



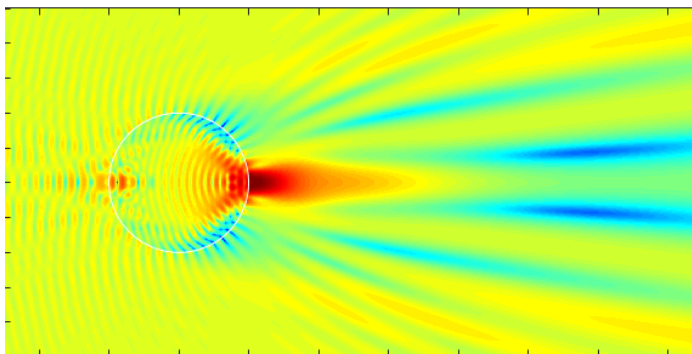
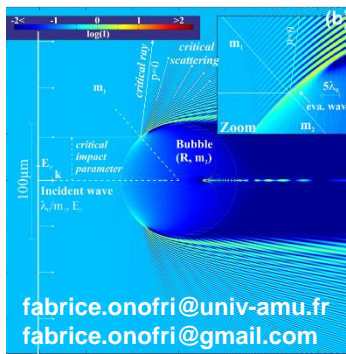
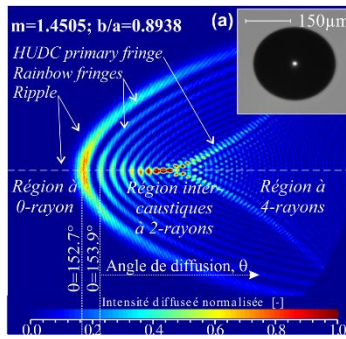
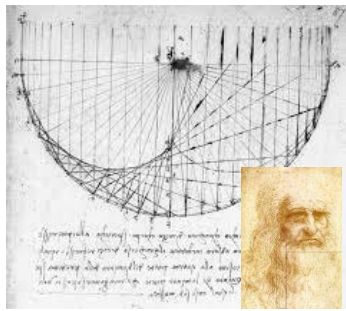
Optical catastrophes and singularities in relation with particle systems

Fabrice R.A. Onofri

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National Center for Scientific Research & Aix-Marseille Université
Marseille, France

with collaborations with **F. Lamadie** (CEA), **K-F Ren** (CORIA), **M. Sentis** (PhD & CEA)



Worshop, August 15-16th, 2018

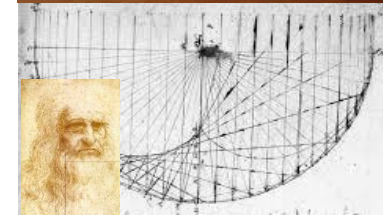
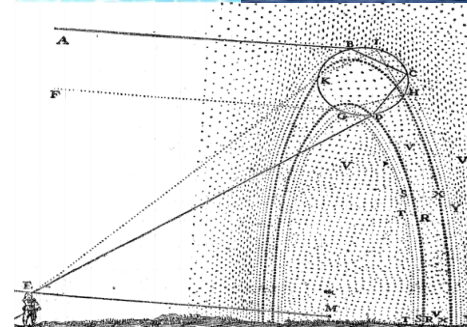


Xidian university

Caustics and optical singularities

From antiquity to nowadays

- Water caustics (...)
- Hot mirrors (Archimedes)
- Rainbow (al-Farisi, Theodoric of Freiberg, Descartes)
- Cup caustic (L. da Vinci)
- Diffraction (Grimaldi, Young, Fresnel)
- Optical catastrophes (R. Thom, V-I. Arnold, M. Berry...)

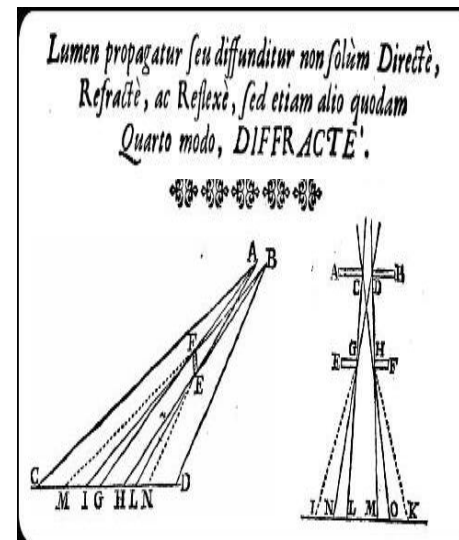


L. da Vinci, ~1520

R. Descartes, 1637

From nature to the laboratory

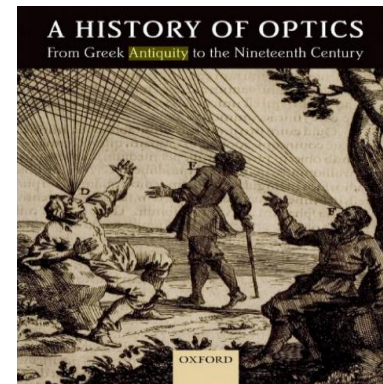
- Huge relative intensity
- Structural stability (wave dislocation lines: singularities of phase, wave vortices & zeros)
- Highly sensitivity to control parameters of interest
- Complete Geometrical Optics features
- Numerical alternative to electromagnetic approaches
- The most studied are connected to the shape and composition of droplets rainbows and diffraction (e.g., G.B. Airy, JR Fraunhofer, G. Davis)



F-M. Grimaldi, 1664



Archimedes, ~-212



e.g. O. Darrigol, 2012

Caustics and optical singularities

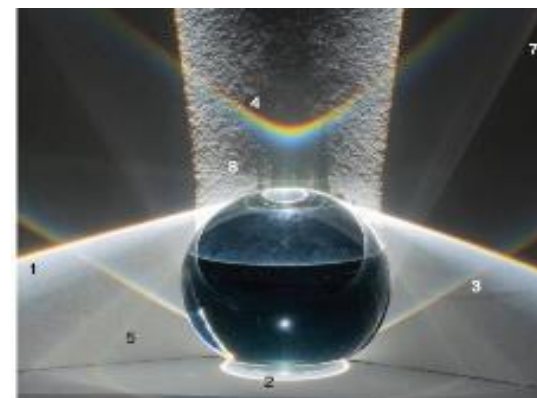
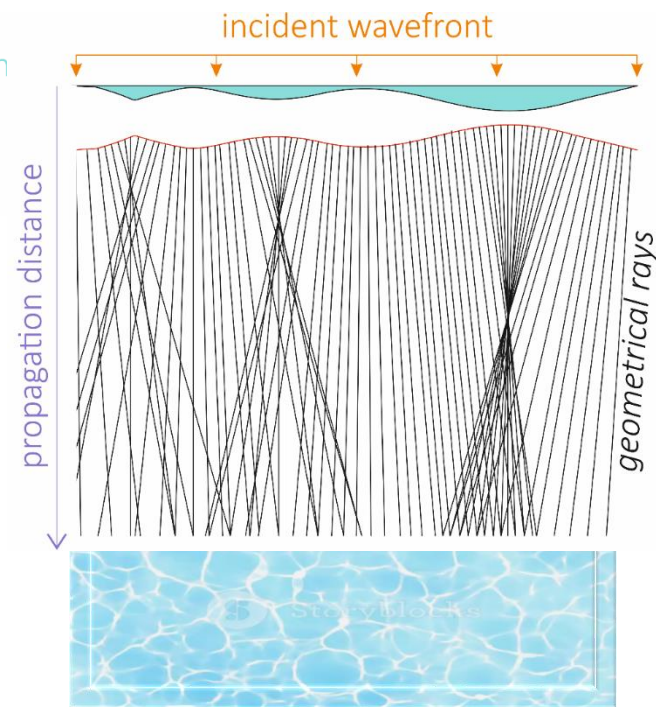
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- Optical catastrophes (R. Thom)

From Nature to the laboratory

- Huge relative intensity
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phase screen
 (object)
 transmitted
 wavefront



P-E Ouellette, JOSA A. 35(2018)

Caustics: optical catastrophes forms & classes in brief!!

Diffraction integral for optical catastrophes (*Berry, Phil. Trans. R. Soc. A 291, 1979*):

$$E_s \propto k^\beta \iint_{\text{Apert.}} E_0 e^{ikP} dx dy$$

$P(x, y; a, b, c, m, \dots)$: « potential » (“phase function” real or virtual) induced by the particle;
 x, y, \dots : state variable (spatiales/angulars); a, b, c, m : control parameters; β : Arnold index

Classification (*R. Thom, InterÉditions, 1977; V.I. Arnold, Springer 1986*)

e.g. 1st Rainbow produced by a droplet that is

- Spherical : A_2 fold caustic with one parameter and one control variable: $P \sim x^3 + a x$; $\beta=1/6$

(cf. *Theories of Young, and Airy 1838*)

- Oblate: D_4^+ umbilic hyperbolic (the “wave”) with parameters a, b & c and two output variables: x & y :

$$P \sim x^3 + y^3 + a x y + b x + c y; \beta=1/3$$

(cf. *Marston, Nye, Nature 312-1984*)



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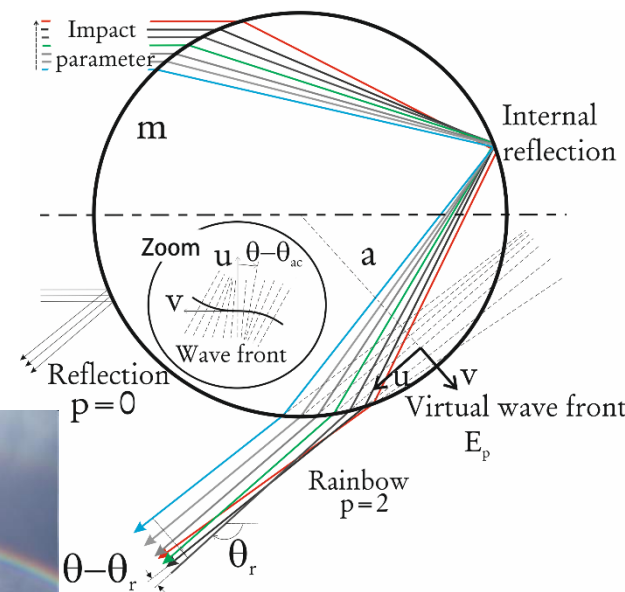
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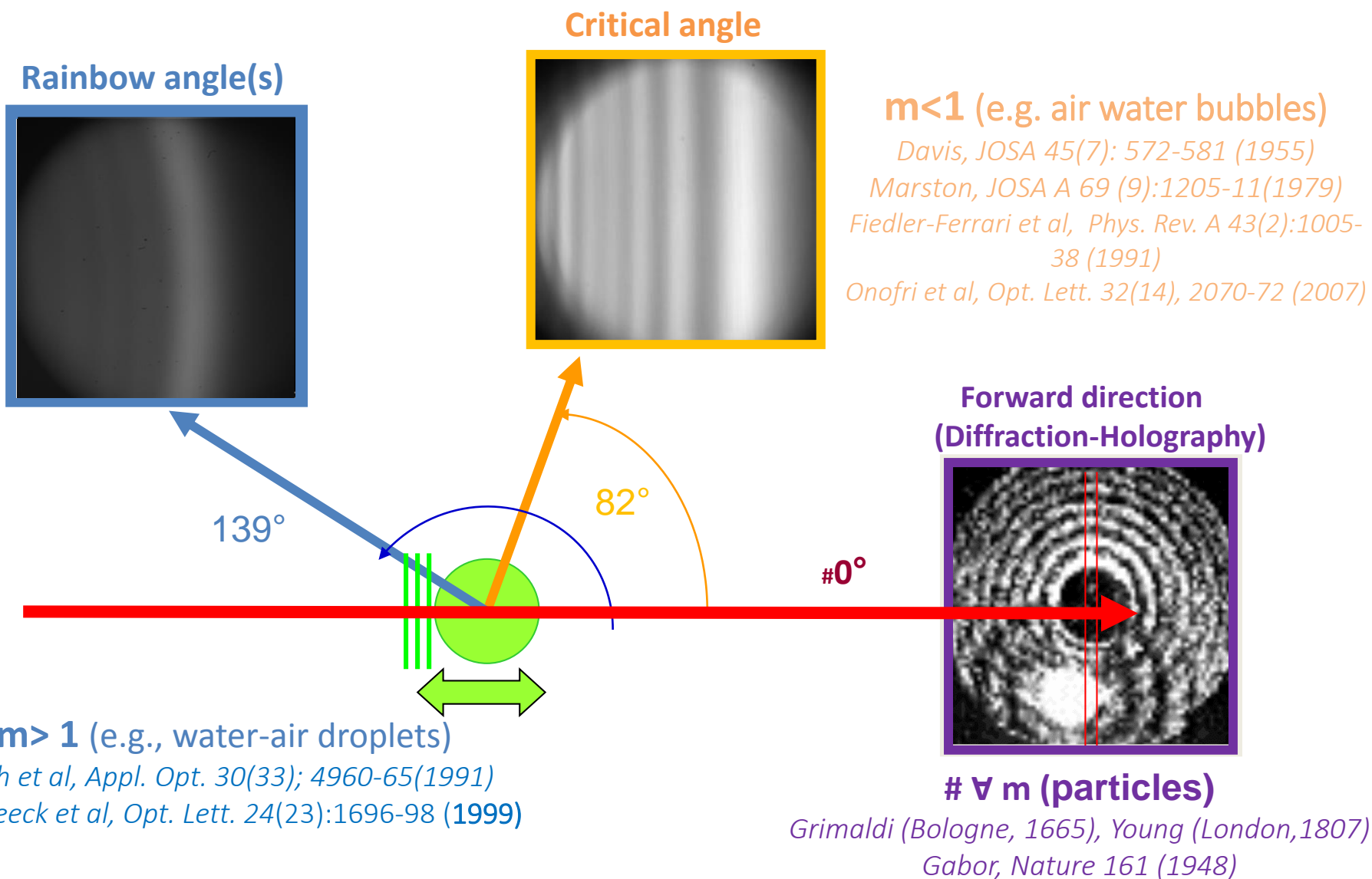
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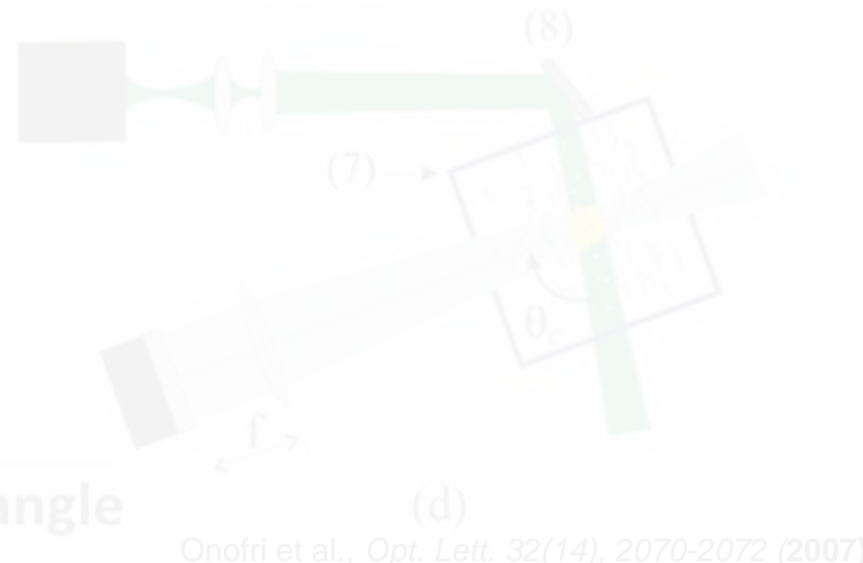
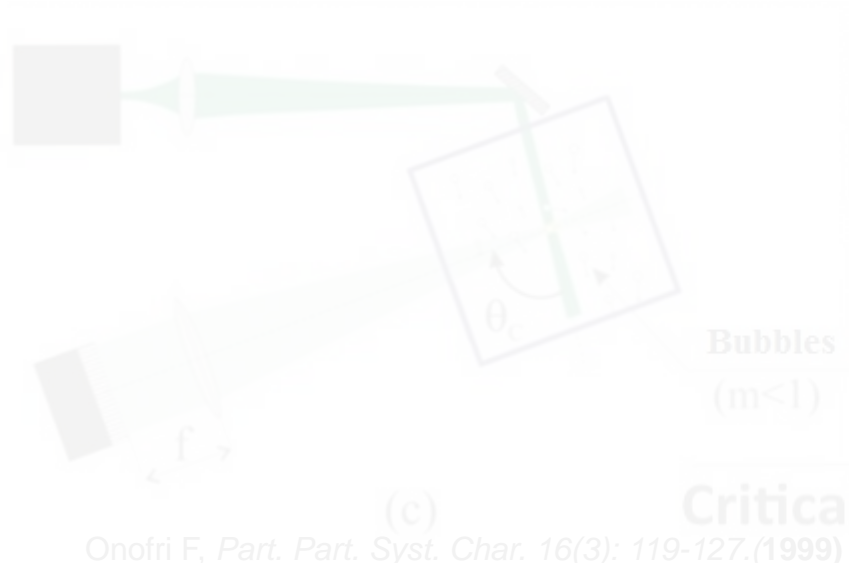
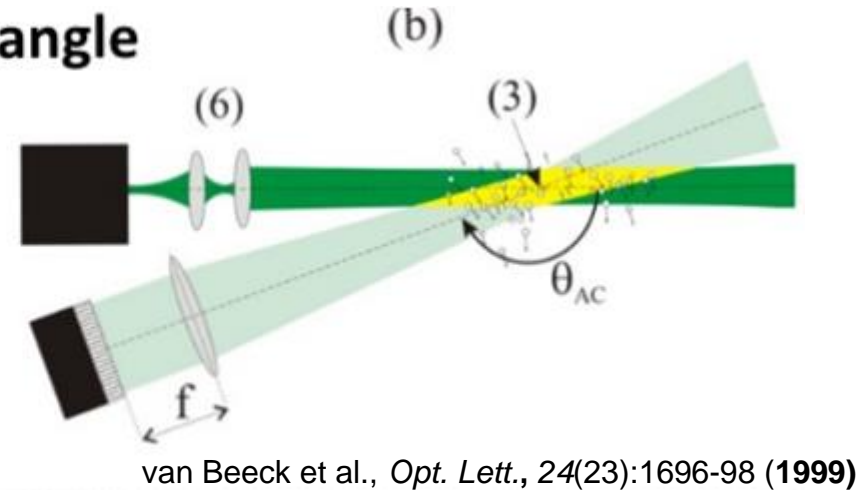
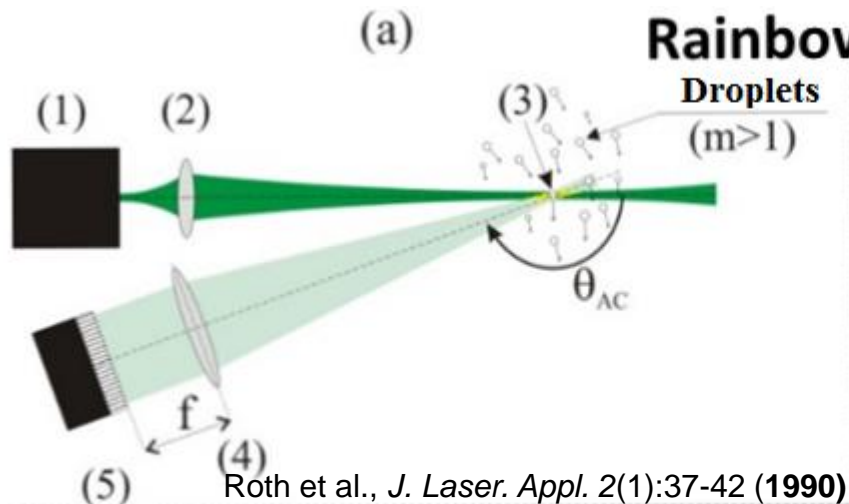


