully understood. Interesting data could be brought by velocity measurements inside the grid sweep 1101 oscillating motion induced by the grid, turbulence, and mean 1102 flow structures.

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