



**HAL**  
open science

## Improving traffic network performance with road banning strategy: A simulation approach comparing user equilibrium and system optimum

Mostafa Ameli, Jean-Patrick Lebacque, Ludovic Leclercq

### ► To cite this version:

Mostafa Ameli, Jean-Patrick Lebacque, Ludovic Leclercq. Improving traffic network performance with road banning strategy: A simulation approach comparing user equilibrium and system optimum. *Simulation Modelling Practice and Theory*, 2020, 99, 22p. hal-02381648

**HAL Id: hal-02381648**

<https://hal.science/hal-02381648v1>

Submitted on 31 May 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Improving traffic network performance with road banning strategy: a simulation approach comparing user equilibrium and system optimum

Mostafa Ameli<sup>a,b</sup>, Jean-Patrick Lebacque<sup>a</sup>, Ludovic Leclercq<sup>b,\*</sup>

<sup>a</sup>Univ. Gustave Eiffel, IFSTTAR, GRETTIA, Paris, France

<sup>b</sup>Univ. Gustave Eiffel, Univ. Lyon, ENTPE, IFSTTAR, LICIT, F-69518, Lyon, France

---

## Abstract

The purpose of this study is to compare network loadings related to different network equilibria by a simulation-based framework. The direct comparison of path flows or trajectory patterns is hard to achieve so here we propose a more aggregate approach based on the comparison of demand level breakpoints. A breakpoint is a demand threshold value that leads to significant changes in path flow loading. More specifically, we set in this paper a demand breakpoint when the list of effective route alternatives differs by at least one path. This is for example the case when one route is no longer considered for one equilibrium while being used by some vehicles in the second one. We are going to investigate both static and dynamic network loading while scanning all demand levels to identify the breakpoints. We focus on discrete demand formulation and choices and use a trip-based traffic simulator.

This study analyzes the breakpoints for the solution of three popular equilibrium conditions: User equilibrium (UE), System optimum (SO) and Boundary Rational User Equilibrium (BRUE). First, we investigate breakpoints on a well-known network (Braess) in the static case in order to better define this concept. Second, breakpoints are investigated on a real network (Lyon, France) where dynamic travel times are provided by a microscopic traffic simulator. When the breakpoints are obtained for a given scenario, we focus on identifying demand ranges where some paths are not used in SO while being travelled in UE or BRUE. Following the concept of Braess paradox, this permits to design banning strategies at some key locations in the network to prevent some alternatives from being used and thus to improve the system performance. We show by simulation that such a strategy is effective, which demonstrates the importance of breakpoint identification.

*Keywords:* traffic, simulation, trip-based assignment, user equilibrium, system optimum, boundary rational user equilibrium, variational demand, switch equilibrium, breakpoint

---

## 1. Introduction

The traffic assignment problem has been studied for more than five decades and a large variety of analytical and simulation-based models and/or algorithms have been developed to find network equilibria. Network equilibria are defined based on travel costs. Cost can be expressed as a function of time or money or a combination of both. When all users experience the minimum possible path cost given the network constraints and other users' path choice, the solution is called User Equilibrium (UE) (Wardrop, 1952). Network operators aim to minimize the sum of all user costs. The solution, in this case, is called System Optimum (SO) (Patriksson, 2015). The users are looking for UE while the system wants to achieve the SO. The difference of the total travel cost between UE and SO is called Price of Anarchy (PoA) (Roughgarden, 2005). There are several researches about how to reduce PoA in the literature (Youn et al. (2008), Colini-Baldeschi et al. (2017)). The goal of this study is to investigate the evolution of PoA depending on the level of demand by introducing a new concept named demand level breakpoints. This concept permits to identify demand level ranges where PoA is high because of some paths which are used in UE whereas they are not used in SO.

---

\*Corresponding author. Tel. : +33 (0)4 72 04 77 16

Email address: ludovic.leclercq@ifsttar.fr (Ludovic Leclercq)

Such situations are known in the literature as the Braess paradox (Frank, 1981). While being highly documented this paradox remains very hard to detect in the real field as the level of demand plays a crucial role (Askoura et al., 2011). This paper aims to go one step further in a better identification of these situations by introducing systematic methods to determine the demand level breakpoints. At the end, we will show how the identification of breakpoints enables to design efficient control strategies at the network level that consist in banning some routes when critical demand level values are experienced.

Path costs can be estimated based on models or simulators. Traffic assignment can be classified into three main groups: Static Traffic Assignment (STA), semi-dynamic and Dynamic Traffic Assignment (DTA) (Bliemer et al., 2017). Since the 1950s there is much research about finding the assignment solutions for UE and SO. In this paper, based on reviews of Szeto and Wong (2012), Wang et al. (2018) and following Mahmassani (2001), a simulation based approach is preferred because the results are easy to interpret and relatively close to reality (Sundaram et al., 2011).

Identifying situations in which the Braess paradox holds and induces the breakpoints means in practice investigating the qualitative differences between UE and SO equilibrium. The UE acts as a proxy for the current network situation whereas SO reproduces the optimal situations with smart traffic control (Ehrgott et al., 2015). Moreover, it is well-known that users are not always taking the shortest paths (Szeto and Lo, 2006) because they lack a perfect knowledge of the traffic conditions (Mahmassani and Chang, 1987; Delle Site, 2018) or because they also favor other criteria when choosing their travel path (Abdel-Aty et al., 1997; Zhou et al., 2017). To investigate the robustness of breakpoint definition while relaxing perfect UE assumptions, we are going to calculate also Boundedly Rational User Equilibrium (BRUE). BRUE can be considered as a relaxation of UE where users try to optimize their own benefit up to a point but stop the process where they are satisfied with the current solution (indifference bound). Equilibrium is achieved when all user costs are within the boundary around the UE solution (Di and Liu, 2016; Han et al., 2015b).

In this paper, we consider UE, SO and BRUE independently and try to investigate the relation between these three equilibria for different demand levels. To the best of our knowledge the studies in the literature focus mainly on finding the path flow distributions over the global network related to each equilibrium. However, analyzing the output of traffic assignment models in order to be used for improving the transportation system is also important. Here, we would like to go further and cross-compare trip patterns. The questions are what is the next step after finding the equilibrium? How can we analyze the network equilibrium? Can we design a strategy to move the system from user equilibrium toward the system optimum? We are going to directly answer these questions by studying demand level breakpoints and apply some banning and rerouting strategy. We attempt to design new traffic management methods based on the idea of incenting users to change paths, so that the network moves closer to SO.

Generally, the travel demand is not fixed even in the short term. There are few studies that focus on the impact of different levels of demand on the UE (see e.g. Wie et al. (2002); Szeto and Lo (2004); Han et al. (2011); D'Ambrogio et al. (2009); Han et al. (2015a)). First we define breakpoints and analyze them. Then, for each level of demand, and given a fixed active paths set, we try to analyze how we can determine the critical users for rerouting. Critical users are the users who, when they change their path, have more impact than others on moving the equilibrium towards SO conditions. There are four main points that we aim at investigating:

- The impact of one origin-destination demand level on the network equilibria.
- Critical users who can have maximum impact on moving UE and BRUE toward SO by change their path.
- Demand level ranges that we can apply control strategies to follow the SO assignment by respecting the network UE or BRUE.
- Does the banning strategy work in order to shift the network UE/BRUE towards an equilibrium with lower total travel costs?

The next section presents the definition of the breakpoint and solves a simple static traffic model in order to define the breakpoints in the Braess network. In section 3, the trip-based simulator is presented first. Second, we explain the dynamic equilibrium model and how we calculate the breakpoints for DTA. The network for numerical experiments and the process of breakpoint detection are presented in the section 4. We consider two test cases on the dynamic network. The control strategy and the obtained results for the first test case are discussed in section 5. The second test case and the application of breakpoint detection and ban strategy to the second test case are presented in section 6. Finally, we state concluding remarks and introduce future directions of work in the "Conclusion" section.

## 2. Breakpoint definition

Traffic assignment provides the path flow distribution for all Origin-Destination (OD) pairs, i.e. how many users take each possible path from each origin to each destination. There is a path set which contains all the active paths from each origin to each destination. A path chosen by at least one user is an active path. The active path set is a component of the optimal path flow distribution defined for each equilibrium: UE, SO and BRUE. For a given origin and destination pair  $w$ , the active path set for a given equilibrium can be the same at different levels of demand. We define the breakpoints as demand levels where we observe a change in the active path set (e.g. one new path in or/and one current path out). We will first study the breakpoints in the static case on the classical Braess network (Braess et al., 2005). As explained in the introduction, the Braess paradox arises when UE and SO have difference path set. Determining the breakpoint for both equilibria will permit to easily identify such situations.

The classic Braess network with linear cost functions in the static case is shown in figure 1. There are five links with cost functions  $t_{ij}$  for the link  $ij$  connecting node  $i$  to node  $j$ . The flow of the link  $ij$  is  $f_{ij}$  and there are 3 alternative paths from origin node 1 to destination node 4: path 1 (1-3-4), path 2 (1-2-4) and path 3 (1-3-2-4). Therefore the cost functions ( $C_k$ ) of paths  $k$  are as follow:

$$C_1(\pi) = t_{13} + t_{34} \quad (1)$$

$$C_2(\pi) = t_{12} + t_{24} \quad (2)$$

$$C_3(\pi) = t_{13} + t_{32} + t_{24} \quad (3)$$

Where  $\pi_k$  denotes the flow of path  $k$  and  $\pi = (\pi_k)_{k=1,2,3}$ .

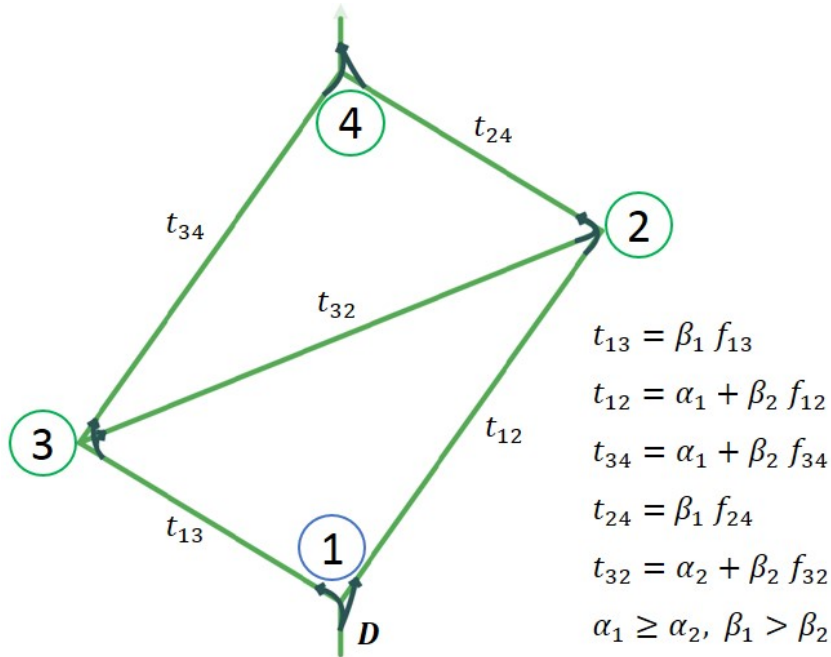


Figure 1: Classic Braess network with the link cost functions and assumptions

The two assumptions in figure 1 ( $\alpha_1 \geq \alpha_2, \beta_1 > \beta_2$ ) guarantee that by increasing the demand, there exist three cases of UE (see e.g. Pas and Principio (1997)). They also ensure that the path 3 is the cheapest free flow path, due to

lower coefficients  $(\alpha_2, \beta_2)$  in the path cost function. This is a critical condition for the Braess paradox to appear. The different demand levels will be defined in relation with the structure of the equilibrium (active paths). Consequently, for UE we will have three scenarios. According to the definition of the UE, the conditions of static UE can be stated mathematically as follows:

$$C_p - C_w^* \geq 0 \quad ; \forall p \in P_w \quad (4)$$

$$\pi_p(C_p - C_w^*) = 0 \quad ; \forall p \in P_w \quad (5)$$

$$\pi_p \geq 0 \quad ; \forall p \in P_w \quad (6)$$

Where  $C_w^*$  is the minimum travel cost for origin-destination pair  $w$ .  $P_w$  is the set of possible paths for  $w$  and  $\pi_p$  denotes the flow on path  $p$ .

The path flow distribution is a  $[1 \times 3]$  vector  $\pi$  that contains the flow value for three paths. Note that the feasible path flow vector is defined as:  $\Pi \triangleq \{\pi : \pi \geq 0, \sum_{p \in P_w} \pi_p = D_w\}$ . So the active path set is the set of path  $p \in P_w$  such that  $\pi > 0$ . The path flow distribution for UE is a function  $UE(D)$  of demand level  $D$  and can be shown to be as follows:

$$UE(D) = \begin{cases} [0, 0, D] & 0 \leq D < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}, \\ \left[ \frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}, \frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}, \frac{2(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)D}{\beta_1 + 3\beta_2} \right] & \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} \leq D < \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}, \\ \left[ \frac{D}{2}, \frac{D}{2}, 0 \right] & D \geq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} \end{cases} \quad (7)$$

Consequently, there are two breakpoints for UE:  $D = \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$  and  $D = \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$ .