The more investigated & used for particle syst. characterization



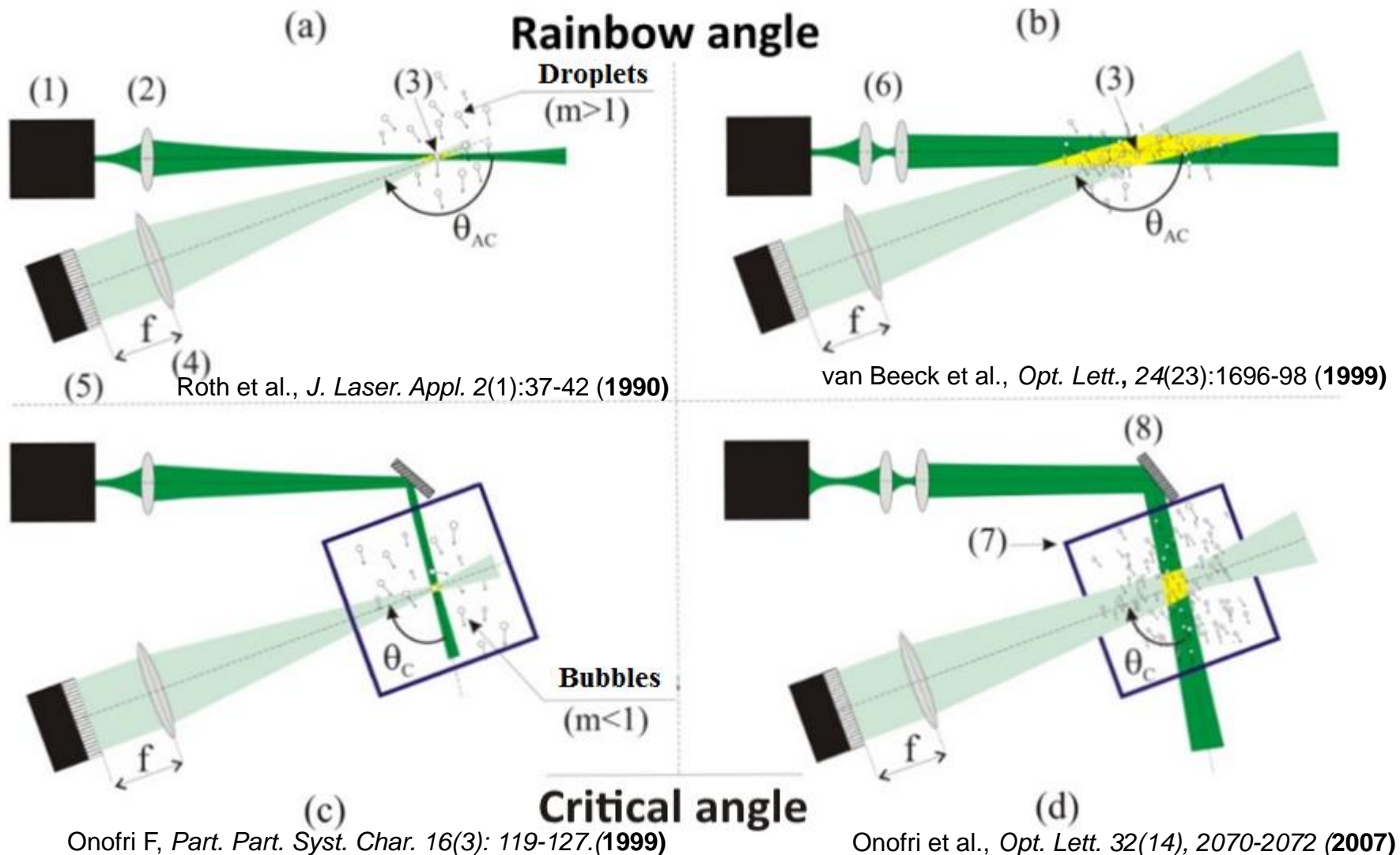
Particle systems: caustics are only observed in the far-field!

Setups for sizing and particle refractive index characterization

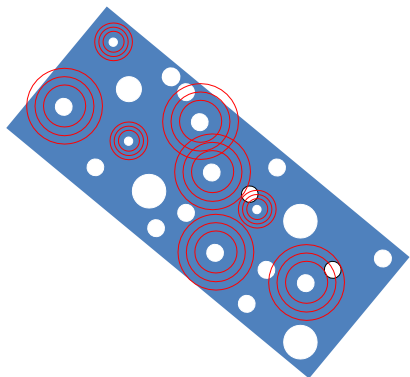


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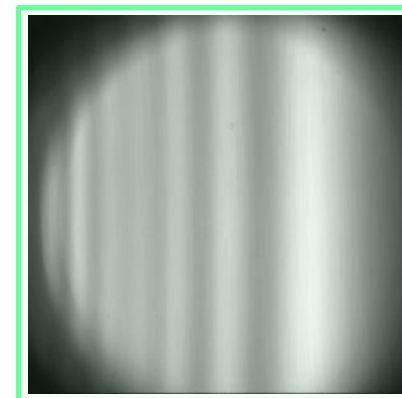
Setups for sizing and particle refractive index characterization



Particle systems: underlying hypotheses...



$$\langle I(\theta) \rangle_m \propto NV \int_{D_{\min}}^{D_{\max}} I(\theta) f(D) dD =$$



Integration- collective scattering

Single scattering (*Optically dilute media*)

Incoherent scattering (*Random positioning*)

Stochastic media (*Large numbers...*)

Ergodicity (*No size-velocity correlation...*).

No size-composition correlation

Inverse and regularization methods

Dependent methods (analytic distribution)

Independent methods (distribution derivatives, cut-off frequencies,...)

Light scattering models

Geometrical optics (GOA)

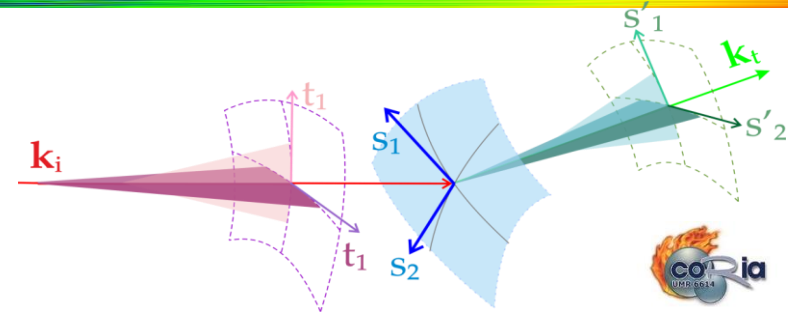
Physical optics (POA)

Electromagnetic theories (LMT, CAM,...)

Rainbow and Critical scattering: recent advances

GOA accounting for rays'

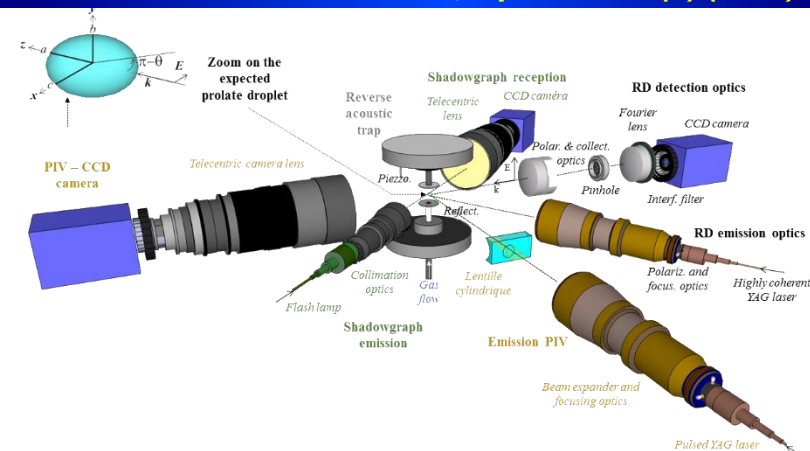
- Direction and amplitude,
- Polarization and various phase terms (...), and
- Local curvature of the wave front (i.e. van de Hulst's divergence)



Ren et al., *Opt. Lett.* 36(3) (2011)

Well-controlled experiments

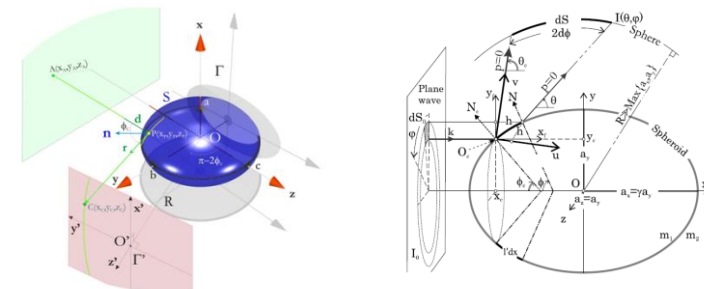
- Particles in acoustic levitation (liquids/solids in gas)
- Drop and bubbles on demand (gas-liquid, liquid-liquid)
- Clouds of monodispersed particles (gas-liquid, liquid-liquid)



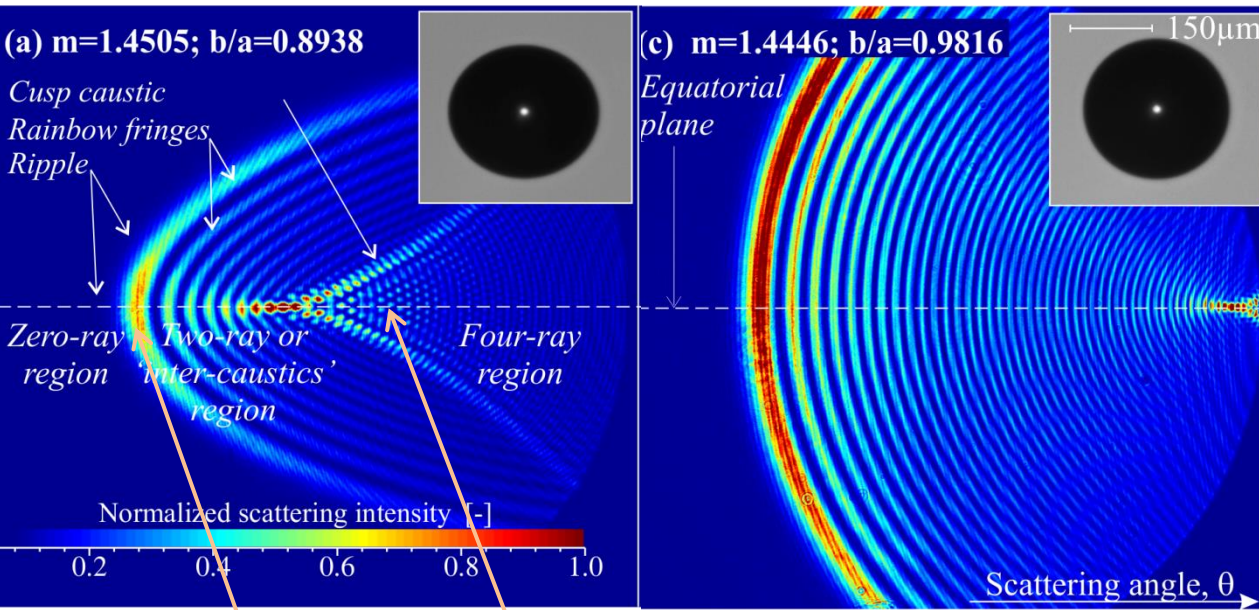
see also Yu, H. et al., *Opt. Express* 21 (2013)

Physical approximations for spheroidal particles

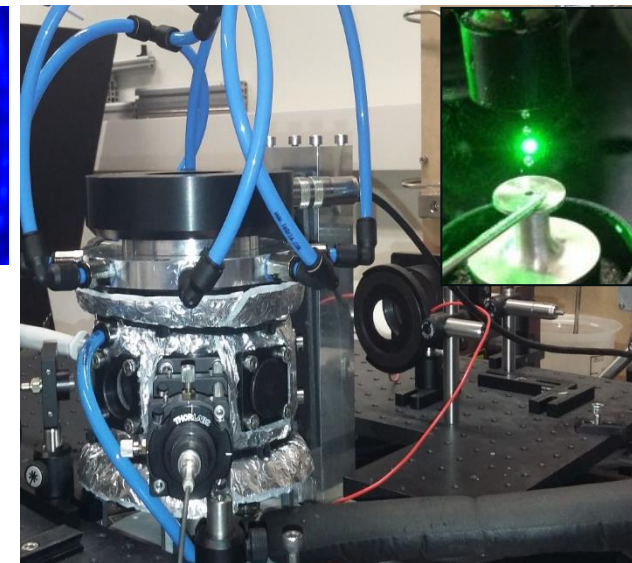
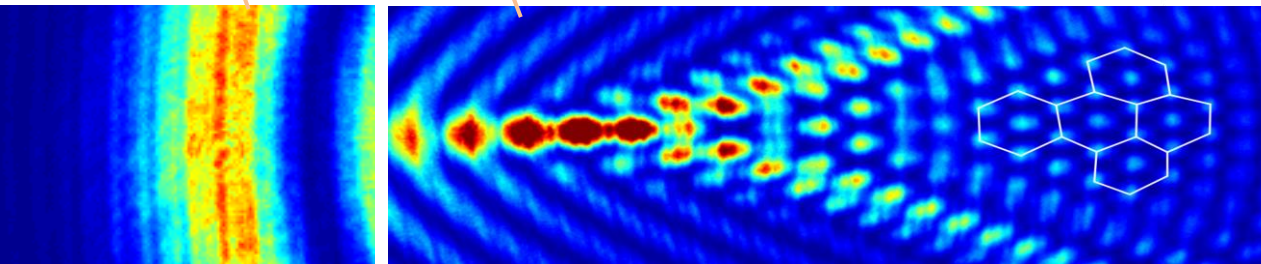
- Generalization of Marston's approximation (*JOSA*, 79)
- Accounting for tunneling phase, total reflection,...
- Heterogeneous particles (suspension, stratified)
- Coupling with GOA and other POAs



