Adaptive leader-follower formation control of mobile robots following arbitrary reference trajectories

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Mohamed Maghenem Antonio Loría Elena Panteley *†

Abstract— **We address the problem of leader-follower formation control of nonholonomic vehicles with parametric uncertainty. The controller that we propose applies to diverse kinds of reference trajectories, including set-point stabilisation, persistently-exciting trajectories, and converging ones and ensures uniform global asymptotic stability in closed loop. In particular, for the first time in the literature, we establish uniform convergence of the parameter estimation errors in such context. Also, some original simulation results illustrate our theoretical findings.**

Index terms— **Formation control, persistency of excitation, Lyapunov design, nonholonomic systems**

I. Introduction

Tracking and set-point stabilisation control of nonholonomic vehicles is a topic that has not ceased to attract the regard of the control community even if, in the interval of a few decades, a tremendous bulk of literature on the subject has already been produced —see, *e.g.*, [1]–[4]. Contemporary problems such as formation control of autonomous swarms of vehicles, consensus control, and other scenarios of co-operative systems communicating over networks maintain the interest for the basic tracking and set-point stabilisation control of nonholonomic systems .

From an academic viewpoint, these systems are attractive because their analysis imposes considerable challenges depending on the nature of the reference trajectories. Roughly speaking, these may be rich in a persistency-of-excitation ([5]) sense, as *e.g.*, in [6], [7], they can be "converging", as in [3] and [8]–[10] or, simply, correspond to a set-point [2]. Most often, the controllers designed for a type of reference trajectory does not apply for another; designing a universal controller that applies indistinctly to the case of persistent or converging trajectories is a very challenging problem that has been little addressed in the literature. In the case of one leader and one follower it has been studied in [8] and in [10]–[14].

In [8] several scenarios of tracking control (circular paths, straight-line paths, vanishing trajectories) are covered. In [10] it is assumed that the reference trajectory is either persistently exciting or integrable —note that integrability imposes certain speed of convergence. In both references, however, it is assumed that the vehicles are velocity-controlled (that is, only the kinematics model is considered). The framework laid in [12] is very general as it applies to chain-form systems, which is a class that includes the unicycle model, most often used. "Full" models of torque-controlled vehicles, that is, which include the Lagrangian dynamics, are considered in [13] and [14]. In the former the convergence of the error positions to a steady-state error, albeit under parametric uncertainty, is established. The convergence to zero of the same errors is guaranteed by the controller reported in [14] provided that either the forward reference velocity is separated from zero (whence persistently exciting) or the angular reference velocity is separated from zero and the forward velocity is integrable in norm. In [11] a robust controller that guarantees the stronger properties of uniform global asymptotic stability and integral input to state stability for the kinematics closed-loop equation is proposed.

The simultaneous tracking and robust stabilisation control problem for groups of robots ($N > 2$), has been addressed in [15]–[17]. In [15] it is established that the formation-errors converge to an arbitrarily small compact ball centred at the origin; the controller is centralised hence, it is assumed that the leader's velocities are accessible to *all* the agents in the network. In [16] and [17] it is established, for the first time, uniform global asymptotic stability for the closed-loop system. The main results in this paper, build upon the latter two references to address the problem of adaptive control and closed-loop identification. We consider vehicles modelled by a unicycle kinematics and a Lagrangian-dynamics equation and we suppose that the lumped parameters in the latter are unknown. Under these conditions, we establish, for the first time in the literature, uniform global asymptotic stability for the closed-loop system of a swarm of nonholonomic vehicles in closed loop with a unique controller that is capable of stabilising both converging and exciting trajectories. From a technical viewpoint, we also emphasise that the main statement implies uniform parametric error convergence.

The rest of the paper is organised as follows. In the next section we recall the equations of the unicycle model and formulate the control problem. In Section III we present, without proof, some statements that appear in [17]. In Section IV we present our main results. Original simulation tests that illustrate our theoretical findings are provided in Section V and we conclude with some remarks in Section VI.