Let us now investigate the network under BRUE. The boundary rational model is one behavioral model used to relax the perfect rational hypothesis in the definition of UE by considering an indifference band ( $\epsilon$ ). There are two main differences in the definition of breakpoints between the BRUE and UE:

- The path flow distribution for BRUE not only depends on the demand level, it also depends on the  $\epsilon$  value for the indifference band for this origin and destination pair.
- The path flow distribution is not unique in some scenarios, so for the Braess network there are two convex sets (simplices) of flow vectors  $\phi_1$  and  $\phi_2$  which can satisfy the conditions of BRUE:

$$\phi_1 = \{\pi \geq 0 : C_i(\pi) - C_j(\pi) \leq \epsilon; i, j \in \{1, 2\}, i \neq j\} \quad (8)$$

$$\phi_2 = \{\pi \geq 0 : C_i(\pi) - C_j(\pi) \leq \epsilon; i, j \in \{1, 2, 3\}, i \neq j\} \quad (9)$$

Note that [equation 8](#) is obtained by assuming that both paths are used. In [equation 9](#),  $\phi_2$  is a set in which all three paths can be active. If the indifference band is equal to zero ( $\epsilon = 0$ ) the BRUE and UE assignment are the same. Due to the non-uniqueness of the BRUE, for a given  $\epsilon$ , an active path has a cost lower or equal to the minimum travel cost path plus  $\epsilon$ . The  $\epsilon$ -BRUE path flow pattern can be shown to be given by:

$$\pi_p > 0 \rightarrow C_p - C_w^* \leq \epsilon \quad ; \forall p \in P_w \quad (10)$$

The path flow distribution for BRUE is a function of the demand level and the band value  $BRUE(D, \epsilon)$  is given by:

$$BRUE(D, \epsilon) = \begin{cases} [0, 0, D] & 0 \leq D < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} - \frac{\epsilon}{\beta_1 + \beta_2}, \\ \phi_1 \cup \phi_2 & D > \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}, \\ \phi_2 & \text{o.w.} \end{cases} \quad (11)$$

For a detailed calculation of [equation 11](#) readers can refer to [Di et al. \(2014\)](#). Therefore, there are two breakpoints but they depend on the  $\epsilon$  value:  $D = \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} - \frac{\epsilon}{\beta_1 + \beta_2}$  and  $D = \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}$ .

The third considered equilibrium is SO. The SO path flow distribution is based on the second principle of [Wardrop \(1952\)](#), i.e. minimizing the total travel cost. Therefore, mathematically the goal is to minimize the total travel cost as a function of demand and flow vector:

$$\min T(D, \pi) = \sum_{p \in P_w} \pi_p C_p \quad (12)$$

Given our assumptions on coefficient values for the Braess network ([figure 1](#)), three scenarios that can occur as in the UE case. The objective function in each scenario is given by:

$$T(D) = \begin{cases} (2\beta_1 + \beta_2)D^2 + \alpha_2 D & \text{if only path 3 used,} \\ (\beta_1 + \beta_2)(\pi_1^2 + \pi_2^2) + \alpha_1 D & \text{if path 1 and 2 used,} \\ (\beta_1 + 2\beta_2)(\pi_1^2 + \pi_2^2) + [\alpha_1 - \alpha_2 - 2(\beta_1 + \beta_2)D](\pi_1 + \pi_2) & \text{if all paths used and } \pi_3 = D - (\pi_1 + \pi_2) \\ + 2\beta_2 \pi_1 \pi_2 + (2\beta_1 + \beta_2)D^2 + \alpha_2 D & \end{cases} \quad (13)$$

Therefore and mathematically, the path flow distribution for SO is defined as follow:

$$SO(D) = \begin{cases} [0, 0, D] & 0 \leq D < \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}, \\ \left[ \frac{-\frac{\alpha_1 + \alpha_2}{2} + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}, \frac{-\frac{\alpha_1 + \alpha_2}{2} + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}, \frac{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)D}{\beta_1 + 3\beta_2} \right] & \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)} \leq D < \frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2}, \\ \left[ \frac{D}{2}, \frac{D}{2}, 0 \right] & D \geq \frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2} \end{cases} \quad (14)$$

There are two breakpoints for SO:  $D = \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}$  and  $D = \frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2}$ . There is a fixed relationship between breakpoints of SO and UE which is shown in [equation 15](#). The relationship between BRUE's breakpoints and the breakpoints of SO is depending on the value of  $\epsilon$ .

$$BP_{i,SO} = \frac{1}{2} BP_{i,UE} ; \forall i \in \{1, 2\} \quad (15)$$

$$BP_{1,SO} = \frac{1}{2} \left[ BP_{1,BRUE} + \frac{\epsilon}{(\beta_1 + \beta_2)} \right] ; BP_{2,SO} = \frac{1}{2} \left[ BP_{2,BRUE} - \frac{\epsilon}{(\beta_1 - \beta_2)} \right] \quad (16)$$

Where  $BP_{i,j}$  is the  $i$ th breakpoint of the equilibrium  $j \in \{UE, SO, BRUE\}$ .

As aforementioned, existing studies usually finish the breakpoints analysis here (finding the path flow distribution for equilibria). We are now going to analyze the equilibria based on breakpoints.

### 2.1. Analysis based on breakpoints

The path flow distribution for UE, SO and BRUE is shown in [figure 2](#) to present the optimal path set of two different types of paths in the Braess network. It is not easy to present the BRUE in the path flow distribution diagram because the  $\epsilon$  allowance for cost implies that for each path there is a set of possible flow values in the BRUE path flow distribution around the UE flow value. Thus in [figures 2\(a\)](#) and [2\(b\)](#), we represent the BRUE solution in terms of the maximum range of flows at equilibrium. [Figure 2\(a\)](#) presents the demand-flow diagram for path 1 and 2 in UE, SO and BRUE situations. Breakpoints are shown on the demand axis. (a $\leftrightarrow$ b) presents the active path set for UE and SO and also possible active path set for BRUE depending on the value of  $\epsilon$ . The same demand-flow diagram for path 3 is shown in [figure 2\(b\)](#). Note that the breakpoints of BRUE,  $\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} - \frac{\epsilon}{\beta_1 + \beta_2}$  and  $\frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}$ , are equal to UE breakpoints when  $\epsilon = 0$ .

The first breakpoint occurs when  $D = \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}$  for analyzing the UE solution and SO. Determining this level of demand as a breakpoint means that the active path set of one of the equilibria is changed. Before this breakpoint, the active path set of both equilibria has the same path:  $P_{UE} = P_{SO} = \{3\}$ . In other words, from  $D = 0$  to  $D = \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}$ ,

the equilibrium under both disciplines refers to the same active path set. Then after this breakpoint, only paths 1 and 2 should be used under SO discipline while path 3 is the only active path for UE. From the first breakpoint of the SO ( $BP_{1,SO}$ ) to the first breakpoint of the UE ( $BP_{1,UE}$ ), the flow is distributed differently for UE and SO. For UE all users are still taking the third path but for SO some of them start to take the paths 1 and 2 to minimize the total travel time. It means that for demand level higher than the  $BP_{1,SO}$ , paths 1 and 2 will be added to the active path set in SO solution. The active path sets for UE and SO are fixed after  $BP_{1,SO}$  until  $BP_{1,UE}$  is detected at  $D = \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$ . Where paths 1 and 2 are used under UE discipline. The active path set is the same for UE and SO when the demand level is  $\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} \leq D < \frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2}$ . Path 3 is not in the active path set for SO but used in UE solution when we pass the  $BP_{2,SO}$  and the demand is in range  $\frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2} \leq D < \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$ . Therefore the  $BP_{2,UE}$  is  $D = \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$  where the active path set of UE is changed. In Braess network, after the last breakpoint,  $D \geq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$ , the UE and SO path flow distribution will be the same.

In general, the BRUE path flow distribution is close to UE but the solution is not symmetric with respect to paths 1 and 2 as in SO and UE. In the BRUE-SO analysis,  $D = \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}$  is the first breakpoint as in the UE-SO only if  $\epsilon \leq \frac{\alpha_1 - \alpha_2}{2}$ . If  $\epsilon > \frac{\alpha_1 - \alpha_2}{2}$ , the first breakpoint is located in the lower demand level with respect to the first SO breakpoint. Path 1 and/or path 2 are added to the path set after the first breakpoint. For higher demand levels, by taking into account  $\epsilon$ , the BRUE-SO breakpoints are same as the UE-SO breakpoints until the last breakpoint, when path 3 exits the active path set. If  $D \geq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$ , the BRUE and SO path flow distribution will be the same only if  $\epsilon = 0$ . It means, for  $\epsilon > 0$ , the last breakpoint will be  $\frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}$ . All paths are active for BRUE and path 3 is not active for SO while  $\frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} \leq D < \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}$ . Consequently, the path flow distribution is the same for BRUE and SO when  $D \geq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}$ . Note that BRUE accepts 3 non zero path-flows for values of  $D < \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} - \frac{\epsilon}{\beta_1 + \beta_2}$  when  $\epsilon > \alpha_1 - \alpha_2$ . Moreover, if  $\epsilon \leq \frac{\alpha_1 - \alpha_2}{2}$  and  $D < \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}$ , the solution is the same for all 3 equilibria. Finally, the mathematical formulas for path flow distributions for the 3 equilibria (UE, SO and BRUE) are shown in table 1. This table is obtained by merging the equations 7, 11 and 14 in order to jointly investigate the breakpoints when demand is increasing.

This study by breakpoint analysis attempts to identify ranges of demand where we have a qualitative difference between UE/BRUE and SO. We are looking in particular for situations where some paths are used in UE/BRUE while not in SO. This corresponds to demand ranges with breakpoints as boundaries, because breakpoints are identifying changes in the active paths for each equilibrium. Then, we can apply a control strategy in order to improve the performance of the network. At each range of demand between two breakpoints, we know the active path set of SO and the network state (UE or BRUE). So at the range of demand where  $P_{UE/BRUE} \neq P_{SO}$ , changing the path set may improve the system. For instance, when  $\frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} \leq D < \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2} + \frac{\epsilon}{\beta_1 - \beta_2}$  path 3 is not used under SO discipline. So in this demand interval the system controller can ban path 3 or design other control strategies in order to prevent users from travelling on path 3, and the system will move towards the SO state. In other ranges of demand the network controller can induce a certain number of users to travel by specific paths, in order to get closer to SO. Such users are critical for an efficient control strategy.

The breakpoint analysis helps the system controller to evaluate the possibility of applying control strategies in order to improve the system performance. In the sequel, we focus on the banning strategy in which, by banning one link or by preventing users from turning at one intersection, the controller can ban one path which is in the active path set of the UE solution but does not belong to the SO active path set. An alternate possibility, which will be explored in future work, consists in using guidance. In urban networks, transportation system controllers can guide (e.g. by Advanced Traveler Information System (ATIS)) the limited number of users that are equipped (Klein et al., 2018). Generally, it will be useful to know how many users need to change their path in order to move the network from one equilibrium on the user side (UE or BRUE) to one equilibrium on the system side (SO).

This part of the study has attempted to explain the path flow distribution breakpoints concept and to define them in a simple manner using the static assignment on the Braess network. In the following parts of the paper, we present the dynamic trip-based framework to find the breakpoints and identify the critical user(s) for rerouting and critical path(s) for banning.

