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Straight contact lines on soft solids and broken contact lines on hard substrates.

Laurent Limat¹

¹Laboratoire Matière et Systèmes Complexes, UMR 7057, Université Paris Diderot, Paris

Corresponding author: laurent.limat@univ-paris-diderot.fr

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I Introduction

Despite decades of studies, wetting dynamics on a solid is still the focus of intense debates [1, 2]. The perhaps simplest wetting dynamics experiment consists in observing the shape of a drop sliding down an incline [3, 4]. Recent works have shown that a conical tail develop at the rear when the velocity is large enough, the structure of both the flow and the interface being well described [5-7] by an appropriate generalization of the so-called hydrodynamical model of wetting [2]. On the other hand, depending on the liquid used, the slip cut-off assumed in these approaches can be of reasonable magnitude (polymer lengths for silicon oils) [3,7] or on the contrary, completely unreasonable, i.e. extremely small (water and mercury) [8-9]. In the first part of this talk, I suggest that this surprising result could be explained by assuming some extra-dissipation at small scale [1,12], that could be represented by a contact line friction in the framework of the combined model [10,11] that mixes the hydrodynamic approach [2] with thermally activated jumps [1]. In this framework, the large scale flow structure would be solely governed by hydrodynamics, while what happens at small scale involve an apparent cut-off modified by the contact line friction, and that can indeed be very small, when this line friction is sufficiently large.

Another subject of passionate debates is also presently the static of wetting on soft solids [13-19]. In the 80's, Shanahan [32] developed a description in which the distortions of the solid due to contact line assumed a logarithmic profile based on a balance between liquid surface tension and elastic stresses developing in the solid. At short scales, this profile was supposed to be cut by yield and plasticity. On the other hand, more recent approaches reveal that the surface tension of the solid, neglected in this initial approach, can not be omitted [15-18], especially when the substrate is very deformable, as happens for tenuous compounds such as gels [14,19]. In the second part of this talk, I will show that Shanahan approach can be modified [17] by adding this effect to the surface equilibrium equations. The same kind of logarithmic profile is obtained, but with a cut-off that is now ruled by the elastocapillary length built upon surface tension and elastic modulus. At small scale, the logarithmic divergence is replaced by the more classical condition of Neumann equilibrium of the three surface tensions at contact line, in agreement with other recent works [18].

II Drops sliding down a plane revisited with a microscopic friction line.

Typical pictures of a drop sliding down a plane and of its conical tail are reproduced on fig. 1, where the notations are also precised. In the lubrication limit, the flow inside the tail results from a balance between viscous losses and the gradient of capillary pressure linked to the conicity. This yields a relationship linking the capillary number $Ca = \eta V / \gamma$ (V drop velocity, η liquid viscosity, γ surface tension of the liquid) to the cone angles Ω and ϕ , that reads [5]:

$$\Omega^3 \approx \frac{35}{16} Ca \tan^2 \phi \quad (1)$$

To obtain some predictions on these angles, one needs to precise the microscopic modeling of wetting at the contact line. I here propose to use the "combined model" [10-11], mixing both the bulk hydrodynamic dissipation with a more microscopic line friction, which reads on the inclined contact lines:

$$\theta^3 = (2\Omega / \sin \phi)^3 = \theta_m^3 - 9Ca \text{Log}(b/a) \sin \phi \quad (2-a)$$

$$\theta_m^2 = \theta_r^2 - ACa \sin \phi \quad (2-b)$$

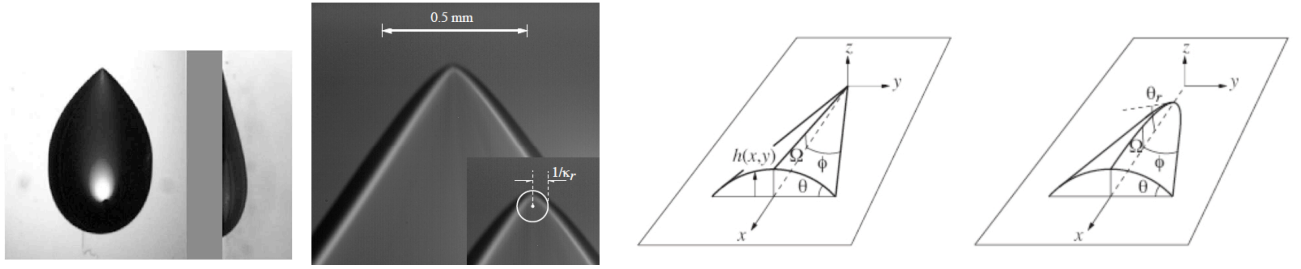


Fig. 1: From left to right, top and side views of a silicon oil drop sliding down a inclined plane, under partial wetting, magnified vision of the cone tip at the drop rear, rounded at small scale $R=1/\kappa_r$ [7], and notations used here.

