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# Convertible delta-wing aircraft for teaching and research<sup>\*</sup>

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**Abstract:** This paper presents our initiative on the design and building of a convertible aircraft for fault tolerant control. The main advantage of convertible over conventional aircraft is that they can take-off and land vertically, practically eliminating the need for a long runway. At the same time, fixed wings allow to improve energy efficiency with respect to multi-copters. To satisfy these objectives, we have chosen a bi-rotor delta-wing design which is described in the first part of the paper. The paper gives a description of the main educational activities of this project emphasizing its multidisciplinary nature. Indeed, one important aspect of this project is that we have had the chance to work with students from various specialities (such as mechanical engineering, embedded systems, avionics and automatic control) on bachelor and master's levels. Moreover, the end result of this project should provide us with a very interesting experimental test-bench for teaching and research purposes. Finally, this activity is also the opportunity for our students to develop their soft skills such as project management and communication.

*Keywords:* convertible aircraft, autonomous vehicles, non-linear systems, experimental test-bench

## 1. INTRODUCTION

In recent years, there has been an increased interest in the field of vertical take-off and landing (VTOL) aircraft with a special emphasis on those equipped with fixed-wings (see Bapst et al. (2015); Gryte et al. (2018); Airimitoiaie et al. (2018)). VTOL fixed-wing aircraft have the advantage of an increased autonomy with respect to multi-rotors. They are also called convertible because of their capability to fly as multi-copters during take-off, landing and hover and as regular aircraft during fast-forward flight. Figure 1 shows the basic principle of a convertible aircraft. The one presented here is equipped with 6 propeller-engines mounted two by two on three pairs of wings making it similar to a hexa-copter during slow speed flight manoeuvres (configuration (a) in Fig. 1). Nevertheless, the wings can tilt to a horizontal position and allow the aircraft to benefit from aerodynamic lift during high speed forward flight (configuration (b) in Fig. 1). One advantage of this tilt-wing configuration is that the main body stays in an horizontal position during the entire flight, which can be useful for the transportation of certain objects.

Being located in one of the most important centres for aerospace engineering in France, the University of Bordeaux has a long history in the development of aeronautics related educational programs. Among these, automatic control is well represented with two Masters of Science, one

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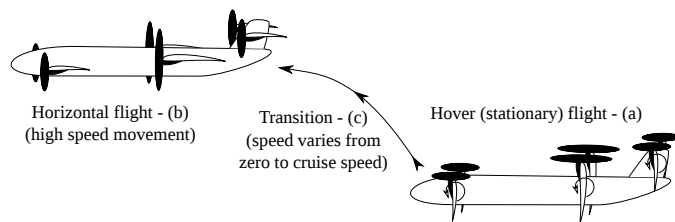


Fig. 1. Hover (a), horizontal flight (b), and transition (c) for a tilt-wing convertible aircraft.

on aeronautics maintenance and another one on complex systems engineering.

Our endeavour is to improve the teaching of automatic control in these educational programs and the convertible aircraft project presented in this paper offers this possibility. Furthermore, it shows that automatic control cannot be dissociated from other fields of engineering such as mechanical, embedded systems, and avionics and can only enrich them.

The starting point of this program has been the MICA (Model Identification and control of a Convertible Aircraft) research project that focuses on the fault tolerant guidance in the context of convertible aircraft (see MICA (2016)). The main idea of the fault detection and isolation (FDI) and fault tolerant control (FTC) that we are aiming to provide is based on the theory of flat systems developed in Lévine (2009). Previous research using this approach has shown its suitability for the design of guidance strategies (Morio et al., 2010) and for FDI (Martínez Torres et al., 2013).

This paper presents our teaching activities around this subject. It presents our experience in the design and building of the convertible. It then describes the main challenges from an automatic control point of view and the educational benefits that can be expected from running a project with this complexity.

The paper is organized as follows. In section 2, are presented the main activities of this project emphasizing its multidisciplinary nature. The non-linear modelling of the convertible aircraft is described in Section 3. This represents the first part of the autopilot control law design. Concluding remarks are given in Section 4.

