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1 A general model for welding of ash particles in volcanic systems
2 validated using *in situ* x-ray tomography

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14 *sintering; porosity; synchrotron; surface tension; tomography; tuffisite; jet engine; obsidian*

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16

17 **Abstract:**

18 **Welding occurs during transport and deposition of volcanic particles in diverse settings,**
19 **including pyroclastic density currents, volcanic conduits, and jet engines. Welding rate**
20 **influences hazard-relevant processes, and is sensitive to water concentration in the melt.**
21 **We characterize welding of fragments of crystal-free, water-supersaturated rhyolitic**
22 **glass at high temperature using *in-situ* synchrotron-source x-ray tomography.**
23 **Continuous measurement of evolving porosity and pore-space geometry reveals that**
24 **porosity decays to a percolation threshold of 1 – 3 vol.%, at which bubbles become**
25 **isolated and welding ceases. We develop a new mathematical model for this process that**
26 **combines sintering and water diffusion, which fits experimental data without requiring**
27 **empirically-adjusted parameters. A key advance is that the model is valid for systems in**
28 **which welding is driven by confining pressure, surface tension, or a combination of the**
29 **two. We use the model to constrain welding timescales in a wide range of volcanic settings.**
30 **We find that volcanic systems span the regime divide between capillary welding in which**
31 **surface tension is important, and pressure welding in which confining pressure is**
32 **important. Our model predicts that welding timescales in nature span seconds to years**
33 **and that this is dominantly dependent on the particle viscosity or the evolution of this**
34 **viscosity during particle degassing. We provide user-friendly tools, written in Python™**
35 **and in Excel™, to solve for the evolution of porosity and dissolved water concentration**
36 **during welding for user-defined initial conditions.**

37

38 Key points:

- 39 (1) First *in situ* determination of the welding rates of hydrous magma.
40 (2) A new mathematical model that couples volatile mass transfer with welding kinetics.
41 (3) This work provides a model relevant for ignimbrite and tuffsite welding

42 **1. Introduction**

43 Magma fragments into particles during explosive volcanic activity. Subsequent welding of
44 these particles can occur at the base of hot pyroclastic density currents (Walker 1983; Branney
45 et al. 1992), at the walls of volcanic conduits (Gonnermann and Manga 2003; Rust et al. 2004;
46 Gardner et al. 2017), in tuffisite veins (Tuffen et al. 2003; Kendrick et al. 2016; Gardner et al.
47 2018), in the hot zone of jet engines (Giehl et al. 2016), and when lightning strikes volcanic
48 ash in the air or on the ground (Cimarelli et al. 2017; Mueller et al. 2018). Despite this wide
49 range of welding scenarios, there has been little work on the physics of welding of volcanic
50 droplets, beyond simple empirical, semi-empirical, or scaling approaches (Friedman et al.
51 1963; Riehle 1973; Sparks et al. 1999; Quane and Russell 2005a; Russell and Quane 2005;
52 Vasseur et al. 2013; Wadsworth et al. 2014).

53 Welding involves a reduction of inter-particle pore space (Branney and Kokelaar 1992;
54 Sparks et al. 1999; Quane and Russell 2005a; Vasseur et al. 2013). Porosity is therefore a
55 convenient metric for tracking the degree of welding, and has been used to rank the ‘grade’ of
56 a welded deposit (Quane and Russell 2005b; Wright and Cashman 2014). Theoretical models
57 for the evolution of porosity as a function of time in a welding system have been proposed
58 (Frenkel 1945; Mackenzie and Shuttleworth 1949) but they do not account for the complexities
59 of welding in magmatic systems, which include non-isothermal behavior, disequilibrium of
60 dissolved volatile species, and the effect of a confining pressure that pushes the particles
61 together. Non-isothermal behavior is important because welding in nature may occur as the
62 particles cool (e.g. at conduit margins, within ejected ballistic bombs, or in ignimbrites) or
63 follow more complex heating and cooling pathways (e.g. in a jet engine). Disequilibrium of
64 volatile species – particularly of water – is important because the solubility changes as the
65 pressure and temperature environment of the particles changes, driving diffusion in or out of
66 the particles during welding (Sparks et al. 1999; Gardner et al. 2018). In the case of water, this

67 has a strong impact on the viscosity of the particle (Hess and Dingwell 1996) affecting welding
68 rate (Grunder et al. 2005; Gardner et al. 2018, 2019). Confining (or lithostatic) pressure
69 resulting, for example, from the weight of aggrading particles at the base of a pyroclastic
70 density current, is important because it provides a stress that pushes the droplets together,
71 accelerating welding. Previous theoretical and quantitative models for welding have focused
72 on cases where welding is driven by surface tension alone (Wadsworth et al. 2016) or, where
73 pressure is considered, have relied on scaling arguments (Sparks et al. 1999) or purely
74 empirical correlations (e.g. Riehle 1973).

75 We develop a general and versatile mathematical framework for welding that can be used
76 to predict the textural evolution of a welding pack of particles in a wide range of natural
77 settings. We perform and analyze experiments conducted under non-isothermal, disequilibrium
78 conditions to validate the model.

79

80 **2. A theoretical model for droplet welding dynamics**

81 **2.1 Viscous welding under arbitrary pressure**

82 Previous work on non-volcanic welding (or ‘sintering’) of spherical particles has shown that
83 in the viscous state (i.e. when particles are droplets), surface-tension-driven welding is well
84 described by a ‘vented bubble model’, in which the inter-droplet porosity is abstracted as a
85 system of spherical bubbles in liquid shells, which are ‘vented’ so that the gas can escape as
86 the bubbles shrink (Mackenzie and Shuttleworth 1949; Wadsworth et al. 2016). The geometric
87 assumptions of the vented bubble model are most valid for highly polydisperse particle
88 distributions (Wadsworth et al. 2017b) of the sort typical in nature, and the approximation
89 becomes increasingly accurate as welding progresses because, as the droplets coalesce, the
90 microstructural geometry continuously diverges from ‘droplet-like’ towards ‘bubble-like’ –
91 that is, there is a topological inversion of the pore space (Wadsworth et al. 2017a). We start

92 from the assumption that the vented bubble model also applies to initially-angular particles.
 93 The conceptual steps in the geometric abstraction from an ash pack to a system of vented
 94 bubbles are shown in Figure 1.