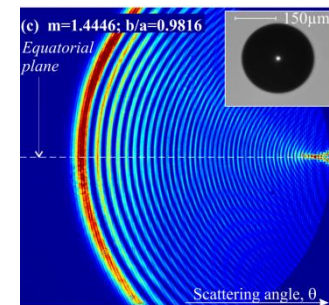
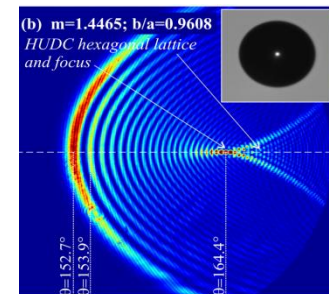
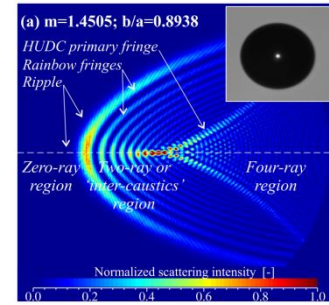
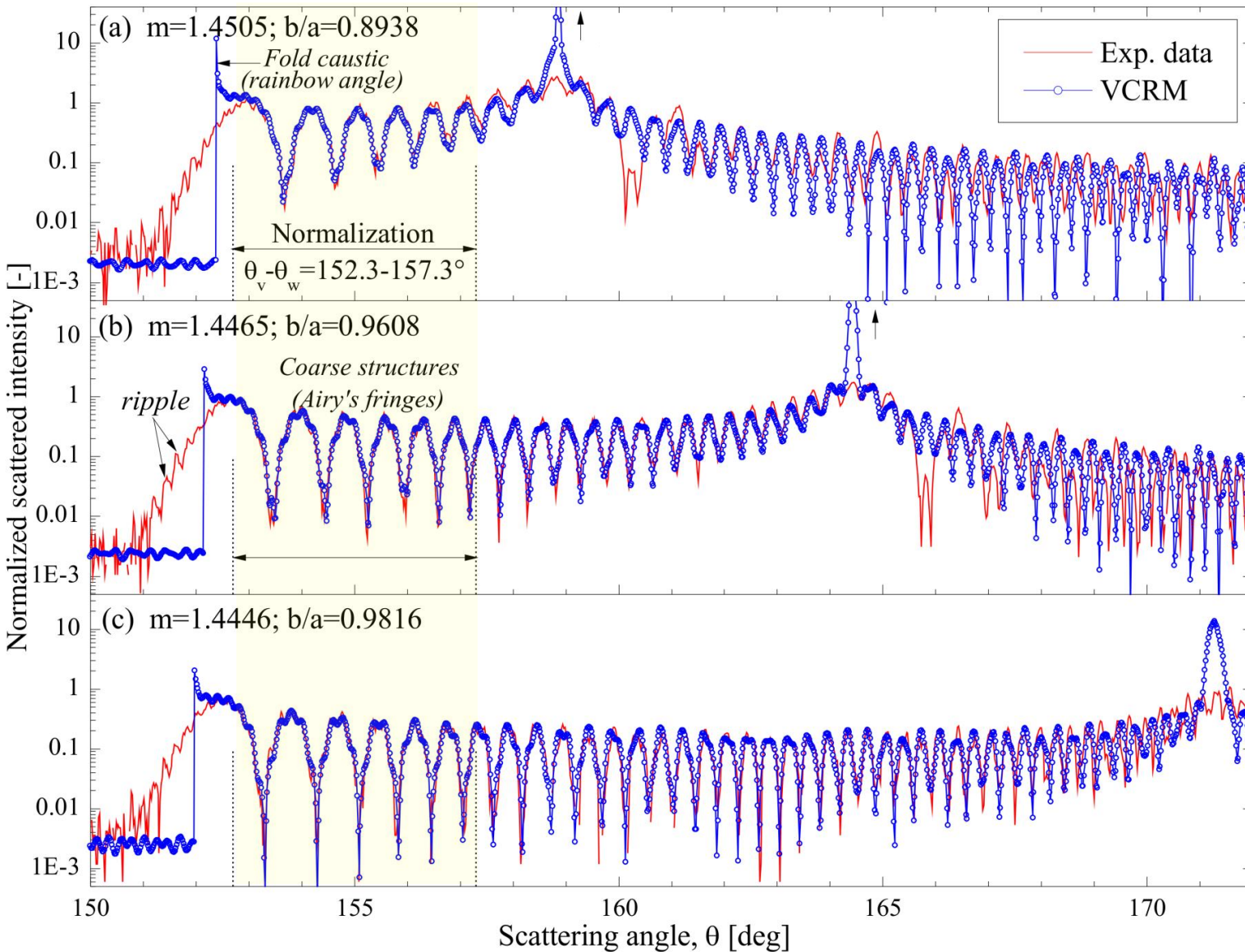
Rainbow: spheroid droplet evaporating in an acoustic trap



Evaporating chamber

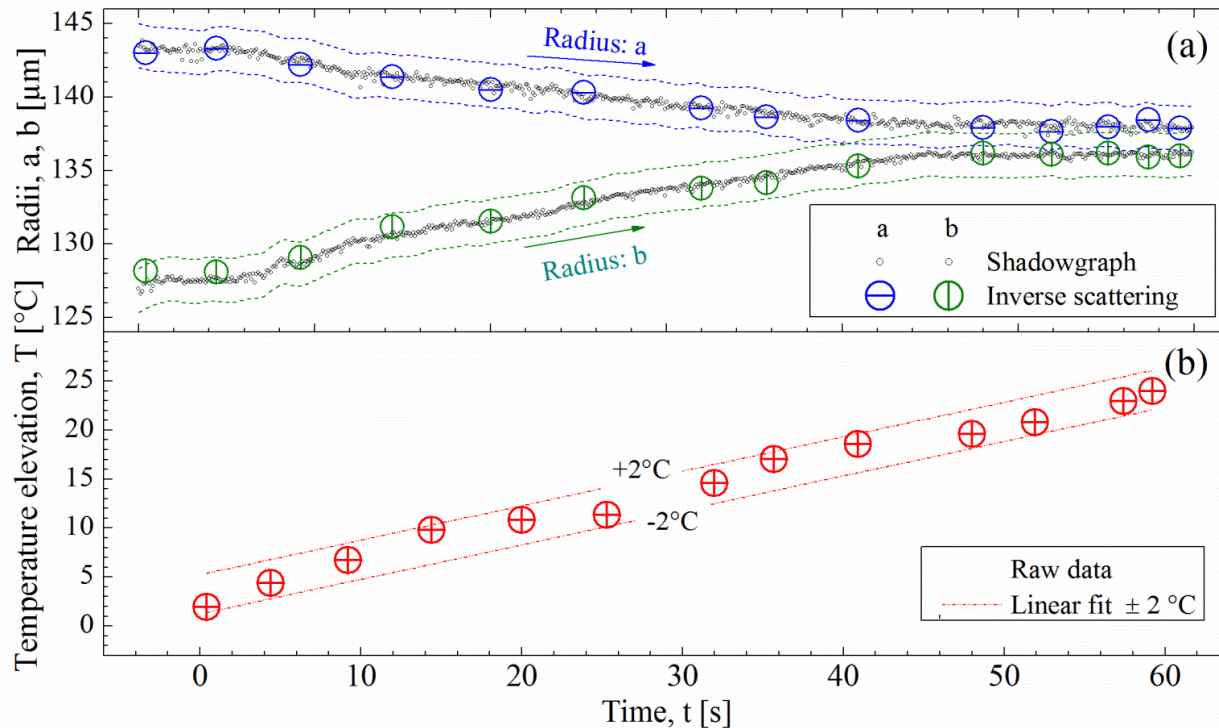
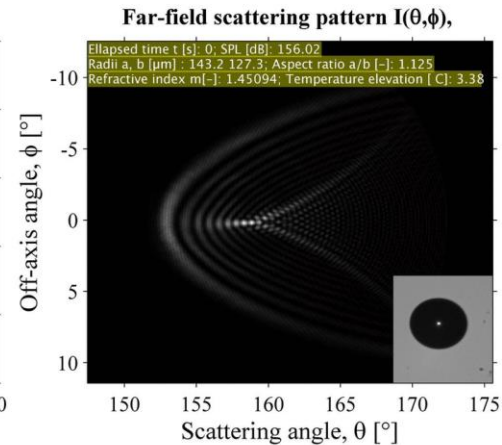
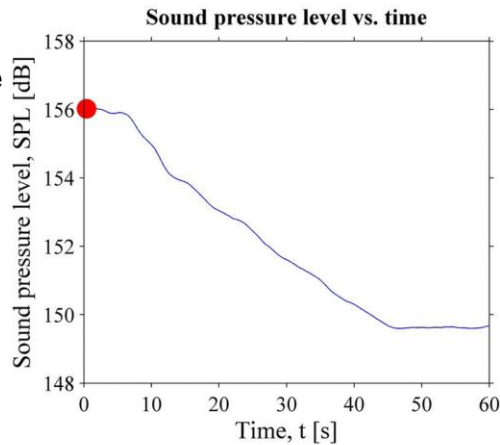


Rainbow: acoustic levitation and heating of an oil droplet



Rainbow: acoustic levitation and heating of an oil droplet

The droplet becomes more and more spherical when the sound pressure level is decreased in the trap



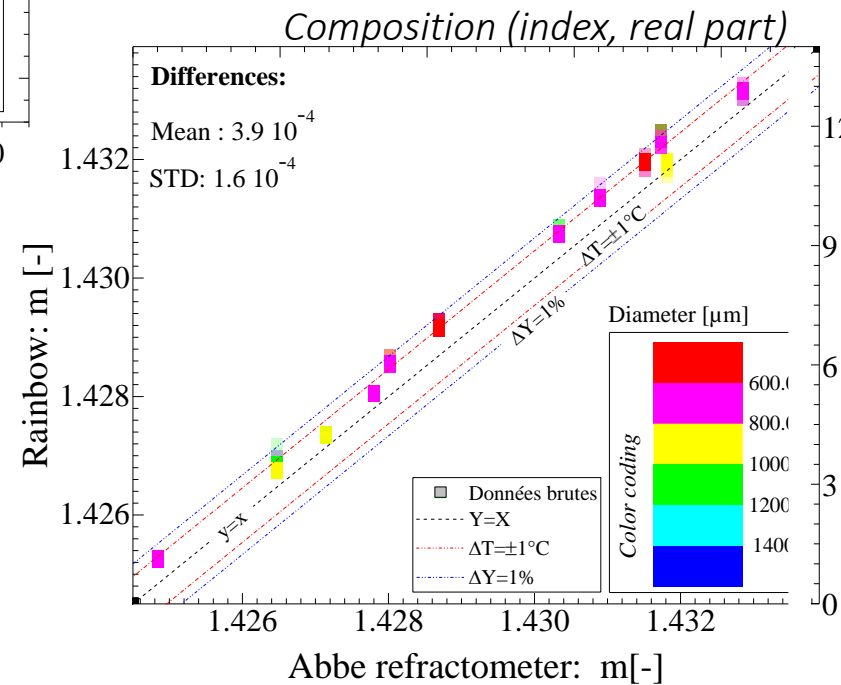
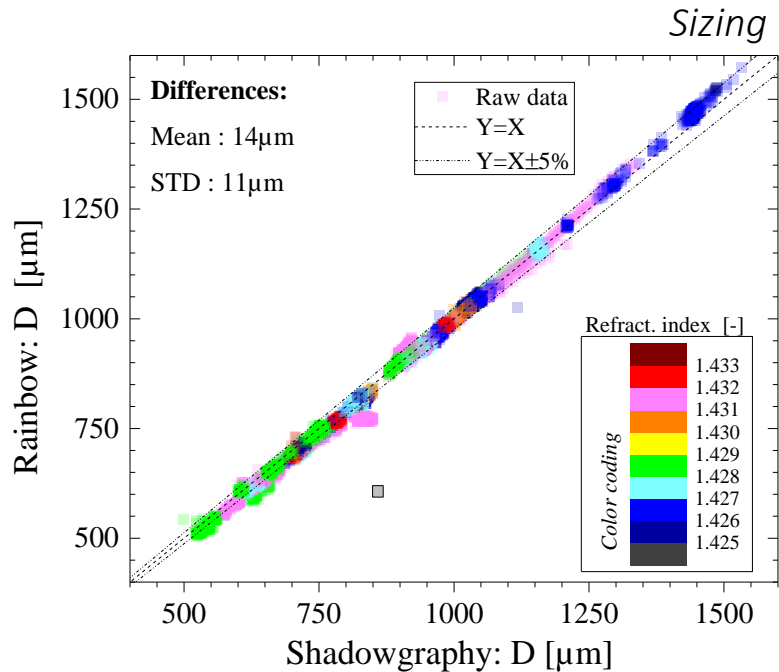
The two principal radii of the droplet converge as droplet becomes more and more spherical

This (almost) non evaporating droplet is heated

Rainbow? e.g. Characterization of liquid-liquid droplets (weak index: #1.06)



*Extraction column
for Nuclear-fuel
recycling/reprocessing*



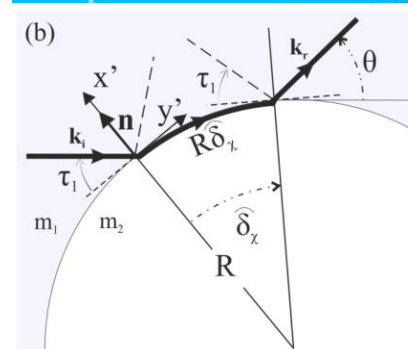
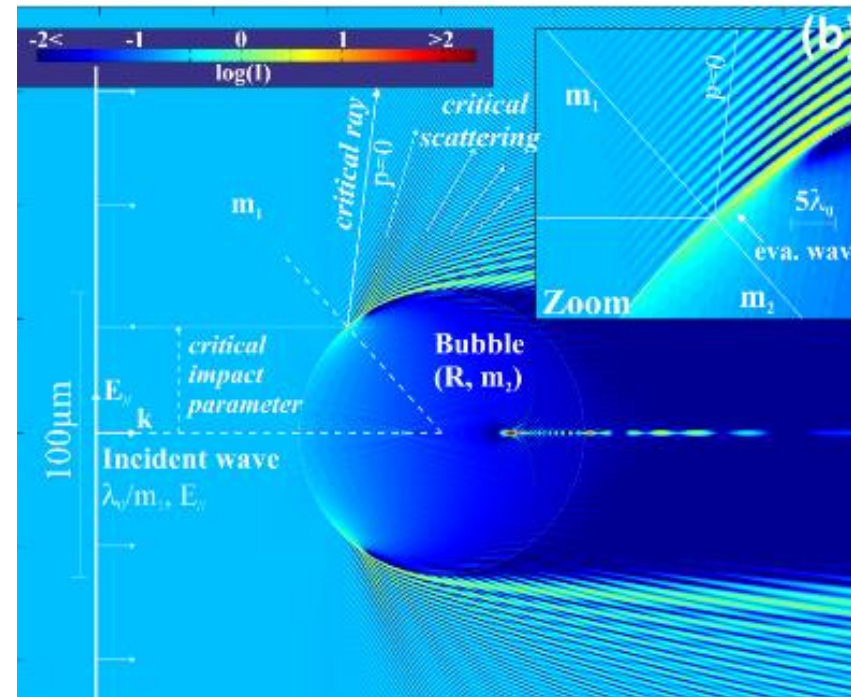
Critical scattering: spheroids & coupling with GPOA

Reflection of wave ($p=0$) at the vicinity of the critical angle as a diffraction phenomena