II. Model and problem formulation

Consider N nonholonomic vehicles moving on the plane, with kinematics modelled by the unicycle equations

$$\dot{x}_i = v_i \cos(\theta_i) \tag{1a}$$

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$$\dot{y}_i = v_i \sin(\theta_i) \quad (1b)$$
$$\dot{\theta}_i = \omega_i \qquad i \in \{1, \dots N\} \quad (1c)$$

where $x_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ correspond to the Cartesian coordinates of a point on the ith vehicle with respect to a fixed reference frame and $\theta_i \in \mathbb{R}$ denotes the vehicle's orientation. In turn, the forward velocity $v_i = [\dot{x}_i + \dot{y}_i]^{1/2}$ and the angular velocity ω_i result from the Lagrangian equations

$$M_i \dot{\eta}_i + C_i(\eta_i)\eta_i = u_i, \quad (2a)$$
$$\eta_i := [v_i \ \omega_i]^\top, \quad (2b)$$

where M_i and C_i are, respectively, the inertia and the Coriolis-and-centrifugal-forces matrices, which are given by

$$M_i = \begin{bmatrix} m_{i1} & m_{i2} \\ m_{i2} & m_{i1} \end{bmatrix}, \quad C_i(\eta_i) = \begin{bmatrix} 0 & c_i \omega_i \\ -c_i \omega_i & 0 \end{bmatrix}, \quad (3)$$

and the control inputs are $u_i := B_i \tau_i$ where, in turn, $B_i \in \mathbb{R}^{2\times 2}$ is a full rank constant matrix of known coefficients and τ_i is the vector of input torques at the wheels —*cf.* [3].

Let such group of vehicles communicate in a leader-follower fashion that is, assume that for each $i \leq N$, the ith robot receives the states of exactly one leader, labelled $(i-1)$; the Nth vehicle having no follower and the leader to the first vehicle being a fictitious unicycle with kinematics

$$\dot{x}_r = v_r \cos(\theta_r) \quad (4a)$$
$$\dot{y}_r = v_r \sin(\theta_r) \quad (4b)$$
$$\dot{\theta}_r = \omega_r. \quad (4c)$$

The leader-follower formation control problem consists in the vehicles acquiring and maintaining a specified physical formation relative to one another and following reference trajectories generated by a fictitious robot. From a control-theory viewpoint, this can be stated as a stabilisation problem as follows.

Let v_r, ω_r be given piece-wise continuous functions mapping $\mathbb{R}_{\geq 0} \to \mathbb{R}$ that represent the forward and angular reference velocities respectively and let $z_r := [x_r,\ y_r,\ \theta_r]^\top$ denote the position and orientation reference coordinates which result from (4). A given relative formation may be designed by imposing certain desired Cartesian distances d_{xi} and $d_{yi} > 0$ between each leader-follower couple, that is,

$$p_{\theta i} := \theta_{i-1} - \theta_i,$$
$$p_{xi} := x_{i-1} - x_i - d_{xi},$$
$$p_{yi} := y_{i-1} - y_i - d_{yi}.$$

Then, transforming the error coordinates $(p_\theta,\, p_x,\, p_y)$ of the leader vehicle from the global coordinate frame to local coordinates fixed on the vehicle, we define $e_i := [e_{\theta i}\ e_{xi}\ e_{yi}]^\top$,

$$\begin{bmatrix} e_{\theta i} \\ e_{xi} \\ e_{yi} \end{bmatrix} := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_i) & \sin(\theta_i) \\ 0 & -\sin(\theta_i) & \cos(\theta_i) \end{bmatrix} \begin{bmatrix} p_{\theta i} \\ p_{xi} \\ p_{yi} \end{bmatrix}, \quad (5)$$

which satisfy

$$\dot{e}_{\theta i} = \omega_{i-1} - \omega_i \quad (6a)$$
$$\dot{e}_{xi} = \omega_i e_{yi} - v_i + v_{i-1}\cos(e_{\theta i}) \quad (6b)$$
$$\dot{e}_{yi} = -\omega_i e_{xi} + v_{i-1}\sin(e_{\theta i}), \quad (6c)$$

where v_{i-1} and ω_{i-1} are, respectively, the forward and angular velocities of the leader vehicle. In (6) we set $v_0 := v_r$ and $\omega_0 := \omega_r$ where v_r and ω_r.

