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► **To cite this version:**

Habib Dimassi, Saim Hadj Said, Antonio Loria, Faouzi Msahli. Adaptive state estimation for a class of nonlinear systems: a high gain approach. 2019 19th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA), Mar 2019, Sousse, Tunisia. pp.359-364, 10.1109/STA.2019.8717203 . hal-02368246

HAL Id: hal-02368246

<https://hal.science/hal-02368246>

Submitted on 9 Mar 2020

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Adaptive state estimation for a class of nonlinear systems: a high gain approach

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Abstract—In this paper, we propose a new adaptive estimation approach for a class of uncertain nonlinear systems such that the classical restrictive observer matching condition is not verified. That is, it is assumed that the relative degree of the outputs w.r.t the uncertain parameters vector is greater or equal than two. To solve this problem, we generate auxiliary outputs and we construct a high gain observer in cascade with an adaptive observer to achieve the objective of states and uncertain parameters reconstruction. Based on a Lyapunov analysis, we establish the convergence of both estimation and adaptation errors. Theoretical results are validated via some numerical simulations for an example of a nonlinear mechanical system.

Index Terms—Adaptive observer, high gain observers, observer matching condition, unknown parameters.

I. INTRODUCTION

Joint states and unknown parameters estimation of nonlinear systems is an important problem in automatic control. Different structures of adaptive observers have been employed in many applications such as fault-tolerant control, fault reconstruction, identification and adaptive synchronization of chaotic systems.

In [1], Zhang have developed an adaptive observer for a class of multi-inputs multi-outputs linear time-varying systems under the persistency of excitation hypothesis. In [2], the authors have proposed adaptive observers for a class of observable nonlinear MIMO systems with linear and/or nonlinear parameterization. In [3], an adaptive estimation approach was designed for a class of nonlinear systems where nonlinear and regressor terms depend only on the output signal, then based on sliding modes techniques, the approach was generalized to the case where the system is corrupted by additive perturbations. In [4], an adaptive observer scheme was applied to particular systems (including chaotic systems) and applied to solve the problem of synchronization of chaotic systems under the persistency of

excitation assumption by exploiting the richness of chaotic systems.

In [5] and [6], classical structures of adaptive observers have been addressed to a class of systems verifying the Lipschitz condition under the persistency of excitation assumption. Two important assumptions are needed for the synthesis of the latter adaptive observers: the minimum phase and the relative degree one hypothesis. The latter assumption is the well-known restrictive observer matching condition which considerably limits the applicability of the latter class of adaptive observers. In particular, the relative degree one assumption is not verified in the case of nonlinear mechanical systems where the outputs are the linear and the angular positions whereas the unknown parameters are present in the dynamics of linear and angular velocities.

In this research work, we consider the problem of adaptive estimation for nonlinear systems such that the relative degree one condition is not verified. Our main objective consists in designing a new adaptive observer which estimates both the states and the unknown parameters for a class of nonlinear systems with relative degree at least equal to two. Inspired by the work of [7] where authors have applied the idea of using auxiliary outputs to relax the relative degree one hypothesis for the sliding mode observer design problem, we solve in this paper the problem of states and unknown parameters reconstruction and we present an adaptive observer in cascade with a high gain observer (whose the purpose is to estimate the generated auxiliary outputs). An appropriate adaptation law is also designed based on a σ -modification technique to ensure the parametric convergence. Based on a Lyapunov analysis, we show that both the estimation and the adaptation errors converge to a compact set that may be diminished by an appropriate choice of the design parameters.

The remainder of this paper is organized as follows. In section II, we state the problem. In section III, we present our new estimation approach based on a high gain observer in cascade with an adaptive observer and we establish the

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convergence of estimation and adaptation errors based on an appropriate Lyapunov function. In section IV, theoretical results are confirmed by numerical simulations for an example of a single link robot arm rotating in a vertical plane. Finally, in section V, we make some concluding remarks .

Notation. We use the following notations throughout the paper. $|\cdot|$ denotes the absolute value for scalars, the induced norm for matrices and the euclidean norm for vectors.

I_n and 0_n represent respectively the identity and the zero matrices of size equal to n .

$\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denote the minimal and the maximal eigenvalues of a matrix X .