where θ_r is the static receding contact angle, b a macroscopic size (typically the drop size, at which the conical tail has to match), a is a small-scale cut-off, below which a line friction of microscopic origin develops, modeled by the constant A . In a limit of small capillary number and/or slender cone of low ϕ value, these equations lead to the following equation linking ϕ to Ca :

$$\frac{Ca}{\theta_r^3} = \frac{2\phi}{35 + 18\phi^2 \text{Log}(b/a')} \quad \text{with} \quad a' = a \exp(-A\theta_r/6) \quad (3)$$

As one can see, these equations are the same as those developed in [6], but in which a' plays the role of an apparent cut-off in a slip length modeling that can become extremely small in the limit of a strong microscopic slip constant A . The same kind of argument can be developed for the modeling of the curvature radius $R=1/\kappa_r$ of the contact line at the corner tip [7], where one has to match the large scale interface slope on the symmetry axis Ω to the same modeling of the contact line at the scale of the corner tip width R/ϕ^2 . Roughly, one has: $\Omega^3 = (\theta_r^2 - ACa)^{3/2} - 9Ca \text{Log}(2R/a\phi^2)$, which finally yields, again in the small Ca limit:

$$R \approx (a'\phi^2/2) \exp(\theta_r^3/9Ca) \quad (4)$$

which is the same form as the one used in [8], but with the same apparent cut-off used in eq.(3). Finally let us conclude with the even more simple case of a straight contact line receding normally to itself, in the vicinity of which the local slope can be written in two forms:

$$\theta^3(x) = (\theta_r^2 - ACa)^{3/2} - 9Ca \text{Log}(x/a) \approx \theta_r^3 - 9Ca \text{Log}(x/a') \quad (5)$$

where x is the distance to the contact line. In all the cases considered, straight contact line, curved contact line at the tip of the cone, inclined contact line on each cone side, the spatial structure is ruled by the "hydrodynamical" model of wetting, i.e. by a balance between capillarity and viscous bulk dissipation, but with an apparent slip length that is modified by the extra friction introduced microscopically by other mechanisms. It is thus not efficient to oppose the hydrodynamical model to others (and in particular to the chemical model of wetting), as all describe in fact complementary aspects of the same puzzle.

III Shanahan approach of soft wetting revisited with substrate surface tension.

When one considers a soft, deformable substrate, statics and dynamics of the contact line is made more complex by the substrate deformation (see fig. 2). For simplicity, I will here focus on the simple case of an incompressible substrate, whose dry and wet surface tension are equal: $\gamma_s \approx \gamma_{SL}$, i.e. with a macroscopic Young equilibrium contact angle close to $\pi/2$. If one writes qualitatively the balance of forces in the vertical direction,

$$\gamma \sin \theta_0 \approx -2\gamma_s \frac{\partial \xi}{\partial x} - 2\pi\mu x \frac{\partial \xi}{\partial x} \quad (6)$$

where the last term stands for the elastic contribution (estimated near the solid surface) $\sigma \sim \mu \partial \xi / \partial x$ (μ elastic shear modulus), one gets the following estimate for the vertical displacement of the substrate surface

$$\xi(x) \approx \frac{\gamma \sin \theta_0}{2\pi\mu} \text{Log} \frac{b+l_s}{|x|+l_s} \quad (7)$$

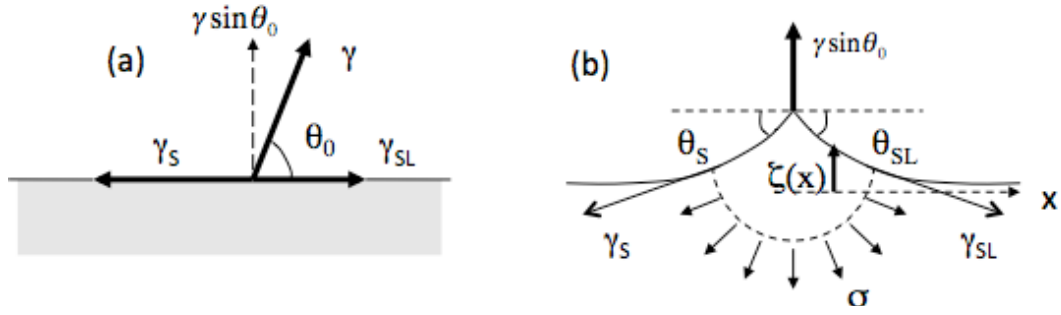


Fig. 2: (a) Decomposition of Young equilibrium into a normal and transverse balance of tensions; (b) distortions induced by the normal component, that is moderated by both elastic stresses and substrate surface tensions.