Thus, the leader-follower formation control problem consists in designing control inputs $u_i := [u_{i1}\ u_{i2}]^\top$, with $i \in \{1 \dots n\}$, such that

$$\lim_{t\to\infty} e_i(t) = 0 \qquad \forall i \in \{1 \dots N\}. \quad (7)$$

hold for the system (1)–(2a).

From a control-theory perspective, however, having (7) as objective is somewhat under-challenging since it does not comprise any stability nor uniformity properties. Instead, we solve the following problem. Let $\hat{\Theta}_i$ be the estimate of $\Theta_i := [m_{i1}\ m_{i2}\ c_i]^\top$ and let $\tilde{\Theta}_i := \hat{\Theta}_i - \Theta_i$. Given a piece-wise continuous function $\mathbb{R}_{\geq 0} \to \mathbb{R}^2$, $\eta_r := [v_r\ \omega_r]^\top$ that generates, through (4), feasible trajectories $t \mapsto z_r$, design virtual control laws v_i^* and ω_i^* and a control law u_i such that, defining,

$$\tilde{v}_i := v_i - v_i^*, \quad \tilde{\omega}_i := \omega_i - \omega_i^*, \quad \text{and} \quad \tilde{\eta}_i := [\tilde{v}_i\ \tilde{\omega}_i]^\top, \quad (8)$$

the origin for the closed-loop system, $\big\{ (e_i, \tilde{\eta}_i, \tilde{\Theta}_i) = (0,0,0) \big\}$ is uniformly globally asymptotically stable.

Such leader-follower formation control problem for arbitrary feasible reference trajectories is beyond reach, even in the one-leader-one-follower scenario [18]. In this paper we address it for a fairly general class of references that includes set-points, vanishing trajectories (so-called robust stabilisation), and persistently-exciting trajectories. More precisely, we assume that reference velocities satisfy *either* of the following mutually exclusive conditions:

$$\int_t^{t+T} |\eta_r(\tau)|^2 d\tau \geq \mu \qquad \forall\, t \geq 0 \quad (9a)$$
$$\lim_{t\to\infty} |\eta_r(t)| = 0. \quad (9b)$$

The main contribution of this paper is a unique adaptive controller that ensures uniform global asymptotic stability in the space of the closed-loop system and, in the case that the lumped parameters of M and C are unknown, the property continues to hold, provided that the reference trajectories satisfy (9a). Note that this includes the uniform convergence of the parameter estimation errors.

III. Leader-follower formation control

The rationale of the control scheme consists in applying recursively an adaptive controller for the one-leader-one-follower scenario that is, it relies on the premise that leader-follower formation control may be regarded as a cascade of successive master-slave couples. Then, for each robot the controller is designed following a basic backstepping procedure. First, virtual control laws v_i^* and ω_i^* are defined for the unicycle model in error coordinates, (6). Then, a tracking controller at the dynamics-equations level, (2), is designed so that $\eta_i \to \eta_{i-1}$, even in the case of parametric uncertainty. For each vehicle, the control u_i depends on the ith vehicle's state-variables and its leader's only. For clarity of exposition, we present first the solution for one arbitrary pair of robots, at the kinematic level —*cf.* [11].

