# Local Spot Noise for Procedural Surface Details Synthesis : Supplementary Material 

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## 1 Extended Results

In this section we show Figures (10.a), (10.c) and (10.d) and all spot noise patterns and textures used to achieve these composite textures. Spot noise values are shown between 0 and 1, dark parts correspond to negative amplitudes and clear parts to positive ones.


This result uses two procedural textures. The first consists in a local spot noise with a single elliptical gaussian per kernel with multiple impulses. We just use the partial derivatives with this pattern. The Second pattern is similar to the "Hive" pattern shown in the paper with a larger gaussian scale. We use an overstated amplitude as a blending between the two procedural textures generated with the Heitz \& Neyret [1] method.


The spot noise pattern used in this result consists in a single isotropic gaussian with a small scale per kernel with multiple impulses per cell. Like Figure (10.a) the amplitude is exaggerated to blend between the two procedural textures.


This spot noise pattern consists in a single anisotropic gaussian with a small scale and a random rotation per kernel with multiple impulses per cell.

## 2 Solution for Gaussian partial derivatives convoluted by another Gaussian

We present the 2D closed-form solution used in our method in Section 6. We start by some notations used in the derivation, followed by some closed form integrals for 2D Gaussians. Finally we compute the integral of the product of a Gaussian function with the partial derivative of another 2D Gaussian.

## Notations :

Given a covariance matrix $\boldsymbol{\Sigma}$, a mean vector $\mu$ and a vector $\mathbf{x}$, we note the elements as:

$$
\boldsymbol{\Sigma}_{i}^{-1}=\left(\begin{array}{cc}
a_{i} & b_{i} \\
b_{i} & c_{i}
\end{array}\right) \quad \mu_{\mathbf{i}}=\binom{\mu_{i x}}{\mu_{i y}}
$$

$$
\mathbf{x}=\binom{x}{y}
$$

To alleviate the derivation we use the following notation for 2D Gaussians:

$$
g_{i}(\mathbf{x})=g\left(\mathbf{x} ; \lambda_{i}, \mu_{\mathbf{i}}, \mathbf{\Sigma}_{\mathbf{i}}\right)
$$

## Partial derivatives of a 2D Gaussian :

Using Equation (8) The partial derivatives of a 2D Gaussian according to $x$ and $y$ are given by :

$$
\begin{aligned}
& \frac{\partial g\left(\mathbf{x} ; \lambda_{i}, \mu_{\mathbf{i}}, \boldsymbol{\Sigma}_{\mathbf{i}}\right)}{\partial x}=-\left(a_{i}, b_{i}\right)^{T} \cdot\left(\mathbf{x}-\mu_{\mathbf{i}}\right) g\left(\mathbf{x} ; \lambda_{i}, \mu_{\mathbf{i}}, \boldsymbol{\Sigma}_{\mathbf{i}}\right) \\
& \frac{\partial g\left(\mathbf{x} ; \lambda_{i}, \mu_{\mathbf{i}}, \boldsymbol{\Sigma}_{\mathbf{i}}\right)}{\partial y}=-\left(b_{i}, c_{i}\right)^{T} \cdot\left(\mathbf{x}-\mu_{\mathbf{i}}\right) g\left(\mathbf{x} ; \lambda_{i}, \mu_{\mathbf{i}}, \boldsymbol{\Sigma}_{\mathbf{i}}\right)
\end{aligned}
$$

## Integral of a 2D Gaussian times $x$ or $y$ :

Integrating a given multivariate Gaussian $g$ times $x$ or $y$ over $\mathbb{R}^{D}$ yields :

$$
\begin{aligned}
& \int_{\mathbb{R}^{2}} x \cdot g(\mathbf{x} ; \lambda, \mu, \boldsymbol{\Sigma}) d \mathbf{x}=\mu_{x} \lambda(2 \pi)|\boldsymbol{\Sigma}|^{1 / 2} \\
& \int_{\mathbb{R}^{2}} y \cdot g(\mathbf{x} ; \lambda, \mu, \boldsymbol{\Sigma}) d \mathbf{x}=\mu_{y} \lambda(2 \pi)|\boldsymbol{\Sigma}|^{1 / 2}
\end{aligned}
$$

## Derivation:

We start by computing the integral of the product of the partial derivative according to $x$ of a 2D Gaussian $g_{1}(\mathbf{x})$ and another 2D Gaussian $g_{2}(\mathbf{x})$ :

$$
\begin{aligned}
\int_{\mathbb{R}^{2}} \frac{\partial g_{1}(\mathbf{x})}{\partial x} g_{2}(\mathbf{x}) d \mathbf{x} & =\int_{\mathbb{R}^{2}}-\left(\left(a_{1}, b_{1}\right)^{T} \cdot\left(\mathbf{x}-\mu_{\mathbf{1}}\right)\right) g_{1}(\mathbf{x}) g_{2}(\mathbf{x}) d \mathbf{x} \\
& =\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot\left(\mu_{\mathbf{1}}-\mathbf{x}\right)\right) g_{1}(\mathbf{x}) g_{2}(\mathbf{x}) d \mathbf{x} \\
& =\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}-\left(a_{1}, b_{1}\right)^{T} \cdot \mathbf{x}\right) g_{1}(\mathbf{x}) g_{2}(\mathbf{x}) d \mathbf{x}
\end{aligned}
$$