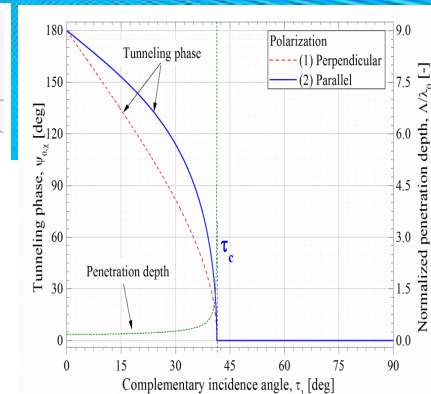
$$S = \underbrace{S_0 + S_1 + S_2 + S_{2'} + S_3 + \dots + S_\infty}_{\text{Debye's series}}$$

LMT
POA
CAM

$$S_p = \frac{-ika}{2} F_p \exp[i\gamma_p]$$



Goos-Hänchen
angular shift



Tunneling phase
(e.g. Yu et al., Particuol. 6, 2008)

Critical scattering: spheroids & coupling with GPOA

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Debye's series

POA
CAM

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Spherical bubble [Marston, 1979]

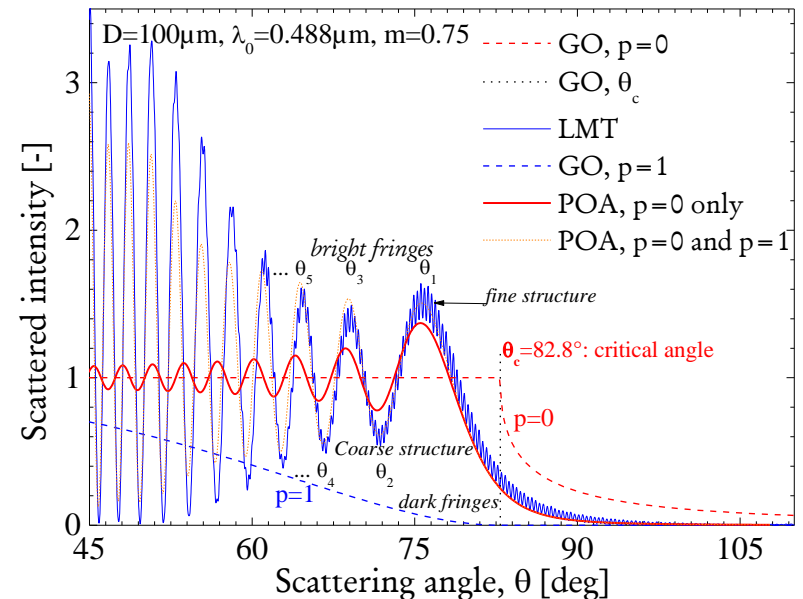
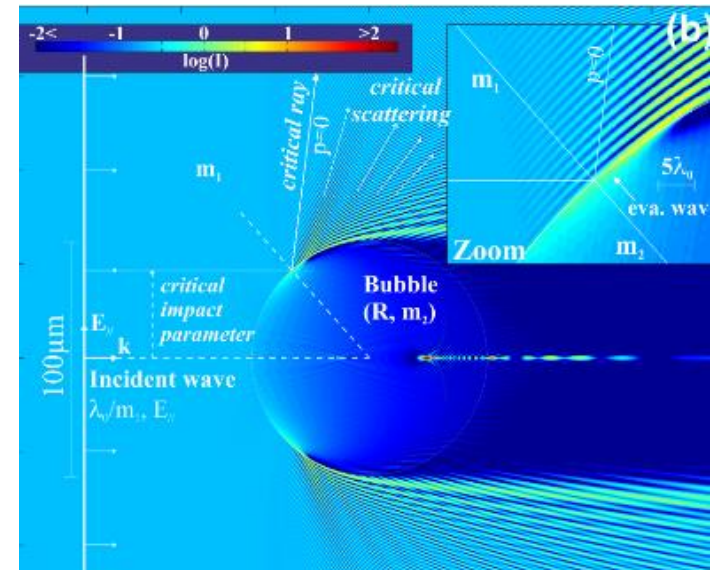
- POA ($p=0$ without Goos-Hänchen angular shift)
- Refracted rays ($p=1$ without tunneling phase)

$$S_0 \propto \int_0^\omega \cos\left(\frac{\pi}{2} z^2\right) dz + i \int_0^\omega \sin\left(\frac{\pi}{2} z^2\right) dz$$

Fresnel's integrals

$$\omega = \sin(\phi_c - \phi) \sqrt{(a/\lambda) \cos \theta_c}$$

Depends on particle composition and size



Critical scattering: spheroids & coupling with GPOA

Spheroid: generalization of the Marston's POA

- Total reflection generates a weak-caustic (the Fresnel's coefficients are only once derivable)
- Its virtual wave front is parabolic (fold)

$$I_{0,\chi}(\theta, \phi) = I_0 \left(\frac{a_{c,\parallel} a_{c,\perp}}{8R^2} \right) |S_{0,j}(\theta, \phi)|^2$$

$$\text{with } S_{0,j}(\eta) = \frac{e^{\frac{i k \eta^2}{4\alpha} - i \frac{\beta_j}{A^{1/2}} w^{1/2}}}{A} \int_{-w}^{\infty} e^{-i \left(\frac{\pi}{2} z^2 \right)} e^{-i \left(\frac{\beta_j}{A^{1/2}} (z+w)^{1/2} - \frac{\beta_j}{A^{1/2}} w^{1/2} \right)} dz$$

Onofri et al., Opt. Lett. 37(22):4780-4782 (2012)

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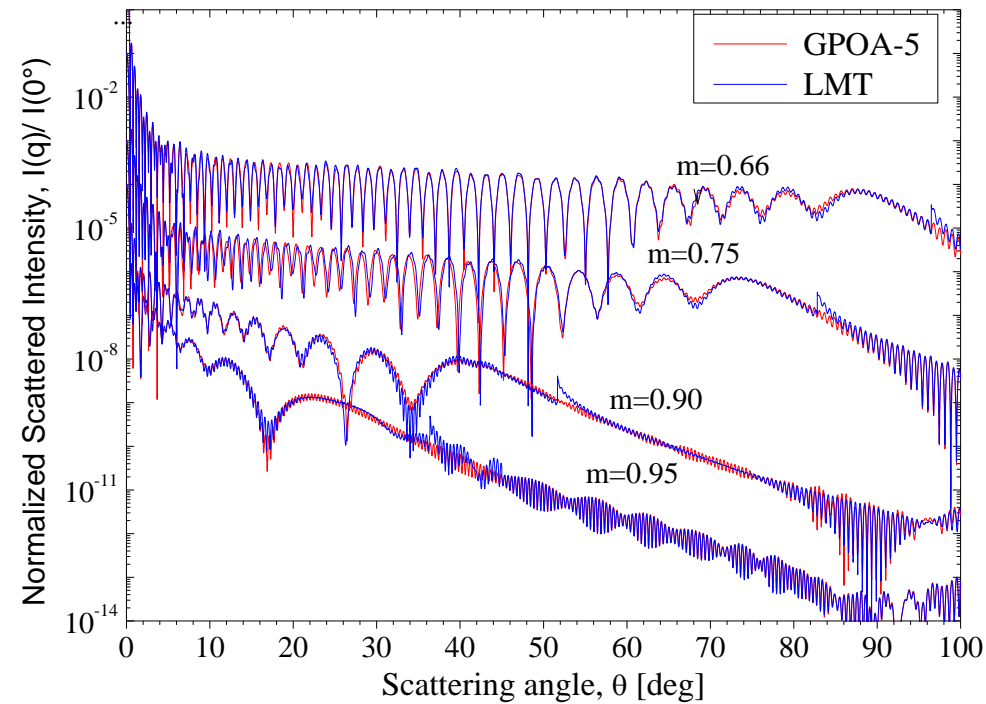
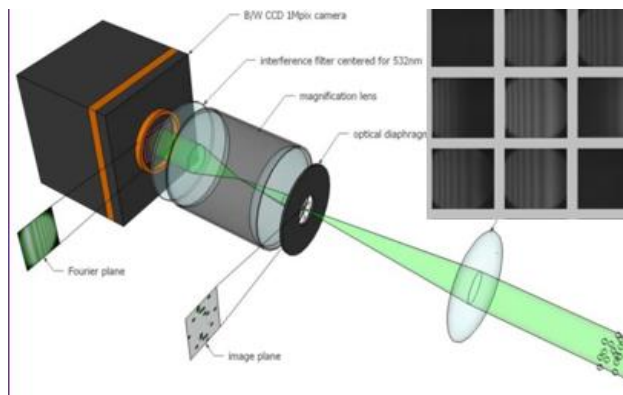
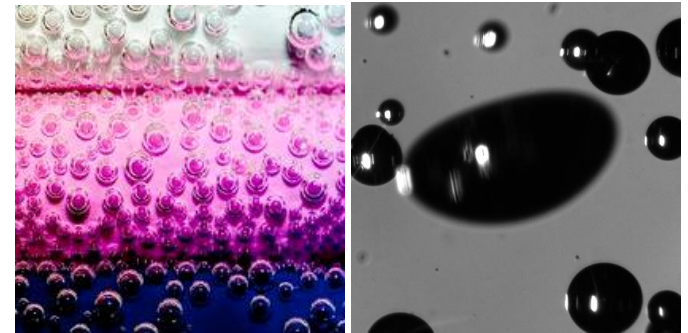
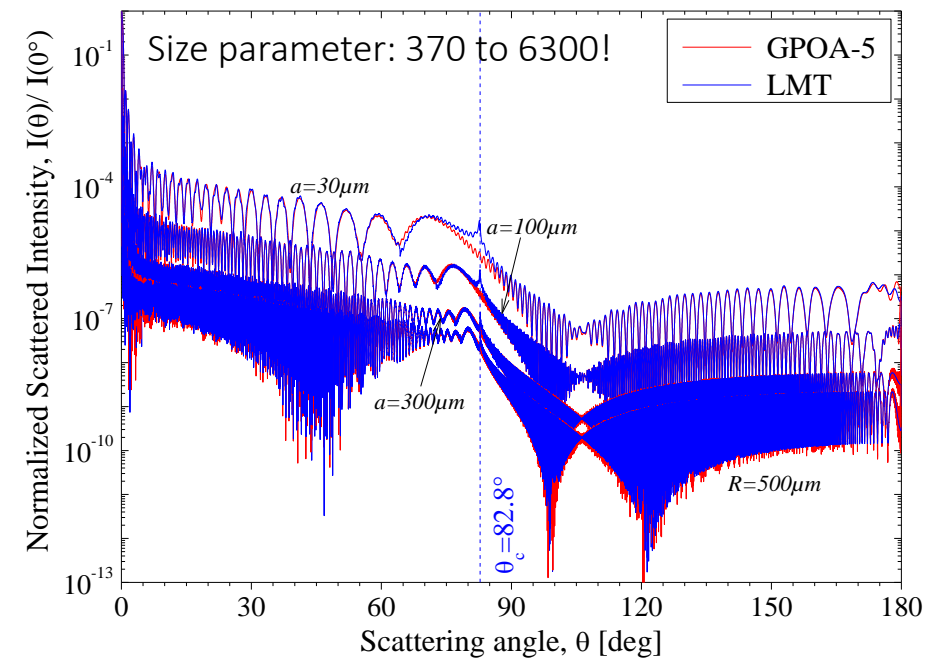
Onofri et al., *Opt. Lett.* 37(22):4780-4782 (2012)

Accounting for other wave effects and ray orders

- We account for the Goos-H nchen angular shift in the POA
- As well as GOA rays ($p=1, \dots, +\infty$) with tunneling phase
- And forward diffraction (Franhoffer,...) with an incoherent summation

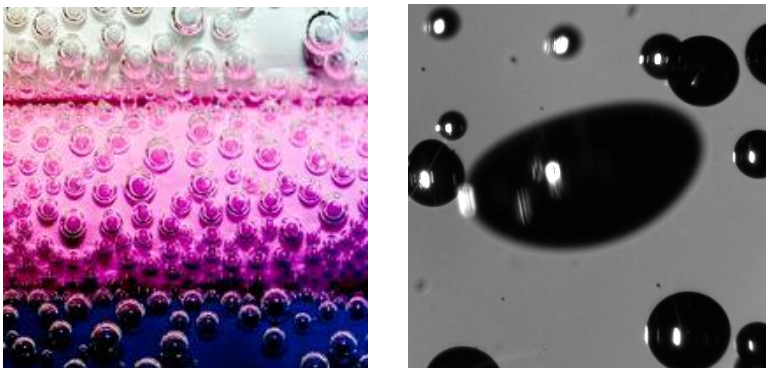
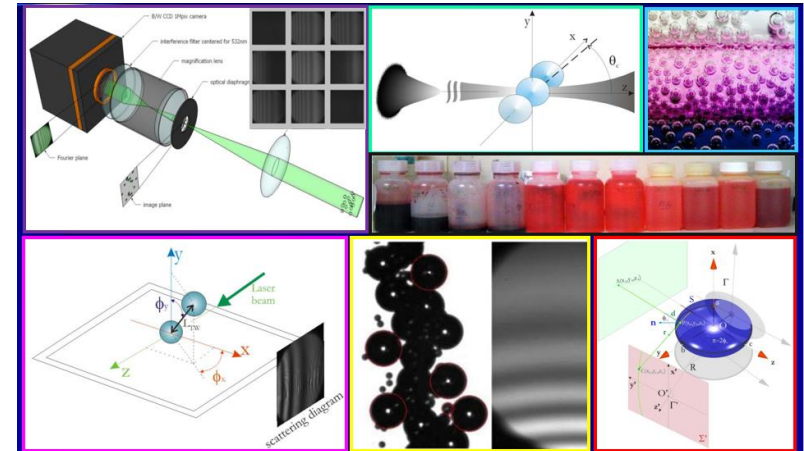
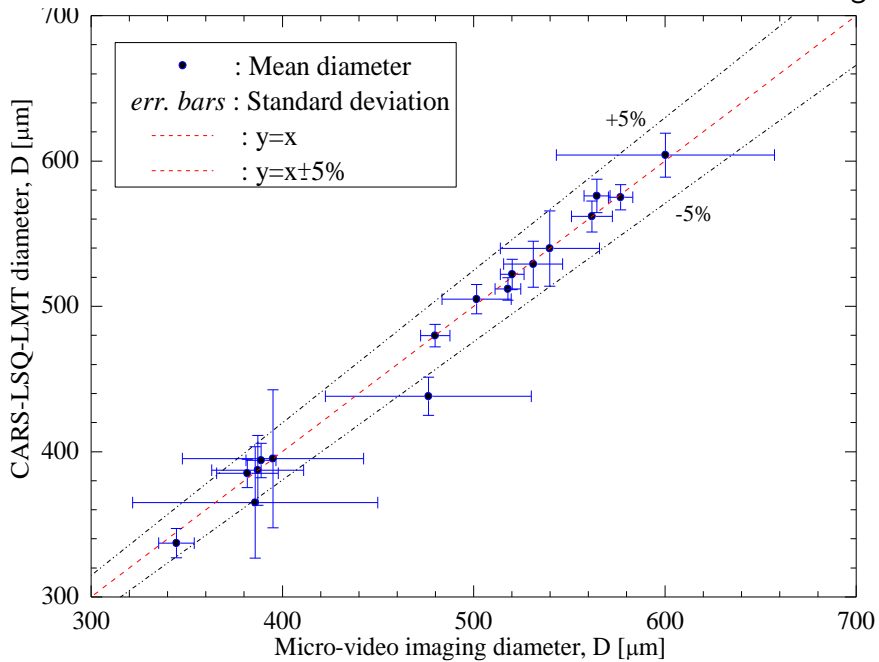
$$I_{\chi}(\theta, \phi) = \begin{cases} \left| S_{0,\chi,t}^{GOA}(\theta, \phi) + \sum_{p=1}^{\infty} S_{p,\chi}^{GOA}(\theta, \phi) \right|^2 + |S^{F-POA}(\theta, \phi)|^2, & \text{if } \theta \in [0, \theta_{\max}^{F-POA}]; \\ \left| S_{0,\chi,t}^{GOA}(\theta, \phi) + \sum_{p=1}^{\infty} S_{p,\chi}^{GOA}(\theta, \phi) \right|^2, & \text{if } \theta \in]\theta_{\max}^{F-POA}, \theta_2]; \\ \left| S_{0,\chi,\delta_{\chi}}^{M-POA}(\theta, \phi) + \sum_{p=1}^{\infty} S_{p,\chi}^{GOA}(\theta, \phi) \right|^2, & \text{if } \theta \in]\theta_2, \theta_c]; \\ \left| \sum_{p=0}^{\infty} S_{p,\chi}^{GOA}(\theta, \phi) \right|^2, & \text{if } \theta \in]\theta_c, \pi]. \end{cases}$$

