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# REASONING ABOUT CONDITIONS IN STIT LOGIC

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## Abstract

In this paper we propose a logical formalization of the legal concepts of suspensive and resolutive conditions within the STIT approach to action. At the technical level, our proposal consists in extending the STIT language with a special operator that allows us to represent the concept of a presumption. This enables us to model the retroactive effect of conditions.

## 1 Introduction

This paper will provide a logical analysis of legal conditions, namely, future and uncertain events on which a legal arrangement is dependent, according to a juristic act, such as a contract or will.

First, we shall introduce notions of conditioned disposition, condition and conditioned legal arrangement. Then we shall distinguish different kinds of conditions. In particular we shall distinguish suspensive and resolutive conditions, which postpone and revoke respectively a legal arrangement. We shall also distinguish non-retroactive and retroactive conditions. The first concern a legal arrangement existing from the time of the condition and the second a legal arrangement pre-existing to the condition.

Our formalization of the notion of legal condition is based on STIT logic, a well-known logic of action introduced in philosophical logic by Belnap et al. [5]. The

reason why we use STIT is that it offers a clear account of time, action and their combination. These are fundamental constituents of the notion of legal condition. In order to capture retroactive effects of conditions, we will extend the basic STIT theory by the concept of presumption. The resulting framework will allow us to represent two complementary aspects of a retroactive condition: (a) the institutional past differs depending on the realization of the retroactive condition, and (b) until a suspensive condition is realized (or a resolutive condition fails to be realized) it is presumed that the conditioned arrangement does not obtain (or does obtain).

The paper is organized as follows. Section 2 is devoted to the conceptual background of our work by discussing the relevant philosophical theories of conditions. In Section 3, the STIT framework is presented. In Section 4, it is used to formalize the notion of condition. In Section 5, we conclude.

## 2 Legal conditions

In this section we shall introduce the concept and the regulation of conditions, describing the phenomenon that will be formalized logically in the next section.

### 2.1 Conditional dispositions, conditions, conditioned positions

Our analysis will address *conditional dispositions* included in juridical acts, namely, in those performative declarations —such as contracts or wills— through which private parties establish legal arrangements, e.g., they transfer property and create or remove obligations. A *conditional disposition* makes a legal arrangement, the *conditioned arrangement*, dependent upon a future and uncertain event, the *condition*.

To avoid ambiguities, we shall reserve the term “condition” to denote the future and uncertain conditioning events. We shall explicitly speak of a “conditioned arrangement” to denote an institutional outcome whose existence is dependent upon the realization of the condition, and of a “conditional disposition” for the juristic act (or part of it) establishing a conditional arrangement.

Note that a conditional disposition is no descriptive statement, it rather is a performative one, meant to constitute the conditioned arrangement in case the condition will be happen. Consider, for instance the following example of a conditional disposition, from Roman law: “I shall give you 100 sesterces, on condition that the ship arrives from Asia” (the parties to the transport contract, agree that only if the ship arrives the fee for transport is to be paid). In this conditional disposition, the proposition “the ship arrives from Asia” is the condition, and the obligation to give you 100 sesterces is the conditioned arrangement. The conditioned arrangement and the condition can also be stated in separate statements: Statement 1: I shall give

you 100 sesterces. Statement 2: Statement 1 shall have effect only on condition that the ship arrives from Asia. Or also “This contract shall only have effect if the ship arrives from Asia”.

The conditioned arrangement that is constituted by this conditional disposition (the obligation of the promisor and the right of the promisee, both dependent on the ship’s arrival) will (a) become pure or unconditioned, if the condition takes place or (b) terminate, if the condition becomes impossible.

Conditioned arrangements may consist in any kind of legal outcome that can be constituted by private parties: the creation, the elimination, the modification or the transfer of any legal position, such as an obligation or a property right. A conditioned legal right may be the object of a transaction, being sold, purchased, donated, etc. The effectiveness of such a transaction, however, will remain subject to the verification of the condition.

The regulation of a conditional disposition goes back to Roman law. Justinian’s Institutes (Book III, Section XIV, [22]), in discussing stipulations (*stipulationes*), i.e., legally binding promises, classifies them as being pure, with a deadline, or under condition: in the first case, performance can be requested immediately; in the second, performance can only be requested at the established date; in the third case, performance can only be required if the condition obtains. The structure of the third case (which is a suspensive conditioned disposition) is as follows that “A promise is made conditionally, when the obligation is made dependent on an uncertain occurrence, so that the promise is binding if something happens or does not happen” (Book III, Section XV).

The notions of a conditional disposition can also be found in modern civil codes. For instance, Article 1335 of the Italian civil code states that “The parties to a contract can make the efficacy or the resolution of the contract, or of a single agreement, dependent upon a future and uncertain event.” Similarly, according to the Draft Common Frame of Reference, a project for a Common Civil Code for the EU: “[t]he terms regulating a right, obligation or contractual relationship may provide that it is conditional upon the occurrence of an uncertain future event, so that it takes effect only if the event occurs (suspensive condition) or comes to an end if the event occurs (resolutive condition).”[21]

Also the common law addresses conditional dispositions, using different terminologies (see [12]).

## 2.2 The effects of conditions

Here, following the civil law tradition, we shall limit our analysis to conditional dispositions in private juridical acts. We do not address the conditional connections

established by authoritative legal norms, in legislation or judicial rulings, though it may be argued that similar logical structure may also characterise the institutional outcome of authoritative conditional declarations.