II. CONTEXT AND PROBLEM FORMULATION

Consider the following class of uncertain nonlinear systems,

$$\begin{cases} \dot{x} = Ax + B\phi(x, u) + B\psi(x, u)\theta + Eu \\ y = Cx \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^r$, $\theta \in \mathbb{R}^q$ are respectively the state, the measured output, the unmeasured state, the known input vector and the unknown constant parameters vector. A , B , C and E are constant matrices of appropriate dimensions (B , C and D are assumed to be of full rank). Let us now consider the following classical assumptions usually considered in the literature of adaptive and unknown inputs observers.

Assumption 1: The nonlinear functions ϕ and ψ are once continuously differentiable and satisfy the Lipschitz condition in x , uniformly with respect to u with Lipschitz constants K_ϕ and K_ψ respectively. That is, $\forall x, \hat{x} \in \mathbb{R}^n$,

$$|\phi(x, u) - \phi(\hat{x}, u)| \leq K_\phi |x - \hat{x}| \quad (2)$$

$$|\psi(x, u) - \psi(\hat{x}, u)| \leq K_\psi |x - \hat{x}| \quad (3)$$

Assumption 2: The solutions to the system (1) as well as the input function u are assumed to be uniformly bounded.

Assumption 3: The invariant zeros of the system model given by the triple (A, B, C) are in the open left-hand complex plane. That is,

$$\text{Rank} \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} = n + \text{Rank}[B] \quad (4)$$

for each complex number λ such that $\text{Re}(\lambda) \geq 0$

Assumption 4: The observer matching hypothesis is satisfied. That is,

$$\text{Rank}[CB] = \text{Rank}[B]$$

Under assumptions 1–4, the problem was solved in the literature of adaptive control and different design procedures have been proposed such that the regressor function g satisfy a particular persistency of excitation condition, i.e, such that the latter function is sufficiently rich in frequency – see for instance [5] and [6].

In this research work, we focus our interest on the restrictive relative degree one assumption 4 that clearly limits the applications of the classical adaptive observers like those

developed in [5] and [6]. Particularly, the observer matching 4 is not satisfied for several mechanical systems such as the inverted pendulum, flexible robots and self balancing robots for which the only available measurements are linear and angular positions whereas unknown parameters appear in the dynamics of the velocities. The main purpose of this paper is to solve the problem of states and unknown parameters estimation for a class of nonlinear Lipschitz systems such that the relative degree of the outputs w.r.t the unknown parameters vector are greater or equal to two.

III. A HIGH GAIN OBSERVER BASED ADAPTIVE OBSERVER IN A CASCADE CONFIGURATION

In [7], a new sliding mode observer design method based on a high gain approach was proposed in the case where the relative degree one assumption is not satisfied. The authors of the latter paper have proposed a method based on the idea of using auxiliary outputs in order to relax the 4. Inspired from this approach, we propose a new adaptive estimation approach using a high observer in cascade with an adaptive observer and we prove the convergence of the state estimation error as well as the parametric convergence based on a Lyapunov stability analysis.

The adaptive observer that we propose is described by the following dynamics:

$$\dot{\hat{x}} = A\hat{x} + B\phi(\hat{x}, u) + B\psi(\hat{x}, u)\hat{\theta} + Eu + \bar{L}(z_h - H\hat{x}) \quad (5)$$

where

$$H = \begin{bmatrix} C_1 \\ \vdots \\ C_1 A^{q_1-1} \\ \vdots \\ C_p \\ \vdots \\ C_p A^{q_p-1} \end{bmatrix}$$

is an augmented auxiliary matrix constructed based on the output matrix $C = [C_1, \dots, C_p]^T$ such that $\text{Rank}(HB) = \text{Rank}(B)$ where q_i ($1 \leq q_i \leq r_i$) are appropriate integers to be selected, for $i = 1, \dots, p$ and r_i is the relative degree of the i^{th} output $y_i = C_i x$ w.r.t the unknown parameters vector θ . The auxiliary matrix H represents the new output matrix for which the pair (B, H) satisfy the relative degree one assumption. Since the auxiliary output $z = Hx$ is not available, we construct a high gain observer whose the main objective is to estimate the auxiliary outputs. The latter observer is described by the following dynamics

$$\dot{z}_{hi} = a_i z_{hi} + \Gamma_i c_i (z_i - z_{hi}) + b_{i2} u \quad (6)$$

where $z_h = [z_{h1}^T \dots z_{hp}^T]^T$ with $z_{hi} = [z_{hi1} \dots z_{hiq_i}]^T$ represents the estimated auxiliary output.