95 We extend the vented bubble model to include a confining pressure that acts alongside
 96 surface tension stress to drive welding, where we use the term *confining* pressure to refer to an
 97 isotropic pressure acting to push the particles together – equivalently, when viewing the
 98 particles as viscous droplets, an isotropic pressure in the continuous liquid phase of the
 99 coalescing droplets. In their supplementary material, Wadsworth et al. (2016) derive the vented
 100 bubble model from the model of Prousevitch et al. (1993) for bubble growth in magma by
 101 setting the bubble pressure inside the associated liquid shell to be equal to the gas pressure
 102 outside the shell at all times. Here, we relax that assumption, and instead set the pressure
 103 difference to a value ΔP . Neglecting inertia, the full equation for the inter-droplet porosity ϕ
 104 with time t is then

$$\frac{d\phi}{dt} = -\frac{3\Delta P}{4\mu}\phi - \frac{3\Gamma}{2\mu a_i} \left(\frac{\phi_i}{1-\phi_i}\right)^{1/3} \phi^{2/3}(1-\phi)^{1/3}, \quad \text{Eq. 1}$$

106
 107 where ΔP is the difference between the confining pressure on the liquid droplets P_l and the
 108 pressure of the interstitial gas P_g , such that $\Delta P = P_l - P_g$, μ is the particle viscosity, Γ is the
 109 interfacial tension between the particles and the gas, a_i is the initial size of the bubble, and ϕ_i
 110 is the initial porosity when welding starts. A derivation of Eq. 1 from the Rayleigh-Plesset
 111 equation is given in the Supplementary Information.

112 Eq. 1 can be cast in dimensionless form by normalizing time to a characteristic capillary
 113 timescale $\lambda = \mu a_i / \Gamma$, such that $\bar{t} = t / \lambda$, normalizing pressure to a capillary pressure scale $P_c =$

114 $2\Gamma/a_i$, such that $\bar{P} = \Delta P/P_c$, and normalizing porosity to its initial value, such that $\bar{\phi} = \phi/\phi_i$,

115 yielding

116

$$\frac{d\bar{\phi}}{d\bar{t}} = -\frac{3}{2} \left[\bar{P}\bar{\phi} + \left(\frac{1 - \phi_i\bar{\phi}}{1 - \phi_i} \right)^{1/3} \bar{\phi}^{2/3} \right], \quad \text{Eq. 2}$$

117

118 where a bar above a parameter denotes that it has been rendered dimensionless. The first term

119 within the square brackets represents the contribution of the confining pressure, the second

120 term represents the contribution of the capillary (Laplace) pressure.

121 The dimensionless time \bar{t} can be generalized to account for non-isothermal

122 temperature–time history, which is especially useful for natural magmatic scenarios. This is

123 achieved by accounting for the change in viscosity μ as temperature varies, via

124

$$\bar{t} = \frac{t}{\lambda} = \frac{\Gamma}{a_i} \int_{t_i}^t \frac{1}{\mu} dt \quad \text{Eq. 3}$$

125

126 where t_i is the time at which the welding process starts. In casting Eq. 3 this way, we assume

127 that Γ is a constant, independent of time during welding. In reality, Γ is dependent on both

128 water concentration and temperature, however, variations in surface tension are negligible

129 compared with variations in liquid viscosity arising from the effects of both temperature and

130 water concentration (discussed later). Together, Eqs 2 & 3 represent a universal description of

131 isotropic, viscous particle welding, derived from micromechanical first principles. In the case

132 where $\bar{P} = 0$, this approach has been validated against experimental data across a large range

133 of temperatures (Wadsworth et al. 2016). While other models exist, they are either less easy to

134 use, requiring a switch-point between two competing processes (Prado et al. 2001), or they rely

135 on bulk properties of the system, which have to be empirically determined and are therefore
136 less general as they are not constructed from the micromechanics involved (Olevsky 1998;
137 Quane and Russell 2005a).

138 The parameter a_i can be difficult to measure, or even define, for what is a complex,
139 interconnected pore network (Figure 1). We use a relationship between a_i and the distribution
140 of particle sizes $F(R)$ in a pack of particles or droplets (Lu and Torquato 1992), which is
141 described in detail elsewhere (Wadsworth et al. 2016, 2017b). The relationship relies on
142 knowledge of ϕ_i , and the moments of the distribution of R , denoted $\langle R^n \rangle$, which can be
143 grouped into a polydispersivity factor $S = \langle R \rangle \langle R^2 \rangle / \langle R^3 \rangle$. The output from this is a pore size
144 distribution $f(a_i)$ that relates to the particle size distribution $F(R)$ and ϕ_i , and is described in
145 the Supplementary Information along with an account of how this is used in conjunction with
146 Eqs 1-2 using convolution techniques (Wadsworth et al. 2017b).

147

148 **2.2 Accounting for diffusion of volatiles during welding**

149 In the model formulation above, the viscosity μ is assumed to be dependent on temperature, T ,
150 only. However, viscosity also depends on the concentration of water dissolved in the melt,
151 which, in nature, may vary during welding. Volcanic particles formed at fragmentation may be
152 super-saturated in dissolved water (Giachetti and Gonnermann 2013), and that super-saturation
153 can grow as the particles ascend rapidly to lower pressures up-conduit without time to fully re-
154 equilibrate (Gardner et al. 2017). Similarly, the solubility of water increases as the particles
155 cool, which may cause them to re-hydrate (McIntosh et al. 2014; Ryan et al. 2015). Mass
156 diffusion of water in or out of particles may occur on timescales similar to the timescale of
157 welding (Sparks et al. 1999; Gardner et al. 2017, 2018, 2019); consequently, we anticipate that
158 diffusion of water can affect the rate of welding through its impact on melt viscosity.

159 In order to account for diffusion, we must define the geometry of the internal welding
160 system. The internal geometry of welding systems is complex and evolves from an initial state
161 of particles in a gas continuum, to bubbles isolated in a liquid continuum. A rigorous solution
162 for mass diffusion through this evolving geometry would require an approach that explicitly
163 resolves both the fluid motion and the diffusion. However, as for the welding model, we
164 simplify the problem by abstracting the geometry. For the purposes of diffusion modeling we
165 assume that the particles remain spherical and simply apply Fick's 2nd law in spherical
166 coordinates,

167

$$\frac{\partial C}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial C}{\partial r} \right),$$

Eq. 4

168

169 where C is the concentration of water in the melt particle, D is its diffusivity (which depends
170 on temperature and local water concentration), and r is the radial position from the particle
171 center. We adopt the assumption that the initial concentration of dissolved water C_i is uniform
172 throughout the particle at the onset of welding, giving the initial condition $C = C_i$ for all r at
173 $t = 0$. At all later times the water concentration at the surface of the particle is given by the
174 equilibrium solubility C_e at the current conditions of gas pressure P_g and temperature T , giving
175 the boundary condition $C = C_e(t)$ at $r = R$ for $t > 0$. We define a zero-flux boundary
176 condition at the center of the particle: $\partial C / \partial r = 0$ at $r = 0$.

177

178 To account for the effect of variable water concentration in the particle we determine a
179 spatial average by integrating C over $0 < r < R$; this integral is $\langle C \rangle = \int_0^1 C d\bar{r}$, where $\bar{r} =$
180 r/R . We then use $\langle C \rangle$ to compute an average viscosity $\langle \mu \rangle$ which is used in Eqs 1-3 in place of
181 μ . This approach results in an effective coupling between the diffusion model (Eq. 4) and the
welding model (Eqs 1-3). In the Supplementary Information, we describe the numerical

182 solution of these equations in detail. We note here that our assumption of spherical particles
183 undergoing diffusion of volatiles may be invalid at large polydispersivity for cases where the
184 smallest particles are in equilibrium while the largest particles are far from equilibrium. This
185 is discussed elsewhere (Gardner et al. 2019).