The purpose of conditional dispositions in private juridical acts is to address future contingencies. The parties want to establish a legal arrangement that only fits a particular future situation (the condition), but they are uncertain on whether this situation will obtain. Therefore, they make their arrangement dependent on the existence of that situation. For instance, a person that is likely to get a job in a city can make a rental contract the legal effects of which (the landlord's obligation to provide the house, and the tenant's obligation to pay the rent for the other) are conditioned on that person's getting the job. If the prospective tenant does not get the job, neither party should be due to perform their obligations.

For a conditioned legal position to exist as such, it is essential that the condition is uncertain. If a legal effect is dependant upon an event which is certain, even though the time in which it takes place is uncertain (e.g., a person's death), then the event constitutes a deadline rather than a condition. The uncertainty of conditions is assumed to be "objective": for the condition to be uncertain, at a certain point in time, it is sufficient that it is not humanly possible to anticipate with certainty whether the event will happen or not.

### **2.3 Suspensive and resolutive conditions**

There are two different types of conditions in all legal orders since the times of Ancient Roman Law: suspensive and resolutive ones.

A suspensive conditional disposition makes the constitution of a legal arrangement dependent on the occurrence of a condition. This means that the legal arrangement only becomes effective if the condition occurs.

Consider for instance the case in which a developer acquires a piece of land from the owner for a certain price, under the suspensive condition that within a year a building permit is issued. If the building permit is issued, then the land is transferred to the developer and the developer will be obliged to pay the price. If this is not the case, the transfer will not take place, so that the land will remain with the owner, and the developer will have no obligation to pay the price.

As another example, consider an installment sale where the transfer of the property (e.g., of a vehicle) is subject to the suspensive condition of the payment of all installments. In this case only if the payment of the last installment is completed the ownership is transferred to the buyer. If this is not the case, the property remains with the seller. This arrangement is useful for the seller in case the buyer defaults: rather than having to compete with other creditors, the seller will simply claim back

what still belongs to her.

A resolutive conditional disposition constitutes a legal arrangement and makes the termination of a legal arrangement dependent on the occurrence of the condition. Consider the case in which a developer acquires a piece of land from a seller, under the resolutive condition that the developer does not obtain a building permit within a year (the sale will be cancelled if this negative condition avers). If the resolutive condition is not met (the permit is given) the developer continues to own what he has purchased. If the condition is met (the permit is not given within the deadline) the transfer is cancelled, and the seller's ownership is restored. As another example, consider the case in which a buyer purchases a property from a seller under the resolutive condition that the seller gives back the whole price paid by the buyer, plus an additional sum (an interest). In this case if the seller gives back the whole amounts due, the transfer is annulled; if he does not, the transfer remains.

## 2.4 Retroactivity

A condition is retroactive when the occurrence of the conditioned legal arrangement (for suspensive conditions) or its cancellation (for resolutive conditions) is assumed to take place at a time that precedes the realization of the condition (usually, at the time in which the conditional disposition was enacted).

Let us assume that at a time  $t_0$  a contract is enacted according to which 1 sells a piece of land to 2, subject to the retroactive suspensive condition that a building permit is issued.<sup>1</sup> If the permit is issued at the subsequent time  $t_1$  (e.g., one year after  $t_0$ ) the transfer is assumed to have taken place at  $t_0$ . If the condition were not retroactive, the transfer would be assumed to take place only at the time in which the condition takes place, namely at  $t_1$ .

The retroactivity of a condition affects subsequent transactions dependant on the conditioned arrangement. Let us assume that 1 sells the land to 2 at  $t_0$ , under the suspensive condition that the building permit is granted, 2 unconditionally sells to 3 at  $t_1$ , and the permit is issued at  $t_2$ . The retroactivity of the suspensive condition means that 2's sale to 3 is effective, since it is assumed that 2 owned the property at the time  $t_1$  of 2's sale to 3. On the other hand, if the permit were not granted within the deadline, 2's sale to 3 would be ineffective, since it would be assumed that 2 did not own the property at  $t_2$ .

In the case of a retroactive suspensive condition, while the condition is still pending —i.e., before the condition either is realized, or definitely fails— the law assumes that the conditioned legal arrangement has not been constituted. For instance, in

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<sup>1</sup>We assume in our examples a legal regime, such as Italian law, in which the transfer of property in a sale contract does not require the delivery of the thing (as it is the case in German law).

our example, it is assumed that no transfer has taken place, i.e., that the seller has remained the legitimate owner. Therefore, while the condition is pending, seller 1 can exercise the rights that pertain to an owner, e.g., maintain his possession of the land and use all legal remedies against trespassers and other infringers of property rights.

If a resolutive condition is retroactive, the conditioned arrangement is cancelled from the beginning. Let us assume that at time  $t_0$  a contract is executed according to which 1 sells a piece of land to 2, subject to the retroactive resolutive condition that the contract will be cancelled if a building permit is not provided within a year. In this case it is assumed that the transfer takes place immediately (that 2 becomes the owner at  $t_0$ ). However, if the permit is not issued within a year (the condition is realized), the transfer is retroactively cancelled (it is assumed that 1 has remained the owner without interruption).

Let us assume that 1 sells to 2 under the retroactive condition at  $t_0$ , 2 unconditionally sells to 3 at  $t_1$ , and the condition is realized at  $t_2$  (a year elapses without the building permission being issued). The retroactivity of the condition means that it is assumed that the original transfer from 1 to 2 is ineffective, so that 2 was not the owner of the land at  $t_1$ , when he sold it to 3. Consequently, the transfer from 2 to 3 is also ineffective.