## 2. PROJECT MAIN ACTIVITIES

The biggest challenge and at the same time to most exciting aspect of this project is its multidisciplinary nature. It has been a great opportunity to get involved many students from various disciplines of engineering. To coordinate their activities, we have opted for an organisation based on technological bricks, which implies that each group of students has the responsibility of proposing a solution for a smaller problem related to this project. This allows us to increment in steps our expertise for being able to complete the entire project.

There are four main activities in this project: (i) mechanical system design and build, (ii) numerical wind-tunnel analysis by computational fluid dynamics (CFD), (iii) embedded system design and programming, and (iv) development of automatic control algorithms for the autopilot.

The desired concept of convertible aircraft for this project is a 6-rotors VTOL similar to the one shown in Fig. 1. Nevertheless, for reasons of dependency on the results from parallel activities, it is not possible to do all four previous activities directly on this configuration. As a first step of the project, we have thus decided to: (i) build a quad-copter drone from parts available on the market in order to work on the embedded system (while the convertible aircraft is not finished); and (ii) build a delta-wing aircraft in order to analyse the manufacturing difficulties on a design that is simpler than a 6-rotors / 3 tilt-wings aircraft. We will give more details about these activities in the following.

### 2.1 Computer aided design (CAD) of the mechanical system

The first step in the construction of any modern aircraft is the design of a 3-dimensional computer model. Various CAD software can be used, such as Catia<sup>®</sup>, Solidworks<sup>®</sup> (commercial) or FreeCAD<sup>®</sup> (free). CAD modelling has become a very important part in aircraft manufacturing as it allows to highly reduce the cost and time before marketing of a new product.

The use of CAD software for the design and analysis of various components has the advantage of being more rapid than the construction itself. Modern CAD software benefit from complex functionalities which allow to do various tests such as the analysis of the resistance of the system, the computation of the inertia matrix, the mass, etc. In

our aircraft maintenance master's program, students are trained in the use of CAD software for the design of various aircraft components.

The same is true for this project. Let us consider for example the design of the tilt-wing mechanism.<sup>1</sup> This component of the convertible aircraft should satisfy the following criteria:

- have a self-lock mechanism in order to avoid spending energy to counteract wind forces while maintaining the wings at a desired angle with respect to the aircraft body;
- the complete tilting of the wing should be done sufficiently fast in order to avoid introducing additional delays in the control system;
- the effect of the wing tilting on the aircraft inertia matrix should be minimized.

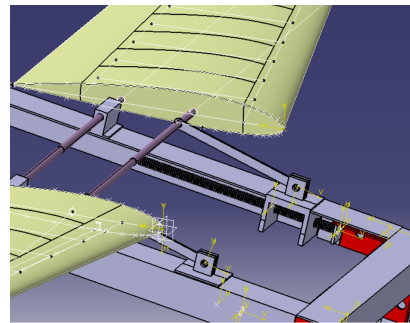


Fig. 2. 3D design of tilt-wing mechanism using infinite screw.

An initial design of the tilt-wing mechanism using infinite screws is shown in Fig. 2. The advantage of this solution is that the effect of wind is highly reduced and very little energy is needed to keep the wing at a desired angle. Nevertheless, this kind of mechanism might not be sufficiently fast to a change of angle request from the autopilot.

### 2.2 Numerical wind-tunnel analysis

Another important step in the design of an aircraft is the wind-tunnel analysis of the aerodynamic coefficient functions. As we do not have a convenient facility for this, we have chosen to do a numerical wind-tunnel analysis instead. This implies using CFD algorithms to evaluate the flow of air around the aircraft by numerically solving the fluid flow equations. This is a very complex task and is usually done under some approximations using dedicated CFD software.