Critical scattering: full GOPA model vs. LMT

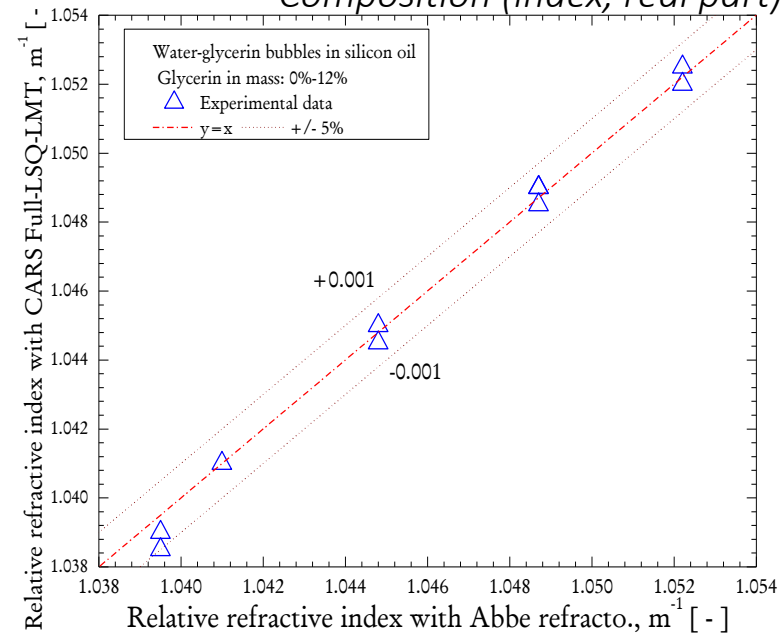


Critical scattering: experimental application

Sizing

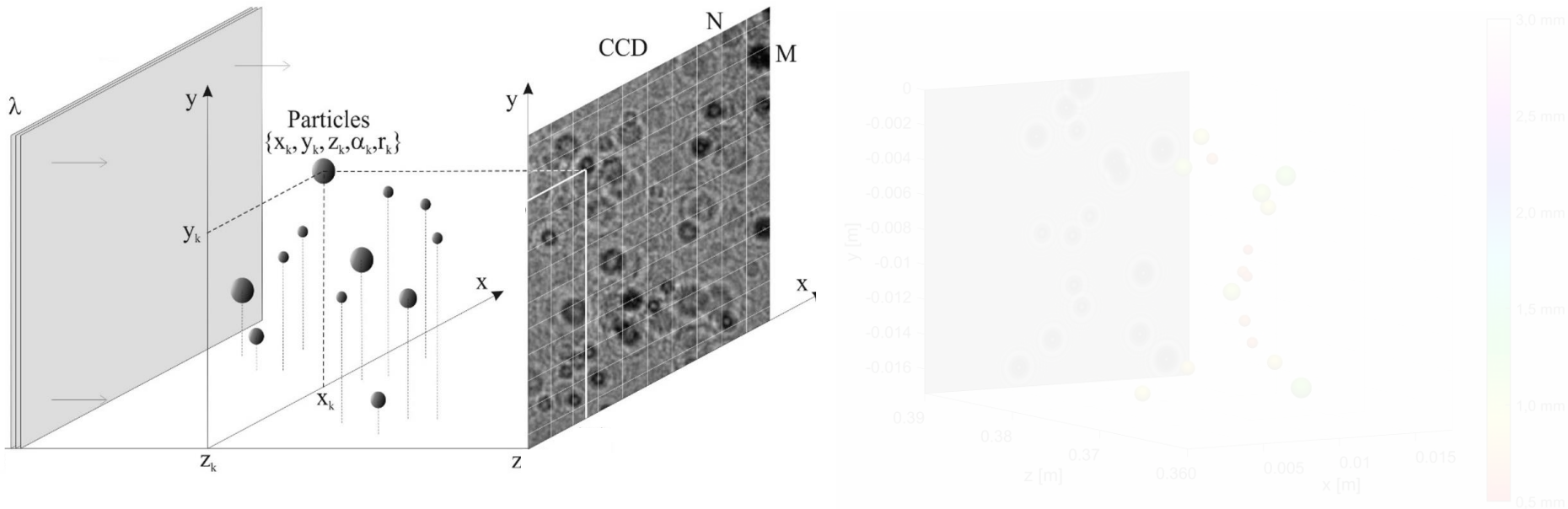


Composition (index, real part)



Digital In-line Holography (DIH) : classical method (free space)

Classical digital in-line holography: Gabor configuration with a quadratic sensor



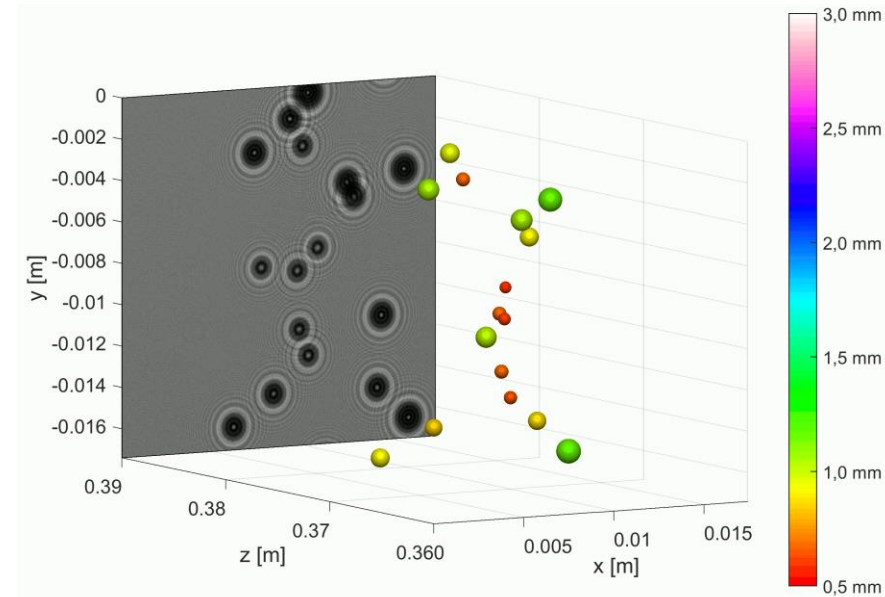
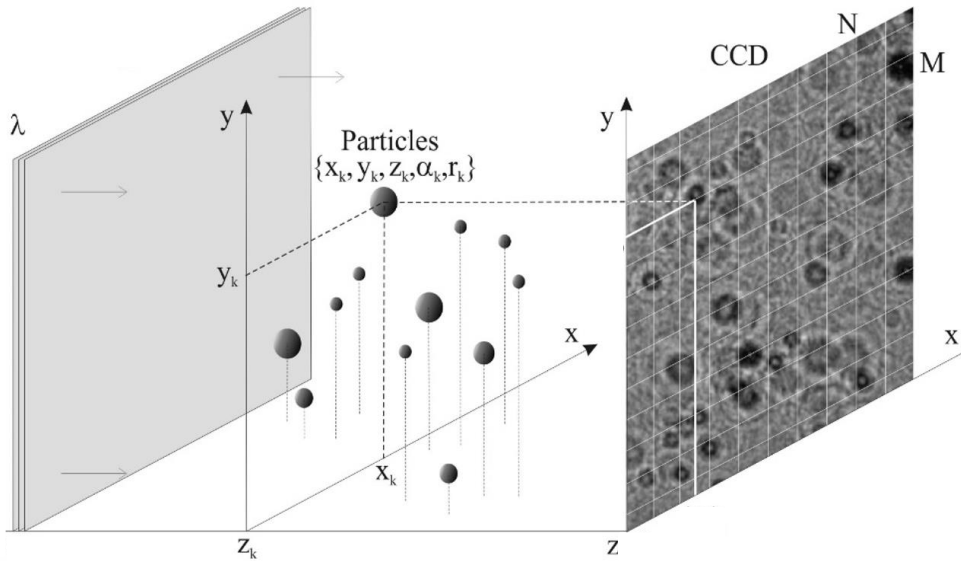
3D Numerical restitution with scalar diffraction theory/theories:

$$E_r(x, y) \approx \frac{\varepsilon C}{2} \left\{ 1 + \left[E_p^* \otimes h_{z_r - z_s} \right](x, y) + \left[E_p \otimes h_{z_r + z_s} \right](x, y) \right\}$$

Reconstruction distance
Fresnel or Rayleigh-Sommerfeld propagators (impulsive)
Phase function of the particle
Twin image

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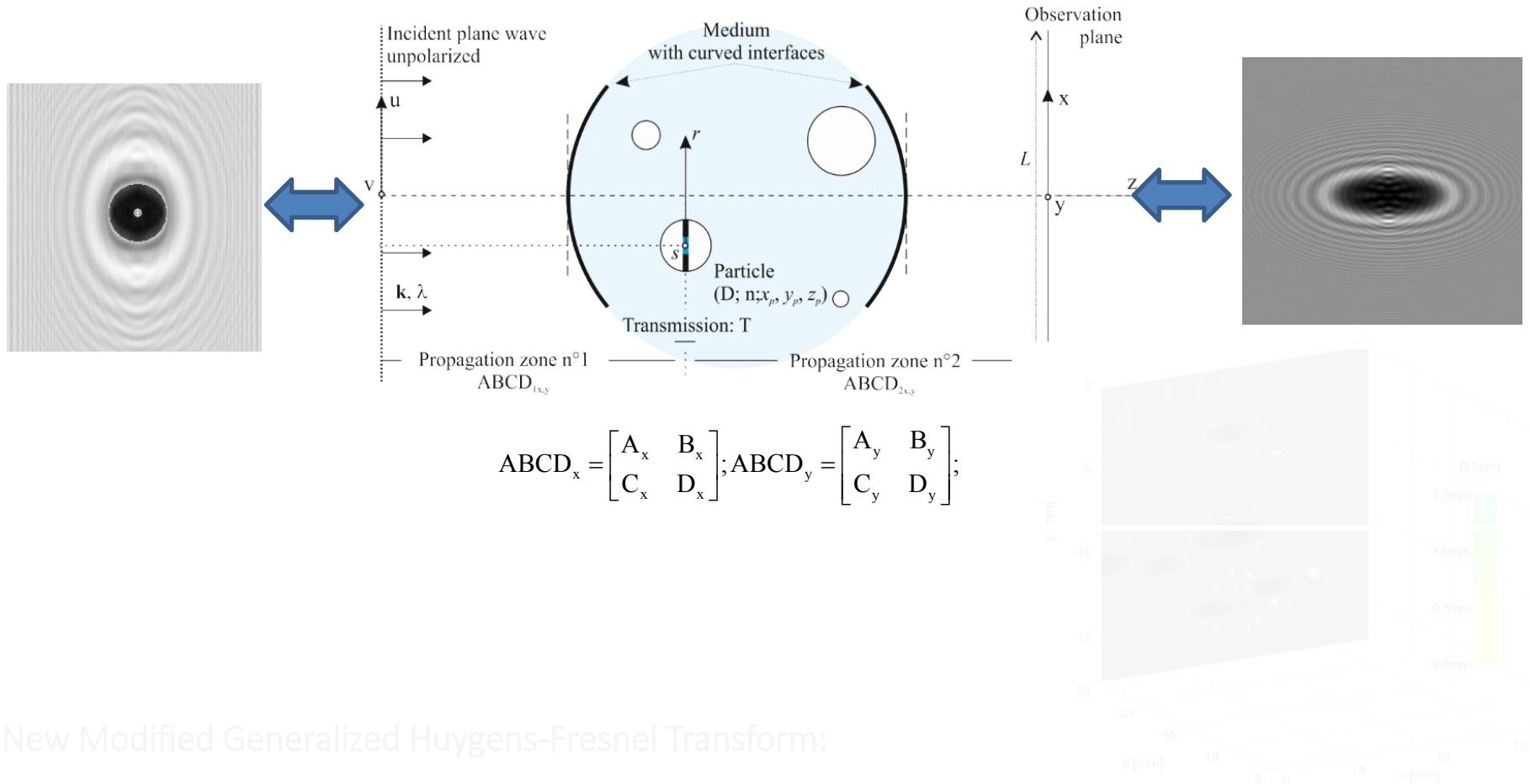
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Reconstruction distance
Fresnel or Rayleigh-Sommerfeld propagators (impulsive)
Phase function of the particle
Twin image

DIH: new method for astigmatic media (size and velocity)

Small digression : Digital in-line holography in astigmatic media (i.e. pipe flow!)

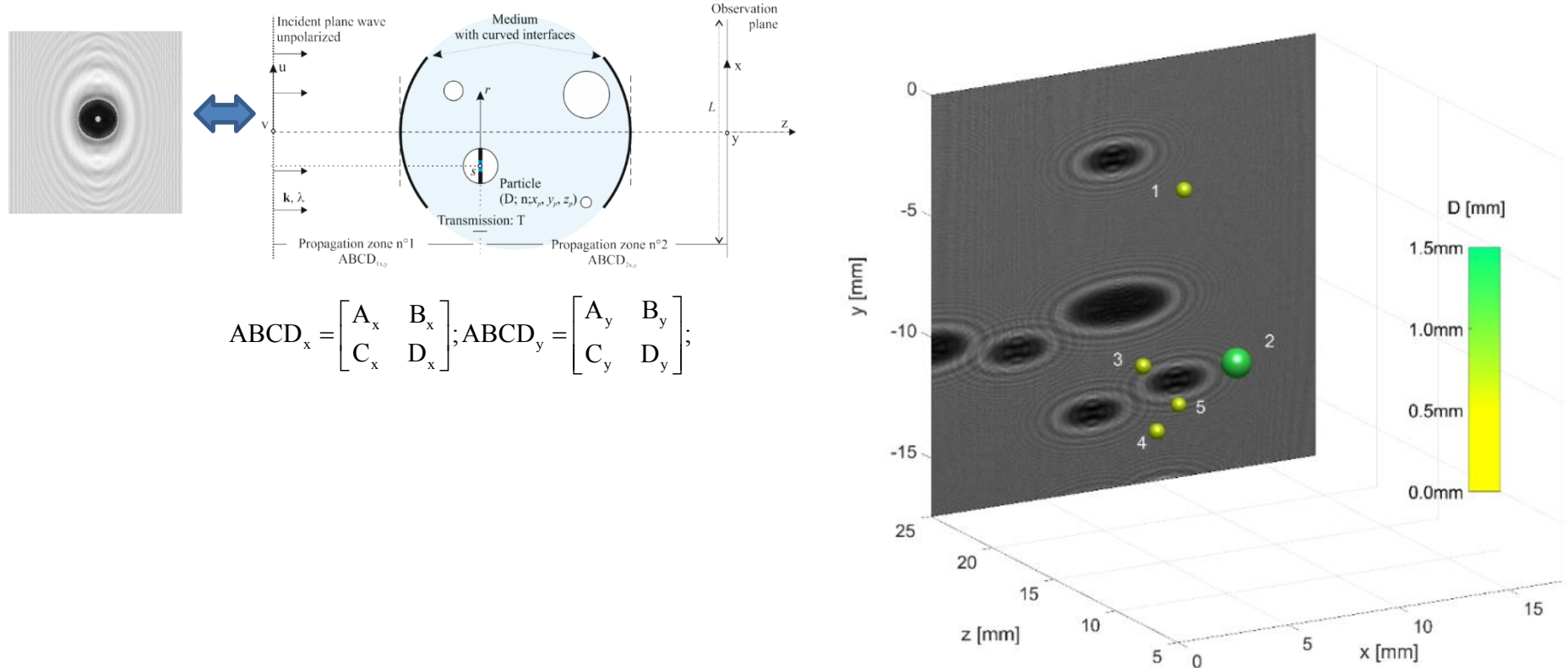


New Modified Generalized Huygens-Fresnel Transform:

$$E_r(x,y) \approx \frac{\varepsilon c}{2} \frac{\exp[ik(z_r - z_s)]}{i\lambda A_x A_y \sqrt{B_x B_y}} \exp\left(i\pi \frac{C_y x^2}{\lambda A_x}\right) \exp\left(i\pi \frac{C_y y^2}{\lambda A_y}\right) \left\{ 1 + [E_p^* \otimes h_{z_r - z_s}] \left(\frac{x}{A_x}, \frac{y}{A_y} \right) + \dots \right\}$$

DIH: new method for astigmatic media (size and velocity)

Small digression : Digital in-line holography in astigmatic media (i.e. pipe flow!)