A. One-leader-one-follower tracking control

Consider an arbitrary pair of leader-follower vehicles, labelled i and $i-1$, for which (7) must hold. We define

$$\begin{aligned} v_i^* &= v_{i-1}\cos(e_{\theta i}) + k_{xi}e_{xi} && \text{(10a)}\\ \omega_i^* &= \omega_{i-1} + k_{\theta i}e_{\theta i} + k_{yi}e_{yi}v_{i-1}\phi(e_{\theta i}) \\ &\quad + \rho_i(t)k_{yi}p_i(t)|e_{xyi}|, && \text{(10b)}\end{aligned}$$

where $e_{xyi} := [e_{xi}\ e_{yi}]^\top$, the function $p_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is assumed to be once continuously differentiable, bounded, and with bounded derivative $\dot{p}_i$, $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is the so-called sinc$(\cdot)$ function,

$$\phi(x) = \frac{\sin(x)}{x},$$

and k_{xi}, k_{yi}, $k_{\theta i}$ are positive constants.

Roughly speaking, akin to [10], the controller is constructed as a combination of two control laws; one that ensures tracking of rich trajectories satisfying (9a) and another one that ensures tracking of vanishing ones, satisfying (9b). More precisely, the first three terms on the right-hand side of (10b) guarantee the achievement of the tracking control goal of persistently-exciting trajectories, while the fourth one ensures tracking of converging trajectories. Then, the function ρ_i is designed to smoothly "weigh" the effect of one term or the others relatively to the trajectory to be tracked: ρ_i is required to be approximately null (thereby enforcing the action of the third term in (10b)) to track persistently exciting trajectories and it is required to remain separated from zero to favour the tracking control of vanishing trajectories. To that end, we define $\rho_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ as the solution of

$$\dot{\rho}_i := -F\big(\eta_{i-1}(t)\big)\rho_i \qquad (11)$$

where $F : \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ is a piece-wise continuous function defined as

$$F(\eta) := \begin{cases} 0 & \text{if } \eta \ \in (0, \frac{\mu}{2T\bar{\eta}}] \\ \alpha(|\eta|) & \text{otherwise} \end{cases} \qquad (12)$$

where $\bar{\eta} \geq |\eta|_\infty$, $|\varphi|_\infty := \text{ess}\sup_{t\geq 0}|\varphi(t)|$, and $\alpha \in \mathcal{K}$. The interest this definition is that $F(\eta_r(\cdot))$ is persistently exciting, *i.e.*, there exist μ_1 and $T_1 > 0$ such that

$$\int_t^{t+T_1} F\big(\eta_r(s)\big)^2 ds \geq \mu_1, \quad \forall t \geq 0 \qquad (13)$$

under (9a) and $F(\eta_r(\cdot))$ is integrable, *i.e.*,

$$\int_0^\infty F\big(\eta_r(s)\big)ds \leq \beta, \qquad (14)$$

under (9b).

Under these conditions, we have the following statement on the convergence of the tracking errors.

Proposition 1 Let $i \leq N$ be arbitrarily fixed and consider the system (6) with state e_i, exogenous signal $\eta_{i-1} = [v_{i-1}, \omega_{i-1}]$ such that

$$\max\left\{|\eta_{i-1}|_\infty, |\dot{\eta}_{i-1}|_\infty\right\} \leq \bar{\eta}_{i-1}, \qquad (15)$$

and inputs ω_i and v_i. Consider the virtual control laws (v_i^*, ω_i^*) as given by (10), (11)–(14), with the functions p_i and $\dot{p}_i$ being bounded and persistently exciting. Then, if $\tilde{v}_i := v_i - v_{i-1}$ and $\tilde{\omega}_i := \omega_i - \omega_{i-1}$ are bounded, the trajectories exist on $[t_\circ, \infty)$. Furthermore,

1) if (9a) holds with η_r replaced by η_{i-1}, the system is integral input-to-state stable with respect to the input $\tilde{\eta}_i$. Consequently, if $\tilde{\eta}_i$ tends to zero and is square integrable, the limit in (7) holds.
2) If, alternatively, (9b) holds with η_r replaced by η_{i-1} the system is small input-to-state stable with respect to the input $\tilde{\eta}_i$ and if $\tilde{\eta}_i$ converges to zero the limit in (7) holds. □