During this project we have used either XFLR5 (Deperrois, 2013) or OpenFOAM (Jasak, 2009). The former is a simplified CFD analysis software implementing the lifting line theory and vortex lattice method and it is only accurate at small angles of attack. OpenFOAM allows to analyse complex shapes under more general fluid flow conditions. It can be used to analyse aerodynamic flow at any angle of attack and for various speeds of the aircraft. Nevertheless, properly configuring a CFD analysis is not

<sup>1</sup> There exist two types of convertible aircraft, tilt-wing or tilt-rotor, depending on whether the wing tilts entirely or just the propeller-engine part. In this project we have chosen a tilt-wing configuration.

an easy task and it gets more complicated as one aims to obtain more reliable results.

One of the tasks that has been done using XFLR5, was the configuration of the delta-wing aircraft. The objective was to configure the dimensions in order to improve the flight characteristics of this aircraft. XFLR5 allowed to obtain also an estimation of the inertia matrix and of the aerodynamic coefficients for small angles of attack. The obtained design is shown in Fig. 3.

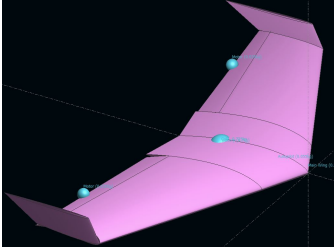


Fig. 3. XFLR5 model of the delta-wing aircraft.

A first test case for OpenFOAM was to evaluate a simple 2-dimensional symmetric NACA 0012 wing profile, which is the one chosen for the delta-wing. To assess the numerical wind-tunnel results, these have been compared with Sheldahl and Klimas (1981). Figure 4 shows the comparison for the lift coefficient and for a Reynolds number  $Re=5e5$ . From this figure, one can see that the numerical wind-tunnel results are close to the experimental wind-tunnel data up to  $25^\circ$  and they estimate correctly the stall Angle of Attack (AoA) which is an important information for the design of the autopilot control algorithms. At higher AoAs, the numerical wind-tunnel results underestimate the lift coefficient.

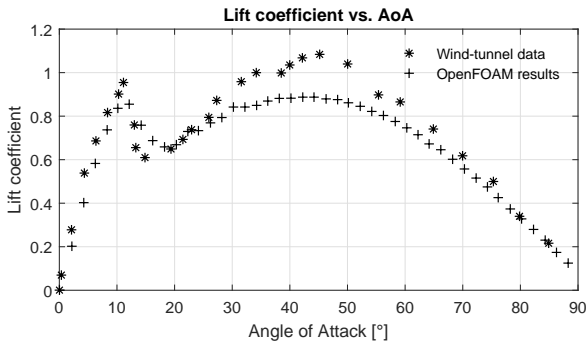


Fig. 4. Comparison of wind-tunnel data from Sheldahl and Klimas (1981) with OpenFOAM results for the NACA 0012 profile for  $Re=5e5$ .

### 2.3 Embedded systems design

The hardware part of the autopilot for our project is based on a Raspberry Pi3 with a Navio2 hat (Emlid, 2019) (see photo in Fig. 5). The software is based on the open-source ArduPilot flight stack. To begin working on the autopilot code, we have started by using a classical quad-copter drone. The main advantage is that quad-copters are easy to build using off-the-shelf components. Another advantage is that the quad-copter can be effectively piloted using PID controllers and a decoupling between roll, pitch, yaw and altitude movements. We have mainly worked with 3<sup>rd</sup> year Bachelor students on this part. In order to design

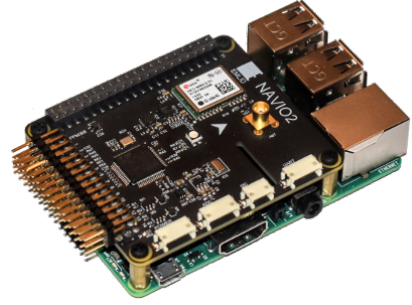


Fig. 5. Photo of Raspberry Pi 3 with Navio2 hat used for implementing the drone autopilot.