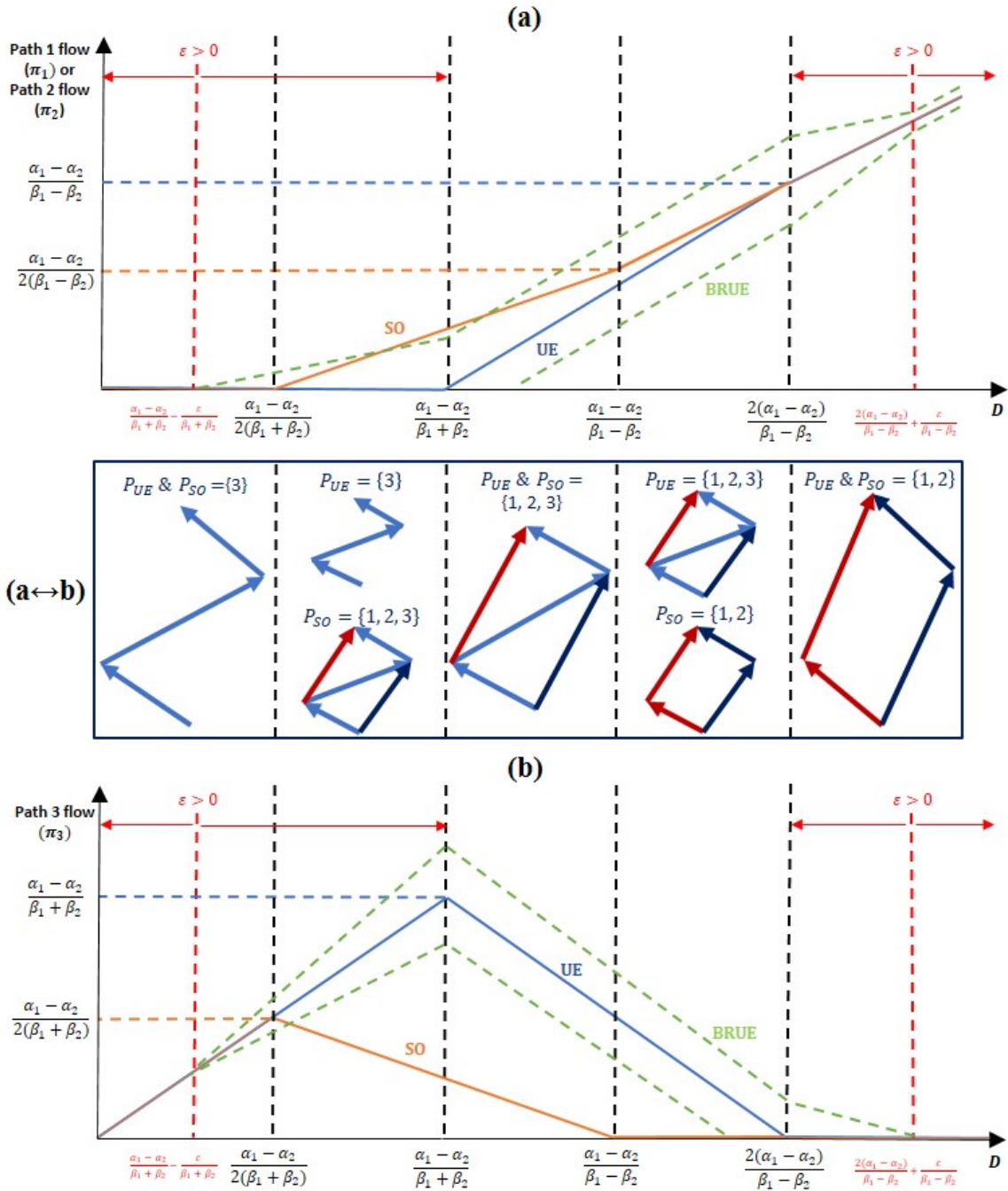


Figure 2: (a): Path 1 or Path 2 flow-demand diagram for UE, SO and BRUE. (a↔b): Possible path set of optimal solution [ $P_x$  = Possible path set of optimal solution for equilibrium  $x$ , where  $x$  stands for UE or SO]. Note that for BRUE, it depends on the  $\epsilon$ . (b): Path 3 flow-demand diagram for UE, SO and BRUE.

Red dash lines in figures (a) and (b) presents the value of breakpoints in BRUE path flow distribution based on the given  $\epsilon$ . It can change in ranges that are specified by the red arrows.



Table 1: Path flow distribution in Braess network for UE, SO and BRUE

Demand level (D)	User Equilibrium (UE)	System optimum (SO)	Band Value ( $\epsilon$ )	Boundary rational UE
$0 \leq D \leq \frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)}$	[0, 0, D]	[0, 0, D]	$\epsilon < \alpha_1 - \alpha_2 - (\beta_1 + \beta_2)D$	[0, 0, D]
			$\epsilon > \alpha_1 - \alpha_2 - (\beta_1 + \beta_2)D$	$\varphi_2$
$\frac{\alpha_1 - \alpha_2}{2(\beta_1 + \beta_2)} \leq D \leq \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$	[0, 0, D]	[0, 0, D]	$\epsilon < \alpha_1 - \alpha_2 - (\beta_1 + \beta_2)D$	[0, 0, D]
			$\epsilon > \alpha_1 - \alpha_2 - (\beta_1 + \beta_2)D$	$\varphi_2$
$\frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} \leq D \leq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$	[ $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{2(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)D}{\beta_1 + 3\beta_2}$ ]	[ $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)D}{\beta_1 + 3\beta_2}$ ]	All values	$\varphi_2$
			All values	$\varphi_2$
$\frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2} \leq D \leq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$	[ $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{2(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)D}{\beta_1 + 3\beta_2}$ ]	[ $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{-\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)D}{\beta_1 + 3\beta_2}$ , $\frac{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)D}{\beta_1 + 3\beta_2}$ ]	All values	$\varphi_2$
			All values	$\varphi_2$
$D \geq \frac{2(\alpha_1 - \alpha_2)}{\beta_1 - \beta_2}$	[ $\frac{D}{2}$ , $\frac{D}{2}$ , 0]	[ $\frac{D}{2}$ , $\frac{D}{2}$ , 0]	$\epsilon < -2(\alpha_1 - \alpha_2) + (\beta_1 - \beta_2)D$	$\varphi_1 \cup \varphi_2$
			$\epsilon > -2(\alpha_1 - \alpha_2) + (\beta_1 - \beta_2)D$	$\varphi_2$

### 3. Simulation-based dynamic network equilibrium

The concept and the application of breakpoint detection are presented in the static case. This section presents the dynamic simulator, network equilibrium model and tools to calculate the equilibria in the dynamic case.

#### 3.1. Dynamic traffic simulator

In this work, we use the Symuvia<sup>1</sup> platform for trip-based dynamic simulation in order to calculate travel times in the network for any given path flow distributions for all OD pairs. Symuvia gives access to the position, speed and acceleration of each vehicle (user) on the network. It is a microscopic simulator based on the Lagrangian resolution of the LWR (Lighthill Whitham Richards) model (Leclercq et al., 2007) which is the conservation law with respect to traffic density. Vehicle movements at the microscopic scale are governed by a set of rules, including car-following modeling (Leclercq, 2007a,b), lane-changes by using a macroscopic theory of vehicle lane-changing inside microscopic models (Laval and Leclercq, 2008) and specific movements based on noise modeling at intersections (Chevallier and Leclercq, 2007). It has a simulation time-step equal to 1 second, travel time information is aggregated at the link level every 1 minute. The travel demand is given (dynamic OD pair demand) and users' routes are determined by a dynamic traffic assignment model, which guides each vehicle in the network on the route that optimizes its travel time to its initially assigned destination based on some specific equilibria discipline (UE, SO and BRUE are considered here).

#### 3.2. Network equilibrium model

Consider a network  $G(N, A)$  with a finite set of nodes  $N$  and a finite set of directed links  $A$ . The demand is time-dependent. The period of interest (planning horizon) of duration  $H$  is discretized into a set of small time intervals indexed by  $\tau$  ( $\tau \in T = \{\tau_0, \tau_0 + \sigma, \tau_0 + 2\sigma, \dots, \tau_0 + M\sigma\}$  and  $\tau_0 + M\sigma = H$ ).  $\sigma$  is the duration of the time intervals. In an interval  $\tau$ , Travel Time (TT) and traffic conditions do not change. The main notations to introduce in the dynamic equilibrium model are as follows:

$W$ : OD pairs, subset of origin  $\times$  destination nodes,  $W \subset N \times N$ .

$a$ : index of link,  $a \in A$ .

$w$ : index of origin and destination pair,  $w \in W$ .

$P_{w,\tau}$ : set of paths for  $w$  in departure time  $\tau$ .

$P_{w,\tau}^*$ : set of shortest paths for  $w$  in departure time interval  $\tau$ .

$p$ : index of path,  $p \in P_{w,\tau}$ .

$p^*$ : index of shortest path,  $p^* \in P_{w,\tau}^*$ .

$D_w$ : total demand for  $w$  pair.

$C_{p,\tau}$ : travel cost of path  $p$  in departure time  $\tau$ .

$C_{w,\tau}^*$ : minimum travel cost of OD pair  $w$  in departure time  $\tau$ .

$\hat{C}_{p,\tau}$ : marginal travel cost of path  $p$  in departure time  $\tau$ .

$\hat{C}_{w,\tau}^*$ : minimum marginal travel cost of OD pair  $w$  in departure time  $\tau$ .

According to the definition of the time interval in DTA and based on the study of (Sbayti et al., 2007), the conditions of dynamic UE can be mathematically restated from equations 4, 5 and 6:

$$C_{p,\tau} - C_{w,\tau}^* \geq 0 \quad ; \forall w \in W, p \in P_{w,\tau}, \tau \in T \quad (17)$$

$$\pi_{p,\tau}(C_{p,\tau} - C_{w,\tau}^*) = 0 \quad ; \forall w \in W, p \in P_{w,\tau}, \tau \in T \quad (18)$$

$$\pi_{p,\tau} \geq 0 \quad ; \forall p \in P_{w,\tau}, \tau \in T \quad (19)$$

<sup>1</sup>Note that Symuvia is an open source simulator that will be made available starting winter 2020.  
<http://www.licit-lyon.eu/themes/realisations/plateformes/symuvia/>

The equilibrium condition for BRUE is:

$$\pi_{p,\tau} > 0 \rightarrow C_{p,\tau} - C_{w,\tau}^* \leq \epsilon \quad ; \forall p \in P_{w,\tau} \quad (20)$$

In order to find the equilibrium we are using an iterative algorithm. The Method of Successive Average (MSA) has been presented for the first time by [Robbins and Monro \(1951\)](#). In this study, we use the modified MSA in simulation-based DTA. Consider  $i$  as the index of iteration in the optimization process and  $\pi_{p,\tau}^i$  as the number of users on path  $p$  in departure interval  $\tau$  and iteration  $i$  of the optimization. In iteration  $i$ , MSA swaps a fraction  $\frac{1}{i+1}$  of users on each path with higher cost value than the lowest cost path, to the lowest cost path(s). The direction of the optimization process is defined by an all-or-nothing assignment vector ( $y_{p,\tau}^i$ ) for each OD pair at departure interval  $\tau$ . With this step size, path assignments  $\pi_{p,\tau}^i$  at iteration  $i$  are updated in order to obtain the path assignments  $\pi_{p,\tau}^{i+1}$  for iteration  $(i+1)$  as follows:

$$\pi_{p,\tau}^{i+1} = \frac{i}{i+1} \pi_{p,\tau}^i + \frac{1}{i+1} y_{p,\tau}^i \quad ; \forall p \in P_{w,\tau} \quad (21)$$

The main drawback of MSA is that it swaps users from all non-lowest cost paths without considering the gap between path costs. In fact, there is no priority for expensive paths to swap more users. In this study, we use MSA ranking by [Sbayti et al. \(2007\)](#) to remove this drawback for trip-based DTA. The idea of MSA ranking is first, rank the users by the experienced TT then swap a maximum number of users based on MSA method. Maximum of number of swaps  $NS^i$  is observed when we have an empty lowest cost path for swapping. It means:

$$NS^i = \frac{1}{i+1} D_w \quad (22)$$

This method swaps the user from the most expensive paths so it accelerates convergence and provides the best quality solution for the trip-based problem. With a large number of users traveling between many OD pairs and with many paths per OD pair, ranking the users is very costly and has no justification in a large-scale problem. But here with the sub-area network it is efficient to find the UE, SO and BRUE with this ranking method.

### 3.4. Definition of SO for dynamic case

In the static case, we focused on specific OD pair in mono-OD network but here we are going to focus on specific OD pairs in the network with multiple OD pairs. Therefore we consider two different definitions for SO: Global SO and Local SO.

#### 3.4.1. Global SO

The SO conditions in the dynamic case are mathematically stated in equations 23, 24 and 25 based on marginal TT. The path marginal TT is the extra travel time that will be added to the path TT if an extra user is assigned to the path at the current time interval. The SO conditions state that if the path flow is positive, then the experienced path marginal TT should be equal to the minimum path marginal travel times ([Sbayti et al., 2007](#)).

$$\hat{C}_{p,\tau} - \hat{C}_{w,\tau}^* \geq 0 \quad ; \forall w \in W, p \in P_{w,\tau}, \tau \in T \quad (23)$$

$$\pi_{p,\tau} (\hat{C}_{p,\tau} - \hat{C}_{w,\tau}^*) = 0 \quad ; \forall w \in W, p \in P_{w,\tau}, \tau \in T \quad (24)$$

$$\pi_{p,\tau} \geq 0 \quad ; \forall p \in P_{w,\tau}, \tau \in T \quad (25)$$

Computing the path marginal TT analytically is very costly. In [Leclercq et al. \(2016\)](#) it has been shown that using the simulation-based approach to compute the path marginals is also very costly, even in a simple grid network. Therefore, we use the simulator to compute a surrogate model for the marginal TT. We used three methods for calculating the SO and consider the minimum total TT as the SO path flow distribution. The first method updates the link marginal

every time a vehicle is exiting the link by calculating the marginal variation since the last exit the link. Finally, link marginal are averaged using the 1 minute window. Consequently, the path marginal TT can obtained by:

$$\hat{C}_{p,\tau} = \sum_{a \in A_p} \left[ \frac{\sum_{t=\tau}^{\tau+\sigma} \hat{C}_{a,t}}{\sigma} \right] \quad (26)$$

Where  $\hat{C}_{a,\tau}$  is the marginal TT of link  $a$  at second  $t$ ,  $\sigma$  is the length of each time interval and  $A_p$  is the subset of link set  $A$  which defines path  $p$ . In fact, the surrogate function by Peeta and Mahmassani (1995) considers the sum of link marginal TTs as the actual function. In the SO problem, we aim to minimize the cost in equation 26 for all users of each OD pair. We also consider two methods based on the observation of trajectories at the link level. The observation process of link marginal travel time for both methods (second and third) is the same but in the second method we use the average operator for link marginal TT and for the third one, median operator is used in order to calculate the link marginal TT. Finally, the path marginal TT will be the sum of link marginal TTs of that path like the first method. The last two methods are well defined in Yildirimoglu and Kahraman (2018). For each scenario of SO, the simulation is executed three times with these methods and the path flow distribution by minimum total TT is considered as the SO solution.