Whether conditions are by default retroactive or not depends upon the applicable national orders. Whereas French and Italian law have retroactivity by default, German law assumes that conditions are non-retroactive. Legal systems also contain regulations that address the time in which the condition is still pending, namely, it is still uncertain whether the condition will be realized. For instance, the party that has purchased a property under a suspensive condition, may take some legal initiatives to limit the risk that the property is destroyed before the realization of the condition.

## 2.5 How to model retroactivity

In the history of legal thinking two main theories have been proposed for the retroactivity of legal conditions (for a historical discussion on the retroactivity of conditions, see [2]).

The first theory, that goes back to the medieval jurist Bartolus, views the retroactive effect of conditions as based on a fiction: the law makes the fictive (false but binding) assumption that the condition, once realized, was effective at the time in which the conditional disposition was enacted. For all practical purposes lawyers and citizens have to reason and behave pretending to be in the fictitious history in which the conditioned arrangement held since that time. In the example above,



even if in reality 1's sale to 2 could not be effective at  $t_0$  and so at  $t_1$ , we pretend to be in a fictitious history in which the sale from 1 to 2 took place at  $t_0$ .

The second theory—which goes back to G.W. Leibniz [14, 1]—views the retroactive effect of conditions as immediate and real. Thus, in case the condition should obtain, the conditioned arrangement holds (in suspensive condition) or does not hold (in resolutive conditions) from the time in which the conditional disposition was executed. Leibniz advanced this view in his early legal work (see [3]), when he was only 20 year old, but his approach to legal conditions is consistent with his later philosophy. This approach may indeed be linked to the principle of sufficient reason [18] that characterizes his mature metaphysics: everything that happens is determined by a chain of reasons (or causes) though such reasons may be inaccessible to human cognition, while being known to God. This is the case also for the future realization of a condition: if the condition will take place in the future, it is determined, from a divine perspective, that it will take place, and so its retroactive effect is immediate. However, the realization of the condition remains contingent, when viewed from the human stance. This idea is concisely expressed by the following famous statement in the *Monadology* (section 22): “And as every present state of a simple substance is a natural consequence of its preceding state, so is its present pregnant with the future.” [19, 96]. In the *Theodicy* (section 360), the idea is developed as follows:

It is one of the rules of my system of general harmony, that the present is big with the future, and that he who sees all sees in that which is that which shall be. What is more, I have proved conclusively that God sees in each portion of the universe the whole universe, owing to the perfect connection of things. He is infinitely more discerning than Pythagoras, who judged the height of Hercules by the size of his footprint. There must therefore be no doubt that effects follow their causes determinately, in spite of contingency and even of freedom, which nevertheless coexist with certainty or determination. [19, 97]

A third ontology of retroactivity is also possible, namely the view that, as time goes by, the past may change. Before the realization of the retroactive suspensive condition, the legal effect does not hold in the time interval between the enactment of the suspensive conditional disposition and the verification of the condition. After the verification of the condition, the legal effect holds also in that interval. Thus, in this approach, the truth of a proposition about the past has different truth values depending on the time in which the proposition is asserted (see [9, 8], who address retroactive legislation).

Here we will shall capture retroactivity through a branching-time model corresponding to Leibniz's approach to legal conditions, i.e., with the idea of the immedi-

ate effect of retroactive conditions. In our model, the histories in which a retroactive legal disposition —making arrangement  $\psi$  dependent upon condition  $\varphi$ — is enacted at a time  $t$ , differ depending on whether the condition takes place. If the suspensive condition  $\varphi$  happens at a subsequent time  $t'$ , the conditioned legal arrangement  $\psi$  holds from  $t$ . If the suspensive condition  $\varphi$  fails to happen, the conditioned legal arrangement  $\psi$  never holds.

Thus, a history  $h_1$  in which  $\varphi$  happens is paralleled by a history  $h_2$  in which  $\varphi$  fails to happen that is undivided from  $h_1$  up to the time  $t'$  of the realization of the condition. The two histories differ on the legal arrangements that are dependent on the conditions (and on the implications of these arrangements). In this situation, there is an indeterminacy concerning the conditioned arrangement, that is only removed at the time of the realization (or failure) of the condition.

We shall complement Leibniz's idea of the immediate effect of retroactive conditions, with the following observation. While the realisation of a retroactive condition still is a contingent future event, the law addresses this indeterminacy through a presumption: regardless of whether the conditioned effect holds or does not hold (depending on the realisation of the condition), the law presumes —i.e., it assumes for all practical purposes— that the conditioned arrangement does not hold (for suspensive conditions) or holds (for resolutive conditions).

This means that while it is possible that an arrangement under suspensive condition already holds (this is the case in those histories in which the condition will be realized), at the time in which the condition is still pending, we have to presume that the arrangement does not hold.

Similarly, while it remains possible that the arrangement under resolutive condition does not really hold, at the time in which the condition is still pending we have to presume that the arrangement does hold.

Assume, for instance that at  $t_0$ , 1 sells a piece of land to 2, under the suspensive condition that a building permit is granted, and that the permit will indeed be granted, at the subsequent time  $t_1$ . At  $t_0$  it is still uncertain whether the permit will be granted or not. In such a context, we shall say that the transfer of property (the suspended legal effect) already takes place at  $t_0$ , but that at  $t_0$ , while the condition still is uncertain, the law presumes that effect has not taken place, since it has chosen to take into consideration (to presume) only this possibility.