We remind that the product of two Gaussians $g_{1}$ and $g_{2}$ yields another Gaussian $g_{3}$ yielding:

$$
\begin{align*}
\int_{\mathbb{R}^{2}} \frac{\partial g_{1}(\mathbf{x})}{\partial x} g_{2}(\mathbf{x}) d \mathbf{x} & =\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}-\left(a_{1}, b_{1}\right)^{T} \cdot \mathbf{x}\right) g_{3}(\mathbf{x}) d \mathbf{x} \\
& =\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{1}\right) g_{3}(\mathbf{x}) d \mathbf{x}-\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mathbf{x}\right) g_{3}(\mathbf{x}) d \mathbf{x} \tag{1}
\end{align*}
$$

The first part of equation 1 can be evaluated using the closed-form integral of a Gaussian :

$$
\begin{align*}
\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}\right) g_{3}(\mathbf{x}) d \mathbf{x} & =\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}\right) \int_{\mathbb{R}^{2}} g_{3}(\mathbf{x}) d \mathbf{x} \\
& =\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2} \tag{2}
\end{align*}
$$

For the second part of equation 1 we develop the dot product between $\left(a_{1}, b_{1}\right)^{T}$ and $\mu_{1}$ :

$$
\begin{aligned}
\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mathbf{x}\right) g_{3}(\mathbf{x}) d \mathbf{x} & =\int_{\mathbb{R}^{2}}\left(a_{1} x+b_{1} y\right) g_{3}(\mathbf{x}) d \mathbf{x} \\
& =\int_{\mathbb{R}^{2}}\left(a_{1} x\right) g_{3}(\mathbf{x}) d \mathbf{x}+\int_{\mathbb{R}^{2}}\left(b_{1} y\right) g_{3}(\mathbf{x}) d \mathbf{x} \\
& =a_{1} \int_{\mathbb{R}^{2}} x g_{3}(\mathbf{x}) d \mathbf{x}+b_{1} \int_{\mathbb{R}^{2}} y g_{3}(\mathbf{x}) d \mathbf{x}
\end{aligned}
$$

By using the two closed form integrals previously introduced it yields:

$$
\begin{align*}
a_{1} \int_{\mathbb{R}^{2}} x g_{3}(\mathbf{x}) d \mathbf{x}+b_{1} \int_{\mathbb{R}^{2}} y g_{3}(\mathbf{x}) d \mathbf{x} & =\left(a_{1} \mu_{3 x}+b_{1} \mu_{3 y}\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2} \\
& =\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{3}}\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2} \tag{3}
\end{align*}
$$

Now by substituting 2 and 3 in 1 we obtain:

$$
\begin{aligned}
\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}\right) g_{3}(\mathbf{x}) d \mathbf{x}-\int_{\mathbb{R}^{2}}\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mathbf{x}\right) g_{3}(\mathbf{x}) d \mathbf{x} & =\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2}-\left[\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{3}}\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2}\right] \\
& =\left[\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{1}}\right)-\left(\left(a_{1}, b_{1}\right)^{T} \cdot \mu_{\mathbf{3}}\right)\right] \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2} \\
& =\left(\left(a_{1}, b_{1}\right)^{T} \cdot\left(\mu_{\mathbf{1}}-\mu_{\mathbf{3}}\right)\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2}
\end{aligned}
$$

Finally we obtain :

$$
\int_{\mathbb{R}^{2}} \frac{\partial g_{1}(\mathbf{x})}{\partial x} g_{2}(\mathbf{x}) d \mathbf{x}=\left(\left(a_{1}, b_{1}\right)^{T} \cdot\left(\mu_{\mathbf{1}}-\mu_{\mathbf{3}}\right)\right) \lambda_{3}(2 \pi)\left|\Sigma_{3}\right|^{1 / 2}
$$

To retrieve the convolution we substitute $\mu_{\mathbf{1}}$ by $\mathbf{x}-\mu_{\mathbf{1}}$

$$
\int_{\mathbb{R}^{2}} \frac{\partial g_{1}(\mathbf{x}-\mathbf{t})}{\partial x} g_{2}(\mathbf{t}) d \mathbf{t}=-\left(\left(a_{1}, b_{1}\right)^{T} \cdot\left(\mathbf{x}-\left(\mu_{\mathbf{1}}+\mu_{\mathbf{3}}\right)\right)\right) \lambda_{3}(2 \pi)\left|\boldsymbol{\Sigma}_{\mathbf{3}}\right|^{1 / 2}
$$

By following the same process, the prefiltered partial derivative according to $y$ is

$$
\int_{\mathbb{R}^{2}} \frac{\partial g_{1}(\mathbf{x}-\mathbf{t})}{\partial y} g_{2}(\mathbf{t}) d \mathbf{t}=-\left(\left(b_{1}, c_{1}\right)^{T} \cdot\left(\mathbf{x}-\left(\mu_{\mathbf{1}}+\mu_{\mathbf{3}}\right)\right)\right) \lambda_{3}(2 \pi)\left|\boldsymbol{\Sigma}_{\mathbf{3}}\right|^{1 / 2}
$$

## References

[1] Eric Heitz and Fabrice Neyret. High-Performance By-Example Noise using a Histogram-Preserving Blending Operator. Proceedings of the ACM on Computer Graphics and Interactive Techniques, 1(2):Article No. 31:125, August 2018.

