

# Congestion, diseconomies of scale and subsidies in urban public transportation

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Online appendix v1

This online appendix describes the methodology used to estimate the value of several key parameters. The main parameters of interest are displayed in Table A.1 below.

Table A.1: Key estimates for the Piccadilly line

Parameter	Label	Value
$H_0$	Minimum safe headway (s)	111.8
$\delta$	Marginal dwelling time (s/user)	0.43
$A$	Maximum WTP (£)	14.12
$B$	Slope of WTP (£/user.km <sup>-1</sup> .h <sup>-1</sup> )	- 0.0040
$\alpha_C$	Maximum crowding penalty (£/trip)	1.65
$c_K$	Capital cost parameter (£/seat-km)	0.0425
$c_O$	Operating cost parameter (£/train.h)	1431.3

## A. System operating characteristics

To estimate the minimum safe headway  $H_0$  and the marginal dwelling time  $\delta$ , we use Equation (4) in the paper that characterizes the maximum feasible frequency as a linear function of demand  $N$ :

$$F = F_0(1 - \delta d_M N).$$

This constraint arises when demand exceeds a certain threshold, denoted by  $\hat{N}$  in the paper. Below this threshold ( $N \leq \hat{N}$ ), the operated frequency is expected to increase with demand, while above this threshold ( $N \geq \hat{N}$ ) the maximum frequency constraint becomes active, so that frequency is expected to decrease with demand as a result of congestion between vehicles.

In order to estimate Equation (4), we combine two datasets provided by Transport for London (TfL). The first dataset records realized frequencies for the Piccadilly tube line on a hourly basis for years 2013 and 2014.<sup>3</sup> The second dataset provides the number of entries per quarter of hour for all stations of the London underground for the same years. To rebuild the realized demand for the Piccadilly line in the westbound direction:

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<sup>3</sup> Data have been obtained from Transport for London ([https://www.whatdotheyknow.com/request/tube\\_trips\\_per\\_hour\\_by\\_line\\_and](https://www.whatdotheyknow.com/request/tube_trips_per_hour_by_line_and)).

### C. Crowding penalty

We use the paper by Whelan & Crockett (2009) which estimates the time multipliers associated with sitting and standing positions in the UK. Using their results and a capacity of 228 seats + 114 square meters available to stand, we can compute average time multipliers as a function of the load factor. We recall that a load factor equal to 1 means that 684 passengers are in the train (228 seating + 456 standing with a passenger density of 4 pax/m<sup>2</sup>). We linearly approximate the relationship between the load factor and the average time multiplier and find a slope of 0.7283 (see Figure A.1).<sup>7</sup> This means that the average time multiplier increases by 0.7283 when the load factor increases by 1.

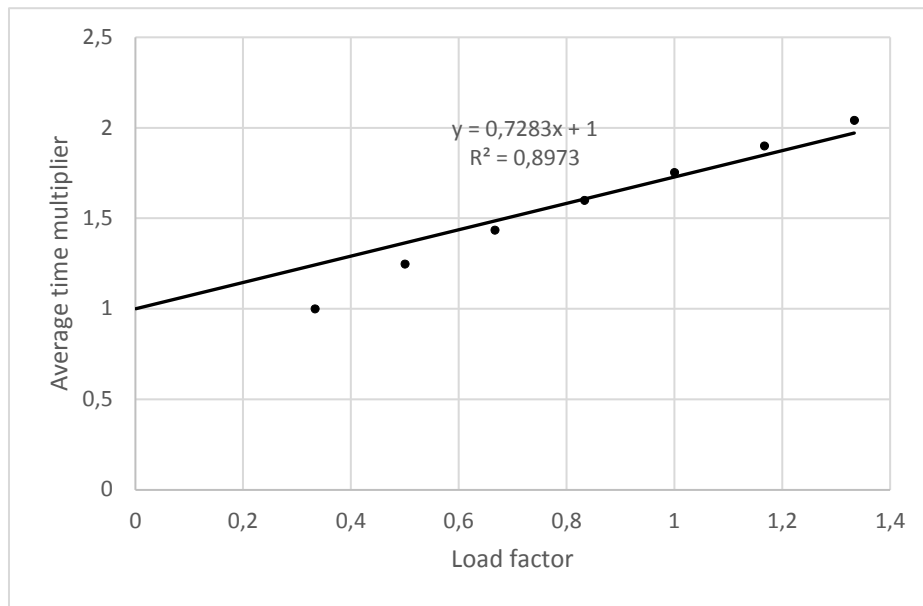


Figure A.1: Relationship between average time multiplier and load factor

The crowding cost is given by  $\alpha_C l_C$ , where  $l_C$  is the load factor and  $\alpha_C$  the maximum crowding penalty. The average trip distance on the Piccadilly line is 10.21 km<sup>8</sup> and the average speed during the morning peak is 33 km/h<sup>9</sup>. The average travel time is therefore about 18.6 minutes. The crowding cost per trip is  $l_C \times 0.7283 \times 7.33 \times \frac{18.56}{60} = l_C \times 1.65$  £/trip. It equals 1.65 £ when the load factor equals 100%.

### D. Vehicle capital and operating costs

Vehicle capital and operating costs are introduced in Equation (8):

$$C_{TA}(X, Z, s) = c_K s X + c_O Z.$$

We assume that the vehicle capital costs  $c_K s X$  are proportional to vehicle-kilometers  $X$  and vehicle capacity  $s$ , whereas other operating costs  $c_O Z$  only depend on vehicle-hours  $Z$ , and are independent of vehicle size.

<sup>7</sup> We constrain the intercept to be equal to 1, because the time multiplier equals 1 when the train is empty.

<sup>8</sup> Source: RODS 2017.

<sup>9</sup> Source: Inter station Database, available from

[https://www.whatdotheyknow.com/request/distance\\_between\\_adjacent\\_underg](https://www.whatdotheyknow.com/request/distance_between_adjacent_underg)