#### 3.4.2. Local SO

The idea of the local SO is to look for a SO to be achieved only by the users of specific OD pair(s), while all other users choose the path based on the UE framework. This permits to study control strategies that are focusing on a single OD pair in the network while other users for other OD pairs are not targeted. Therefore, conditions 23, 24 and 25 for SO are applied to the users of specific OD pair(s) who are aiming to achieve SO and conditions 17, 18 and 19 for UE are applied to the other users in the network (who choose their paths according to UE). Finally we will have a mixed equilibrium in the network which is called "ME" solution in this study.

## 4. Dynamic test case

As we are now using a simulator to derive dynamic TT, we can investigate a more complex network configuration than the Braess network. We are then now considering a sub-area of the Lyon full regional network. It is a network of the Lyon 6 district (figures 3(a)). It is a Braess-like network when considering a specific OD pair that corresponds to travel from the west ("Quai de Serbie") to the east of the network ("6 Avenue Verguin"). In this network we are looking for the breakpoints at various demand levels. Note that the travelers of other OD pairs load the complete network, in order to represent the peak half an hour of the network based on the study of Krug et al. (2018). The simulation and optimization are carried out for each level of demand with a 30 minutes horizon. The demand pattern has been set up to adjust the regular level observed in this area during the peak hour. We select the 3 most likely routes from each origin to destination to define the set of path candidates. They are shown in figures 3(b). The network of Lyon 6 has: 430 nodes, 786 links, 26 origins, 24 destinations and 3732 trips for OD pairs excluding the demand of "Quai de Serbie" to "6 Avenue Verguin" (for more details see Appendix A).

We search for the three equilibria in the complete network. It means that the optimization process is executed for all OD pairs and users in the network and we try to analyze the breakpoints by increasing the demand level on the specific OD pair. A simulation-based DTA is used to find the UE, SO and BRUE at every level of the demand on the three predefined paths. The departure time for the test users is spread uniformly in the 30 minutes duration of the horizon. Moreover, the users with other OD pairs travel with fixed departure time. According to the scale of the network, for calculating the BRUE assignment, we consider  $\epsilon = 0.1 C_{p,\tau}$ . It means users will be satisfied if they perceive a maximum ten percent more than the shortest path cost.

#### 4.1. Breakpoint detection in the dynamic case

As mentioned earlier, we solve the problem with different demand levels. Demand of the specific "Quai de Serbie to Verguin" OD pair is increased from one user to 1439 users (maximum demand based on the study of Krug et al. (2018)) over the simulation time horizon = 30 min. We focus on the travels from the specific "Quai de Serbie to Verguin" OD pair with 3 paths like the Braess network in order to detect the breakpoints which are expected to

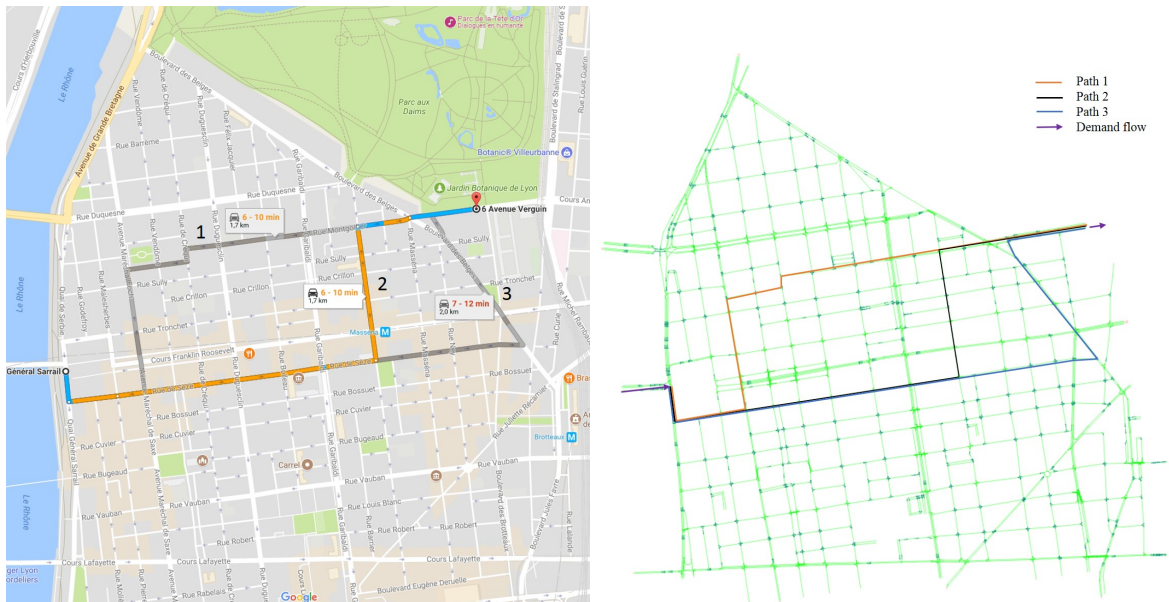
exist in the real network. The departure time is fixed for all users who do not travel between the specific OD pair. The distribution of departure time for this OD pair is uniform. The optimization process has been carried out for all equilibria and the full network. All experiments for UE and SO are reproducible and lead to a unique solution but for BRUE the solution depends on the first network initialization (starting point in optimization process) and the value of  $\epsilon$ . Note that we will present one instance for the BRUE path flow distribution for breakpoint detection with all-or-nothing initialization and  $\epsilon = 0.1C_{p,\tau}$ .

The solution space is not continuous because of the trip-based simulation (flows have integer values). Therefore, for each experiment, we have three integer numbers as the flow on each path in the vector of assignment. In order to represent and analyze the breakpoints in a continuous space, we draw the flow distribution diagram by making a (piece-wise) linear regression with  $R^2 < 0.9$  on the integer data. The demand-flow diagram for each path is presented in figure 4(a)-(c). Moreover, the results of breakpoint detection are presented in figure 4(d). In each experiment, each equilibrium is calculated and the breakpoint occurs when at least one path enters or exits the active path set of one equilibrium. The breakpoint analysis will be carried out by comparing one equilibrium from travellers' point of view (UE or BRUE) and one equilibrium from system point of view (SO or ME).

#### 4.2. Breakpoint analysis

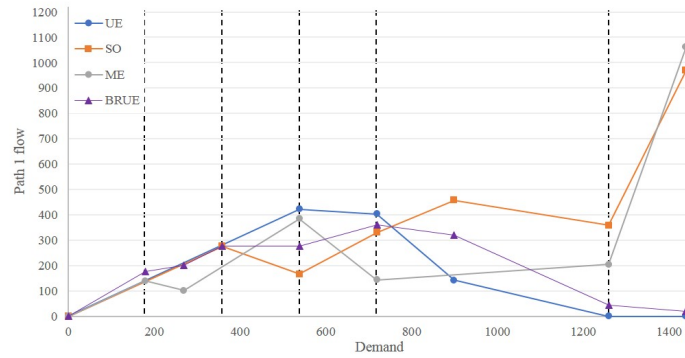
According to the figure 4, the SO active path set contains paths 1 and 2 at low demand level until the first SO breakpoint is reached at the demand level of 359 users where path 3 enters the active path set. Then, path 3 exits the active path set at a demand level of 719 users where the second breakpoint is detected. The active path set remains the same for all demand levels bigger than 719 users. Therefore, SO path flow distribution has two breakpoints. On the other hand, UE path flow distribution has two breakpoints at a demand level of 539 travellers where path 2 enters to the active path set and at a demand level of 1259 travellers where path 1 exits the active set; path 1 is not used beyond this level. Consequently, the breakpoint analysis for UE-SO contains 4 breakpoints. It is remarkable that path 3 is not being used at most demand levels in the SO path flow distribution but it always belongs the active path set of UE for all demand levels.

The ME path flow distribution is obtained by applying UE discipline to users of all OD pairs except the users of the specific OD pair (figure 3(b)) who travel under SO discipline. ME has paths 1 and 2 in the active path set until the only breakpoint of ME where path 3 enters the path set. According to the UE breakpoint detection, the analysis

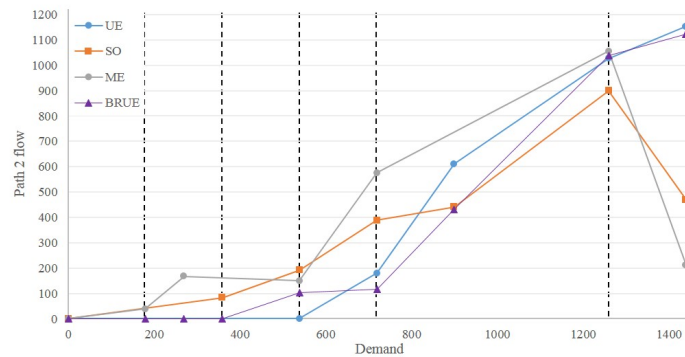


(a) Origin, destination and paths (Mapping data ©Google 2018). (b) Lyon 6 network consider by this study for one OD pair test case

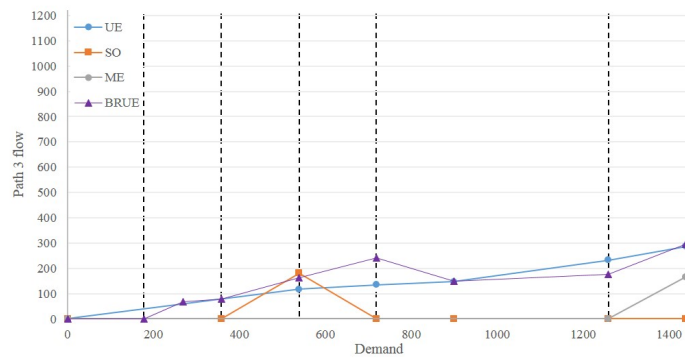
Figure 3: Network of Lyon 6



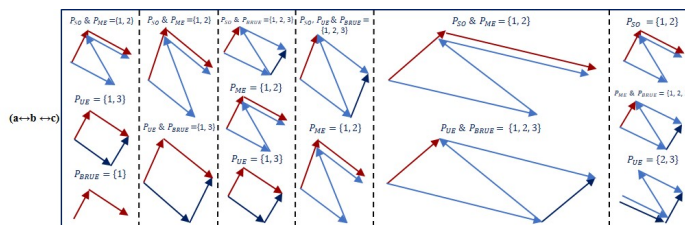
(a) Path 1 flow-demand diagram.



(b) Path 2 flow-demand diagram.



(c) Path 3 flow-demand diagram



(d) Active path sets between breakpoints for four equilibria

Figure 4: Paths flow-demand diagram for UE, BRUE, SO and ME. Breakpoints are presented by black vertical dash lines on total demand axis.

of UE-ME breakpoint involves two breakpoints. Path 3 is not used at most demand levels as in the SO path flow distribution.

As mentioned previously the BRUE scenario is one instance from BRUE solution set. There are two breakpoints in this scenario. The first one is where path 3 starts to be used and the second one happens where path 2 enters to the active path set. All three paths are used beyond the second breakpoint. There are 3 breakpoints with respect to the SO breakpoints for BRUE-SO breakpoint analysis. Note that in BRUE, path 3 is used at all demand levels beyond the first breakpoint. In the BRUE-ME breakpoint analysis, there are three breakpoints. The active path set for both equilibria is same when demand is below the first breakpoint where path 3 enters the BRUE active path set. Path 2 enters the active path set of BRUE at the second breakpoint and the last breakpoint is the BRUE breakpoint.

Here, we focus on one specific OD pair to detect the breakpoints in the real network. We compare the breakpoints of different equilibria. We identify the range where the active path sets of two equilibria are different. This range is used for applying control strategies in order to push the system from the current state to SO.