### 3 Logical framework

In this section, we present the language and the semantics of the logic STIT-P (STIT logic with presumption). STIT logic (the logic of *seeing to it that*) by Belnap et al.

[5] is one of the most prominent formal accounts of agency. It is the logic of sentences of the form “the agent  $i$  (or the group of agents  $G$ ) sees to it that  $\varphi$  is true”. Two variants of STIT have been studied in the literature: ‘atemporal STIT’ and ‘temporal STIT’. At the syntactic level, the former corresponds to the family of languages for expressing properties of individual and group agency with no temporal operators. Notable examples are the languages studied by [4, 17, 11, 10]. The latter corresponds to extensions of atemporal STIT languages by temporal operators for expressing properties of agency in connection with time such as the temporal operator ‘next’ of linear temporal logic [6, 16]<sup>2</sup> as well as future and past tense operators of basic tense logic [13, 15, 23].

STIT-P consists in extending the language of the temporal STIT presented in [16] by a new modal operator for ‘presumption’.

### 3.1 Language

Let  $Atm$  be a countable set of atomic propositions denoting atomic facts and let  $Agt$  be a finite set of agents. The language of STIT-P, denoted by  $\mathcal{L}_{STIT-P}(Atm, Agt)$ , is defined by the following grammar in Backus-Naur form (BNF):

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid X\varphi \mid Y\varphi \mid [i]\varphi \mid [Agt]\varphi \mid \Box\varphi \mid P\varphi$$

where  $p$  ranges over  $Atm$  and  $i$  ranges over  $Agt$ . The other boolean operators are defined from negation and conjunction in the usual way.

The formulas  $X\varphi$ ,  $Y\varphi$ ,  $[i]\varphi$ ,  $[Agt]\varphi$ ,  $\Box\varphi$  and  $P\varphi$  have to be read as follows:

- $X\varphi$ : “ $\varphi$  will be true in the next state along the current history”,
- $Y\varphi$ : “ $\varphi$  was true in the previous state along the current history”,
- $[i]\varphi$ : “agent  $i$  sees to it that  $\varphi$ , regardless of what the others choose”,
- $[Agt]\varphi$ : “all agents together see to it that  $\varphi$ ”,
- $\Box\varphi$ : “it is historically necessary that  $\varphi$ ”, and
- $P\varphi$ : “ $\varphi$  is legally presumed to be true”.

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<sup>2</sup>The main feature of the language studied by [6] is that the temporal operator ‘next’ and the agency operator are fused into a single operator. In the languages studied by [20, 16] they are kept separated.

The duals of the operators  $\Box$ ,  $\langle i \rangle$  and  $\langle \text{Agt} \rangle$  are defined as follows:

$$\begin{aligned}\Diamond\varphi &\stackrel{\text{def}}{=} \neg\Box\neg\varphi \\ \langle i \rangle\varphi &\stackrel{\text{def}}{=} [i]\varphi \\ \langle \text{Agt} \rangle\varphi &\stackrel{\text{def}}{=} [\text{Agt}]\varphi\end{aligned}$$

where  $\Diamond\varphi$ ,  $\langle i \rangle\varphi$  and  $\langle \text{Agt} \rangle\varphi$  have to be read, respectively, “it is historically possible that  $\varphi$ ”, “ $\varphi$  may result from agent  $i$ ’s actual choice” and “ $\varphi$  may result from the agents’ actual choices”.

### 3.2 Semantics

Different semantics for STIT have been proposed in the literature (see [7] for a recent systematic analysis and comparison of these different semantics). The original semantics of STIT by Belnap et al. [5] is defined in terms of  $BT+AC$  structures: branching-time structures ( $BT$ ) augmented by agent choice functions ( $AC$ ). A  $BT$  structure is made of a set of moments and a tree-like ordering over them. An  $AC$  for an agent  $i$  is a function mapping each moment  $m$  into a partition of the set of histories passing through that moment, a history  $h$  being a maximal set of linearly ordered moments and the equivalence classes of the partition being the possible choices for agent  $i$  at moment  $m$ .

Kripke-style semantics for STIT have been proposed by [17] for non-necessarily discrete time and by [16] for discrete time. We here present a semantics for STIT-P based on the following concept of agentive structure with discrete time that is well-suited to formalize the concept of condition given its explicit representation of discrete time. It turns out that this semantics and the discrete-time Kripkean semantics for temporal STIT presented in [16] are equivalent relative to the language  $\mathcal{L}_{\text{STIT-P}}(\text{Atm}, \text{Agt})$  under consideration. On the conceptual side, the main difference between this semantics for STIT and Belnap et al.’s  $BT+AC$  semantics is that the former takes the concept of *history* as a primitive instead of the concept of *moment* and defines: (i) a *moment* as an equivalence class induced by a certain equivalence relation over the set of histories, and (ii) an agent  $i$ ’s set of *choices* at a moment as a partition of that moment.