The proof is omitted here due to space constraints; interested readers may see [16]. The statement in Proposition 1 is significant for several reasons. Firstly, it establishes small and integral input-to-state stability for the closed-loop system; these are robustness properties that allow to consider the kinematics and dynamics control loops separately, that is as two systems in cascade —*cf.* [19]. In turn, it may be established that if not only $\tilde{\eta}_i \to 0$, but $\{\tilde{\eta}_i = 0\}$ is UGAS, then, the origin for the overall closed-loop system is also UGAS —see Proposition 2 below. Finally, the robustness statement of Proposition 1 allows to extend this result to the case of successive pairs of leader-follower couples, that is, to solve the leader-follower formation-and-tracking control problem. This is presented in the following section.

Proposition 2 (UGAS of the full model) Consider the system (1), (2) under the action of *any* controller u_i guaranteeing uniform global asymptotic stability of $\{\tilde{\eta}_i = 0\}$ and that $\tilde{\eta}_i \in \mathcal{L}_2$. Then, under the conditions of Proposition 1, the origin $(\tilde{e}_i, \tilde{\eta}_i) = (0, 0)$ is uniformly globally asymptotically stable. □

The statement of Proposition 2 relies on a cascaded-systems stability argument. Let u_i be a given controller for the dynamics equations (2), depending on the leader and follower's states, as well as on the virtual control laws (10). Then, by a suitable change of coordinates the closed-loop equations take the generic form

$$\dot{\tilde{\eta}}_i = F_{\eta i}(t, \tilde{\eta}_i, e_i). \qquad (16)$$

Next, we replace e_i in (16) by complete trajectories $e_i(t)$ so the overall closed-loop equations cover a cascaded form

$$\begin{aligned} \dot{e}_i &= F_{ei}(t, e_i) + G_{ei}(t, e_i)\tilde{\eta}_i && (17)\\ \dot{\tilde{\eta}}_i &= \tilde{F}_{\eta i}(t, \tilde{\eta}_i) && (18)\end{aligned}$$

where $\tilde{F}_{\eta i}(t, \tilde{\eta}_i) := F_{\eta i}(t, \tilde{\eta}_i, e_i(t))$ —*cf.* [19], [20, p. 627]. Equation (17) is a compact representation of Equations (6).

Now, after Proposition 1, if (9a) holds the system (17) is integral-input-to-state stable while, if (9b) holds it is small input-to-state stable. On the other hand, either of these conditions implies the so-called 0-UGAS property, that is, uniform global asymptotic stability for $\dot{e}_i = F_{ei}(t, e_i)$. On the other hand, the origin for (18) is also UGAS. Thus, after [21, Lemma 2], the origin $(e_i, \tilde{\eta}_i) = (0, 0)$ is uniformly globally asymptotically stable if the solutions of (17) are uniformly globally bounded. The latter follows under condition (9a), from the integral-input-to-state-stability property and the assumption that $\tilde{\eta}_i \in \mathcal{L}_2$ in the case of persistently-exciting reference trajectories and under condition (9b) and the property of small input-to-state stability (see Proposition 1) in the case of vanishing trajectories.

B. Leader-follower formation tracking control

We consider next a network of autonomous vehicles ($N \geq 2$) that communicate according to a spanning-tree graph and which are required to follow a reference fictitious vehicle following reference trajectories that satisfy either (9a) or (9b). We have the following.