Param.	Symbol	Value
Mass	$m$	1.6 kg
Local acceleration of gravity	$g$	9.8 m/s <sup>2</sup>
Length of arm	$l$	0.375 m
Inertia on $x^b$ axis	$I_{xx}$	3.35e-2 kg · m <sup>2</sup>
Inertia on $y^b$ axis	$I_{yy}$	3.73e-2 kg · m <sup>2</sup>
Inertia on $z^b$ axis	$I_{zz}$	4.39e-2 kg · m <sup>2</sup>
Propeller thrust coefficient	$k$	6.9e-6 kg · m
Propeller torque coefficient	$d$	7.57e-7 kg · m <sup>2</sup>

Table 1. Parameters of the quad-copter.

the PID control law, a quad-copter model was used. Its parameters are given in Table 1. For the inertia coefficients, students used the parallel axis theorem and the formula for the pendulum natural frequency:  $\omega_n = \sqrt{\frac{mgr}{I}}$ , where  $r$  defines the length of the pendulum and  $I$  represents the coefficient of inertia.

## 3. NON-LINEAR MODEL AND AUTOPILOT DESIGN

In this section, we describe more precisely the results that have been obtained at this point for the fourth activity of this project which is focused on the autopilot control law design design. There are two important challenges. The first one is to obtain a reliable non-linear dynamic model of the convertible aircraft and the second one is the design of the control law. The next subsections will focus on the non-linear model of the bi-motor delta-wing convertible aircraft.

### 3.1 Basic assumptions and notations

We make the assumption that the Earth is flat and fixed. As such, the ground fixed north-east-down frame ( $\mathcal{I}$ ) is considered to be inertial. For the modelling of the convertible aircraft, three other reference frames are used: the vehicle-carried normal Earth frame ( $\mathcal{O}$ ), the body frame ( $\mathcal{B}$ ) and the aerodynamic frame ( $\mathcal{A}$ ). All coordinate systems are considered right-handed, and their axis are denoted using  $x, y, z$  with superscripts  $i, o, b$  or  $a$  depending on the frame.

Euler angles are used to represent rotations between reference frames. The rotation matrices are denoted as  $R$  with a two small-case letters subscript. The first letter indicates the final frame and the second gives the original frame, such as <sup>2</sup>

$$R_{ob} = R_z(\psi(t))R_y(\theta(t))R_x(\phi(t)) \quad (1)$$

<sup>2</sup> The parentheses ( $t$ ) are dropped out in some of the following equations to save space.

$$= \begin{bmatrix} c\psi c\theta & -s\psi c\theta + c\psi s\theta s\phi & s\psi s\theta s\phi & s\psi s\theta c\phi \\ s\psi c\theta & c\psi c\theta + s\psi s\theta s\phi & -c\psi s\theta s\phi & -c\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi & \end{bmatrix} \quad (2)$$

for the rotation of a vector from frame  $\mathcal{B}$  to frame  $\mathcal{O}$ , where  $\psi$ ,  $\theta$ , and  $\phi$  are, respectively, the yaw, pitch and roll angles. In the previous equations, cos, sin, and tan trigonometric functions are abbreviated using their first letter. The elementary rotations are given by

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}, \quad R_y(\theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix},$$

$$R_z(\psi) = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Similarly, the rotation from frame  $\mathcal{A}$  to frame  $\mathcal{B}$  is given by

$$R_{ba} = R_y(-\alpha(t))R_z(\beta(t)) = \begin{bmatrix} c\alpha c\beta & -c\alpha s\beta & -s\alpha \\ s\beta & c\beta & 0 \\ s\alpha c\beta & -s\alpha s\beta & c\alpha \end{bmatrix}, \quad (3)$$

where  $\alpha(t)$  represents the angle of attack (AoA) and  $\beta(t)$  is called the side slip angle. In the absence of wind, the kinematic speed is the same as the aerodynamic speed and is denoted  $V_a^a(t) = [v_a(t) \ 0 \ 0]^T$ . The superscript  $a$  added to the vector notation  $V_a(t)$  indicates that its components are represented in the aerodynamic frame  $\mathcal{A}$ . The same convention is used with superscripts  $o$  or  $b$  for any vector when its components are given in, respectively, frame  $\mathcal{O}$  or  $\mathcal{B}$ .