$$b_{i2} = [C_i E \dots C_i A^{q_i-1} E], \quad c_i = [1, 0, \dots, 0],$$

$$a_i = \begin{bmatrix} 0 & I_{q_i-1} \\ 0 & 0 \end{bmatrix}, \quad \Gamma_i = \left[\frac{\gamma_{i1}}{\varepsilon} \dots \frac{\gamma_{iq_i}}{\varepsilon^{q_i}} \right]^T \text{ with } \varepsilon \text{ and } \gamma_{ij} \text{ (for } j = 1, \dots, q_i) \text{ are design terms to be determined later.}$$

The estimated auxiliary outputs obtained from the high gain observer are employed by the main adaptive observer (5) whose the purpose is to join states and unknown parameters reconstruction.

Moreover, $\hat{\theta}$ is an adaptive parameter updated online following the adaptation law:

$$\dot{\hat{\theta}} = \delta \psi(\hat{x}, u)^T \bar{M}(z_h - H\hat{x}) - \sigma \hat{\theta} \quad (7)$$

The constants δ and σ are positive real numbers imposed by the designer. The design matrices \bar{L} and \bar{M} are regular matrices of appropriate dimensions such that for some symmetric positive definite matrices \bar{P} and \bar{Q} , we have

$$(A - \bar{L}H)^T \bar{P} + \bar{P}(A - \bar{L}H) = -2\bar{Q} \quad (8)$$

$$B^T \bar{P} = \bar{M}H \quad (9)$$

$$|B|(K_\phi + K_\psi \theta) < \frac{\lambda_{\min}(\bar{Q})}{\lambda_{\max}(\bar{P})} \quad (10)$$

Note that if 3 is verified for (A, B, C) , so minimum phase hypothesis is also satisfied for the triple (A, B, H) . That is 3 is also verified for the triple (A, B, H) – See [7] and [8].

Remark 1: Condition (10) is restrictive since it depends to the Lipschitz constants of the nonlinear functions f and g which may be of large values and unknown. To relax this condition, the additional adaptation law may be designed [9]:

$$\dot{\hat{\omega}} = |M(z_h - H\hat{x})|^2 - \sigma_2 \hat{\omega} \quad (11)$$

where $\hat{\omega}$ is an adaptive parameter updated online and whose the objective is to compensate for the constant term $\omega = |B|(K_\phi + K_\psi \theta)$. For the simplicity of presentation, in our main theorem and its corresponding proof, we don't consider the latter adaptation law (11).

Now, we state our main theorem

Theorem 1: Consider the nonlinear uncertain system (1) under assumptions 1, 2 and 3. Consider the adaptive observer (5) with adaptation law (7) and the high gain observer (6). Then, there exist positive constants K_0, K_1, K_2 et K_3 such that the state observation error $e(t)$ and the adaptation error $\tilde{\theta}(t)$ are uniformly bounded and converge to the compact set:

$$D_R = \left\{ e, \tilde{\theta} : V(e, \tilde{\theta}) < \frac{(K_2 \varepsilon + K_3)^2}{(K_0 - K_1 \varepsilon)^2} \right\}$$

with a rate at least as fast as $e^{-K_0(t-T(\varepsilon))}$.

Proof 1: The proof is divided in two main parts. First, we show the convergence of the high gain observer (6). Next, we prove the convergence of the adaptive observer (5) based on the results of the first part of the proof.

Convergence of the high gain observer:

Let $z = Hx = [z_1^T, \dots, z_p^T]^T$ with $z_i = [z_{i1}, \dots, z_{iq_i}]^T$ such that $z_{ij} = C_i A^{j-1} x$ for $i = 1, \dots, p$ and $j = 1, \dots, q_i$. If $q_i > 1$, we synthesize the dynamics of z_i as following

$$\begin{cases} \dot{z}_i = a_i z_i + b_{i1} h_i(y, z, u, t, \theta) + b_{i2} u \\ z_{i1} = c_i z_i \end{cases} \quad (12)$$

where (a_i, b_{i1}) is in the canonical form, $h_i(y, z, u, t, \theta) = C_i A^{q_i} x + C_i A^{q_i-1} B[\phi(x, u) + \psi(x, u)\theta]$, $b_{i2} = [C_i E \dots C_i A^{q_i-1} E]$ and $c_i = [1, 0, \dots, 0]$.