New Modified Generalized Huygens-Fresnel Transform (M-GHFT):

$$E_r(x, y) \approx \frac{\epsilon c}{2} \frac{\exp[ik(z_r - z_s)]}{i\lambda A_x A_y \sqrt{B_x B_y}} \exp\left(i\pi \frac{C_y x^2}{\lambda A_x}\right) \exp\left(i\pi \frac{C_y y^2}{\lambda A_y}\right) \left\{ 1 + \left[E_p^* \otimes h_{z_r - z_s} \right] \left(\frac{x}{A_x}, \frac{y}{A_y} \right) + \dots \right\}$$

Digital in-line holography (DIH): principle (classical)

Numerical restitution in free space:

$$E_r(\mathbf{x}, y) \approx \frac{\varepsilon c}{2} \left\{ 1 + \left[\mathbf{E}_p^* \otimes \mathbf{h}_{z_r - z_s} \right](\mathbf{x}, y) + \left[\mathbf{E}_p \otimes \mathbf{h}_{z_r + z_s} \right](\mathbf{x}, y) \right\}$$

$$\begin{aligned}
 &\uparrow \\
 &E_p = 0, \rho = \sqrt{x^2 + y^2} \leq a \\
 &= E_0, \rho > a
 \end{aligned}$$

Classically: the particle is modelled as
pinhole or an opaque disk!

DIH : forward caustic generated by rays $p=1$!

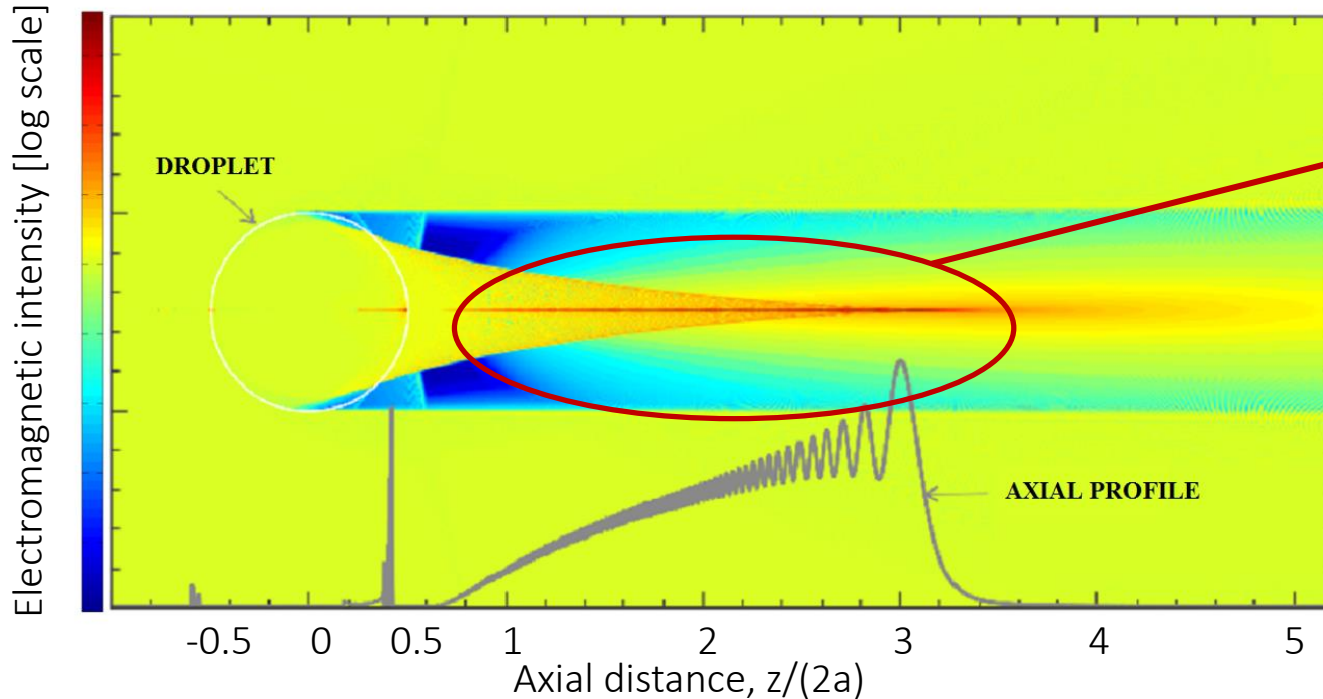
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Classically: the particle is modelled as pinhole or an opaque disk!

Direct calculation (GLMT) of the electromagnetic intensity in the near-field intensity of a 1 mm droplet:



Caustic of rays $p=1$!
(i.e. photonic jet)

DIH : forward caustic, real or virtual, generated by rays $p=1$

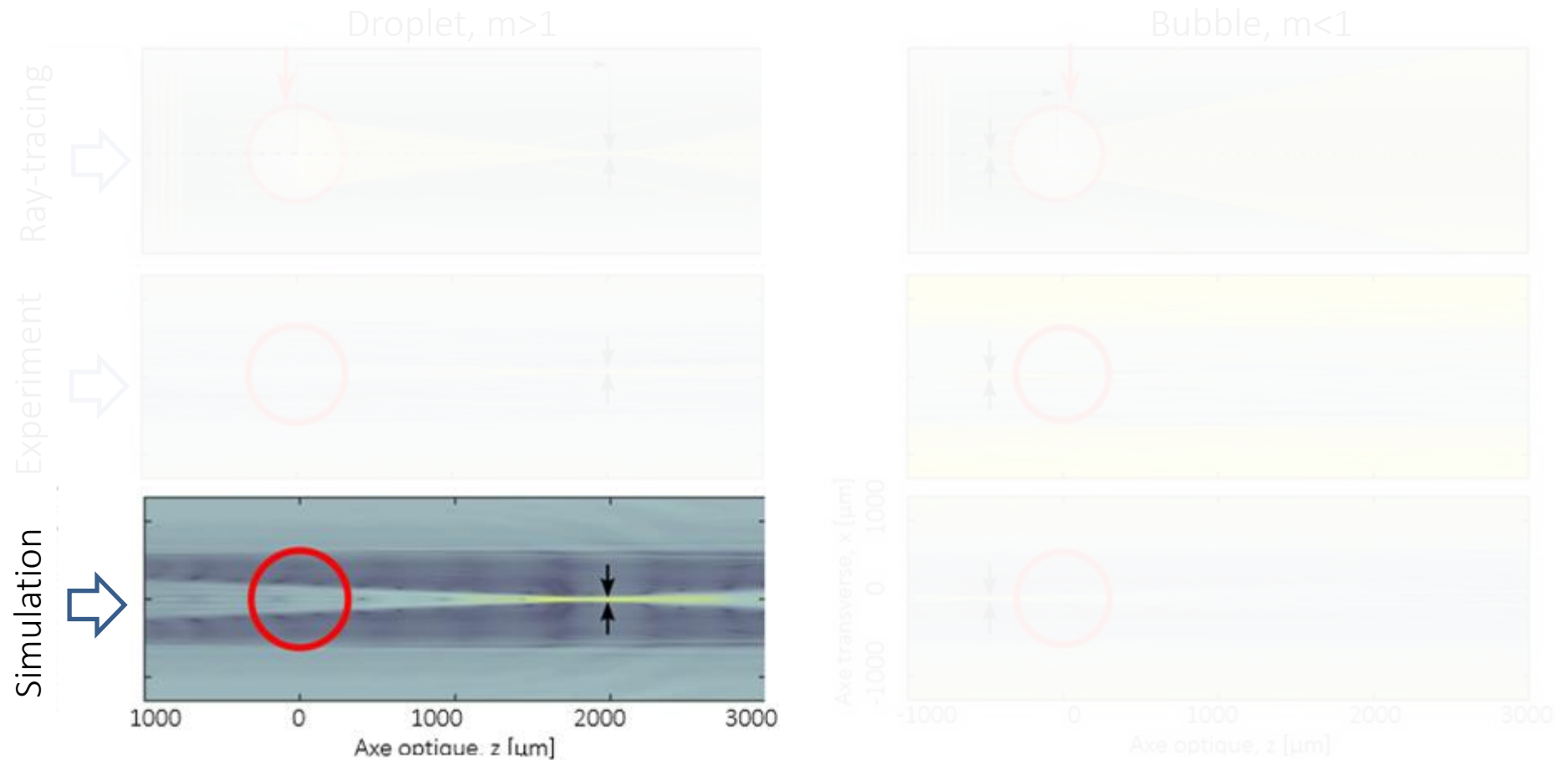
Accounting for the forward caustic generated by the refracted rays ($p=1$):

$$E_p = E_0 \times \exp(j\phi), \rho \leq a$$

$$= 1, \rho > a$$

Phase function of a ball lens:

$$\phi(\rho) = 2ka - \frac{k}{2f} \rho^2, f = ma / [2(m-1)]$$



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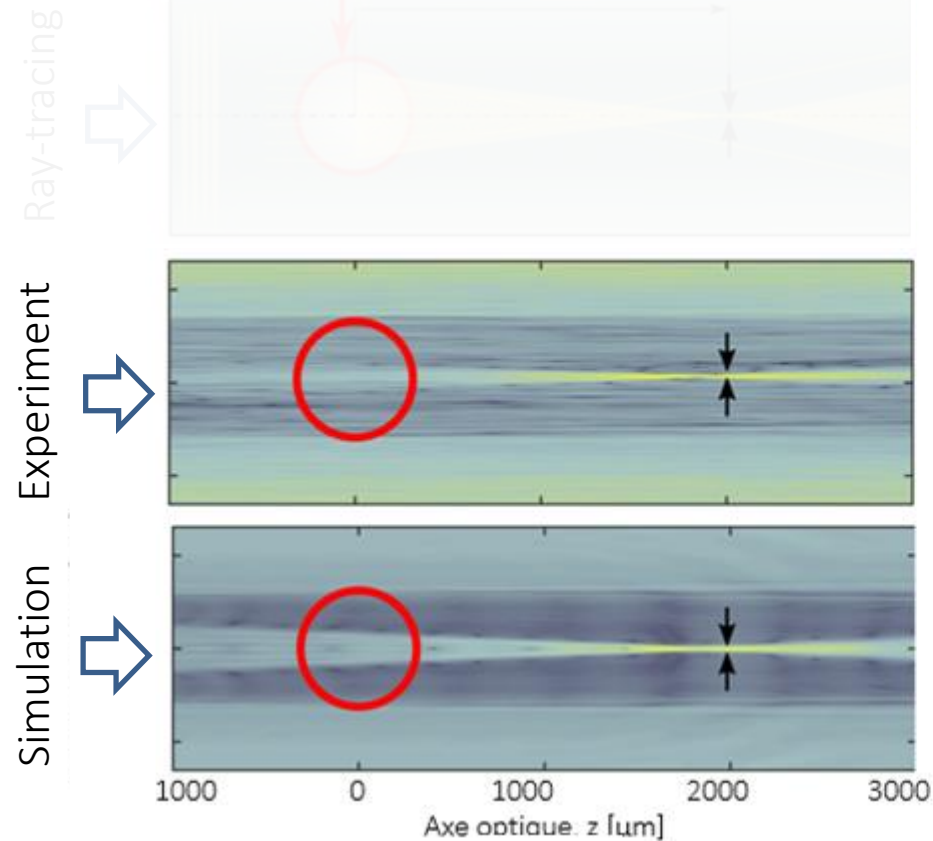
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Droplet, $m > 1$

Bubble, $m < 1$



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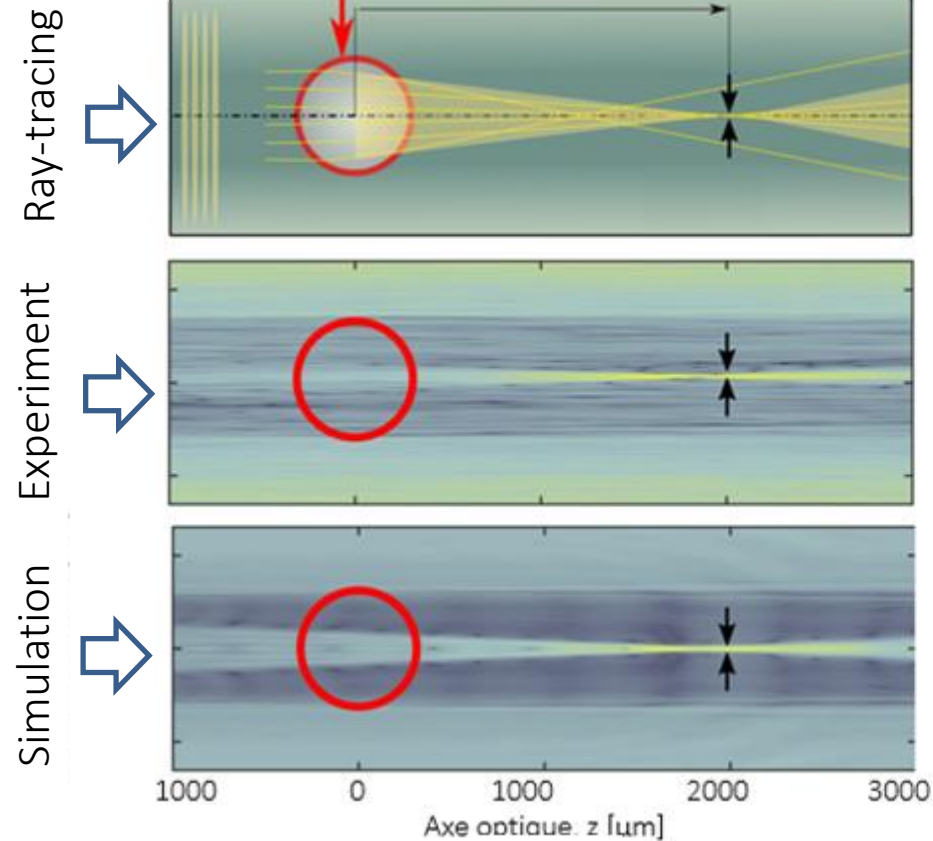
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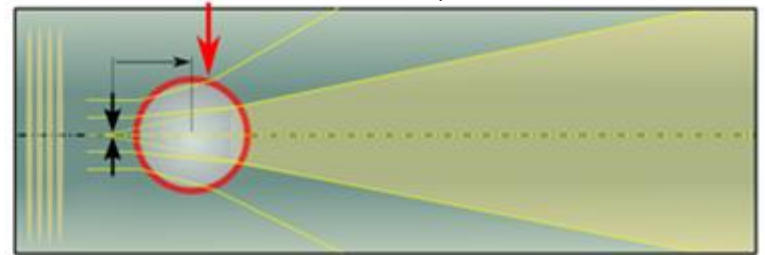
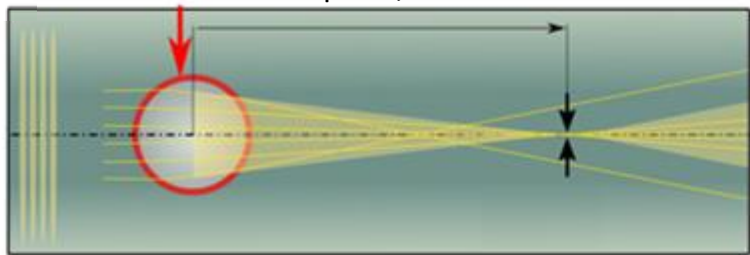
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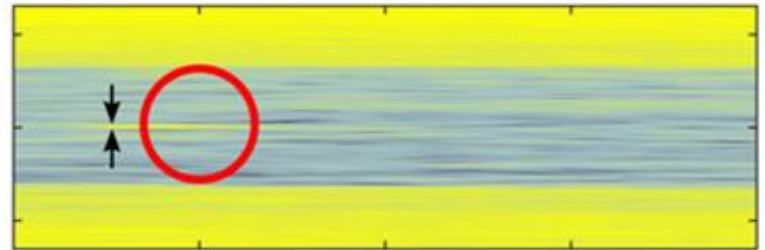
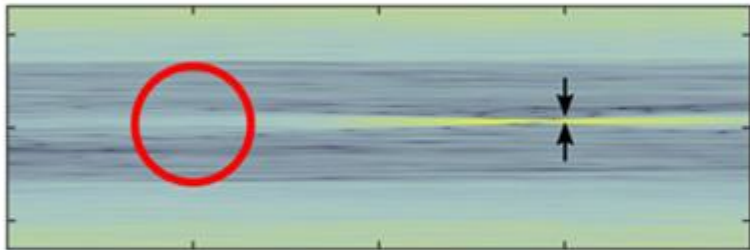
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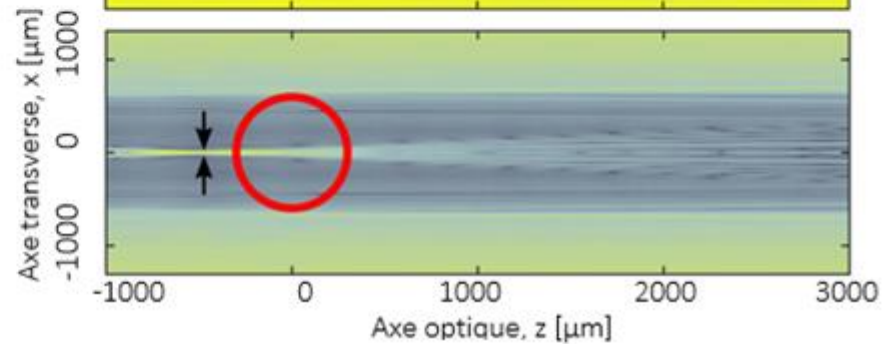
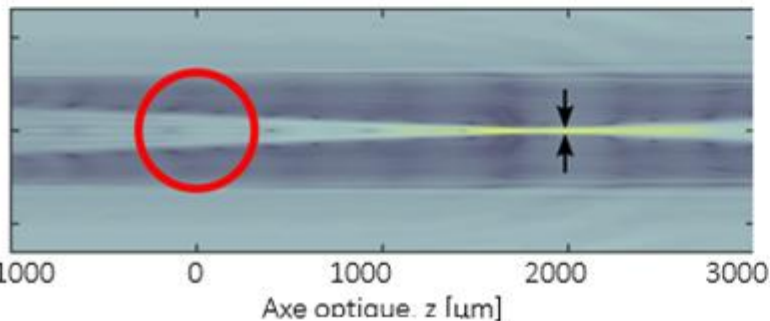
Ray-tracing