An important property of rotation matrices is that they are orthogonal, i.e. they are invertible and the inverse is equal to the transpose ( $R_{yz} = (R_{zy})^{-1} = (R_{zy})^T$ ).

The rotation speed vector of frame  $\mathcal{B}$  with respect to frame  $\mathcal{O}$  is denoted  $\Omega_{bo}$ .

Although very popular, Euler angles are limited by the gimbal lock problem. This manifests itself by the loss of distinguishability between the first and the third rotation angles, when the second one is of  $90^\circ$  (see Castillo et al. (2005)). This is an important issue for us as the conversion from hover to forward flight implies that the conditions for the gimbal lock problem are met. Different solutions exist, such as the use of a second set of vertical Euler angles, rotation matrices, quaternions, modified Rodrigues parameters and more.

### 3.2 System description

Figure 6 shows the delta-wing bi-motor aircraft design considered in this paper. The orientation of the body frame  $\mathcal{B}$  axes can be observed:  $x^b$  points towards the front of the aircraft and belongs to its plane of symmetry alongside  $z^b$  axis, which is perpendicular to  $x^b$  and points towards the Earth when the delta-wing is in forward horizontal flight. The  $y^b$  axis is perpendicular to the plane of symmetry and oriented towards the right of the delta-wing.  $x^b$ ,  $y^b$ , and  $z^b$  form a right-handed orthonormal coordinate system with the origin at the aircraft's centre of gravity (CG).

Each wing has one propeller-engine and one elevon, which can be controlled independently. The centre of rotation of each propeller is situated in the  $(x^b, y^b)$  plane and the

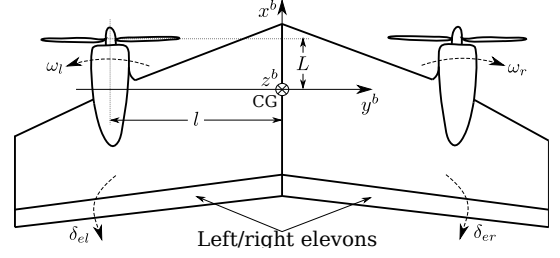


Fig. 6. Delta-wing bimotor aircraft (view from above). distance to the centre of gravity is given by the variable  $L$  on the  $x^b$  axis and the variable  $l$  on the  $y^b$  axis.

There are 4 control inputs:

- $\delta_{el}$ ,  $\delta_{er}$  are the deflection of the left and right elevons;
- $\omega_l$ ,  $\omega_r$  are the rotation speeds of the left and right propeller-engines.

The wings are cut out of extruded polystyrene using a hot-wire technique. The profile used for the wings is NACA0012.

### 3.3 Forces and moments acting on the delta-wing

The main efforts on the aircraft are of gravitational and aerodynamic natures. The later are due either to fixed (surfaces of the left and right wings) or rotating wings (propellers). The gravitational force can be written in frame  $\mathcal{O}$ :

$$G^o = [0 \ 0 \ mg]^T. \quad (4)$$

Aerodynamic forces and moments of fixed wings are most easily represented in frame  $\mathcal{A}$ . It is useful to distinguish between longitudinal and lateral forces and moments. Flight in the vertical plane is influenced by lift and drag forces and the moment around the lateral axis  $y^a$  (the same as  $y^b$  during longitudinal flight):

$$F_{lift}(t) = \frac{1}{2}\rho v_a^2(t) S C_x(\alpha, q, \delta_e), \quad (5)$$

$$F_{drag}(t) = \frac{1}{2}\rho v_a^2(t) S C_z(\alpha, q, \delta_e), \quad (6)$$

$$\tau_m(t) = \frac{1}{2}\rho v_a^2(t) S c C_m(\alpha, q, \delta_e). \quad (7)$$

These are most significantly influenced by the angle of attack  $\alpha(t)$ , the pitch rate  $q(t)$ , and the control  $\delta_e(t) = \delta_{el}(t) + \delta_{er}(t)$  (see also Beard and McLain (2012)). The lateral force and moments, on the other hand, depend on the side-slip angle  $\beta(t)$ , the roll rate  $p(t)$ , the yaw rate  $r(t)$ , and the control  $\delta_a(t) = \delta_{el}(t) - \delta_{er}(t)$ :