186

187 **3. Experimental validation: materials and methods**

188 Our starting material is a metaluminous, tholeiitic natural rhyolitic glass collected from
189 Hrafninnuhryggur, Krafla (Iceland). We crushed the obsidian to a powder using an agate
190 mortar and pestle and then crushed the resultant chips to a fine powder using a concussion ball
191 mill for short durations to prevent the sample heating significantly. The powder was sieved to
192 $< 125 \mu\text{m}$ diameter pieces and the size distribution was measured using a Beckman Coulter
193 LSTM 230 laser refraction particle size analyzer with the measuring range $0.375 - 1000 \mu\text{m}$
194 diameter. The particle size distribution is given in the Supplementary Information, and has a
195 mean radius $\langle R \rangle = 2.2 \times 10^{-5} \text{ m}$.

196 Using a Netzsch Pegasus 404c device for simultaneous thermal analysis, we determined
197 the dissolved volatile concentration that is excess (above solubility) at up to 1325 K to be $C_i =$
198 $0.15 \pm 0.02 \text{ wt.}\%$, by the relative loss of mass during heating, consistent with Tuffen and
199 Castro (2009). This determination was performed on single chips ($n = 6$) from within a few
200 millimeters of the sub-sample of the glass block that was used throughout this study.

201 To solve the governing equations given in our model, we require a parameterization for
202 D , μ , and C_e relevant to the material in question. We use models relevant to the metaluminous
203 rhyolites (Hess and Dingwell 1996; Liu et al. 2005; Zhang and Ni 2010)

204

$$\log_{10}(\mu) = -3.545 + 0.833 \ln(C) + \frac{9601 - 2368 \ln(C)}{T - 195.7 - 32.25 \ln(C)}$$

$$D = C \exp \left[-18.1 + 0.001888P_l - \left(\frac{9699 + 3.626P_l}{T} \right) \right] \quad \text{Eq. 5}$$

$$C_e = \frac{354.941P_w^{0.5} + 9.623P_w - 1.5223P_w^{1.5}}{T} + 0.0012439P_w^{1.5}$$

205

206 where P_w is the partial pressure of water in the interstitial gas phase, and the coefficients given
 207 are valid when T is in K, P_l and P_w are in MPa, and C and C_e are in wt.%. In the Supplementary
 208 Information we independently verify $\mu(T)$ for our material using a parallel plate compression
 209 method (Hess et al. 2007), a calorimetric method using a shift factor of 10.4 (Gottsmann et al.
 210 2002), and a micropenetration method (Hess et al. 1995), which all demonstrate internal
 211 consistency, as well as matching the prediction of Eq. 5 for the measured C_i . We take a value
 212 $P_w = P_l\alpha$ with $\alpha = 0.2$, representing the typical humidity pressure in a laboratory furnace.
 213 Finally, we use $\Gamma = 0.3 \text{ N.m}^{-1}$.

214 We performed two sets of *in situ* high temperature experiments, which used different
 215 methods to image the evolution of a welding pack of obsidian powder, lightly pressed into a
 216 free-standing cylinder with 3 mm diameter: (1) synchrotron-source x-ray tomography,
 217 providing continuous 3-dimensional microstructural data; and (2) optical dilatometry,
 218 providing bulk sample volume changes only. The first set of experiments were performed at
 219 the TOMCAT beamline of the Swiss Light Source at the Paul Scherrer Institute. Cylinders of
 220 obsidian powder were loaded into the imaging window of the x-ray beam path. We used a laser
 221 system (Fife et al. 2012) to heat an alumina sleeve (muffle) placed over the samples, thus
 222 heating the obsidian pack indirectly. The temperatures measured by a pyrometer were
 223 calibrated by comparing the *in situ* welding of a well-studied sample of monodisperse glass
 224 beads with *ex situ* characterization of the same process (Wadsworth et al. 2016), resulting in a
 225 continuous correction for T , and confirming that temperature gradients on the sample scale
 226 were negligible. Full 3-dimensional tomographs were collected at $5.5 \times 10^{-3} \text{ Hz}$ with a spatial

227 resolution of 1.6 μm . The second set of experiments were performed using a Hesse Instruments
228 EM-201 optical dilatometer, which continuously records the silhouette of the sample during
229 heating at 1 Hz. Volume is determined from the silhouette as the solid of revolution.

230 The experiments covered a wide range of conditions, including isothermal
231 experiments at temperatures of 1050 – 1500 K, and linear heating ramps at rates of 0.04 –
232 0.25 $\text{K}\cdot\text{s}^{-1}$. The data from optical dilatometry are confined to porosity $\phi(t)$, while the
233 tomography provides 3-dimensional data that are used to measure both the total porosity $\phi(t)$,
234 and the porosity that is connected across the sample (or segmented domain) $\phi_p(t)$. After the
235 heating experiments, we repeated the thermal analysis step on a few sub-samples, and no mass
236 loss was observed, demonstrating the samples did equilibrate volatiles during the *in situ*
237 experiments.

238 Supplementing our datasets collected at relatively low C_i and a small difference
239 between C_i and C_e (small initial supersaturation), we re-analyse the data from Gardner et al.
240 (2018) and Gardner et al. (2019). These data were collected at high P_g such that the equilibrium
241 water concentration is also high. These data also include particles that hydrate and particles
242 that de-hydrate while also welding.

243 All of the above experiments were conducted without confining pressure, hence $\bar{P} \approx 0$, a
244 situation typical of small-scale laboratory settings. In order to examine the effect of $\bar{P} > 0$, we
245 use data for welding PyrexTM glass presented previously (Friedman et al. 1963). In those
246 experiments the glass was crushed to a particle size around 100 – 250 μm (not specified
247 exactly), and welded under a uniaxial liquid pressure of $P_l = 1.52 \times 10^6$ Pa and $P_l = 3.63 \times$
248 10^6 Pa at temperatures 883 – 943 K. We re-analyze these data using our model. While the
249 same authors provide data for rhyolite particle welding (Friedman et al. 1963), some doubt
250 exists as to the exact pressures used (Sparks et al. 1999) so we choose not to reanalyze those
251 data.

252

253 4. Results, data analysis and model validation

254 *In situ* tomography allows us to render the evolving internal pore space of the samples in 3-
255 dimensions through the welding process. Figure 2 shows a typical experimental result. The
256 pore space is initially fully interconnected, and has a complex geometry between the angular
257 glass fragments. As we heat the sample, the glass particles relax to liquid droplets at high
258 temperature, and the droplet–droplet contacts weld. The porosity decreases with time smoothly
259 and monotonically, and the rate at which it decreases depends strongly on temperature (in the
260 isothermal experiments) or heating rate (in the non-isothermal experiments). The connectivity
261 of the pore space drops during welding from fully connected at the start ($\phi_p/\phi = 1$), to fully
262 closed at volume equilibrium ($\phi_p/\phi = 0$). The porosity at which the connectivity drops to zero
263 is the percolation threshold porosity ϕ_c below which the system is impermeable. We determine
264 this from our experiments as $\phi_c = 0.02 \pm 0.019$, which agrees with theory, simulations (Elam
265 et al. 1984; Vasseur and Wadsworth 2017), and experiments (Wadsworth et al. 2016) in other
266 welding droplet or overlapping sphere systems. This value is far lower than the percolation
267 threshold for bubbly systems, as has previously been noted for the internal geometry of welding
268 systems (Vasseur and Wadsworth 2017).