## 5. Control strategy

The idea of shifting the network from one equilibrium to another requires applying a control strategy in the demand range between two breakpoints, where the active path set remains fixed. When both equilibria have the same path set but different flows, we need to design a strategy to reroute the critical users. The number of critical users, in this case, is the difference between the two flow values of the paths. For instance, in this network, the pattern of each equilibrium (set of active paths) is constant between two breakpoints. The traffic management system can induce a change of path for the critical users. Also, when the difference in flow is high for two equilibria, then promoting users to use one specific path may also be efficient. On the other hand, if the active path set is not the same for the two equilibria (e.g. paths used in UE which are not used in SO), the system can ban some paths (unused in SO) through routing advises. In this study, we highlight the situation when one path is not used in SO or ME while it is being used in UE or BRUE. Note that it is important to consider which discipline is used with respect to a control strategy. If we consider the ME discipline as a reference, the range of banning or the path involved in banning may change. Here, we want to investigate which discipline would be a reference for control strategy? And does banning improve the network performance? The answer to the latter question is connected to the Braess paradox.

In these experiments, according to the breakpoint detection (figure 4(c)) when the demand level is below 359 and when it exceeds 719, the SO flow for the third path is zero. For ME, when the demand is higher than 1259 the flow of path 3 is not zero. Therefore, if we ban path 3 for UE and BRUE scenarios, users will use the two other paths and the performance of the network is changed. The goal of the control methods is to improve the total travel time of the network. We will check that banning is efficient in those ranges of demand where path 3 is unused in SO or ME. Here we present the ban strategy that we apply to the test case.

### 5.1. Ban Strategy (BS)

Banning some links or some turning movements at intersections and optimizing traffic lights' settings are major tools that traffic managers use in order to improve the performance of a traffic network. Here based on the breakpoints analysis we apply the BS to prevent users of specific OD pair(s) from taking a path which has zero flow in the SO/ME framework and non-zero flow in the UE/BRUE solution. Such a strategy may also stop users from other OD pairs to use the banned link or turning at the specific intersection in the network. Therefore, we can have side effects because the route choice of other users is also affected.

In order to choose the best location banning point, we first list all possibilities and then count how many users from other OD pair use that intersection or link. Then we choose the banning point that affects fewer users from other OD pairs. Note that all the simulations with BS are executed in the UE/BRUE framework for all users in the network. We consider one banning point which is presented in figure 5 which prevents the users of the "Quai de Serbie to Verguin" OD pair from using path 3. The banning point prevent all users from going straight forward at the intersection.

### 5.2. Applying BS to one OD pair test case

For the "Quai de Serbie to Verguin" OD pair, we consider one BS based on SO and ME because with both disciplines there are approximately same zone between breakpoints that the level of demand in ME and SO is zero

for path 3 and not zero in UE/BRUE framework. In other words, the BS optimal ranges of demand (identifying by breakpoints) for both references (SO and ME) are similar. We apply the banning strategy for all demand levels to see the impact of the strategy on the different demand levels. Therefore, path 3 is banned for all levels of demand. We remark that the BS scenarios are run in UE framework. The BRUE is also considered for all ban scenarios (BRUE+BS) in order to compare with the BRUE scenario without ban strategy.

The total Travel Time (TT) of the breakpoints for all equilibria are presented in Figure 6. We present it here in order to first show that optimizing the surrogate model instead of the actual value of the path marginal still yields a better total TT than UE and BRUE. Second, in the mixed equilibrium the rerouting of a specific OD pair improves the performance of the network but it also shows that when users of one OD pair switch to the SO solution, other users take advantage and use the capacity of those links which are less used. Third, the total TT of BS is below the total TT of UE. Even the BRUE+BS obtains better performance than BRUE. For ban strategies, we also close a link of path 3 between two breakpoints where path 3 is used in SO, and the result shows that the total TT of this level is higher than UE (dash lines in figure 6). The results of all experiments for one OD pair test case are presented in figure 7. Note that the ban strategy is applied for all demand level so the path 3 flow of BS and BRUE+BS in figure 7(c) is zero for all demand levels. At the demand level at which we should not apply BS, flows and TT are presented by dash lines for BS. The dash lines in figures 7(a) and 7(b) show that between demand levels 359 and 719 the flow assigned to path one and the flow level is higher than SO solution. The BS improves the total TT also after the last breakpoint where the SO solution guides us to ban path 3 and ME contains this path in the active path set. The result for one OD pair test case shows that considering the SO solution as the reference can provide a better control strategy than ME solution in order to improve the total TT of the system. Also, because all users in BS/BRUE+BS are looking for UE/BRUE, we do not expect the total TT for the whole network to be equal to SO TT. But the result shows that the performance of the network is improved with BS in UE and BRUE frameworks. If we would be able also to apply BS to more than one OD pair flow based on the breakpoint analysis we would expect better performance in term of total TT from the traffic network.

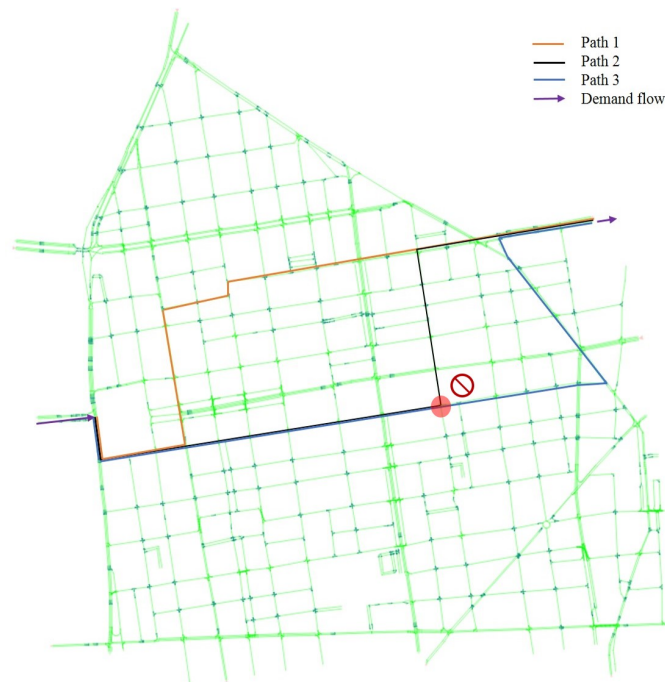


Figure 5: Network of Lyon 6 with banning point



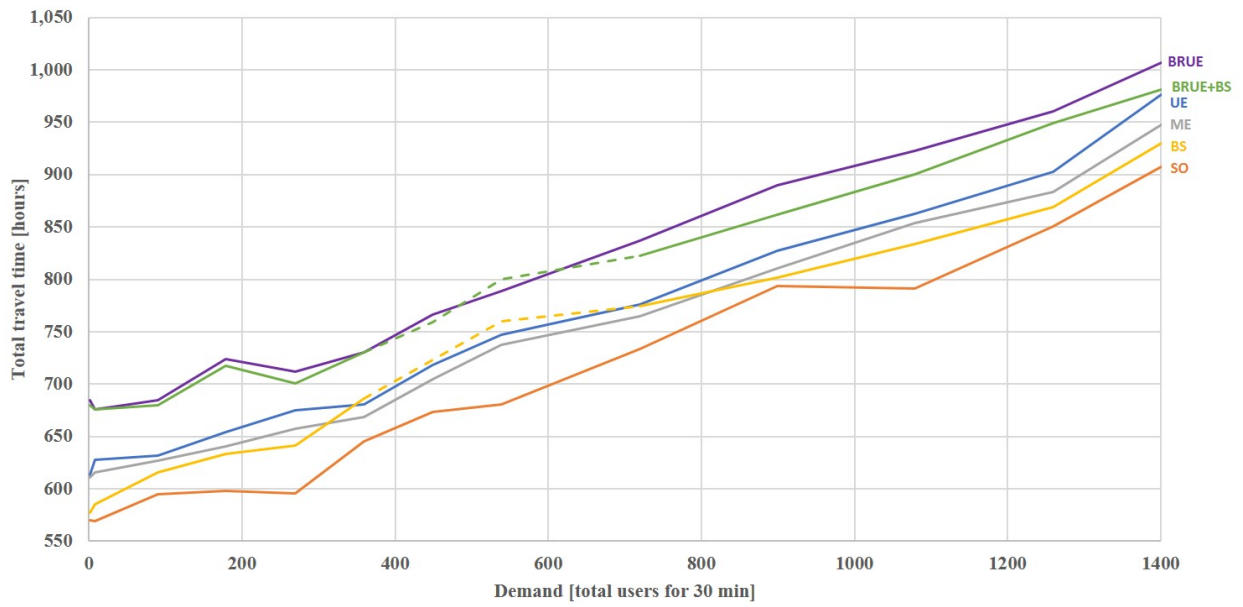
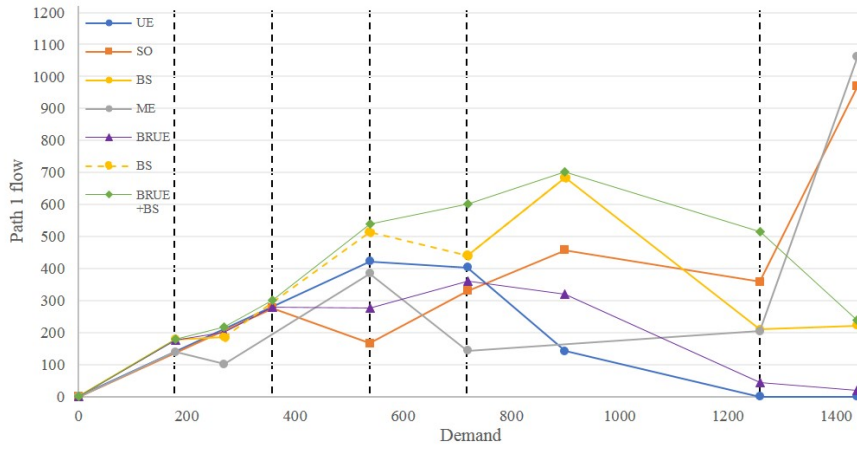
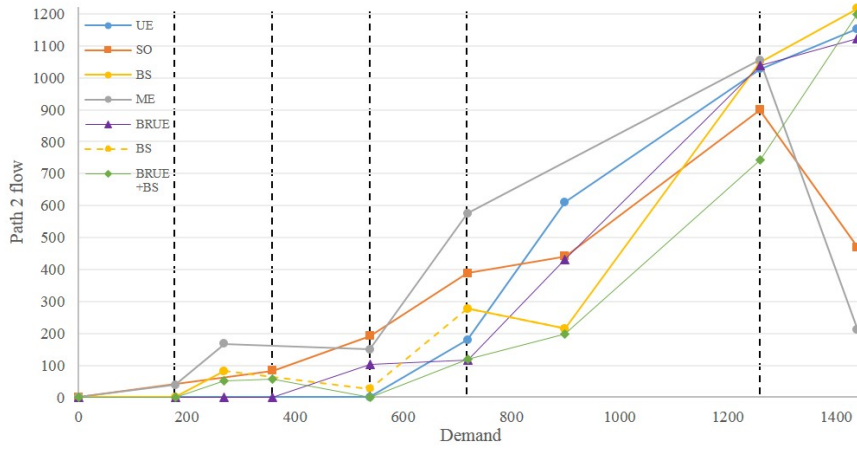


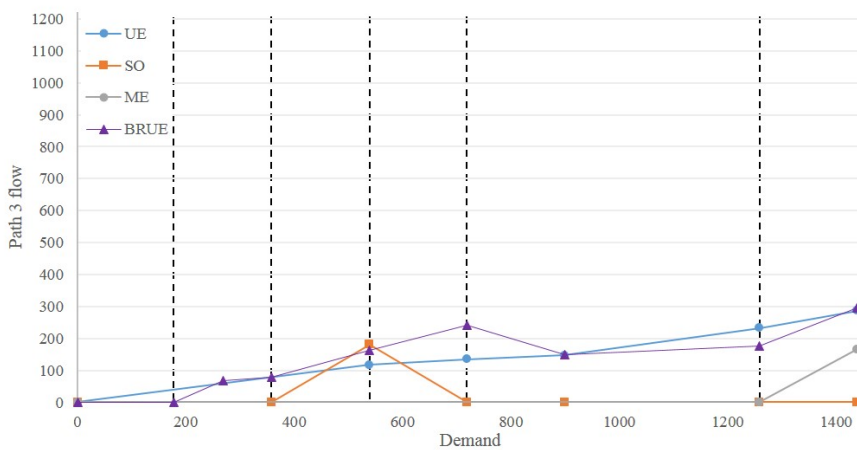
Figure 6: Total TT for each level of demand in one OD pair test case for UE, SO, BRUE, ME, BS and BRUE+BS.



(a) Path 1 flow-demand diagram.



(b) Path 2 flow-demand diagram.



(c) Path 3 flow-demand diagram

Figure 7: flow diagrams in one OD pair test case for UE, SO, BRUE, Mixed Equilibrium (ME), Ban strategy (BS) and the combination of BRUE and Ban Strategy (BRUE+BS). Breakpoints are presented by black vertical dash lines on total demand axis.