Now we show that the state z_{hi} of the high gain observer (6) is an estimate for the state z_i of the latter system.

Let $\eta_i = [\eta_{i1} \dots \eta_{iq_i}]^T$ where

$$\eta_{ij} = \frac{z_{ij} - z_{hij}}{\varepsilon^{q_i-j}}, j = 1, \dots, q_i. \quad (13)$$

Let now $\eta = [\eta_1^T \dots \eta_p^T]^T$ and $D = \text{diag}[D_1 \dots D_p]$ such that $D_i = \text{diag}[\varepsilon^{q_i-1} \varepsilon^{q_i-2} \dots 1]$. So, we have

$$z - z_h = D\eta. \quad (14)$$

Now, we select ε sufficiently small such that $\varepsilon \in (0, 1)$ and γ_j (for $j = 1, \dots, q_i$) such that the matrix

$$\bar{a}_i = \begin{bmatrix} -\gamma_i^T & I_{q_i-1} \\ -\gamma_i q_i & 0_{q_i-1} \end{bmatrix}$$

is Hurwitz, where $\gamma_i = [\gamma_{i1} \dots \gamma_{i(q_i-1)}]$. As a consequence, using the arguments of [7], for the observer (6), there exist a positive real β and a finite time $T(\varepsilon)$ such that for all $t \geq T(\varepsilon)$

$$|\eta(t)| \leq \beta \varepsilon. \quad (15)$$

Moreover, we have $\lim_{\varepsilon \rightarrow +\infty} T(\varepsilon) = 0$.

Convergence of the adaptive observer:

Let $e := x - \hat{x}$ and $\tilde{\theta} := \theta - \hat{\theta}$, respectively, the observation and the adaptation errors corresponding to the considered nonlinear system (1) and the adaptive observer (5).

Differentiating on both sides of the observation error $e(t)$ and using (14), we obtain

$$\begin{aligned} \dot{e} &= (A - \bar{L}H)e + B[\phi(x, u) - \phi(\hat{x}, u)] \\ &\quad + B[\psi(x, u) - \psi(\hat{x}, u)]\theta \\ &\quad + B\psi(\hat{x}, u)\tilde{\theta} + \bar{L}D\eta. \end{aligned} \quad (16)$$

According to the adaptation law (7) and since $\dot{\theta} \simeq 0$, $\tilde{\theta}$ is solution to

$$\begin{aligned} \dot{\tilde{\theta}} &= -\delta \psi(\hat{x}, u)^T \bar{M}(He - D\eta) + \sigma \hat{\theta} \\ &= -\delta \psi(\hat{x}, u)^T \bar{M}(He - D\eta) - \sigma \tilde{\theta} + \sigma \theta \end{aligned} \quad (17)$$

where we have also used the equation (14).

Next, consider the Lyapunov function

$$V(e, \tilde{\theta}) := e^T \bar{P} e + \frac{1}{\delta} \tilde{\theta}^T \tilde{\theta}. \quad (18)$$

The total time derivative of (18) along the trajectories of (16)–(17) yields

$$\begin{aligned}\dot{V} &= e^\top [(A - \bar{L}H)^\top \bar{P} + \bar{p}(A - \bar{L}H)]e \\ &+ 2e^\top \bar{P}B[\phi(x, u) - \phi(\hat{x}, u)] \\ &+ 2e^\top \bar{P}B[\psi(x, u) - \psi(\hat{x}, u)]\theta \\ &+ 2e^\top \bar{P}B\bar{\theta}\psi(\hat{x}, u) + 2e^\top \bar{P}\bar{L}D\eta \\ &- 2\bar{\theta}^\top [\psi(\hat{x}, u)^\top \bar{M}He] - \frac{2\sigma}{\delta}\bar{\theta}^2 + \frac{2\sigma}{\delta}\theta\bar{\theta} \\ &- 2\bar{\theta}^\top [\psi(x, u) - \psi(\hat{x}, u)]^\top \bar{M}D\eta \\ &+ 2\bar{\theta}^\top [\psi(x, u)]^\top \bar{M}D\eta.\end{aligned}$$