in which $l_s = (1/\pi)(\gamma_s/\mu)$ is the elastocapillary length and b some large-scale cut-off. In the limit $\gamma_s = 0$ one recovers the classical Shanahan solution that diverges near $x=0$. The present approach indicates that, when the substrate surface tension is taken into account, this Log-profile is still valid but shift of a distance equal to l_s which "cuts" this divergence at this scale, this one being replaced by Neumann equilibrium of surface that reads here:

$$2\gamma_s \theta_s \approx \gamma \sin \theta_0 \quad (8)$$

In [17], I have developed a more rigorous approach that allowed me to calculate exactly $\zeta(x)$ for $\theta_0 = \pi/2$, the discrepancies with eq.(8) being smaller than a few percents. I also suggest in the same paper a possible extension of the modeling to the case $\gamma_s \neq \gamma_{SL}$, the surface tension mismatch $\delta\gamma = \gamma_s - \gamma_{SL}$ remaining small compared to the mean value. This one suggests the following selection rule for the substrate slopes at the contact line:

$$\left\{ \begin{array}{l} \theta_s = \frac{\gamma \sin \theta_0}{2\gamma_s} \left(1 + \varepsilon \frac{\delta\gamma}{\gamma_s} \right) \\ \theta_{SL} = \frac{\gamma \sin \theta_0}{2\gamma_{SL}} \left(1 - \varepsilon \frac{\delta\gamma}{\gamma_{SL}} \right) \end{array} \right. \quad \begin{array}{l} (9-a) \\ (9-b) \end{array}$$

where ε is a small parameter, that vanishes when the standard linear theory of elasticity is used, and that should be equal to 1/4 if one combines the idea that Young equilibrium should hold at large scale, before to cross-over to Neumann equilibrium very near contact line [18]. I suggest to consider non-linear elasticity with additional quadratic gradient contributions to solve this apparent mismatch. This question of the slope reached by the substrate near contact line is essential to understand wetting hysteresis and more complex wetting behaviours observed on gels and elastomers (stick-slip, wetting transitions induced by swelling [19]).

References

1. J. De Coninck, T. D. Blake, *Ann. Rev. of Mat. Research*, **38**, 1-22 (2008)
2. D. Bonn, J. Eggers, J. Indekeu, J. Meunier, E. Rolley, *Rev. Mod. Phys.* **81**, 739-805 (2009)
3. T. Podgorski, J. M. Flesselles, L. Limat, *Phys. Rev. Lett.* **87**, 036102-036105 (2001)
4. N. Le Grand, A. Daerr, L. Limat, *J. Fluid Mech.* **541**, 293-315 (2005)
5. J. H. Snoeijer, E. Rio, N. Le Grand, L. Limat, *Phys. Fluids* **17**, 072101 (2005)
6. J. H. Snoeijer, N. Le Grand, L. Limat, H. A. Stone, J. Eggers, *Phys. Fluids* **19**, 042104 (2007)
7. I. Peters, J. H. Snoeijer, A. Daerr, L. Limat, *Phys. Rev. Lett.* **103**, 114501 (2009)
8. K. G. Winkels, I. R. Peters, J. Snoeijer, F. Evangelista, M. Riepen, A. Daerr, L. Limat, *Eur. Phys. J. Spec. Topics.* **192**, 195-205 (2011)
9. B. A. Puthenveetil, V. K. Kumar, E. J. Hopfinger, to appear in *J. Fluid Mech.* (2013)
10. P. G. Petrov, J. G. Petrov, *Langmuir* **8**, 1762-1767 (1992)
11. M. de Ruijter, M. Charlot, M. Voué, De Coninck, *Langmuir*, **16** 2363-2368 (2000)
12. E. Rolley, C. Guthmann, *Phys. Rev. Lett.* **98**, 166105 (2007)
13. M. E. R. Shanahan, *J. Phys. D, Appl. Phys* **20**, 945-950 (1987)
14. R. Pericet-Camara, A. Best, H.-J. Butt, E. Bonaccorso, *Langmuir* **24**, 10565-10568 (2008)
15. E. R. Jerison, Y. Xu, L. A. Wilen, E. R. Dufresne, *Phys. Rev. Lett.* **106**, 186103 (2011)
16. S. Das, A. Marchand, B. Andreotti and J. H. Snoeijer, *Phys. Fluids* **23**, 072006 (2011)
17. L. Limat, *Eur. Phys. J. E* **35**, 134 (2012)
18. R.W. Style, E. R. Dufresne, *Soft Matter* **8**, 3177 (2012); Marchand et al. *Phys. Rev. Lett.* **109**, 236101 (2012)
19. T. Kajiyi, A. Daerr, L. Royon, T. Narita, F. Lequeux, L. Limat, *Soft Matt.* **7**, 11425 (2011); **9**, 454-461 (2013)