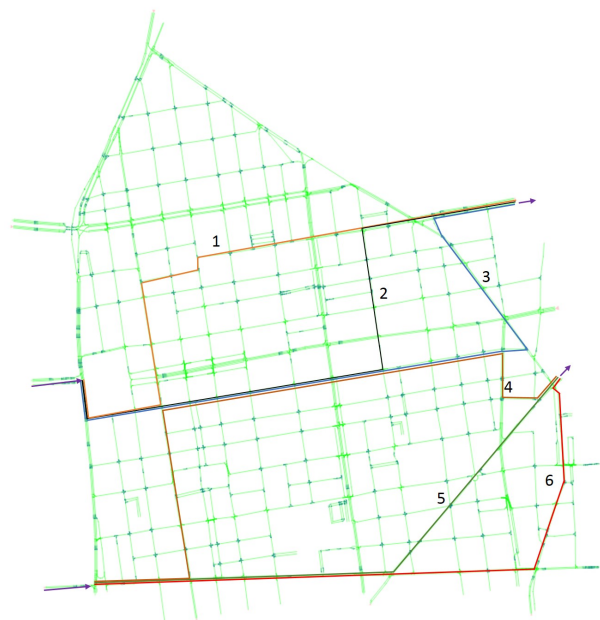
## 6. Two OD pairs numerical experiments

In this section, we apply the breakpoint analysis and the ban strategy based on the breakpoint analysis to the second test case considering jointly two OD pairs in order to show that our framework can be applied to more general configurations. We first identify what are the two most problematic OD pairs in terms of congestion (carry the most demand). Recall that the main idea is through the banning we try to improve the traffic condition of the full network by focusing on these OD pairs. We trigger our banning based on the evolution of demand on these OD pairs.

We consider an experiment which contains two OD pairs in order to analyze the breakpoints by considering the interaction of the two OD pairs. The Lyon 6 network, including the two OD pairs test case, is presented in [figure 8](#). There are 3139 trips for all OD pairs excluding the two targeted OD pairs (for more details see [Appendix A](#)). In order to find the breakpoints, the demand levels of both OD pairs are increased at the same time because both demands come from the same direction and ME discipline is also applied for the users of these OD pairs while the other users follow the UE/BRUE framework. [Figure 8](#) shows that the two OD pairs have many links in common. Taking this fact into consideration, we try to choose the banning intersection in a smart way based on the breakpoint analysis.



(a) Satellite view of Lyon 6, France (Mapping data ©Google 2018).



(b) The paths on the network for two OD pairs test case with banning points.

Figure 8: Network of Lyon 6

### 6.1. Breakpoint analysis

The flow-demand diagrams of all six paths are presented in [Figure 9](#). The demand level of both OD pairs is increased at the same time and the breakpoints are detected from the path flows in order to identify the range of demand for applying the ban strategy. The demand axis shows the inflow of both targeted OD pairs which are equal and increase together for each simulation. The 3 figures on the right ([figures 9\(a\),\(c\) and \(e\)](#)) are related to the first OD pair. They show the breakpoints which are changed because of the impact of the demand level of the second OD pair. The third path is used in the SO solution when the demand level of both OD pairs exceeds than 362. It shows the impact of OD pairs on each other when the demand level is changed. For the second OD pair the results in [figures 9\(b\),\(d\) and \(f\)](#) show the opportunity of banning strategy for path 4 which is almost not used for SO solution. Path 3 and path 4 have many common links and we observe that these links are more used for the UE discipline of the first OD pair which shows that these links are critical for breakpoint detection and ban strategies.

The breakpoint detection is carried out for each equilibrium by considering the active path set and the ban strategy is designed based on the breakpoint analysis. Here, we do not represent the process which is same as in subsections 4.2 and 5.1 and just demonstrate the result of breakpoint analysis and BS designing. The ban strategies based on SO and ME for two OD pairs test case are presented in table 2. For instance, when the path flow is zero for ME and not zero for UE/BRUE, we ban that flow until the next breakpoint that this path is used.

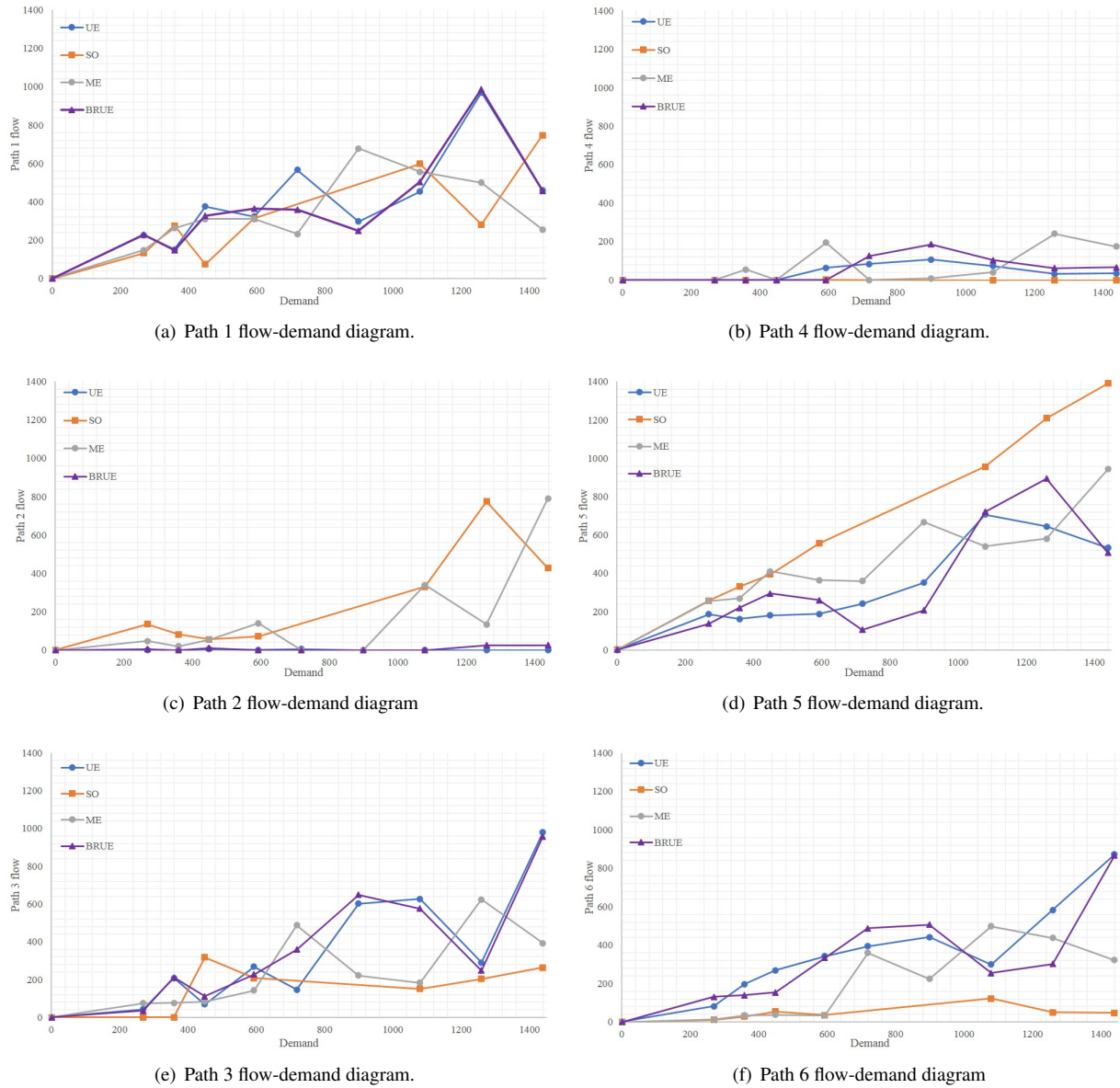


Figure 9: flow diagrams in two OD pairs test case for UE, SO, BRUE, Mixed Equilibrium (ME).

For this case, we consider two ban strategies. The first BS (BS1) is applied based on the SO and the second one (BS2) is designed based on ME path flow distribution (table 2). Four banning points are chosen in order to implement the BS which are presented in figure 10. Point  $\alpha$  is same as in the first scenario and we ban path 3 for the first OD pair and path 4 for the second OD pair. The second point is point  $\beta$  which prevents users from turning left at the intersection and bans path 3 only for the first OD pair. Point  $\gamma$  prevents users from turning left and ban path 4 and

the last banning point  $\delta$  is forbidding users to go straight forward and use path 6. These four banning point(s) are activated based on the breakpoint analysis. When the flows on both paths 3 and 4 are zero in the SO solution, point  $\alpha$  is activated and when the flow of path 3 is zero, point  $\beta$  becomes activated. The point  $\gamma$  allows us to ban path 4 and prevents users from turning left at the intersection. Finally point  $\delta$  becomes active when we need to ban path 6 in order to prevent users from moving straight at the intersection. Considering these banning points we are able to apply the designed ban strategies in each range of demand.

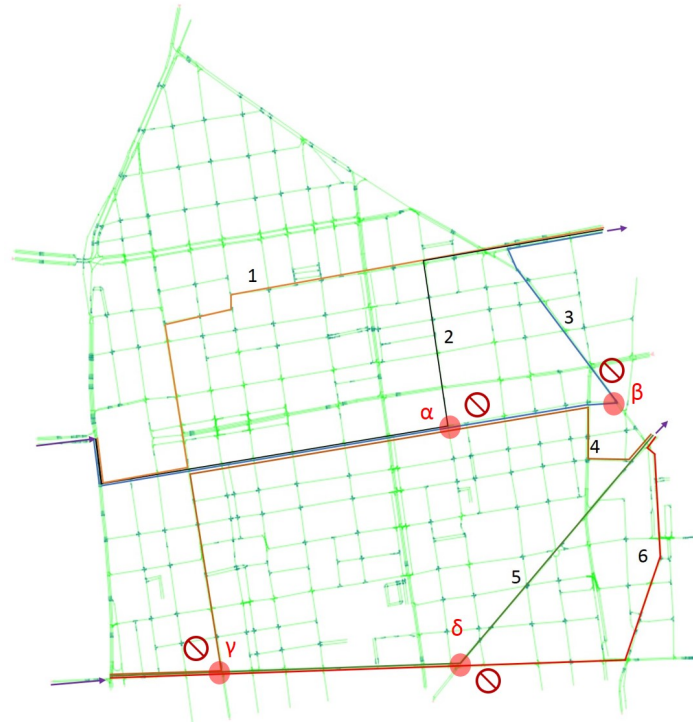


Figure 10: Network of Lyon 6 with banning points for two OD pairs scenario

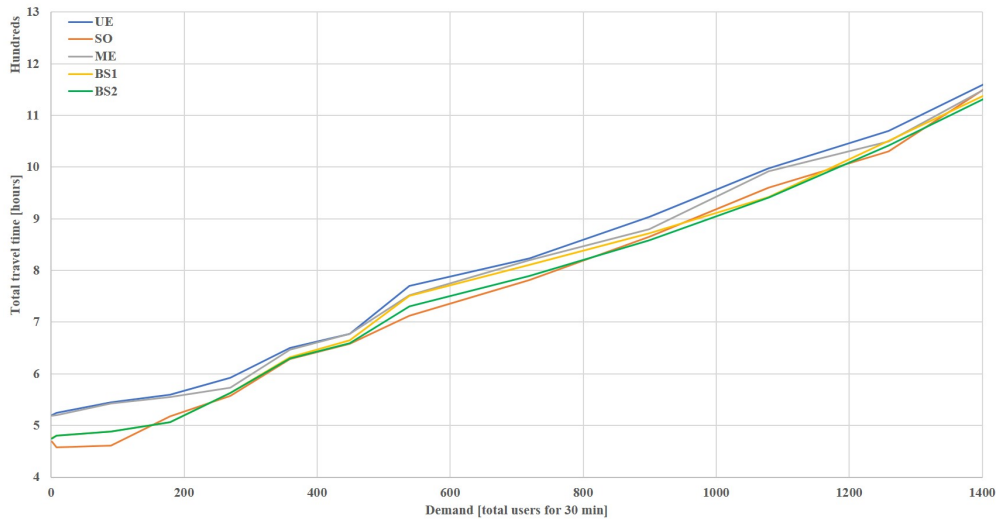
6.2. Applying BS to two OD pairs test case

The results of the Ban strategies are provided for the two OD pairs scenario. Figure 11 presents the evolution of the total TT at different demand levels of the two OD pairs for different disciplines. We present the application of ban strategy in the UE and BRUE frameworks. "BS1" and "BS2" correspond to the UE framework in Figure 11(a) and "BRUE+BS1" and "BRUE+BS2" in Figure 11(b) present the total travel time of ban strategies in BRUE framework.