Proposition 3 Consider the system (1), (2). Let $\eta_r = [v_r \; \omega_r]^\top$ be a given piece-wise continuous function satisfying either (9a) or (9b) and assume that there exists $\bar{\eta}_r > 0$ such that

$$\max\left\{|\eta_r|_\infty,\ |\dot{\eta}_r|_\infty\right\} \leq \bar{\eta}_r. \tag{19}$$

For each $i \leq N$ consider the expressions of v_i^* and ω_i^* as in (10) (with $v_0 := v_r$ and $\omega_0 := \omega_r$) where:

(i) k_{xi}, k_{yi}, $k_{\theta i}$ are positive constants;

(ii) the functions p_i and $\dot{p}_i$ are bounded and persistently exciting.

Then, for any given control laws u_{i1} and u_{i2} guaranteeing that $\tilde{\eta}_i$ is square integrable and converges to zero, the control objective (7) holds.

Furthermore, define $\tilde{\eta} := [\tilde{\eta}_1 \cdots \tilde{\eta}_N]^\top$, $\eta^* := [\eta_1^* \cdots \eta_N^*]^\top$, and $e := [e_1 \cdots e_N]^\top$. If $\{\tilde{\eta} = 0\}$ for (18) is uniformly globally asymptotically stable (UGAS) and $\tilde{\eta} \in \mathcal{L}_2$ then, for the closed-loop system (17)-(18), $\{(e, \tilde{\eta}) = (0, 0)\}$ is also UGAS. Consequently, if $\tilde{\eta} \equiv 0$ then $\{e = 0\}$ for (1) in closed loop with η^* is UGAS. $\square$

The proof of this statement consists in applying recursively the statement of Proposition 1 for each $i \leq N$ that is, for each pair of leader-follower vehicles. Indeed, Proposition 1 guarantees the asymptotic convergence of the formation errors whether the leader velocities are persistently exciting or converging. Therefore, the properties of $(i-1)$th leader's velocities are propagated to the ith follower and, in turn, to the $(i+1)$th vehicle down to the leaf nodes in the graph.

We use $\omega_i = \tilde{\omega}_i + \omega_i^*$ and $v_i = \tilde{v}_i + v_i^*$ in (6), together with (10) to write the error-dynamics equations as

$$\dot{e}_i = A_{v_{i-1}}(t, e_i)e_i + B_{1i}(t, e_i)\rho_i(t) + B_{2i}(e_i)\tilde{\eta}_i, \tag{20}$$

where

$$A_{v_{i-1}} := \begin{bmatrix} -k_{\theta i} & 0 & -v_{i-1}(t)k_{yi}\phi(e_{\theta i}) \\ 0 & -k_{xi} & \varphi_i(t, e_i) \\ v_{i-1}(t)\phi(e_{\theta i}) & -\varphi_i(t, e_i) & 0 \end{bmatrix},$$

$$B_{1i} := \begin{bmatrix} -k_{yi}p_i(t)|e_{xyi}| \\ k_{yi}p_i(t)|e_{xyi}|e_{yi} \\ -k_{yi}p_i(t)|e_{xyi}|e_{xi} \end{bmatrix}, \quad B_{2i} := \begin{bmatrix} 0 & -1 \\ -1 & e_{yi} \\ 0 & -e_{xi} \end{bmatrix}$$

and $\varphi_i(t, e_i) := \omega_{i-1} + k_{\theta i}e_{\theta i} + k_{yi}e_{yi}v_{i-1}\phi(e_{\theta i})$. We stress that these closed-loop equations have the convenient triangular structure

$$\begin{aligned} \dot{e}_N &= A_{v_{N-1}}(t, e_N)e_N + B_{1N}(t, e_N)\rho_N + B_{2N}(e_N)\tilde{\eta}_N \\ &\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (21a) \\ \dot{e}_2 &= A_{v_1}(t, e_2)e_2 + B_{12}(t, e_2)\rho_2 + B_{22}(e_2)\tilde{\eta}_2 \qquad (21b) \\ \dot{e}_1 &= A_{v_r}(t, e_1)e_1 + B_{11}(t, e_1)\rho_1 + B_{21}(e_1)\tilde{\eta}_1 \qquad (21c) \end{aligned}$$

Note that for the ith vehicle the dynamics equations depend on e_i and, through $\eta_{i-1} = [v_{i-1} \; \omega_{i-1}]^\top$, on the states of the vehicles above in the graph, up to the reference vehicle. However, in view of forward completeness (which can be established as in the proof of Proposition 1), for the purpose of analysis the velocities η_{i-1} may be regarded as exogenous signals. This allows us to consider the system as a multi-*cascaded* time-varying one —see [19]. Then, we may invoke Proposition 1 recursively.