We recall that the nonlinear function ψ is once continuously differentiable, then using the boundedness of the trajectory of the considered system (2), there exists a constant Ψ_{\max} such that for all $t \geq 0$, we have:

$$|\psi(x, u)| \leq \Psi_{\max} \quad (19)$$

Next, we use respectively the equations (1), (8), (9), (19), (15) and the fact that $|D| = 1$ from the definition of the matrix D , then we employ the Young's inequality on the term $|\bar{\theta}| |e|$ and we apply the fact that $|e| \leq \frac{\sqrt{V}}{\sqrt{\lambda_{\min}(\bar{P})}}$ and $|\bar{\theta}| \leq \sqrt{\delta}\sqrt{V}$ to finally deduce that:

$$\dot{V} \leq -2K_0V + K_1\varepsilon V + (K_2\varepsilon + K_3)\sqrt{V} \quad (20)$$

where

$$\begin{aligned}K_0 &= \min(\tau, \sigma) \\ K_1 &= 2\beta \max\left(\frac{1}{2\lambda_{\min}(\bar{P})}, \frac{\delta}{2}\right) K_\psi |\bar{M}| \\ K_2 &= \frac{2\beta |\bar{P}\bar{L}|}{\sqrt{\lambda_{\min}(\bar{P})}} + \sqrt{\delta}\beta\Psi_{\max} |\bar{M}| \\ K_3 &= \frac{2\sigma}{\sqrt{\delta}}\theta,\end{aligned}$$

where $\tau := \frac{\lambda_{\min}(Q)}{\lambda_{\max}(\bar{P})} - |B| [K_\phi + K_g\theta]$.

It follows that

$$\dot{V} \leq -K_0V - \sqrt{V} \left[(K_0 - K_1\varepsilon) - (K_2\varepsilon + K_3)\sqrt{V} \right]. \quad (21)$$

Consequently, we can show that $V(e(t), \bar{\theta}(t))$ will be decreasing exponentially fast such that

$$V(e(t), \bar{\theta}(t)) \leq V(T(\varepsilon))e^{-K_0(t-T(\varepsilon))}. \quad (22)$$

and that $e(t)$ and $\bar{\theta}(t)$ converge to the compact set:

$$D_R = \left\{ e, \bar{\theta} : V(e, \bar{\theta}) < \frac{(K_2\varepsilon + K_3)^2}{(K_0 - K_1\varepsilon)^2} \right\}. \quad (23)$$

It is to be noticed that the design parameters ε , δ and σ may be appropriately selected in order to reduce the compact set D_R . Indeed, it may be observed that when fixing ε and σ , the compact set D_R may be diminished by selecting a sufficiently large value of the parameter

δ . On the other hand, when fixing the parameters δ and σ such that $\delta \gg \sigma$, the gain parameter ε may be selected sufficiently small such that $\varepsilon \ll 1$ in order to make the radius R of the compact set D_R as small as possible.

Note also that in our adaptive estimation approach, by applying the σ -modification technique, we avoid the restrictive persistency of excitation assumption: of course, we don't obtain the asymptotic stability, but we obtain satisfactory convergence results: the state observation and adaptation errors converge to a compact set whose the radius may be made as small as possible by choosing appropriate values of the design parameters ε , δ and σ . The σ -modification technique consisting of adding the term $-\sigma\hat{\theta}$ is a classical solution to improve the robustness of adaptation laws usually used in the literature of adaptive control – See for instance [10].

Remark 2: Our adaptive estimation approach may be compared with three classes of adaptive observers. The first one concerns the class of adaptive observers designed for MIMO linear-time varying systems where the pioneering work is presented in [1]). Its design method doesn't need any structural assumptions but is addressed only to a class of time-varying linear systems. The second one is the class of high gain-adaptive observers (See references [2], [11], [12]) which doesn't need to satisfy the minimum phase and relative degree one conditions, however the synthesis concept is mainly based on the use of particular triangular canonical forms which restricts the applicability of these observers. The third one is the class of adaptive observers considered in the references [5] and [6]. The construction of these observers doesn't need particular canonical forms, but is based on the minimum phase and relative degree one assumptions. All these observers are designed under the persistency of excitation assumption. It is to be noticed that the latter class of adaptive observers have a simpler architecture for implementation compared to the other classes of adaptive observers. The main objective of the proposed adaptive observer in this paper is to extend the range of applicability of the third class of adaptive observers ([5] and [6]) by the relaxation of the observer matching condition which constitutes one of the main contributions of this paper.