**Definition 1** (Agentive structure with discrete time). *An agentive structure with discrete time (ASDT) is a tuple*

$$M = (H, (\sim_n)_{n \in \mathbb{N}}, (\sim_{\langle n, i \rangle})_{n \in \mathbb{N}, i \in \text{Agt}}, (\sim_{\langle n, \text{Agt} \rangle})_{n \in \mathbb{N}}, \mathcal{V})$$

where:

- $H$  is a set of histories;
- all relations  $\sim_n$ ,  $\sim_{\langle n,i \rangle}$  and  $\sim_{\langle n, \text{Agt} \rangle}$  are equivalence relations on  $H$  that satisfy the following conditions:
  - (C1) for all  $n \in \mathbb{N}$  and  $i \in \text{Agt}$ :  $\sim_{\langle n,i \rangle} \subseteq \sim_n$ ,
  - (C2) for all  $n \in \mathbb{N}$  and  $h_1, \dots, h_n \in H$ : if  $h_i \sim_n h_j$  for all  $i, j \in \{1, \dots, n\}$  then  $\bigcap_{i \in \text{Agt}} \sim_{\langle n,i \rangle}(h_i) \neq \emptyset$ ,
  - (C3) for all  $n \in \mathbb{N}$ :  $\sim_{\langle n, \text{Agt} \rangle} = \bigcap_{i \in \text{Agt}} \sim_{\langle n,i \rangle}$ ,
  - (C4) for all  $m, n \in \mathbb{N}$  and  $h, h' \in H$ : if  $h \sim_n h'$  and  $m < n$  then  $h \sim_{\langle m, \text{Agt} \rangle} h'$ , and
- $\mathcal{V} : \mathbb{N} \times H \longrightarrow 2^{\text{Atm}}$  is a valuation function for atomic propositions.

An ASDT is defined by a set of histories  $H$ . The truth value of an atomic proposition depends on the time point  $n$  along a given history  $h$ . In particular, proposition  $p$  is true at time point  $n$  along the history  $h$  if and only if  $p \in \mathcal{V}(n, h)$ . The equivalence relation  $\sim_n$  defines the historical alternatives of a history at the time point  $n$ . Specifically, if  $h \sim_n h'$  then, at time point  $n$ ,  $h'$  is a historic alternative of  $h$ .

The equivalence relations  $\sim_n$  also define the set of moments  $Mom$ . In particular,  $Mom = \bigcup_{n \in \mathbb{N}} H / \sim_n$  where  $H / \sim_n$  is the quotient set of  $H$  by the equivalence relation  $\sim_n$ . Elements of  $Mom$  are denoted by  $m, m', \dots$

The equivalence relations  $\sim_{\langle n,i \rangle}$  and  $\sim_{\langle n, \text{Agt} \rangle}$  define, respectively, agent  $i$ 's choices at the time point  $n$  and the collective choices of all agents at time point  $n$ . Specifically, if  $h \sim_{\langle n,i \rangle} h'$  then histories  $h$  and  $h'$  belong to the same choice of agent  $i$  at time  $n$ . If  $h \sim_{\langle n, \text{Agt} \rangle} h'$  then histories  $h$  and  $h'$  belong to the same collective choice of all agents at time  $n$ .

Constraint C1 just means that an agent can only choose among possible alternatives. This constraint ensures that, for every history  $h$ , the equivalence relation  $\sim_{\langle n,i \rangle}$  induces a partition of the moment  $\sim_n(h)$ . An element of this partition is a choice that is possible (or available) for agent  $i$  at moment  $\sim_n(h)$ . Constraint C2 expresses the so-called assumption of independence of agents or independence of choices. Intuitively, this means that agents can never be deprived of choices due to the choices made by other agents. Constraint C3 just says that the collective choice of the grand coalition  $\text{Agt}$  is equal to the intersection of the choices of all individuals. Finally, Constraint C4 corresponds to the so-called property of no choice between undivided histories. It captures the idea that if two histories come together in some future moment then, in the present, each agent does not have a choice between these

two histories. This implies that if an agent can choose between two histories at a later stage, then she does not have a choice between them in the present.

The STIT semantics assumes that agents' choices may have non-deterministic effects, that is to say, it is not necessarily the case that the consequences of the agents' current choices are unequivocally determined. In formal terms, it could be the case that  $h \sim_{\langle n, \text{Agt} \rangle} h'$  and  $h \not\sim_{n+1} h'$ . The latter means that  $h$  and  $h'$  belong to the same collective choice at time  $n$  but do not belong to the same moment at time  $n + 1$ .

The following definition extends the concept of ASDT by the concept of presumption.

**Definition 2** (Agentive structure with discrete time and presumption). *An agentive structure with discrete time and presumption (ASDTP) is a tuple  $M = (H, (\sim_n)_{n \in \mathbb{N}}, (\sim_{\langle n, i \rangle})_{n \in \mathbb{N}, i \in \text{Agt}}, (\sim_{\langle n, \text{Agt} \rangle})_{n \in \mathbb{N}}, \mathcal{P}, \mathcal{V})$  where*

- $M = (H, (\sim_n)_{n \in \mathbb{N}}, (\sim_{\langle n, i \rangle})_{n \in \mathbb{N}, i \in \text{Agt}}, (\sim_{\langle n, \text{Agt} \rangle})_{n \in \mathbb{N}}, \mathcal{V})$  is an ASDT and
- $\mathcal{P}$  is a legal presumption function mapping every moment  $m \in \text{Mom}$  to a non-empty set of histories  $\mathcal{P}(m) \subseteq m$  passing through it.

The idea is that the law, when making a presumption, requires legal reasoners—in particular legal decision-makers—to take into account only a subset of the possible histories (those passing through a given moment), i.e., it requires them to consider only the histories that are compatible with what is presumed. The remaining histories passing through the given moment are excluded from the law's perspective.

Thus, intuitively,  $\mathcal{P}(m)$  is the set of possible histories that the law takes into consideration at moment  $m$ , i.e., the set of histories that are compatible with what law presumes at moment  $m$ . Conversely,  $m \setminus \mathcal{P}(m)$  are the histories that are excluded from the law's consideration, being incompatible with what law presumes at moment  $m$ .