269 In Figures 2g & 3, we compare the results for the *in situ* x-ray tomography and optical
270 dilatometry experiments with the model presented in section 2. For these unconfined
271 laboratory-scale tests, the value of \bar{P} is effectively 0. For the isothermal experiments we solve
272 Eq. 2 with $\bar{P} = 0$, accounting for the diffusion of water out of the particles during welding via
273 Eqs 3 & 4 (Figures 3a & 3b). For the non-isothermal experiments we additionally account for
274 temperature change via Eq. 3 (Figure 3c). In both cases, we find good agreement and a
275 reasonable collapse of the data to the model. This result highlights that, in these experiments,
276 welding rates are influenced by temperature and volatile content, both of which control the

277 particle viscosity and can evolve on the same timescale as the welding; hence, they must be
278 solved explicitly (Eqs 3 & 4). Welding rate also depends on particle size distribution, interfacial
279 tension, and the initial porosity of the packed particles. Particle angularity may subtly affect
280 both the diffusion rate and the sintering rate when compared with the model, which is based on
281 idealised spherical particles, but, given the good agreement between model and data, this effect
282 does not appear to be of first-order.

283 The experimental validation of our simple model for $\bar{P} = 0$ appears to be successful. To
284 extend this to conditions where $\bar{P} > 0$, we re-analyse the results from Friedman et al. (1963)
285 in which anhydrous glass was heated under pressure. In the Supplementary Information we
286 give a detailed description of the methods used in Friedman et al (1963), but note here that Eqs
287 1 and 2 are valid in their experiments. We assume that the uniaxial nature of their applied loads
288 can be accounted for using the Trouton ratio, such that our model for isotropic pressurization
289 can be adapted to uniaxial conditions. For their anhydrous experiments, conducted at $\bar{P} = 100$,
290 we apply Eq. 2 directly. We find good agreement across a wide range of temperature (Figure
291 4), validating our model up to naturally relevant pressures and across the regime boundary $\bar{P} =$
292 1. These conditions represent the state where the volcanic particles are under pressure, but the
293 interconnected gas phase between the particles remain un-pressurized, which is a typical
294 scenario for larger systems in nature. We show that, in this case, the time required for welding
295 to complete is reduced by the elevated confining pressure, as implied by Eq. 2 (note how the
296 data collapse to a model curve to the left of the $\bar{P} = 0$ curve in Figure 4, and are therefore
297 welding more rapidly).

298

299 5. Discussion

300 5.1 Validity of the welding model

301 The welding model agrees well with experimental data across a wide range of conditions,
302 validating the model for application to welding systems: 1) with and without applied pressure;
303 2) under isothermal and non-isothermal conditions; 3) in which dissolved water is in
304 equilibrium or disequilibrium with ambient pressure and temperature conditions. Thus the
305 model is sufficiently general to capture most of the essential features of welding scenarios in
306 magmatic and volcanic systems. There are, however, two limitations. Firstly, the model does
307 not apply directly to welding under high shearing stress. We do use experimental data in which
308 welding particles are under uniaxial compression (Friedman et al. 1963), such that the
309 anisotropy of the pressure applied results in shear stresses internal to the sample (c.f. uniaxial
310 experiments in Quane and Russell 2005; Heap et al. 2015), which we account for via the
311 Trouton ratio. This gives us some confidence that, under minor local shearing within a system
312 that is loaded anisotropically, our model is valid. Nonetheless, we note that validation for
313 shearing systems requires future systematic study over a larger range of better constrained
314 shear stress. This limitation means that our model does not, for example, explicitly predict the
315 formation of fiammé in welded ignimbrites. Secondly, the model assumes that interstitial gas
316 escapes freely from the welding system, and does not apply when gas escape is significantly
317 hindered by the permeability of the connected pore network. We can determine the conditions
318 under which this second limitation is important.

319 The characteristic lengthscale beyond which a viscous system is permeability-limited
320 is the compaction length $L_c = (k_r \mu / \mu_g)^{1/2}$ (Michaut et al. 2009; Kennedy et al. 2016), where
321 k_r is a reference permeability and μ_g is the gas viscosity. If we normalize our system length
322 by L_c we have

323

$$\bar{L} = \frac{L}{L_c} \approx L \sqrt{\frac{\mu_g}{k_r \mu}}. \quad \text{Eq. 6}$$

324

325 If $\bar{L} \gg 1$, permeability is a rate-limiting parameter, with the consequence that P_g may rise in
 326 parts of the system and affect the welding rate (because the welding rate depends is sensitive
 327 to $P_c - P_g$); this regime is termed *compaction welding*. If $\bar{L} \ll 1$, gas escape can occur more
 328 rapidly than welding occurs and the process is not hindered by sluggish gas escape. The
 329 analysis presented via Eqs 1-4 is therefore valid in the regime $\bar{L} \ll 1$.

330

331 5.2 Welding regimes in nature

332 The dimensionless length \bar{L} discriminates between regimes in which welding is or is not limited
 333 by permeable outgassing of the interstitial gas phase. For the non-limited regime ($\bar{L} \ll 1$) in
 334 which our welding model is valid, we can also discriminate between regimes in which welding
 335 is dominated by confining pressure (*pressure welding*), or by capillary pressure arising from
 336 the surface tension (*capillary welding*). From Eq. 2, we see that the confining pressure term
 337 dominates the capillary pressure term when

338

$$\begin{aligned} \bar{P} &\gg \left(\frac{1 - \phi_i \bar{\phi}}{\bar{\phi} - \bar{\phi} \phi_i} \right)^{1/3} && \text{pressure welding,} \\ \bar{P} &\ll \left(\frac{1 - \phi_i \bar{\phi}}{\bar{\phi} - \bar{\phi} \phi_i} \right)^{1/3} && \text{capillary welding.} \end{aligned} \quad \text{Eq. 7}$$

339

340 The dependence on the porosity (via $\bar{\phi}$) arises because capillary stress always tends towards
 341 infinity as bubble radius tends towards zero, such that a system that starts in the pressure
 342 welding regime may end in the capillary regime with no change in the ambient conditions. The
 343 value of the terms on the right hand side of Eq. 7 is equal to unity at the start of welding so, in

344 practice we use $\bar{P} \gg 1$ and $\bar{P} \ll 1$ to discriminate between pressure and capillary welding
345 regimes.