$$F_y(t) = \frac{1}{2}\rho v_a^2(t) S C_y(\beta, p, r, \delta_a), \quad (8)$$

$$\tau_l(t) = \frac{1}{2}\rho v_a^2(t) S b C_l(\beta, p, r, \delta_a), \quad (9)$$

$$\tau_n(t) = \frac{1}{2}\rho v_a^2(t) S b C_n(\beta, p, r, \delta_a). \quad (10)$$

In the previous equations,  $S$  is the area of the delta-wing,  $c$  is the mean chord,  $b$  is the wingspan of the aircraft, and  $\rho$  is the density of the air. The aerodynamic force and moment vectors in frame  $\mathcal{A}$  become  $F_a^a(t) = [-F_{lift} \ F_y(t) \ -F_{drag}]^T$  and  $\tau_a^a(t) = [\tau_l(t) \ \tau_m(t) \ \tau_n(t)]^T$ .



A common representation used for the longitudinal aerodynamic coefficients non-linear functions is given by:

$$C_x(\alpha, q, \delta_e) = C_{x_\alpha}(\alpha) + C_{x_q} \frac{c}{2v_a} q(t) + C_{x_{\delta_e}} \delta_e(t) \quad (11)$$

$$C_z(\alpha, q, \delta_e) = C_{z_\alpha}(\alpha, \text{Re}) + C_{z_q} \frac{c}{2v_a} q(t) + C_{z_{\delta_e}} \delta_e(t) \quad (12)$$

$$C_m(\alpha, q, \delta_e) = C_{m_\alpha}(\alpha) + C_{m_q} \frac{c}{2v_a} q(t) + C_{m_{\delta_e}} \delta_e(t) \quad (13)$$

Lift, drag, and pitch moment are non-linear functions depending primarily on the angle of attack  $\alpha(t)$ . One solution to approximate them (proposed in Phung (2015)) uses sigmoid functions as

$$C_{x_\alpha}(\alpha) = c_{1_x} + c_{2_x} \sin^2(\alpha) \quad (14)$$

$$C_{z_\alpha}(\alpha, \text{Re}) = c_{1_z} \sin(2\alpha) \sigma(\alpha, \alpha_0(\text{Re})) + c_{2_z} \sin(2\alpha) (1 - \sigma(\alpha, \alpha_0(\text{Re}))) \quad (15)$$

$$C_{m_\alpha}(\alpha) = \frac{1}{2} - \sin^2\left(\frac{\pi}{2} + \frac{\alpha}{2}\right). \quad (16)$$

In (15),  $\alpha_0(\text{Re})$  represents the stall angle, which depends on the Reynolds number (Re), and  $\sigma(\alpha, \alpha_0(\text{Re}))$  denotes the sigmoid function:

$$\sigma(\alpha, \alpha_0(\text{Re})) = \frac{1}{1 + e^{\alpha - \alpha_0(\text{Re})}} + \frac{1}{1 + e^{180^\circ - \alpha - \alpha_0(\text{Re})}}.$$

For various Reynolds numbers, the stall angle  $\alpha_0(\text{Re})$  can be obtained either using wind-tunnel or computational fluid dynamics (CFD) studies.