Accordingly, we can define satisfaction as a relation between formulas of the language  $\mathcal{L}_{\text{STIT-P}}(\text{Atm}, \text{Agt})$  and pointed ASDTPs.

In particular, let  $M = (H, (\sim_n)_{n \in \mathbb{N}}, (\sim_{\langle n, i \rangle})_{n \in \mathbb{N}, i \in \text{Agt}}, (\sim_{\langle n, \text{Agt} \rangle})_{n \in \mathbb{N}}, \mathcal{V})$  be a AS-

DTP and let  $(n, h) \in \mathbb{N} \times H$ , then:

$$\begin{aligned}
M, (n, h) \models p &\iff p \in \mathcal{V}(n, h) \\
M, (n, h) \models \neg\varphi &\iff M, (n, h) \not\models \varphi \\
M, (n, h) \models \varphi \wedge \psi &\iff M, (n, h) \models \varphi \text{ and } M, (n, h) \models \psi \\
M, (n, h) \models X\varphi &\iff M, (n+1, h) \models \varphi \\
M, (n, h) \models Y\varphi &\iff \text{if } n > 0 \text{ then } M, (n-1, h) \models \varphi \\
M, (n, h) \models \Box\varphi &\iff \forall h' \in H : \text{if } h \sim_n h' \text{ then } M, (n, h') \models \varphi \\
M, (n, h) \models [i]\varphi &\iff \forall h' \in H : \text{if } h \sim_{\langle n, i \rangle} h' \text{ then } M, (n, h') \models \varphi \\
M, (n, h) \models [Agt]\varphi &\iff \forall h' \in H : \text{if } h \sim_{\langle n, Agt \rangle} h' \text{ then } M, (n, h') \models \varphi \\
M, (n, h) \models P\varphi &\iff \forall h' \in \mathcal{P}(\sim_n(h)) : M, (n, h') \models \varphi
\end{aligned}$$

We say that a formula  $\varphi$  of the language  $\mathcal{L}_{\text{STIT-P}}(\text{Atm}, \text{Agt})$  is satisfiable if there exists an ASDTP  $M$  and a pair  $(n, h)$  such that  $M, (n, h) \models \varphi$ . We say that  $\varphi$  is valid, denoted by  $\models \varphi$ , if  $\neg\varphi$  is not satisfiable.

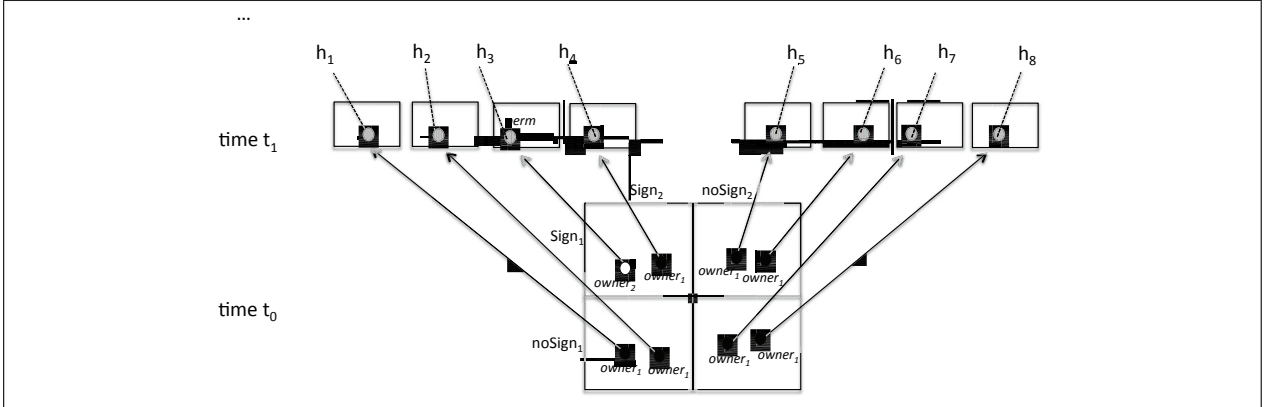


Figure 1: Example of ASDTPs.

Figure 1 illustrates the semantics of the logic STIT-P with the aid of a concrete example where two agents 1 (the seller) and 2 (the buyer) sign a contract. The contract has a retroactive suspensive condition stating that 2 will become the owner of the land ( $owner_2$ ) only if a building permit is issued ( $perm$ ). In the figure, rectangles correspond to moments and arrows represent histories. Black points correspond to histories that are compatible with what law presumes, while white points correspond to histories that are incompatible with what law presumes at a given moment. An agent's set of available choices at a given moment defines a partition of the moment. For instance, at moment  $\sim_{t_0}(h_1)$ , agents 1 and 2 have two choices available, as each of them can decide either to sign or not to sign the contract. Agent 1's choices

correspond to rows (the horizontal partition), while agent 2's correspond to columns (the vertical partition).

At moment  $\sim_{t_0}(h_1)$ , it is uncertain whether the condition *perm* will occur in the next state. Indeed, there are two distinct histories  $h_3$  and  $h_4$  passing through this moment such that the condition *perm* obtains in the former but does not obtain in the latter. In the light of this uncertainty, at moment  $\sim_{t_0}(h_1)$ , it is legally presumed that the condition will not occur and, consequently, that agent 2 is not yet the owner of the land. Indeed, *owner<sub>2</sub>* is false at all histories that are compatible with what law presumes at moment  $\sim_{t_0}(h_1)$ .