346 Given these constraints (\bar{P} and \bar{L}), we can assess the regimes covered by some typical
347 volcanic welding scenarios. For any situation in which welding might occur, we therefore need
348 to know μ , R_i , L , and ΔP (for simplicity, we take a constant $\Gamma = 0.3 \text{ N} \cdot \text{m}^{-1}$ (Wadsworth et al.
349 2016), $\mu_g \approx 10^{-5} \text{ Pa} \cdot \text{s}$, and $k_r = k_i \approx 7 \times 10^{-12} \text{ m}^2$ for packed particles (Wadsworth et al.
350 2017a)). For these dimensional considerations we assume that $a_i = R_i$, because there is usually
351 insufficient information provided to compute a_i explicitly, and we justify this by noting that
352 these values are typically of the same order of magnitude (Wadsworth et al. 2016). In Table 1
353 we give a compilation of estimated values for these parameters gathered from well-studied
354 welded ignimbrites, tuffisites, welded jet engine deposits, and welded obsidian pyroclasts. In
355 each case, these parameters are converted to a quantitative range of \bar{P} and \bar{L} that represents the
356 initial conditions for that particular system, and plotted in Figure 5. *In the case of tuffisites
357 from Volcán Colima, we use the values of particle (droplet) viscosity from Kendrick et al.
358 (2016) which incorporate the effect of crystallinity.*

359 We find that very few systems are in the $\bar{L} \gg 1$ (permeability limited) regime.
360 Exceptions would include welding in particularly large welded ignimbrite systems *if* the
361 emplacement mode is *en masse* (we give the result for the $L \leq 400 \text{ m}$ Bad Step Tuff, for which
362 \bar{L} can exceed unity). However, under the assumption of the progressive-aggradation model for
363 the sedimentation of ignimbrites (Branney and Kokelaar 1992) and an estimated, rising $L \approx 2$
364 m thick welding window (Andrews and Branney 2011), we find that $\bar{L} \ll 1$ is more typical for
365 ignimbrite emplacement. *The en masse and progressive aggradation models for ignimbrite
366 emplacement represent upper and lower bounds on \bar{L} , respectively.*

367 We also find that volcanic welding scenarios span the $\bar{P} = 1$ divide, implying that there
368 are cases for which pressure welding dominates ($\bar{P} > 1$) and cases for which capillary welding

369 dominates ($0 \leq |\bar{P}| < 1$). Pressure welding appears to be typical of tuffisites and ignimbrites,
 370 while capillary welding appears to be typical of the formation of obsidian pyroclasts and of
 371 undesirable welding in the combustion chamber of jet engines (Figure 5). Tuffisites, in
 372 particular, are known to have variable and complex pressure–temperature histories, implying
 373 that they may track through \bar{P} space during their formation and welding (Tuffen and Dingwell
 374 2005; Castro et al. 2012; Saubin et al. 2016). If the exact evolution of pressure and temperature
 375 were known, then our model could be used to determine the degree of welding throughout.

376 For each of the cases presented in Figure 5, we can compute a timescale for the porosity
 377 to reach the equilibrium value $\phi = \phi_c$. For systems welding at any \bar{P} , for $\bar{L} \ll 1$, this timescale
 378 includes contributions from the pressure and capillary components, and can be taken as the
 379 reciprocal of the sum of the characteristic welding rates associated with the confining pressure
 380 and capillary pressure terms

381

$$\lambda_w \approx \left(\frac{\Delta P}{\langle \mu \rangle} + \frac{\Gamma}{\langle \mu \rangle \langle R_i \rangle} \right)^{-1} . \quad \text{Eq. 8}$$

382

383 We use Eq. 8 and the inputs in Table 1 to compute λ_w (in seconds) for each case study example
 384 given. We find that most systems weld over a timescale of 1 second to 1 day. Exceptions,
 385 which require very long timescales to weld, are crystal-rich tuffisites (Kendrick et al. 2016) or
 386 obsidian pyroclasts welding under the lowest temperature and gas pressure conditions expected
 387 (Gardner et al. 2017). In both of those slow-welding scenarios, it is unlikely that welding will
 388 complete before other processes, such as cooling of the particles or deposit, terminate welding.
 389 However, remarkably, the welding timescale for most rhyolitic systems investigated appears
 390 to span a similar range regardless of the \bar{P} of formation; hence $\langle \mu \rangle$, and therefore the degassing

391 and temperature history, is the most important controlling parameter in welding for rhyolitic
392 magmas.

393

394 **5.3 User-friendly computational tools for solving welding problems in volcanic scenarios in** 395 **Python™ and Excel™**

396 As part of this contribution, we provide a downloadable executable file for Linux™ and Mac
397 platforms, which solves the full diffusion–welding problem given here. The executable
398 requires the following user inputs: particle size distribution (as a .txt or .csv file), initial
399 porosity ϕ_i , initial dissolved water concentration C_i , initial temperature, gas pressure P_g ,
400 pressure differential $\Delta P = P_l - P_g$, surface tension Γ , and the spatial resolution for the
401 diffusion solution (we set a default value of 100 steps, which is sufficient for most cases). We
402 additionally allow the user to input a temperature rate, which should be positive for heating,
403 negative for cooling, or zero for isothermal conditions, and which imposes a linear change in
404 temperature. The outputs of this code are the monodisperse or polydisperse solutions for
405 porosity as a function of time and the value of $\langle C \rangle$. Similarly, we provide an editable Excel™
406 sheet for solving our welding code for isothermal or non-isothermal conditions including for
407 polydisperse particles (or droplets), but without diffusion of volatiles. This code is available
408 via VHub (<https://vhub.org/resources/4568>).

409

410

411 **6 Concluding remarks**

412 We present a universal theoretical model of welding of natural volcanic material at relevant
413 volcanic conditions. The model includes the complex effect of syn-welding dehydration,
414 accounts for the effects of confining pressure and capillary pressure, and is valid for both

415 isothermal and non-isothermal conditions. The welding model is grounded in the microphysical
416 behaviour of a welding system, and requires no fitting parameters.

417 We use scaling arguments to assess the validity of the model for natural welding
418 scenarios, and conclude that it can be applied to welding in tuffisites, in volcanic conduits, at
419 the base of aggrading pyroclastic density currents, and in jet engines. Our model predicts that
420 volcanic systems span the divide between the regime in which the capillary stress at particle
421 walls drives welding, and the regime in which the driving pressure for welding is the difference
422 between the liquid and the interstitial gas pressures. We find that in most cases examined here,
423 the permeability of the interstitial gas phase does not limit the welding dynamics. Finally, we
424 find that the total time required for complete welding spans seconds to years, and that the
425 viscosity of the particles, or the evolution of viscosity during particle degassing, is the most
426 variable parameter in nature.

427 The model we present provides a flexible and general tool for investigating welding
428 phenomena across a wide range of volcanically-relevant scenarios. The model solution is given
429 for the specific case of rhyolite welding via a vHub resource.

430

431

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445

446 Author contributions to this work

447 The team at the TOMCAT beamline (PSI, Switzerland) included F.B. Wadsworth (team lead),
448 J. Vasseur, E.W. Llewellyn, K.J. Dobson, F.W. von Aulock, J. Fife, and F. Marone. J. Schauroth
449 and K.J. Dobson processed the 3D datasets and extracted raw data. F.B. Wadsworth, J. Vasseur,
450 and E.W. Llewellyn performed the analysis and model development. F.B. Wadsworth, J.
451 Schauroth, J.E. Gardner, K.-U. Hess, M.J. Heap, H. Tuffen, and D.B. Dingwell provided
452 additional *ex situ* or calibration data for analysis and consulted on the data treatment and the
453 manuscript. F.B. Wadsworth, J. Schauroth, and E.W. Llewellyn led the drafting of a manuscript
454 text with help from all authors. T. Havard compiled data from natural deposits.