For lateral aerodynamics, the following functions are commonly used:

$$C_y = C_{y_0} + C_{y_\beta} \beta + C_{y_p} \frac{b}{2v_a} p + C_{y_r} \frac{b}{2v_a} r + C_{y_{\delta_a}} \delta_a \quad (17)$$

$$C_l = C_{l_0} + C_{l_\beta} \beta + C_{l_p} \frac{b}{2v_a} p + C_{l_r} \frac{b}{2v_a} r + C_{l_{\delta_a}} \delta_a \quad (18)$$

$$C_n = C_{n_0} + C_{n_\beta} \beta + C_{n_p} \frac{b}{2v_a} p + C_{n_r} \frac{b}{2v_a} r + C_{n_{\delta_a}} \delta_a \quad (19)$$

In Beard and McLain (2012) the following equations for the propulsion force and torque of one propeller are proposed, respectively,

$$f_{p_i}(t) = \frac{1}{2} \rho S_{prop} \left[ (k_{motor} \delta_i(t))^2 - v_a^2(t) \right], \quad (20)$$

$$t_{p_i}(t) = k_t [k_\Omega \delta_i(t)]^2, \quad (21)$$

where  $i \in \{l, r\}$  designates either the *left* or the *right* propeller and  $S_{prop}$  is the area swept out by the propeller. The vector of propulsion forces is given, in the body frame  $\mathcal{B}$ , by:

$$F_p^b(t) = [f_{p_l}(t) + f_{p_r}(t) \quad 0 \quad 0]^T. \quad (22)$$

Taking into account also that the two propellers have opposite senses of rotation, the propeller induced torque on the body of the convertible aircraft is given by

$$M_t^b(t) = [t_{p_l}(t) - t_{p_r}(t) \quad 0 \quad 0]^T. \quad (23)$$

Each propulsion force induces also a moment that can be written (in frame  $\mathcal{B}$ ), as:

$$M_{p_i}^b(t) = \overrightarrow{(CG, O_i)}|_{\mathcal{B}} \wedge [f_{p_i}(t) \quad 0 \quad 0]^T. \quad (24)$$

In the previous equation,  $\overrightarrow{(CG, O_i)}|_{\mathcal{B}}$  gives the coordinates of the point of application for each of the two propulsion forces in the Body frame ( $i \in \{l, r\}$ ). Considering a symmetric positioning of the two propellers in the plane

formed by  $x^b$  and  $y^b$  and with distances  $x_p$  on axis  $x^b$  and  $y_p$  on  $y^b$ , then the total moment from the propulsion forces can be written as

$$M_p^b(t) = [0 \quad 0 \quad y_p \cdot (f_{p_l}(t) - f_{p_r}(t))]^T. \quad (25)$$

Let denote the total force vector from propulsion and aerodynamic elements, in frame  $\mathcal{A}$ , as

$$F^a(t) = \begin{bmatrix} X^a(t) \\ Y^a(t) \\ Z^a(t) \end{bmatrix} = F_a^a(t) + R_{ab} F_p^b(t). \quad (26)$$

The total torque vector is given, in frame  $\mathcal{B}$ , by

$$\tau^b(t) = \begin{bmatrix} L^b(t) \\ M^b(t) \\ N^b(t) \end{bmatrix} = \tau_a^b(t) + M_t^b(t) + M_p^b(t). \quad (27)$$

### 3.4 Estimation of aerodynamic coefficient functions

Wind-tunnel aerodynamic coefficients data for the NACA 0012 profile are given in Sheldahl and Klimas (1981) for angles of attack (AoA) ranging from <sup>3</sup> 0° to 180° and different Reynolds numbers in the range  $4 \cdot 10^4$  to  $7 \cdot 10^5$ . These data points can be interpolated to obtain continuous variations with respect to the AoA and the Reynolds number using sigmoid and trigonometric functions as described in (Phung, 2015).

For some common symmetrical NACA profiles, wind-tunnel data are given in Sheldahl and Klimas (1981). Students applied the previously described technique to the lift, drag and pitch moment coefficients. Figure 7 shows a comparison of the wind-tunnel data and the estimation of the lift coefficient. Drag coefficient estimation is compared to the wind-tunnel data in Fig. 8.