## 4 Formalization of conditions

In this section, we use the logic STIT-P to formalize the concepts of suspensive and resolutive conditions.

We focus on conditions occurring in the 'next' state. We will discuss conditions occurring before a certain deadline at the end of the section.

We say that  $\varphi$  is a suspensive condition for  $\psi$  (the effect of the condition), denoted by  $\text{Susp}(\varphi, \psi)$ , if and only if the following requirements are satisfied:

- *Uncertainty requirement*: given the agents' actual choices, it is uncertain whether  $\varphi$  will occur in the next state.
- *Realization requirement*: given the agents' actual choices, if in the next state the condition  $\varphi$  is realized then, from the law's perspective, the condition  $\varphi$  is realized.
- *Conditional requirement*: given the agents' actual choices, the occurrence of  $\varphi$  in the next state is a necessary and sufficient condition for  $\psi$  to become true.

According to the uncertainty and conditional requirements,  $\varphi$  is a condition for  $\psi$  only if  $\varphi$  is a future and uncertain event on which the occurrence of  $\psi$  depends. According to the realization requirement, the law must keep track of the realization of the condition, that is to say, if the condition  $\varphi$  obtains then  $\varphi$  has to be true at all histories that are compatible with what the law presumes. The realization requirement is fundamental to capture the connection between the realization of a condition and its legal effects. Indeed, for a condition to be effective, the law must presume that it obtains and, consequently, that its legal consequences are in place.



This leads to the following formal definition of suspensive condition:

$$\text{Susp}(\varphi, \psi) \stackrel{\text{def}}{=} (\langle \text{Agt} \rangle \text{X}\varphi \wedge \langle \text{Agt} \rangle \neg \text{X}\varphi) \wedge \\ ([\text{Agt}] \text{X}(\varphi \rightarrow \text{P}\varphi)) \wedge \\ ([\text{Agt}] \text{X}(\varphi \leftrightarrow \psi))$$

where formula  $\langle \text{Agt} \rangle \text{X}\varphi \wedge \langle \text{Agt} \rangle \neg \text{X}\varphi$  represents the uncertainty requirement, formula  $[\text{Agt}] \text{X}(\varphi \rightarrow \text{P}\varphi)$  represents the realization requirement and formula  $[\text{Agt}] \text{X}(\varphi \leftrightarrow \psi)$  represents the conditional requirement.

Notice that formula  $\langle \text{Agt} \rangle \text{X}\varphi \wedge \langle \text{Agt} \rangle \neg \text{X}\varphi$  represents *ex-post* uncertainty, i.e., uncertainty after the agents have made their choices and have publicly revealed them. This means that in the example given in Section 3.2, the fact that the building permit is issued becomes a condition for agent 2 to be the owner of the land after agents 1 and 2 have signed the contract. Similarly,  $[\text{Agt}] \text{X}(\varphi \leftrightarrow \psi)$  represents the fact that  $\varphi$  is a necessary and sufficient condition for  $\psi$  only with respect to the agents' current choices. Again in the example of Section 3.2, we assume that the issuing of the building permit is necessary and sufficient for agent 2 to become the owner of the house, under the proviso that 1 and 2 have signed the contract.

Let us consider suspensive conditions with retroactive effects. We say that  $\varphi$  is a suspensive condition with retroactive effect  $\psi$  (the effect of the condition), denoted by  $\text{RetrSusp}(\varphi, \psi)$ , if and only if the following requirements are satisfied.

- *Condition requirement:*  $\varphi$  is a suspensive condition for making  $\psi$  true in the past.
- *Presumption requirement:* it is legally presumed that  $\psi$  is false.

According to the condition requirement, a suspensive condition with retroactive effect  $\psi$  is a special suspensive condition whose effect is  $\text{Y}\psi$ . The presumption requirement is needed since a suspensive condition with retroactive effect  $\psi$  implies the uncertainty whether  $\psi$  is true in the present. Therefore, as explained in Section 2.4, by presuming that  $\psi$  is false the law proceeds *as if* the conditioned legal arrangement has not been constituted. In the example, of Section 3.2, before the building permit is issued it is legally presumed that agent 2 is not yet the owner of the land.

In formal terms, we have:

$$\text{RetrSusp}(\varphi, \psi) \stackrel{\text{def}}{=} \text{Susp}(\varphi, \text{Y}\psi) \wedge \text{P}\neg\psi$$

where formula  $\text{Susp}(\varphi, Y\psi)$  represents the condition requirement and formula  $P\neg\psi$  represents the presumption requirement. It is easy to check that in the structure depicted in Figure 1 the following holds:

$$M, (1, h_1) \models \text{Susp}(\text{perm}, \text{owner}_2) \wedge \text{RetrSusp}(\text{perm}, \text{owner}_2)$$

This means that, at time 1 along the history  $h_1$ , the fact that the building permit is issued is both a suspensive condition for agent 2 to become the owner of the land in the future and a retroactive suspensive condition for agent 2 to become the owner of the land in the present.