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602

Table 1 Constraints underpinning \bar{P} and \bar{L} from natural deposits or scenarios

	Particle viscosity [#] μ (Pa.s)	System length L (m)	Particle radius [§] R_i (m)	Liquid pressure P_l (Pa)	Gas pressure P_g (Pa)	Initial water content** C_i (wt.%)	Emplacement temperature** T (K)	References
Welded ignimbrites								
Bad Step Tuff	10^7 - 10^8	40-400*	10^5 - 10^3	10^6 - 10^7	10^5 - 10^6	0.1-0.2	1273	(Branney et al. 1992)
TL	10^5 - 10^6	10.4-28*	10^5 - 10^3	3.1 - 7.5×10^5	10^5 - 10^6	0.47-0.87	1084-1183	(Sumner and Branney 2002)
Grey's Landing	10^7 - 10^9	2-70*	10^5 - 10^3	10^5 - 10^6	10^5 - 10^6	0.1-0.2	1198-1298	(Andrews and Branney 2011)
Rattlesnake tuff	10^8 - 10^{10}	15-70*	10^5 - 10^3	4.1×10^5 - 10^6	10^5 - 10^6	0.1-0.2	1073-1153	(Streck and Grunder 1995)
Tuffsites								
Chaitén (2008)	10^7 - 10^9	0.005-0.03	1.25×10^{-4} - 2.5×10^{-4}	10^6 - 10^7	4.6×10^6 - 8.1×10^6	0.44-1.2	1023-1098	(Castro et al. 2012; Saubin et al. 2016)
Cordon Caulle 2011-2013	10^8 - 10^9	0.005-0.03	10^6 - 10^3	10^6 - 10^7	4.6×10^6 - 8.1×10^6	0.16-0.25	1168	(Castro et al. 2014)
Colima	10^{10} - 10^{11}	0.001-0.05	10^4 - 10^3	10^5 - 10^6	10^5	0.1-0.2	1213-1253	(Kendrick et al. 2016)
Törfajökull	10^9 - 10^{14}	0.001-0.05	10^5 - 10^3	10^6 - 10^7	4.6×10^6 - 8.1×10^6			(Tuffen and Dingwell 2005)
Obsidian pyroclasts								
Mono craters	10^6 - 10^{12}	0.01-0.02	2×10^{-5} - 1.7×10^{-4}	1.002×10^5	10^5			(Gardner et al. 2017)
Jet engine deposits								
Rhyolitic experimental	10^3 - 10^9	10^{-3} - 10^{-2}	3×10^{-5}	4.0002×10^6	4×10^6	0.1	1148-1848	(Giehl et al. 2016)

[#]The viscosity is either taken from the references for each case study or otherwise is calculated using *Hess and Dingwell*, [1996] with the C_i and T given for most cases (exceptions are the TL ignimbrite and the basaltic example for the jet engine deposits, both of which are calculated using *Giordano et al.*, [2008] and the composition given in the references; for TL, we use the WTL trachyte zone composition because this is the 'lava like' facies (Sumner and Branney 2002)).

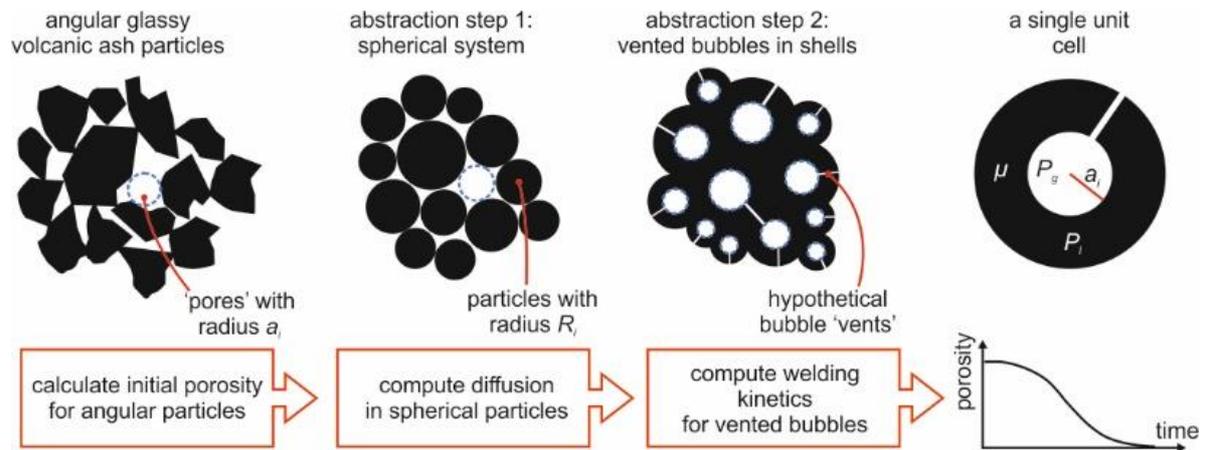
^{*}This system length is assuming *en masse* deposition, but for progressive aggradation we take 2 m for all ignimbrites

[†]We take this liquid pressure to be the hydrostatic loading pressure assuming a density of 2300 kg.m^{-3} . Except for the tuffsite cases, the gas pressure is added to the liquid pressure.

[‡]The upper limit of these gas pressures is given by estimates of dynamic pressures during transport in pyroclastic density currents (Clarke and Voight 2000).

^{**}Note that these parameters are only required if the viscosity is not given directly by the originating authors. The value for C_i is approximated as 0.1-0.2 wt.% if other information is not given.

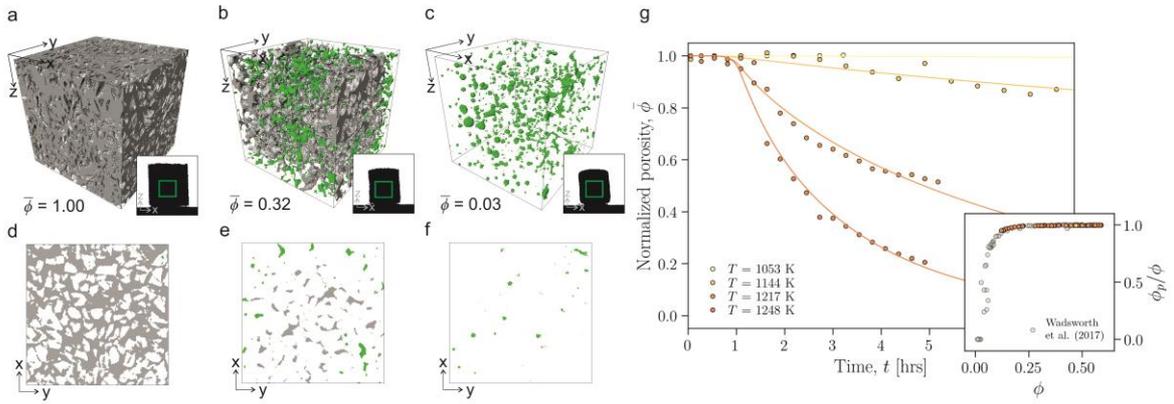
[§]We make the simplifying assumption that $a_i = R_i$ for this scaling analysis.