IV. NUMERICAL SIMULATIONS

In order to illustrate the good performances of the proposed high gain observer based adaptive estimation approach, we consider the nonlinear mechanical system consisting on a single link robot arm rotating in a vertical plane and described by the following differential equation:

$$I\ddot{q} + \frac{1}{2}mgl \sin(q) = u \quad (24)$$

where $I = 0.5$ is the moment of inertia, q is the rotation angle, $g = 9.81$ is the gravity constant, $m = 1$ is the mass, $l = 1$ is the length of the arm and $u(t) = 5(\sin(2t) + \cos(3t))$ is the input torque. The differential equation (24) may be written in the following state model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\theta_2 \sin(y) + \theta_1 u \\ y = x_1 \end{cases} \quad (25)$$

where $x_1 = q$, $x_2 = \dot{q}$, $y = q = x_1$, $\theta_1 = \frac{1}{l}$, $\theta_2 = \frac{mgl}{2l}$ and which may be written in the form of (1) with $x = [x_1, x_2]^T$, $\phi(z, y, u, t) = 0$, $\psi(z, y, u, t) = [-\sin(y), u(t)]^T$, $C = C_1 = [1, 0]$, $B = [0, 1]^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

It is clear that the observer matching hypothesis 4 is not satisfied since $\text{Rank}(CB) \neq \text{Rank}(B)$ and the relative degree of the output y w.r.t the unknown parameters vector θ is equal to 2. We generate the auxiliary output:

$$z = Hx = \begin{bmatrix} z_{11} = C_1 x \\ z_{12} = C_1 A x \end{bmatrix}.$$

Since $C_1 B = 0$, we have

$$\begin{cases} \dot{z}_{11} = C_1 \dot{x} = C_1 A x + C_1 B \psi(z, y, u, t) \theta = C_1 A x = z_{12} \\ \dot{z}_{12} = C_1 A \dot{x} = C_1 A^2 x + C_1 A B \psi(z, y, u, t) \theta \\ z_{11} = y \end{cases} \quad (26)$$

which is of the form of (12) with $h_1(z, y, u, t) = C_1 A^2 x + C_1 A B \psi(z, y, u, t) \theta$, $b_{11} = [0, 1]^T$ and

$$a_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

The high gain observer corresponding to system (26) is given by

$$\begin{cases} \dot{z}_{h1} = a_1 z_{h1} + \Gamma_1 c_1 (z_{11} - z_{h1}) \\ z_{h11} = c_1 z_{h1} \end{cases} \quad (27)$$

where $\Gamma_1 = [\frac{\gamma_{11}}{\varepsilon}, \frac{\gamma_{12}}{\varepsilon^2}]^T$. We select $\gamma_{11} = 6$, $\gamma_{12} = 8$ and $\varepsilon = 0.006$.

Note that, when using the high gain observer, the peaking phenomenon appears in the estimated states. To deal with this undesirable phenomenon, we saturate the signals z_{h1} as follows:

$$z_{h1}^s = \left[S_{11} \text{sat}\left(\frac{z_{h11}}{S_{11}}\right), S_{12} \text{sat}\left(\frac{z_{h12}}{S_{12}}\right) \right]^T$$

We choose $S_{11} = S_{12} = 150$.

We design an adaptive observer of the form of (5) with adaptation law (7). The initial states of the considered system are selected as $x(0) = [1, 1]^T$, the initial states of both high-gain and adaptive observers are fixed to zero and the adaptive parameter is initialized as $\hat{\theta}(0) = [0, 0]^T$. The design parameters are set to $\sigma = 0.01$, $\delta = 100$. To solve the system of equations (8) and (9), one considers the following convex problem [13]:

Minimize ρ subject to

$$\begin{aligned} & \bar{P} > 0 \\ & \bar{P}A - KH + (\bar{P}A - KH)^T < 0 \\ & \begin{bmatrix} \rho I & B^T \bar{P} - \bar{M}H \\ (B^T \bar{P} - \bar{M}H)^T & \rho I \end{bmatrix} > 0 \end{aligned} \quad (28)$$

This problem may be solved by using LMI tools in association with MATLAB. When this problem has a minimum $\rho = 0$, the system of equations (8) and (9) is satisfied and numerical values of \bar{P} , \bar{M} , and $\bar{L} = \bar{P}^{-1}K$ may be obtained. The numerical values of the \bar{M} and \bar{L} are selected respectively as $M = [0, 1]$ and

$$L = \begin{bmatrix} 5 & 0 \\ 1 & 10 \end{bmatrix}.$$

The obtained simulation results are as follows. In Figure (1), we illustrate the estimation of the auxiliary output \hat{z}_{12} by the high gain observer (6). The estimation of the states x_1 and x_2 by the adaptive observer (5) and adaptation law (7) is illustrated in Figures (2) and (3). The estimation of the unknown parameters θ_1 and θ_2 is shown in Figures (4) and (5).