Before considering resolute conditions, we discuss the following properties of suspensive conditions:

$$\models \text{RetrSusp}(\varphi, \psi) \rightarrow \text{Susp}(\varphi, Y\psi) \quad (1)$$

$$\models (\text{Susp}(\varphi, \psi) \wedge X\varphi) \rightarrow X(\psi \wedge P\psi) \quad (2)$$

$$\models (\text{RetrSusp}(\varphi, \psi) \wedge X\varphi) \rightarrow X(Y\psi \wedge PY\psi) \quad (3)$$

The first validity follows straightforwardly from the definition of suspensive condition with retroactive effects. The second and third validities are detachment principles for suspensive conditions: if  $\varphi$  is a suspensive condition for  $\psi$  and  $\varphi$  will occur tomorrow then, tomorrow it has to be the case that  $\psi$  and it has to be presumed that  $\psi$ . Similarly, if  $\varphi$  is a suspensive condition with retroactive effect  $\psi$  and  $\varphi$  will occur tomorrow then, tomorrow it has to be the case that  $\psi$  was true yesterday and it has to be presumed that  $\psi$  was true yesterday.

A resolute condition is nothing but the reverse of a suspensive one. Indeed, a suspensive condition is a condition on the occurrence of which the *constitution* of a legal arrangement depends, while a resolute condition is a condition on the occurrence of which the *termination* of a legal arrangement depends. In formal terms, we have:

$$\text{Resol}(\varphi, \psi) \stackrel{\text{def}}{=} \text{Susp}(\varphi, \neg\psi)$$

where  $\text{Resol}(\varphi, \psi)$  means that  $\varphi$  is a resolute condition for  $\psi$ . Similarly, for the retroactive case, we have:

$$\text{RetrResol}(\varphi, \psi) \stackrel{\text{def}}{=} \text{RetrSusp}(\varphi, \neg\psi)$$

where  $\text{RetrResol}(\varphi, \psi)$  means that  $\varphi$  is a resolute condition with retroactivity with respect to  $\psi$ .

As emphasized at the beginning of the section, the previous definitions of suspensive and resolute conditions are relative to the ‘next’ state. A generalization of

these notions consists in defining suspensive and regulative conditions relative to a deadline  $n$ , that is to say, a fact  $\varphi$  that has to occur before  $n$  steps from now to make  $\psi$  true (for suspensive condition) and to make  $\psi$  false (for regulative condition).

Let us introduce the following inductive definition to represent the occurrence of an event in  $n$  steps from now:

$$X^0\varphi \stackrel{\text{def}}{=} \varphi$$

and for all  $n > 0$ :

$$X^n\varphi \stackrel{\text{def}}{=} XX^{n-1}\varphi$$

where  $X^n\varphi$  has to be read “ $\varphi$  will be true in  $n$  steps from now along the current history”. Furthermore, let us define:

$$X^{\exists n}\varphi \stackrel{\text{def}}{=} \bigvee_{0 \leq m \leq n} X^m\varphi$$

$$X^{\forall n}\varphi \stackrel{\text{def}}{=} \bigwedge_{0 \leq m \leq n} X^m\varphi$$

where  $X^{\exists n}\varphi$  and  $X^{\forall n}\varphi$  have to be read, respectively, “ $\varphi$  will eventually be true before the deadline  $n$ ” and “ $\varphi$  will always be true before the deadline  $n$ ”.

We can now define suspensive condition and regulative condition with deadlines:

$$\text{Susp}(\varphi, \psi, n) \stackrel{\text{def}}{=} (\langle \text{Agt} \rangle X^{\exists n}\varphi \wedge \langle \text{Agt} \rangle \neg X^{\exists n}\varphi) \wedge$$

$$([\text{Agt}] X^{\forall n}(\varphi \rightarrow \text{P}\varphi)) \wedge$$

$$([\text{Agt}] X^{\forall n}(\varphi \leftrightarrow \psi))$$

$$\text{Resol}(\varphi, \psi, n) \stackrel{\text{def}}{=} \text{Susp}(\varphi, \neg\psi, n)$$

where  $\text{Susp}(\varphi, \psi, n)$  means that  $\varphi$  is a suspensive condition for  $\psi$  with deadline  $n$  and  $\text{Resol}(\varphi, \psi, n)$  means that  $\varphi$  is a regulative condition for  $\psi$  with deadline  $n$ . According to the new formulation of the uncertainty requirement, uncertainty is about the occurrence of the condition  $\varphi$  before the deadline. Moreover, according to the new formulation of the conditional requirement, the occurrence of  $\varphi$  at every point before the deadline  $n$  is a necessary and sufficient condition to make  $\psi$  true there.

We have the following generalization of the detachment principle given above:

$$\models (\text{Susp}(\varphi, \psi, n) \wedge X^m\varphi) \rightarrow X^m(\psi \wedge \text{P}\psi) \text{ if } 0 \leq m \leq n \quad (4)$$

According to the previous validity, if  $\varphi$  is a suspensive condition for  $\psi$  with deadline  $n$  and  $\varphi$  will occur at time  $m$  before the deadline then, at time  $m$  it has to be the case that  $\psi$  and it has to be presumed that  $\psi$ .

## 5 Conclusion

We believe that our approach to conditions can be useful not only for computational models of contracts but also for legal doctrine. Our model of retroactivity—as resulting from the conditional possibility of alternative outcomes, coupled with a presumption—provides a clear understanding of the working of retroactive conditions, without the need to postulate a fiction, or to assume backward causality. It provides a better understanding of retroactivity, which may support easier legal solutions of the resulting issues.

As a follow-up of this work, we plan to provide a sound and complete axiomatization for the logic STIT-P presented in Section 3. We also plan to study complexity of model checking for this logic and to come up with a model checking algorithm for it. Indeed, we believe that our formalization of conditions can be exploited in practice via model checking to automatically verify whether, in a certain situation, a certain condition will obtain and/or a certain legal effect will be produced.

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