605

606 **Figure 1.** The development of a diffusion–welding model. The natural system is composed of
 607 angular volcanic particles, which are abstracted to a pack of spherical particles with the same
 608 initial porosity. The diffusion model is used (Eq. 3) to compute the average water content as a
 609 function of time $\langle C \rangle(R_i, t)$, which is converted to an average viscosity $\langle \mu \rangle$. Then we make a
 610 further abstraction to vented bubble geometry and the welding is computed in terms of an
 611 evolution of the total porosity with time $\phi(t)$ using Eqs 1-2, accounting for the polydispersivity
 612 of the initial particle size distribution (Wadsworth et al. 2017b).

613



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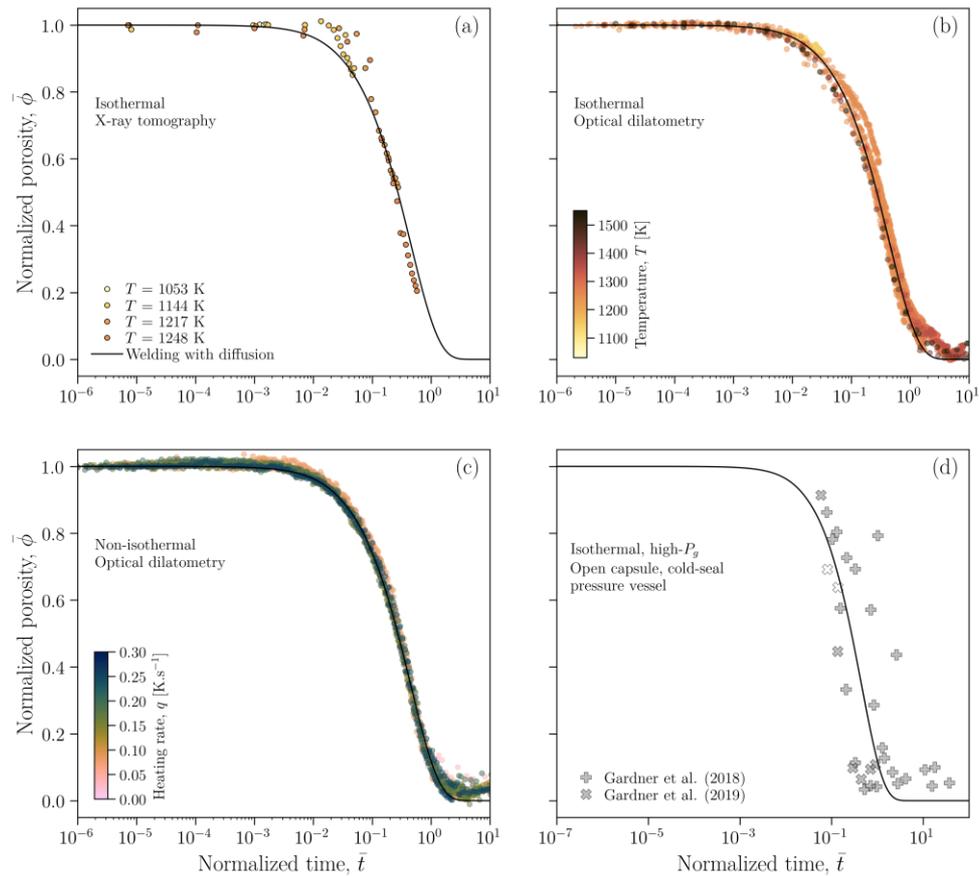
615 [Full page width]

616 **Figure 2.** Constraints of time-dependent welding collected *in situ* using either optical
 617 dilatometry or x-ray tomography. **a-c**, 3-dimensional rendered images of the time-dependent
 618 welding process segmented on the basis of grayscale gradient filtering from continuous, time-
 619 resolved, *in situ* x-ray tomography data. The particle phase is rendered transparent, and the
 620 pore phase is divided into a grey and a green component depending on whether it is connected
 621 across the sample in any direction (grey) or is isolated from connections (green). Box edge
 622 lengths of the sub-volume displayed are 350 μm . Displayed is one representative experiment
 623 performed at $T = 1350 \text{ K}$, for which the dimensionless porosity $\bar{\phi}$ is labelled. Inset in each
 624 panel is the 2-dimensional side-view of an initially cylindrical sample from an experiment at
 625 the same conditions performed in the optical dilatometer (image base length of 5 mm). **d-f**, a
 626 2-dimensional horizontal slice through each of the 3-dimensional rendered images in **a-c** taken
 627 at the midpoint of the z-axis in the sample. **g**, The porosity as a function of time of the obsidian
 628 particles sintered *in situ* using time-resolved x-ray tomography at a range of temperatures
 629 (labelled). The curves represent the solutions to Eqs 1 or 2 with $\bar{P} = 0$ and computing the
 630 time-dependent diffusion of volatiles out of the particles (Eq. 4). *Inset*: the connectivity of the
 631 pore phase with porosity showing the collapse from fully connected to isolated as $\phi \rightarrow \phi_c$

632 during welding. Data are compared with welding in synthetic glass systems (Wadsworth et al.
633 2017a).