Experiment



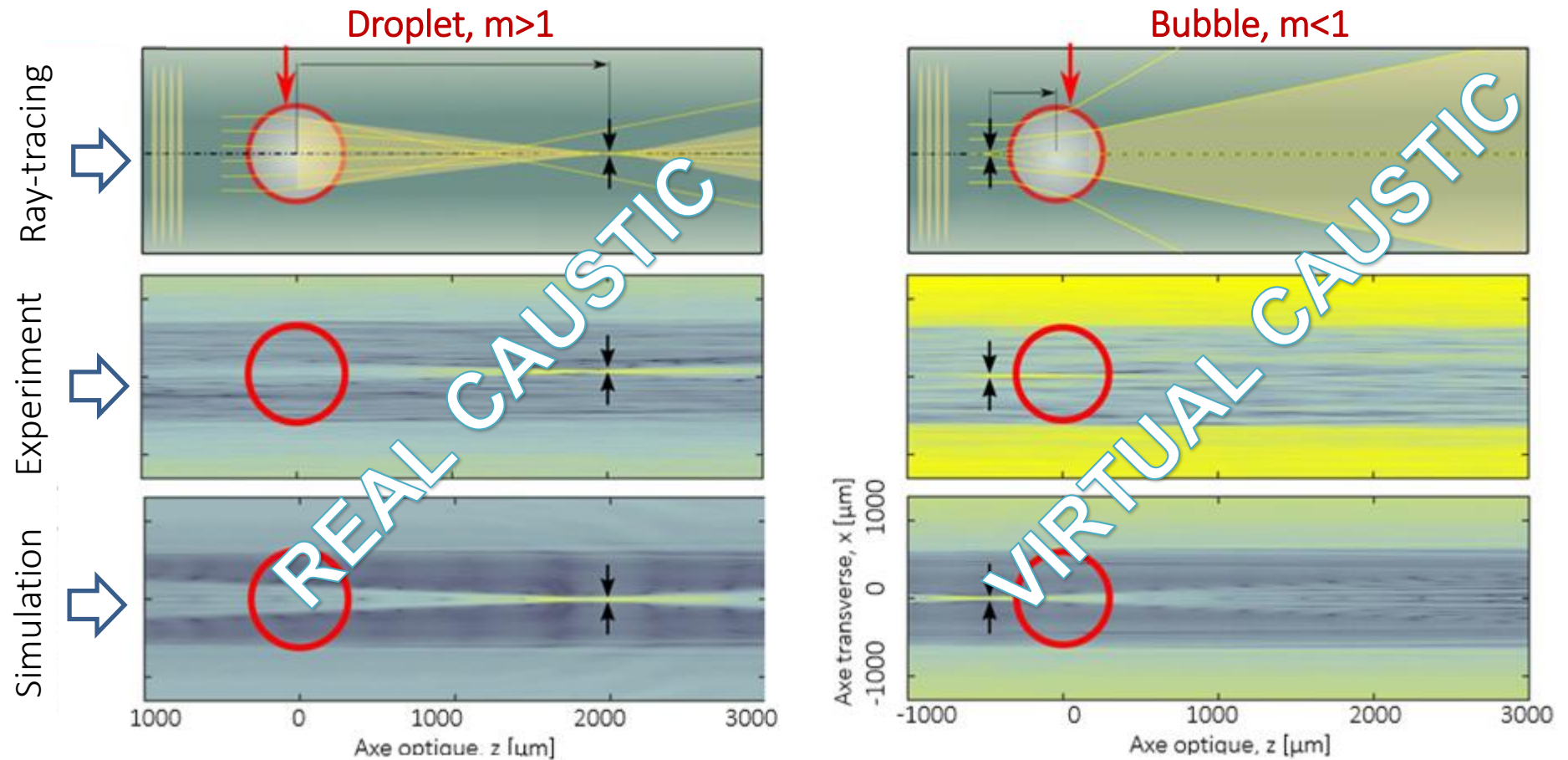
Simulation



DIH : forward caustic, real or virtual, generated by rays $p=1$

Extensive calculations with the Fresnel approximation show that the caustic is the Airy pattern of a lens:

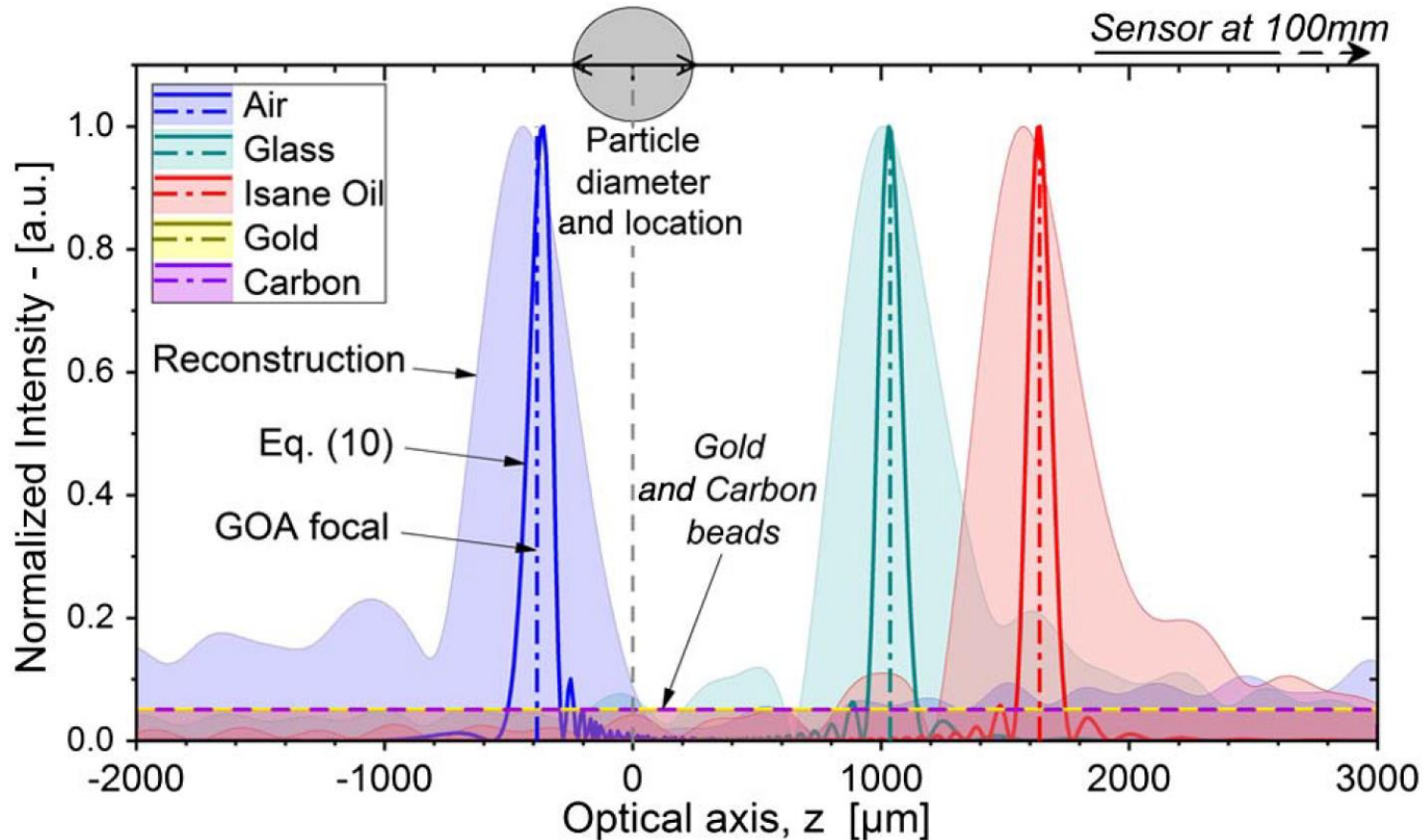
$$E_r(x, y, z_r = z_s - f) = -\frac{\varepsilon c}{2} \frac{\exp[-jk(f+2a)]}{j\lambda f} \exp\left[-\frac{jk\rho^2}{2f}\right] \times \left[2\pi a^2 \frac{J_1(ka\rho/f)}{ka\rho/f} \right] + \dots$$



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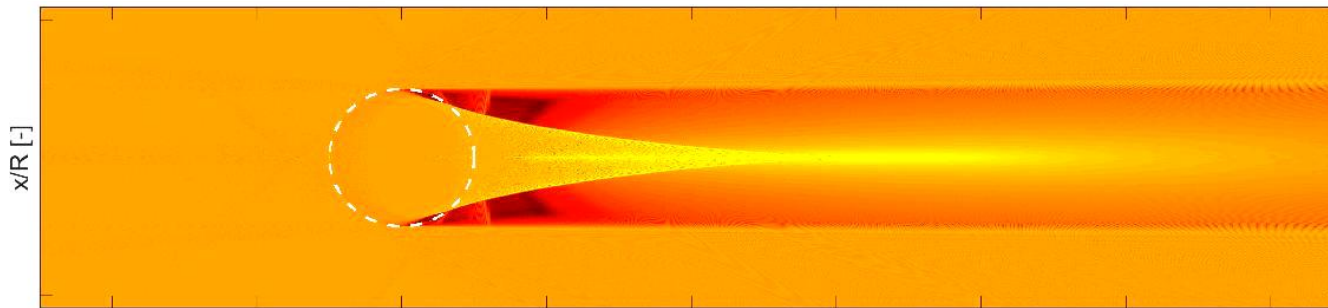
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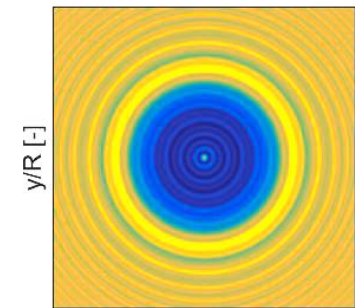
DIH & forward caustic: restitution with proper indicators

Numerical restitution for the recover of the size and position, the locus of its caustic

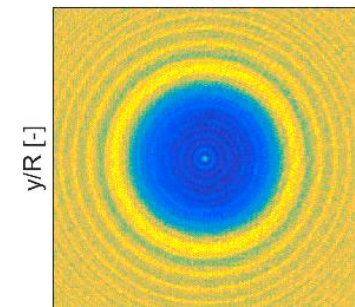
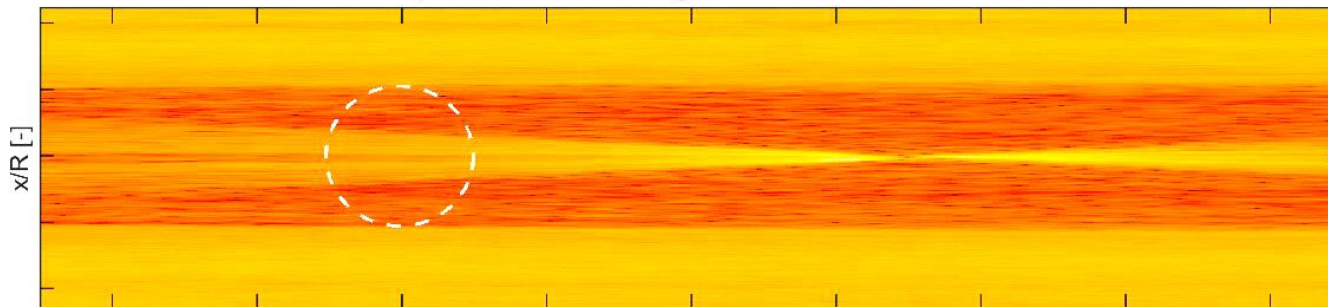
Direct calculation with GLMT



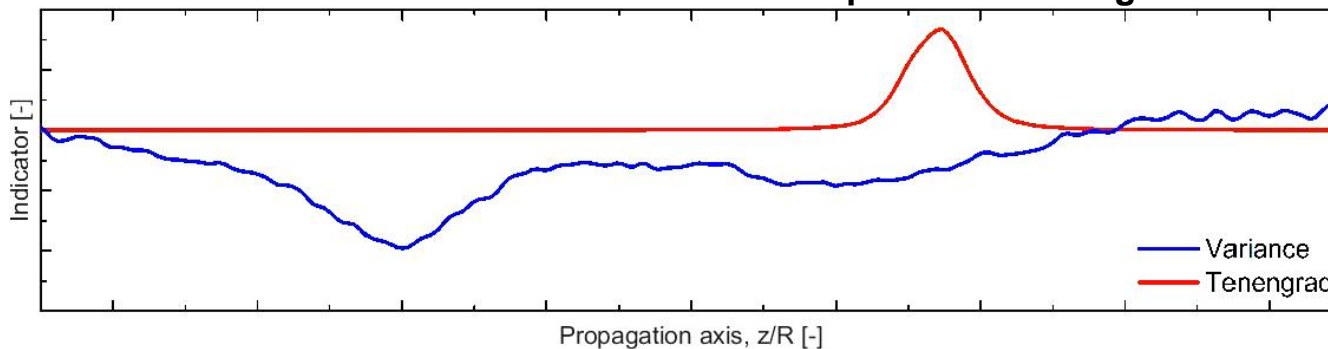
Hologram



Experimental hologram reconstruction



Estimators for the restitution of the experimental hologram



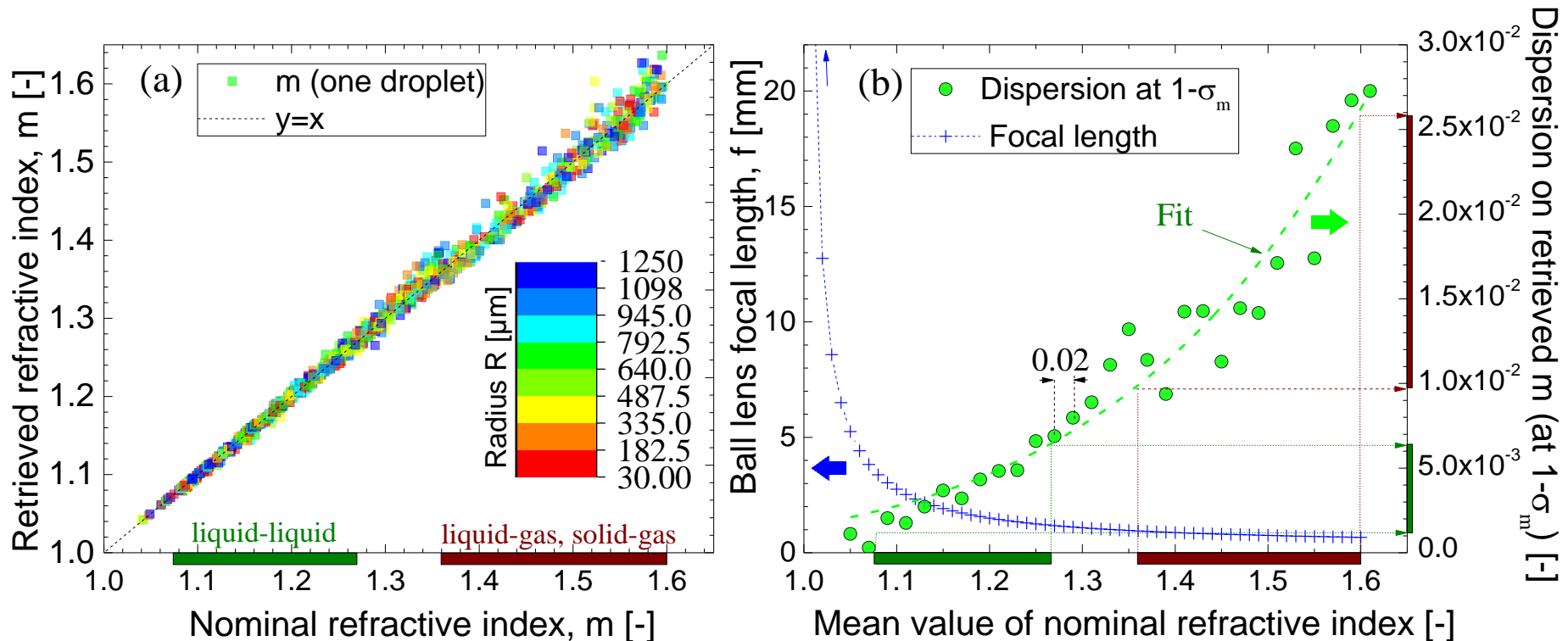
DIH : estimation of the resolution on the refractive index (composition)