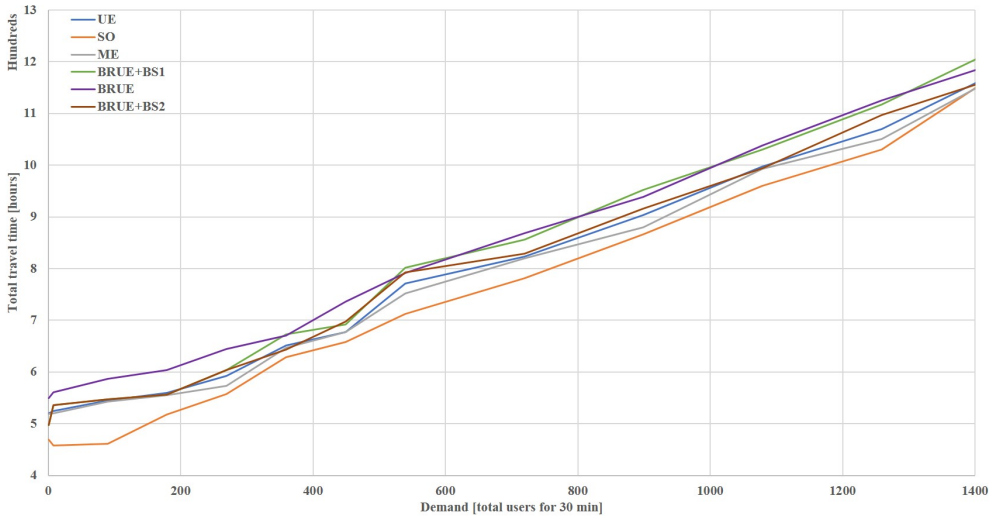
The total TTs for UE, SO, ME, BS1 and BS2 are shown in figure 11(a). The figure shows that BS2 works better than BS1 in the two OD pairs test case, but we should consider that both strategies improve the network performance. BS1 and BS2 also change the network design. Therefore, there is a possibility, while applying BS, to obtain a total TT lower than the SO total TT in some ranges of demand. This has happened in two ranges of demand: [813 – 1187] and [155 – 241], where the total TT of BS is lower than the total TT SO. The second reason for getting lower total TT with

Table 2: The ban strategies for two OD pairs test case

Ban Strategy based on system optimum (BS1)			Ban Strategy based on mixed equilibrium (BS2)		
starting breakpoints	ending breakpoints	Ban	starting breakpoints	ending breakpoints	Ban
0	359	3, 4 and 6	0	269	3, 4 and 6
359	1439	4 and 6	269	539	3 and 6
-	-	-	539	1439	4



(a)



(b)

Figure 11: Total travel time for each level of demand in two OD pairs test case for UE, SO, BRUE, ME, BS1, BS2, BRUE+BS1 and BRUE+BS2.

BS than with SO could be using the surrogate model instead of the analytical path marginal cost in SO calculation. The surrogate model only yields an approximate SO. But the results shows that considering the approximation of the SO solution as a reference is enough to design a control strategy based on breakpoint detection in order to improve the performance of the dynamic traffic network.

The second figure (figure 11(b)) presents the evolution of the total TT for reference disciplines (SO and ME) and BRUE discipline combined with two ban strategies. BRUE+BS1 is the BS based on SO and the second one (BRUE+BS2) is the BS based on ME. ME obtains a better solution for designing the ban strategy in the context of BRUE discipline. The BRUE+BS2 solution is close to UE and better than BRUE in term of total TT. The application of ban strategies to the second test case shows that this strategy can improve the network performance also when we consider more than one OD pair with common links.

### 6.3. Sensitivity analysis

For now, breakpoints for two OD pairs have been derived when the demand level is similar on both side. This situation is quite realistic because, during network loading, we expect that the demand grows in a coordinated way. However, we want to investigate in this subsection what happens if we relax this hypothesis and have different demand levels for each OD pair.

First, we run simulations to derive UE and SO equilibrium on the network for any demand levels on both OD pairs. Figure 12(a) presents the total TT for UE and SO conditions and all demand levels. We draw the planes by using the linear interpolation method (Blu et al., 2004). The total TT in SO is obviously less than UE for all demand levels. Second, we look for breakpoints and path 4 as previous results in section 6.1 show that banning this path at point  $\gamma$  (figure 10) significantly improves the system performance when the demands very coordinated on both OD pairs. In other words, path 4 is the most promising path for the application of banning strategy. The breakpoint detection is carried out by comparing UE and SO solutions and looking for situations when path 4 should not be used in SO while being used in UE. Figure 12(b) presents the demand ranges when the banishment should be applied to path 4 based on such a breakpoint analysis. Note that we look for rectangle areas when all conditions are met and we did not do fine tuning of the area shapes.

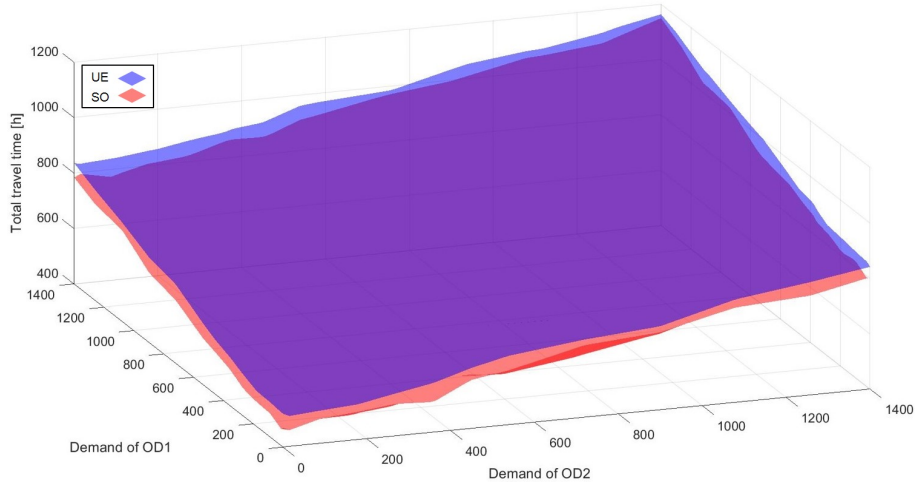
To assess how BS is effective, we first ban path 4 for all demand levels and run again the simulations considering UE equilibrium. We expect that the BS improves the network situation compared to UE without banning within the ranges of demand we have previously identified based on the breakpoint analysis. Figure 13 presents the total TT of UE and UE + banning path 4 (BP4). Two deformed planes are overlapping each other in different demand levels. When the UE plane is placed at the top of BP4 plane, it means by banning path 4 we reduce the total TT.

Figure 14(a) presents the comparison between BS plan (from Figure 12(b)) and the results of banning path 4 for all demand levels of two targeted OD pairs. The green regions are where the system has better performance (less total TT) than UE without banishment, i.e., the banishment is effective. The red regions are the demand ranges where the BS cannot improve (decrease) the total TT of the network. Besides, the heat map in figure 14(b) shows how many hours can be saved or lost by banning path 4 at each level of demand. The results in figure 14 show that breakpoint analysis properly cover the ranges of demand when the BS is effective. Furthermore, it provides a full coverage of the region when the BS is the most effective, i.e. save the more hours, see red regions in figure 14(b).

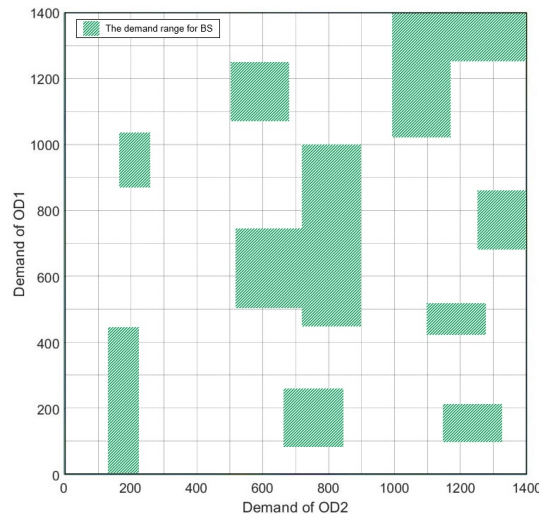
## 7. Conclusion

This paper considers static and dynamic traffic assignment in order to study the impact of different demand levels on three equilibria (User Equilibrium, System Optimum and Boundary Rational User Equilibrium). It defines a breakpoint as a demand level where we observe a change in the active path set of one equilibrium. This study attempts to find the breakpoints and to investigate the possibility to use breakpoint information in order to move from one equilibrium (UE/BRUE) to another (SO). In the static case, we first introduce the process of breakpoint detection and then demonstrate the identification of the situation in which we can improve the network performance by using the ban strategy in the Braess paradox context. For the dynamic case, this paper proposes a novel approach to analyze network DTA equilibrium as a function of demand level. The potential implications of this approach for network suppliers concern the analysis of the network status and the design of ban strategies in order to move from an initial UE or BRUE situation towards SO. Two control strategies are applied based on the breakpoint detection in order to evaluate their impact. The results show that in the dynamic case the banning strategy is efficient and we should apply it in the right range of demand. The mixed equilibrium strategy is designed in order to consider a second reference for designing the ban strategy. The results show that both references can help the design process and the ban strategy improves the network performance. Finally, we have done a sensitivity analysis on the effectiveness of banning a potential path for different demand levels of two OD pairs. The results show that the breakpoint analysis is a powerful tool in order to detect the demand ranges wherein the banning strategy is effective.

The numerical experiments were conducted on a static classic Braess network and on a dynamic real sub network (Lyon 6) in order to examine the equilibrium patterns at different demand levels. In the dynamic network, we consider the full network equilibrium and analyze the pattern when the demand of either one, or two, origin-destination pairs is changed. Note that the experiments of this study have been carried out with homogeneous users. The approach in this paper can be carried over to heterogeneous users and can consider the profile of each user, which allows the study to consider the BRUE in the stable form.



(a) The total TT of UE and SO for different demand levels of joint OD pairs



(b) The output of breakpoint analysis

Figure 12: Breakpoint analysis on path 4 for two targeted OD pairs

The numerical results show that approximating the marginal cost and estimating SO solution yields a breakpoint-based control strategy which improves network equilibrium, so there is a possibility to get better total travel time by improving the approximation of SO. Therefore, the authors plan to improve the calculation method for SO. Thus it will also be possible to investigate if what we observe here in subsection 6.2 (BS total TT lower than SO total TT) is caused by a non-optimal solution or related to network effects (e.g. correlations of the effects between multiple OD pairs). Finally, the results of this study show the existence of the breakpoints, which is to be expected from the static case analysis but has not been analyzed in the dynamic case before. They also show the efficiency of the banning strategy particularly based on mixed equilibrium discipline at the proper level of demand. For future work, the authors are looking for the impact of initialization on the calculation of equilibrium for different demand levels and also apply the breakpoint analysis process to more than two OD pairs while considering of correlation between paths.



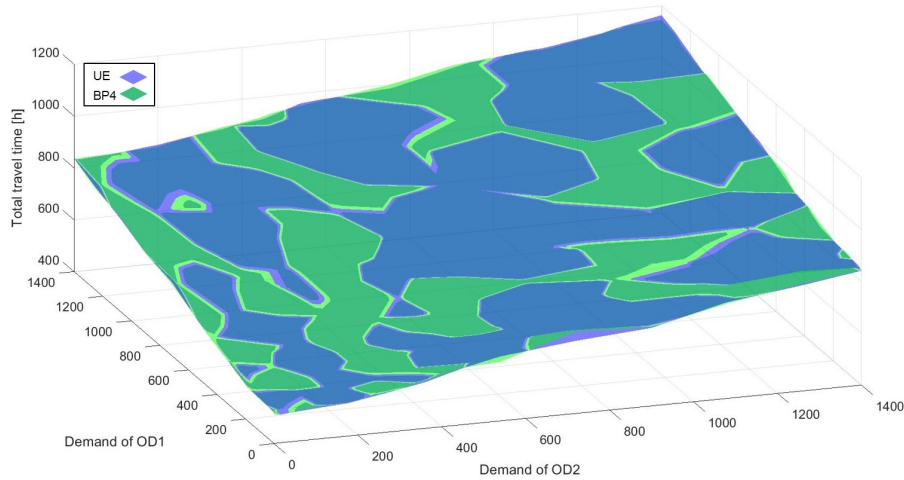
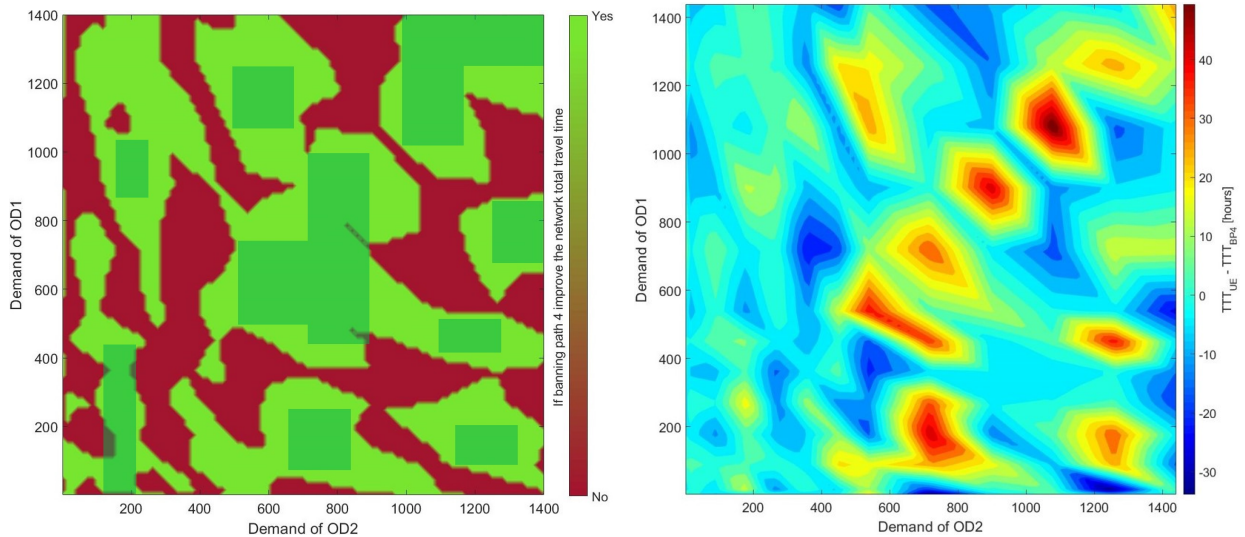