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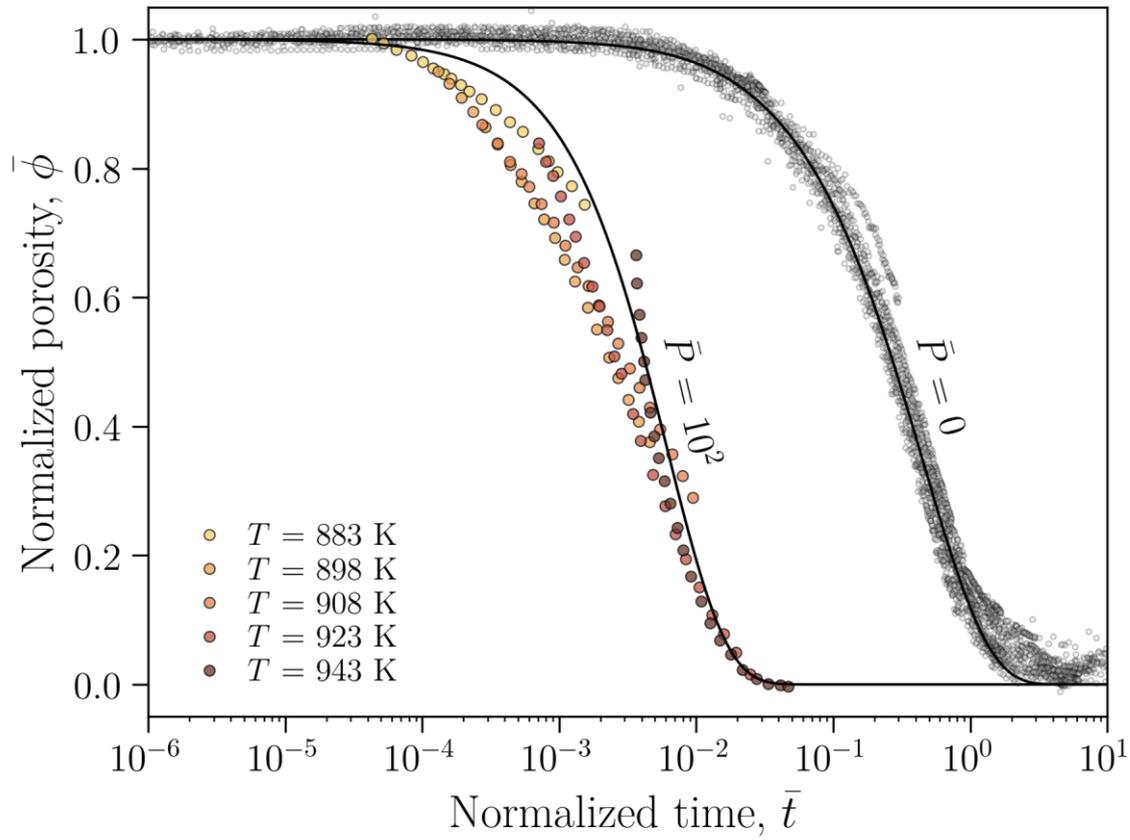


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638 **Figure 3.** Modeling the evolution of the pore phase between the welding droplets. **a**, the
 639 porosity $\bar{\phi}$ as a function of time for each experimental T for the x-ray tomography experiments
 640 only. These data are compared with the results of Eq. 2 (welding) with Eq. 4 (diffusion), which
 641 account for syn-welding degassing of the droplets and which require no fitting parameters.
 642 These data are the same as those presented in Figure 2g. **b**, the same as **a** but for the data
 643 collected using optical dilatometry (i.e. without 3-dimensionl microstructural information)
 644 showing that this technique can be used to capture the bulk decay of porosity with time. **c**, the
 645 same dimensionless plot as in **b** but for non-isothermal experiments at different experimental
 646 heating rates, showing that regardless of the $T(t)$ path taken by the samples, the efficacy of our
 647 model (Eqs 4-6) is robust. In all panels, the dimensionless time is given by Eq. 3. **d**, The data
 648 from Gardner et al. (2018) and Gardner et al. (2019) collected using a high- P_g cold seal vessel,
 649 re-analysed using the diffusion-welding model given here. The filled points represent data for

650 which rhyolite particles are hydrating (from $C_i = 0.15$ wt.% to equilibrium conditions at high
651 P_g and high T) while welding, and the un-filled points represent data for which rhyolite particles
652 are de-hydrating (from $C_i = 2.3$ wt.% to equilibrium conditions at high P_g and high T). See the
653 papers originating the data for more information.
654



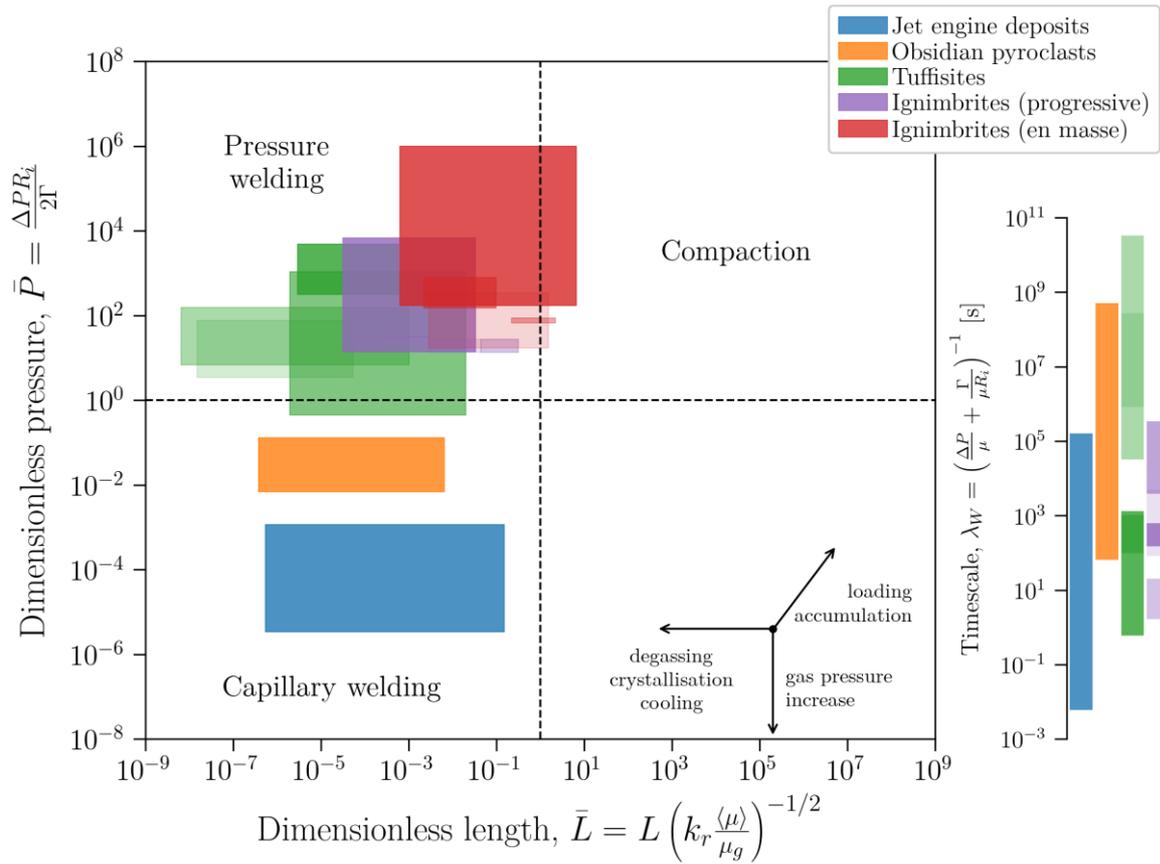
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657 [half-page width or less]

658 **Figure 4.** The effect of pressure on the welding of glassy particles. Shown here are data from
 659 *Friedman et al.*, [1963], in which synthetic glass particles (Pyrex™) are welded under a
 660 pressure equivalent to $\bar{P} = 100$, and at a range of temperatures (labelled). We use the values
 661 of μ , R_i and P_l given in their work, and the curve represents the solution to Eq. 2 without fitting
 662 parameters. Shown for reference is the solution for $\bar{P} = 0$ and all data from Figure 3 given in
 663 grey. In all cases, the dimensionless time is given by Eq. 3.

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670 **Figure 5.** Scenarios and regimes for volcanic welding. A plot of \bar{P} and \bar{L} regimes for volcanic

671 welding with the range of conditions for natural examples given using constraints compiled in

672 Table 1.