GLMT is used to directly simulate a recorded hologram

Restitution is performed with the Fresnel Impulse Transform

Liquid-liquid and gas-liquid systems: particles size between $30\mu\text{m}$ and 1mm

(1000 holograms generated from randomly parametrized particles (x, y, z, D, m))



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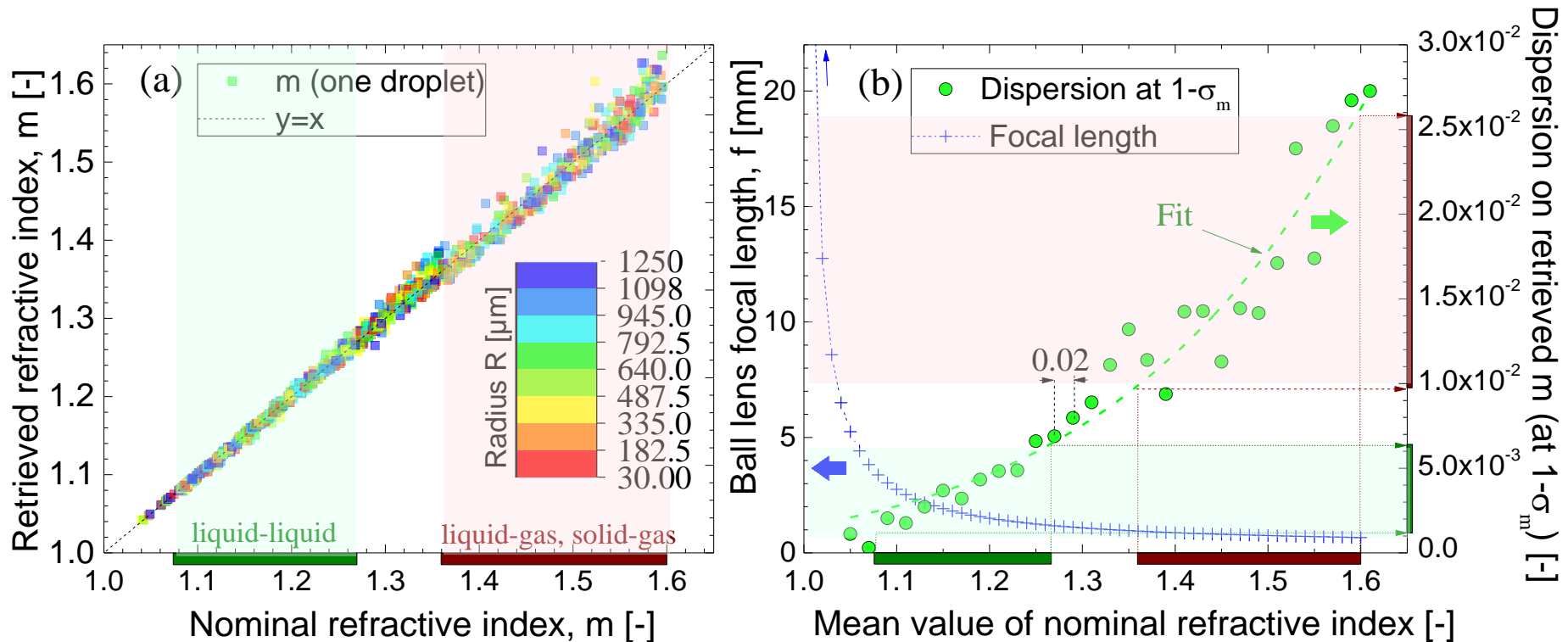
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=> liquid-liquid systems: better than $6 \cdot 10^{-3}$

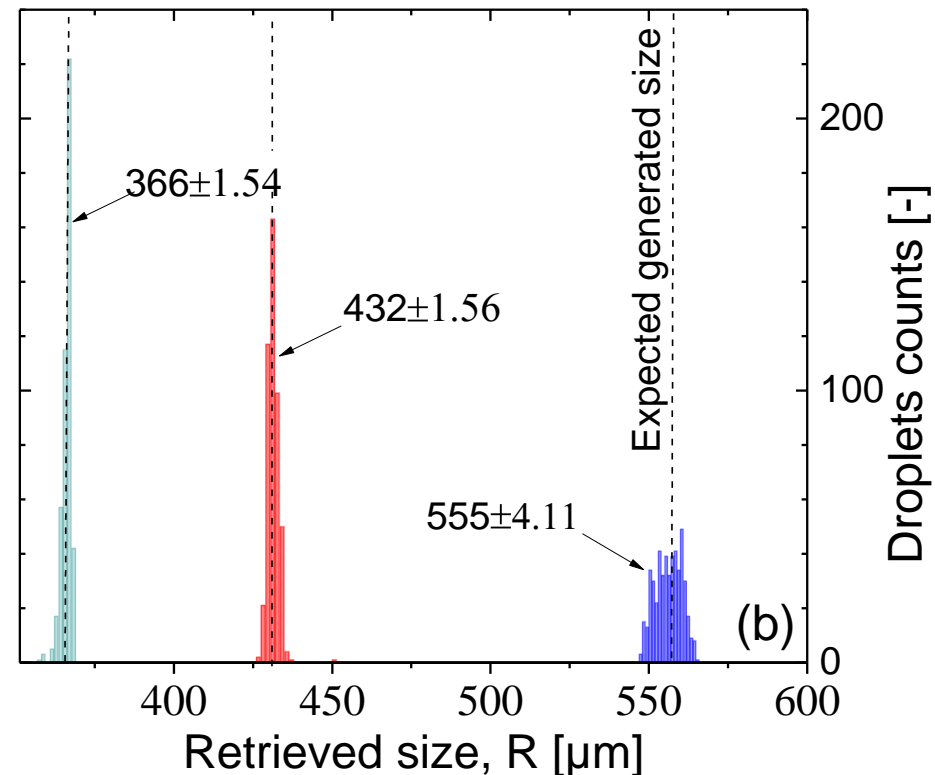
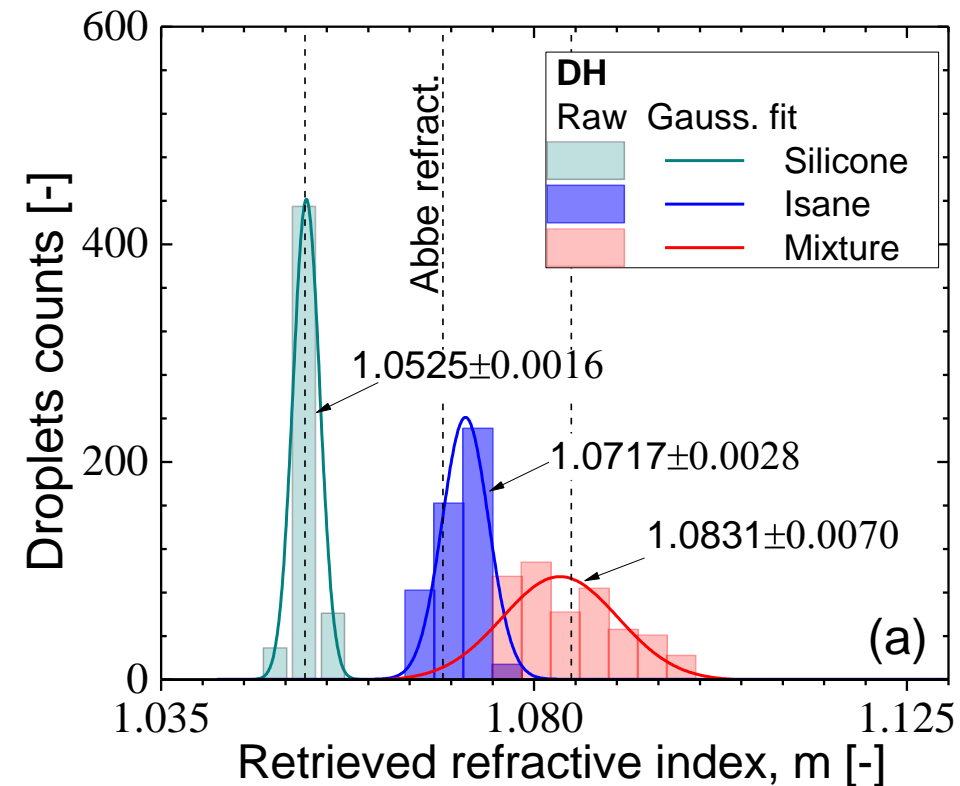
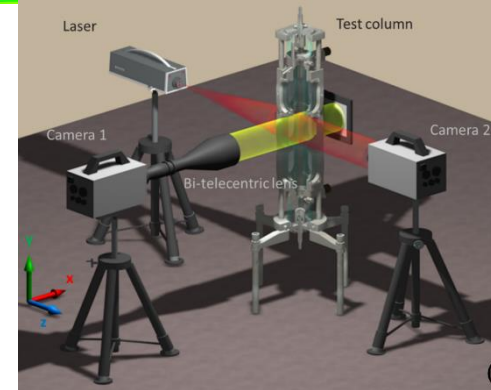
=> liquid or solid-gas systems: better than $2.5 \cdot 10^{-2}$



DIH experimental example 1: resolution on the refractive index & size

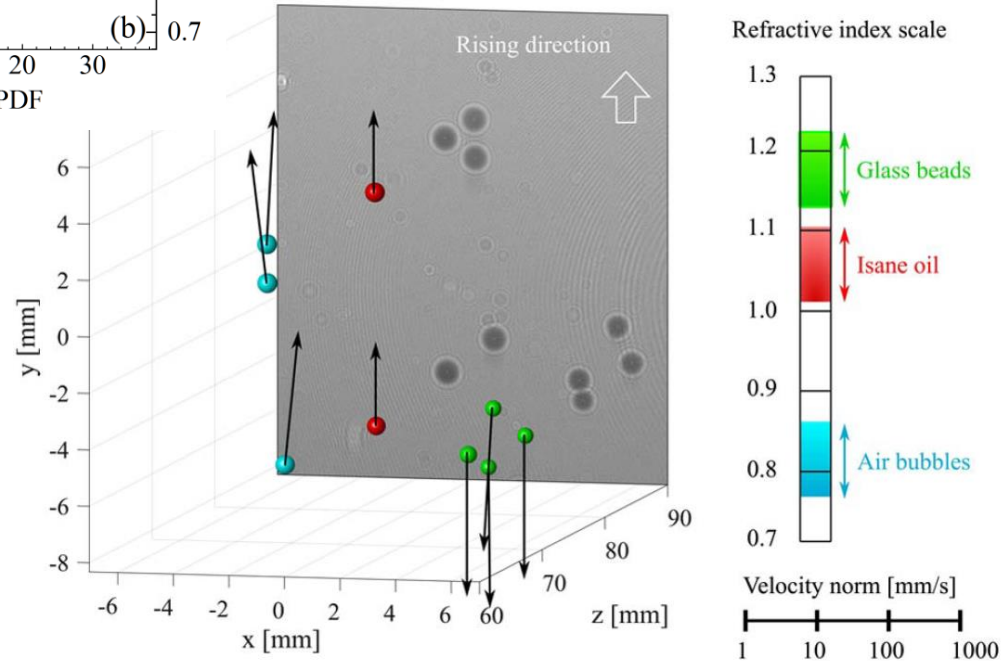
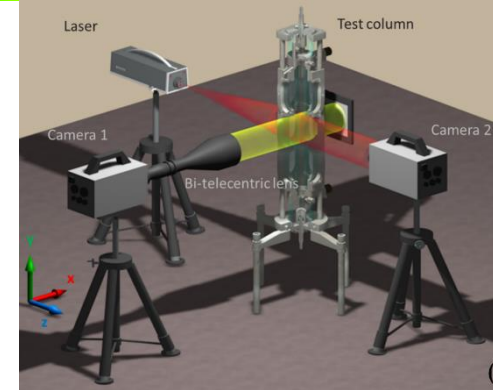
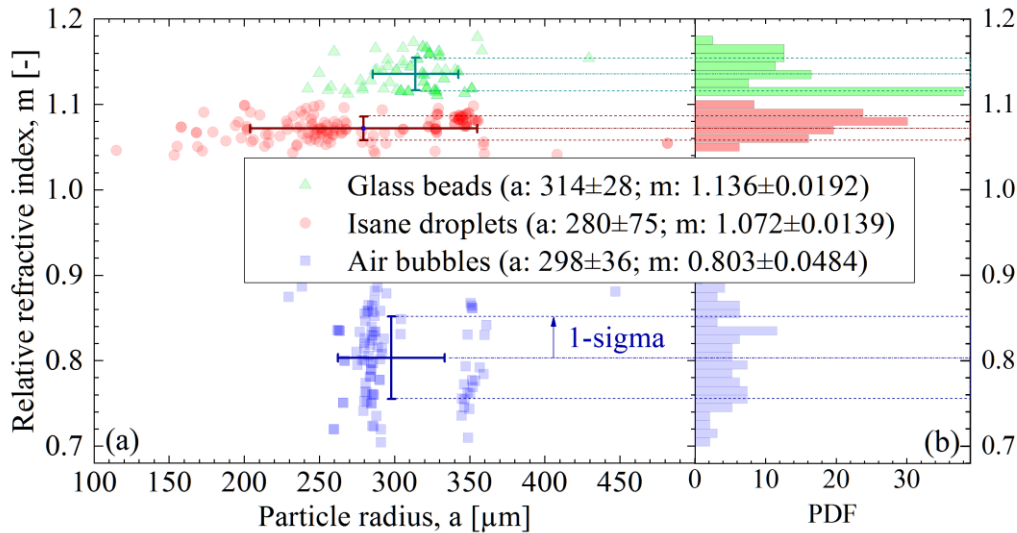
Experimental tests on the resolution achievable on refractive index

- two pure fluids (silicone & isane in water) and one mixture (heptane+isane)
- 500 single droplets for each fluids



DIH experimental example 2: 3D motion dynamic + Size + Composition

Tri-phase flow: rising bubbles & droplets and sinking solid particles



Conclusion et Perspectives

Caustics: an ancient and modern subject...

Physical Asymptotics: alternative to current numerical limits of full electromagnetic approaches (that can hardly manage complex shaped particles of more than a few μm or a few tens μm in the optical range)

Diffraction in the vicinity of the rainbow(s) and critical angles

- Spheroidal particles
- Heterogeneous particles
- 3D dynamics, size and composition
- *Accounting for tilt angles...*
- *Improvement of inverse and regularization methods (i.e. hidden correlations)*
- *Particle concentration !?*

Diffraction in the forward direction and in-line holography

- Optical media with astigmatism
- Recognition of particle material (gas, liquid and solid).
- Estimation of the corresponding refractive index (material composition)
- *Improve the modelling of the phase function (much more to come !)*
- *More complex particle shapes*

