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A scalable estimator of space varying blurs

Application in super-resolution

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Abstract—We propose a scalable method to find a low dimensional subspace of spatially varying blur operators, given a set of noisy observations. This aims to improve the identifiability in blind inverse problems. We then show how to use this subspace estimator to train deep learning architectures for solving super-resolution problems in single molecule localization microscopy.

I. MOTIVATION

Many measurement devices suffer from variations over time and a single calibration is not enough to reflect all their possible states. Our aim here is to calibrate the device by learning a low-dimensional subspace of operators. From a mathematical standpoint, we assume that a collection of L operators $(H_l)_{1 \leq l \leq L}$ are sampled from a convex set $\mathcal{C} \subset \mathcal{H}$, where \mathcal{H} is a vector space of operators. We design two estimators $\hat{\mathcal{H}}$ and $\hat{\mathcal{C}}$ provided that each instance H_l is given with a product-convolution structure. We then show an application to super-resolution microscopy.

II. THE ESTIMATORS

A. Product-convolution

Product-convolution expansions are a family of decompositions that were analyzed recently in [6]. Given a discrete signal $u \in \mathbb{R}^n$, it takes the form

$$H_l(u) = \sum_{k=1}^K \alpha_{k,l} \star (\beta_{k,l} \odot u), \quad (1)$$

where $\alpha_{k,l} \in \mathbb{R}^n$ and $\beta_{k,l} \in \mathbb{R}^n$. This decomposition can be computed efficiently using fast Fourier transforms. It can be shown that the *space varying impulse response* (SVIR) S_l of a linear operator H_l of the form (1) can be written as $S_l = \sum_{k \in K} \alpha_{k,l} \otimes \beta_{k,l}$. Hence, assuming that H_l can be approximated by a product-convolution expansion is equivalent to saying that its SVIR is nearly low-rank.

This assumption covers many practical applications. For instance, a sufficient condition for an operator H_l to be well approximated by such a decomposition is that all the *impulse responses* $(S_l(\cdot, y))_{y \in Y}$ of the operators H_l can be simultaneously encoded in the basis $\text{span}(\alpha_{k,l}, k \in K)$. This is particularly relevant in imaging applications. In addition, we developed numerical strategies to estimate a space varying blur as a product-convolution operator by just observing fluorescent microbeads [1].

B. Subspace estimation

The principle of our approach is to find two orthogonal bases $(e_i)_{1 \leq i \leq I}$ and $(f_j)_{1 \leq j \leq J}$ such that all the operators S_l can be decomposed simultaneously as $S_l \simeq \sum_{i,j} \gamma_{i,j,l} e_i \otimes f_j$ for a well chosen matrix $\gamma_{\cdot,\cdot,l} \in \mathbb{R}^{I \times J}$. The estimate $\hat{\mathcal{H}}$ of the vector space of operators \mathcal{H} can then be identified with

$$E \otimes F \stackrel{\text{def}}{=} \text{span}(e_i \otimes f_j, 1 \leq i \leq I, 1 \leq j \leq J).$$

Given two integers I and J , we aim at solving:

$$\underset{\substack{(e_i)_{i \in I} \in \mathcal{E} \\ (f_j)_{j \in J} \in \mathcal{F}}}{\text{argmin}} \frac{1}{2} \sum_{l \in L} \|\Pi_{E \otimes F}(S_l) - S_l\|_F^2, \quad (2)$$

where $\Pi_{E \otimes F}$ is the projection onto $E \otimes F$ and \mathcal{E} and \mathcal{F} are the sets of orthogonal families. Problem (2) then exactly coincides with the Tucker2 model [7].

Problem (2) is a complex nonconvex problem. Empirically, we observed that the simple High Order Singular Value Decomposition (HOSVD) [4] led to satisfactory results in short computing times. We let $\hat{\mathcal{H}}_L = \hat{E} \otimes \hat{F}$ denote the subspace estimated using L observations. In [5], we provide rates of convergence of $\hat{\mathcal{H}}_L$ to the true subspace $\hat{\mathcal{H}}$ as L increases.

C. Convex hull estimation

If the operators S_l are drawn uniformly over a convex compact set \mathcal{C} , it is known [3] that the convex hull $\hat{\mathcal{C}}$ of those points is a minimax optimal estimator under the Nikodym metric.