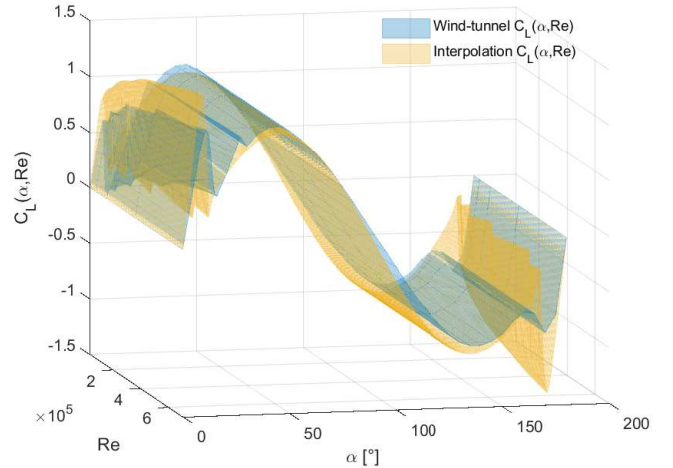


Fig. 7. Lift coefficient wind-tunnel data compared with the interpolation result.

### 3.5 Non-linear dynamic model

There are different ways of choosing the frames for writing the translational and rotational equations of motion. Here, we have chosen to write them both in frame  $\mathcal{B}$  using the Newton-Euler formalism (another solution is given in

<sup>3</sup> For symmetric profiles, this is the same as having  $-180^\circ$  to  $180^\circ$ .

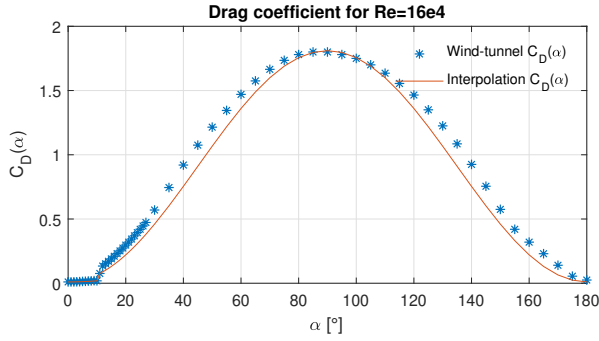


Fig. 8. Drag coefficient interpolation compared to wind-tunnel data for  $Re=16e4$ .

Airimitoiaie et al. (2018)). The complete set of differential equations is given by

$$\dot{\xi} = R_{ob} V_a^b, \quad (28)$$

$$m \frac{dV_a^b}{dt} + \Omega_{bo}^b \wedge m V_a^b = R_{ba} F^a + R_{ob}^T G^o, \quad (29)$$

$$\dot{R}_{ob} = R_{ob} [\Omega_{bo}^b]_{\times}, \quad (30)$$

$$\frac{d(I \Omega_{bo}^b)}{dt} + \Omega_{bo}^b \wedge I \Omega_{bo}^b = \tau^b, \quad (31)$$

where  $\xi = [x \ y \ z]^T$  gives the position of the center of mass of the aircraft relative to frame  $\mathcal{I}$  and  $I$  is the aircraft matrix of inertia

$$I = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}. \quad (32)$$

The elements of  $\Omega_{bo}^b$  are denoted  $\Omega_{bo}^b = [p \ q \ r]^T$  and  $[\cdot]_{\times}$  defines the skew-symmetric operator:

$$[\Omega_{ao}^a]_{\times} = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}. \quad (33)$$

Equations (28)-(31) together with formulation of forces and moments given in Section 3.3 can be used to write a non-linear simulator of the convertible aircraft. This is also the first step before design the autopilot control law.

#### 4. CONCLUSIONS

This paper describes the progress of the convertible aircraft project at the University of Bordeaux with emphasis on the results obtained during student projects. The complexity of this project stands on its multidisciplinary nature. An important aspect is the effective transmission of results between different activities. The final beneficiary of the entire work are the students working on the automatic controller design, which have to integrate information from other student teams into their own work. This can be seen in the modelling part described in this paper. Ongoing work focuses on the design of the autopilot control law using a feedforward-feedback architecture.

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