Figure 13: The total TT of UE and BP4 for different demand levels of targeted OD pairs



(a) The comparison between BS plan by breakpoint analysis and the results of banning path 4 for all demand levels of two targeted OD pairs (b) The difference between UE Total Travel Time (TTT) and the TTT when path 4 is banned for all demand levels of two targeted OD pairs

Figure 14: Comparison between BS by breakpoint analysis and the result of banning path 4 for all demand levels of two targeted OD pairs

## 8. Acknowledgements

We thank the anonymous reviewers for their comments and suggestions that have improved this manuscript. This work has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program. (Grant agreement No 646592 – MAGnUM project)

## Appendix A. Lyon 6 demand description

The total travel demand for Lyon metropolis (Figure A.15) is about one million trips per day (Souche et al., 2016). During peak hour, we should then observe about 100,000 trips. The size of the Lyon 6 network ( $1.72 \text{ km}^2$ ) is about 7% of the full network of Lyon. Therefore, approximately, we should have about 7,000 trips per hour. In this study,

based on the real data (Krug et al., 2018), we have, in total, 4361 trips for half an hour in Lyon 6 network, which shows that our test case is fully consistence with real life pattern.

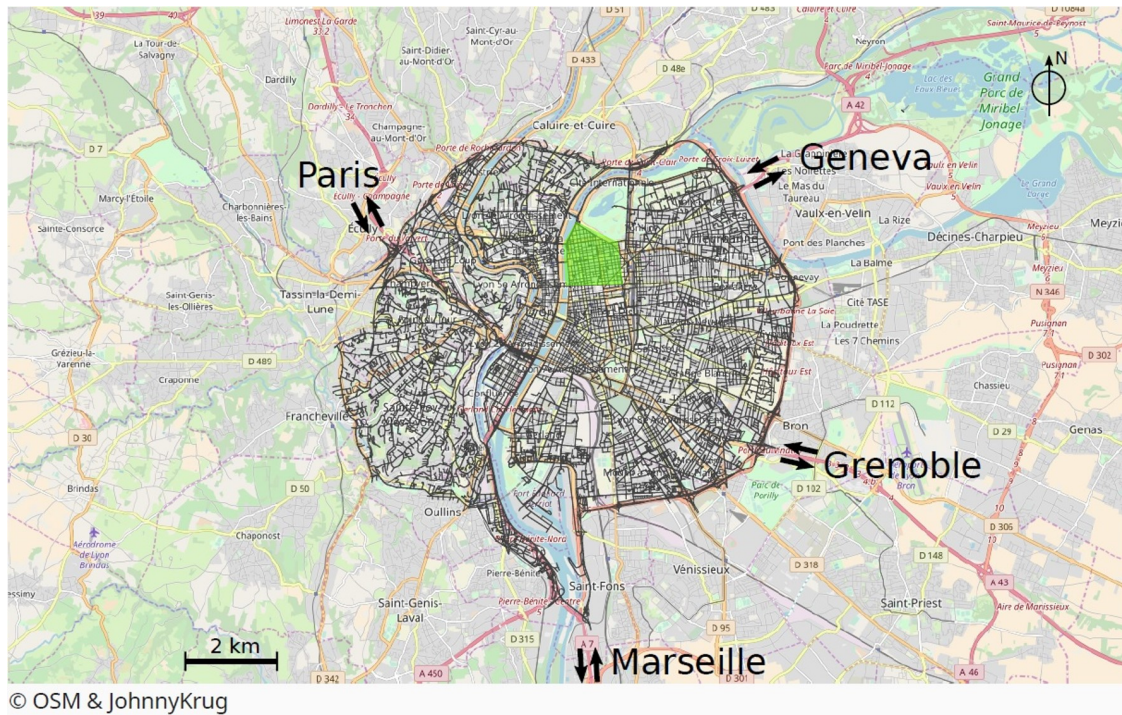


Figure A.15: Full network of Lyon: Lyon 6 is highlighted by green color

## References

- Abdel-Aty, M. A., Kitamura, R., Jovanis, P. P., 1997. Using stated preference data for studying the effect of advanced traffic information on drivers' route choice. *Transportation Research Part C: Emerging Technologies* 5 (1), 39–50.
- Askoura, Y., Lebacque, J.-P., Haj-Salem, H., 2011. Optimal sub-networks in traffic assignment problem and the braess paradox. *Computers & Industrial Engineering* 61 (2), 382–390.
- Bliemer, M. C., Raadsen, M. P., Brederode, L. J., Bell, M. G., Wismans, L. J., Smith, M. J., 2017. Genetics of traffic assignment models for strategic transport planning. *Transport reviews* 37 (1), 56–78.
- Blu, T., Thévenaz, P., Unser, M., 2004. Linear interpolation revitalized. *IEEE Transactions on Image Processing* 13 (5), 710–719.
- Braess, D., Nagurney, A., Wakolbinger, T., 2005. On a paradox of traffic planning. *Transportation science* 39 (4), 446–450.
- Chevallier, E., Leclercq, L., 2007. A macroscopic theory for unsignalized intersections. *Transportation Research Part B: Methodological* 41 (10), 1139–1150.
- Colini-Baldeschi, R., Cominetti, R., Mertikopoulos, P., Scarsini, M., 2017. The price of anarchy in light and heavy traffic: When is selfish routing bad? In: *Book of Abstracts*. p. 39.
- Delle Site, P., 2018. A mixed-behaviour equilibrium model under predictive and static advanced traveller information systems (atis) and state-dependent route choice. *Transportation Research Part C: Emerging Technologies* 86, 549–562.
- Di, X., He, X., Guo, X., Liu, H. X., 2014. Braess paradox under the boundedly rational user equilibria. *Transportation Research Part B: Methodological* 67, 86–108.
- Di, X., Liu, H. X., 2016. Boundedly rational route choice behavior: A review of models and methodologies. *Transportation Research Part B: Methodological* 85, 142–179.
- D'Ambrogio, A., Iazeolla, G., Pasini, L., Pieroni, A., 2009. Simulation model building of traffic intersections. *Simulation Modelling Practice and Theory* 17 (4), 625–640.
- Ehrgott, M., Wang, J. Y., Watling, D. P., 2015. On multi-objective stochastic user equilibrium. *Transportation Research Part B: Methodological* 81, 704–717.
- Frank, M., 1981. The braess paradox. *Mathematical Programming* 20 (1), 283–302.
- Han, K., Friesz, T. L., Szeto, W., Liu, H., 2015a. Elastic demand dynamic network user equilibrium: Formulation, existence and computation. *Transportation Research Part B: Methodological* 81, 183–209.

- Han, K., Szeto, W., Friesz, T. L., 2015b. Formulation, existence, and computation of boundedly rational dynamic user equilibrium with fixed or endogenous user tolerance. *Transportation Research Part B: Methodological* 79, 16–49.
- Han, L., Ukkusuri, S., Doan, K., 2011. Complementarity formulations for the cell transmission model based dynamic user equilibrium with departure time choice, elastic demand and user heterogeneity. *Transportation Research Part B: Methodological* 45 (10), 1749–1767.
- Klein, I., Levy, N., Ben-Elia, E., 2018. An agent-based model of the emergence of cooperation and a fair and stable system optimum using atis on a simple road network. *Transportation research part C: emerging technologies* 86, 183–201.
- Krug, J., Burianne, A., Leclercq, L., 2018. Reconstituting demand patterns of the city of lyon by using multiple gis data sources. Tech. rep., University of Lyon, ENTPE, LICIT.
- Laval, J. A., Leclercq, L., 2008. Microscopic modeling of the relaxation phenomenon using a macroscopic lane-changing model. *Transportation Research Part B: Methodological* 42 (6), 511–522.
- Leclercq, L., 2007a. Bounded acceleration close to fixed and moving bottlenecks. *Transportation Research Part B: Methodological* 41 (3), 309–319.
- Leclercq, L., 2007b. Hybrid approaches to the solutions of the “lighthill–whitham–richards” model. *Transportation Research Part B: Methodological* 41 (7), 701–709.
- Leclercq, L., Laval, J. A., Chevallier, E., 2007. The lagrangian coordinates and what it means for first order traffic flow models. In: *Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17*.
- Leclercq, L., Verchier, A., Krug, J., Menendez, M., 2016. Investigating the performances of the method of successive averages for determining dynamic user equilibrium and system optimum in manhattan networks. In: *DTA2016, 6th International Symposium on Dynamic Traffic Assignment*, pp. 1–p.
- Mahmassani, H. S., 2001. Dynamic network traffic assignment and simulation methodology for advanced system management applications. *Networks and spatial economics* 1 (3), 267–292.
- Mahmassani, H. S., Chang, G.-L., 1987. On boundedly rational user equilibrium in transportation systems. *Transportation science* 21 (2), 89–99.
- Pas, E. I., Principio, S. L., 1997. Braess’ paradox: Some new insights. *Transportation Research Part B: Methodological* 31 (3), 265–276.
- Patriksson, M., 2015. *The traffic assignment problem: models and methods*. Courier Dover Publications.
- Peeta, S., Mahmassani, H. S., 1995. System optimal and user equilibrium time-dependent traffic assignment in congested networks. *Annals of Operations Research* 60 (1), 81–113.
- Robbins, H., Monro, S., 1951. A stochastic approximation method. *The annals of mathematical statistics*, 400–407.
- Roughgarden, T., 2005. *Selfish routing and the price of anarchy*. Vol. 174. MIT press Cambridge.
- Sbayti, H., Lu, C.-C., Mahmassani, H., 2007. Efficient implementation of method of successive averages in simulation-based dynamic traffic assignment models for large-scale network applications. *Transportation Research Record: Journal of the Transportation Research Board* (2029), 22–30.
- Souche, S., Mercier, A., Ovtracht, N., 2016. The impacts of urban pricing on social and spatial inequalities: The case study of lyon (france). *Urban Studies* 53 (2), 373–399.
- Sundaram, S., Koutsopoulos, H. N., Ben-Akiva, M., Antoniou, C., Balakrishna, R., 2011. Simulation-based dynamic traffic assignment for short-term planning applications. *Simulation Modelling Practice and Theory* 19 (1), 450–462.
- Szeto, W., Lo, H. K., 2006. Dynamic traffic assignment: properties and extensions. *Transportmetrica* 2 (1), 31–52.
- Szeto, W., Wong, S., 2012. Dynamic traffic assignment: model classifications and recent advances in travel choice principles. *Central European Journal of Engineering* 2 (1), 1–18.
- Szeto, W. Y., Lo, H. K., 2004. A cell-based simultaneous route and departure time choice model with elastic demand. *Transportation Research Part B: Methodological* 38 (7), 593–612.
- Wang, Y., Szeto, W., Han, K., Friesz, T. L., 2018. Dynamic traffic assignment: A review of the methodological advances for environmentally sustainable road transportation applications. *Transportation Research Part B: Methodological*.
- Wardrop, J. G., 1952. Road paper. some theoretical aspects of road traffic research. *Proceedings of the institution of civil engineers* 1 (3), 325–362.
- Wie, B.-W., Tobin, R. L., Carey, M., 2002. The existence, uniqueness and computation of an arc-based dynamic network user equilibrium formulation. *Transportation Research Part B: Methodological* 36 (10), 897–918.
- Yildirimoglu, M., Kahraman, O., 2018. Investigating the empirical existence of equilibrium conditions. Tech. rep.
- Youn, H., Gastner, M. T., Jeong, H., 2008. Price of anarchy in transportation networks: efficiency and optimality control. *Physical review letters* 101 (12), 128701.
- Zhou, B., Xu, M., Meng, Q., Huang, Z., 2017. A day-to-day route flow evolution process towards the mixed equilibria. *Transportation Research Part C: Emerging Technologies* 82, 210–228.