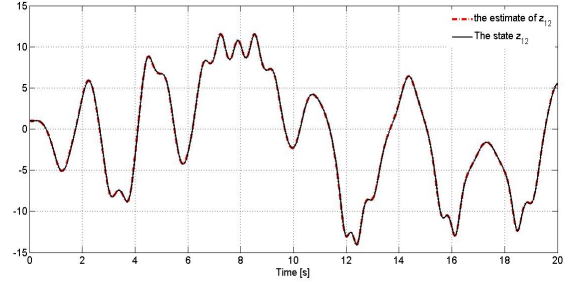


Fig. 1. Evolution of the auxiliary output z_{12} and its estimate \hat{z}_{12} by the high gain observer (6)

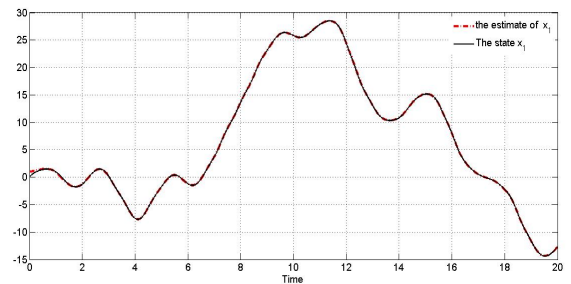


Fig. 2. Evolution of the state x_1 and its estimate \hat{x}_1 by the adaptive observer (5) and adaptation law (7)

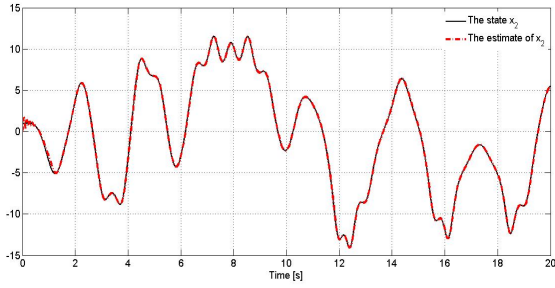


Fig. 3. Evolution of the state x_2 and its estimate \hat{x}_2 by the adaptive observer (5) and adaptation law (7)

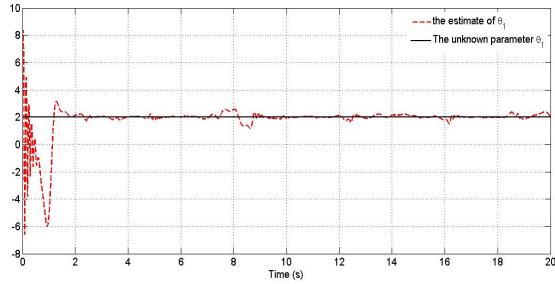


Fig. 4. The unknown parameter θ_1 and its estimate $\hat{\theta}_1$

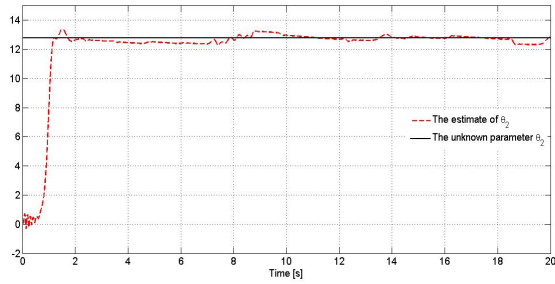


Fig. 5. The unknown parameter θ_2 and its estimate $\hat{\theta}_2$

V. CONCLUSION

In this paper, the problem of joint states and unknown parameters reconstruction is solved for a class of nonlinear systems such that the relative degree of the outputs w.r.t the unknown parameters vector is at least equal to two which means that the so-called observer matching hypothesis is assumed not verified for the considered class of systems. Using a high gain observer and an adaptive observer in a cascade configuration, we established the convergence of both estimation and adaptation errors to a small compact set with radius depending on the design parameters. Theoretical results are validated with some numerical simulations for an example of a single link robot arm rotating in a vertical plane.

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