III. APPLICATION: SUPER-RESOLUTION

Techniques such as single molecule localization (SLM) microscopy, allow to obtain sub-Nyquist images by finding the centroids of PSFs on a low resolution image. Nearly all computational approaches in this field rely on the assumption that the PSF is stationary. This assumption is unsound whenever a large field of view is used. In addition, the calibration is done once for all, while there are often significant variations of the PSFs due to changes of temperature, shifts of optical elements, ...

Recent approaches [2] tackle the SLM problem by training a well chosen deep learning architecture on random images synthesized using an appropriate forward model. This idea can be combined with the estimates above to tackle both the space and time variations of the microscope and handle significantly harder transfer functions than what is currently done. Given the estimate $\hat{\mathcal{C}}$, we can generate random operators with an arbitrary distribution supported on $\hat{\mathcal{C}}$ and then apply them on random discrete measures, allowing to synthesize realistic SLM images.

Using this approach and a realistic family of space varying blurs, we could decrease the average localization error from 0.5 pixels to 0.39 pixels using the recent neural network architecture DeepLoco [2]. A few examples are provided in Fig. 1.

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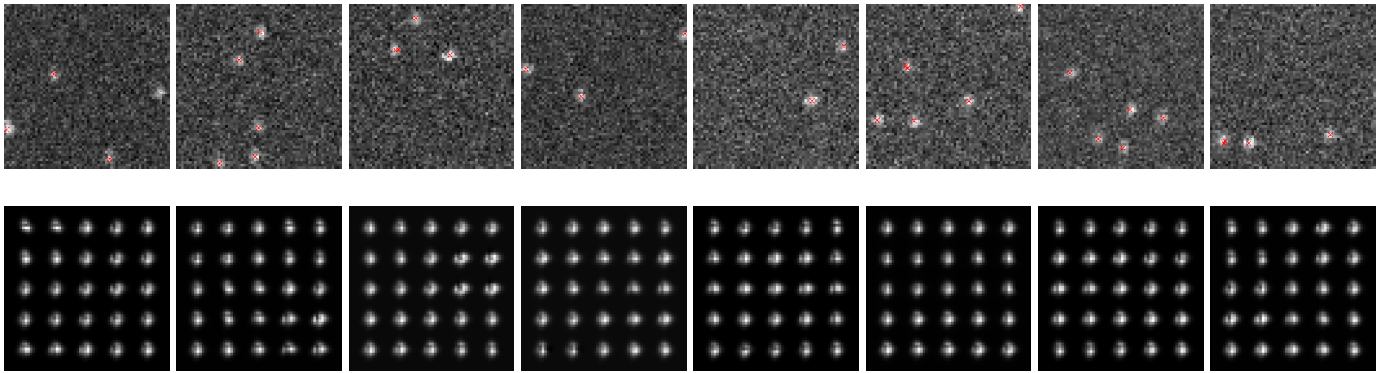


Fig. 1: Top: simulated images of molecules fluorescence. The true locations of the molecules are marked by a white circle. The predictions of DeepLoco are marked by a red cross. Bottom: forward operator corresponding to the above image apply to a grid of point.

REFERENCES

- [1] Jérémie Bigot, Paul Escande, and Pierre Weiss. Estimation of linear operators from scattered impulse responses. *Applied and Computational Harmonic Analysis*, 2017.
- [2] Nicholas Boyd, Eric Jonas, Hazen P Babcock, and Benjamin Recht. Deeploco: Fast 3d localization microscopy using neural networks. *BioRxiv*, page 267096, 2018.
- [3] Victor-Emmanuel Brunel. Methods in estimation of convex sets. <https://arxiv.org/abs/1709.03137>, 2017.
- [4] Lieven De Lathauwer, Bart De Moor, and Joos Vandewalle. A multilinear singular value decomposition. *SIAM journal on Matrix Analysis and Applications*, 21(4):1253–1278, 2000.
- [5] Valentin Debarnot, Paul Escande, and Pierre Weiss. A scalable estimator of sets of integral operators. *preprint*, 2019.
- [6] Paul Escande and Pierre Weiss. Approximation of integral operators using product-convolution expansions. *Journal of Mathematical Imaging and Vision*, 58(3):333–348, 2017.
- [7] Tamara G Kolda and Brett W Bader. Tensor decompositions and applications. *SIAM review*, 51(3):455–500, 2009.