IV. Main results

The previous propositions, whose proofs may be found in [16], establish uniform global asymptotic stability for the unicycle kinematics and set the basis for the following statements, which is the first of its kind in the literature.

Proposition 4 Consider the the system (1), (2) in closed loop with (10) and

$$u_i = M_i\dot{\eta}_i^* + C_i(\eta_i)\eta_i^* - k_{di}\tilde{\eta}_i, \qquad k_{di} > 0. \tag{22}$$

Let condition (19) as well as items (i) and (ii) of Proposition 3 hold. Then, the origin in the state space of the closed-loop system is uniformly globally asymptotically stable. $\square$

Proof: The closed-loop dynamics (16) is

$$M_i\dot{\tilde{\eta}}_i + C_i(\tilde{\eta}_i + \eta_i^*(t, e_i))\tilde{\eta}_i + k_{di}\tilde{\eta}_i = 0, \qquad i \leq N \tag{23}$$

which may be rewritten along complete solutions $e_i(t)$ in the form (18). Then, after the skew-symmetry of $C_i(\cdot)$, we have

$$V(\tilde{\eta}_i) := \tilde{\eta}_i^\top M\tilde{\eta}_i \quad \Longrightarrow \quad \dot{V}(\tilde{\eta}_i) = -2k_{di}|\tilde{\eta}_i|^2,$$

so $\{\tilde{\eta} = 0\}$ is a uniformly (in the initial times $t_\circ$ and in the trajectories $e_i(t)$) globally exponentially stable equilibrium of (23). The result follows from Proposition 3. ■

Let us now assume that the constant lumped parameters in M_i and $C_i(\eta_i)$, denoted $\Theta_i \in \mathbb{R}^m$, are unknown and let $\hat{M}_i$ and $\hat{C}_i$ denote the estimates of the inertia and Coriolis matrices respectively. Let $\hat{\Theta}_i$ correspond to an estimate of Θ_i and consider the controller

$$\begin{aligned} u_i &= \hat{M}_i\dot{\eta}_i^* + \hat{C}_i(\eta_i)\eta_i^* - k_{di}\tilde{\eta}_i, \qquad k_{di} > 0 \quad (24a) \\ \dot{\hat{\Theta}}_i &= -\gamma\Phi_i(t, \dot{\eta}_i^*, \eta_i^*, \tilde{\eta}_i)^\top\tilde{\eta}_i, \quad \gamma > 0 \qquad (24b) \end{aligned}$$

where, for any $i \leq N$, Φ_i is a smooth function implicitly defined by the expression

$$\Phi_i(t, \dot{\eta}_i^*, \eta_i^*, \tilde{\eta}_i)\tilde{\Theta}_i := [\hat{C}_i - C_i]\eta_i^* + [\hat{M}_i - M_i]\dot{\eta}_i^*, \tag{25}$$

where $[\hat{C}_i - C_i]$ is a function of $\eta_i = \tilde{\eta}_i + \eta_i^*$.

Proposition 5 Consider the system (1), (2) in closed loop with (10) and (24). Then, the origin $\{(e_i, \tilde{\eta}_i, \tilde{\Theta}_i) = (0, 0, 0)\}$, for all $i \leq N$, is a uniformly globally asymptotically stable equilibrium point if $\Phi_1(\cdot, \dot{\eta}_r(\cdot), \eta_r(\cdot), 0)$ is persistently exciting. □

Proof: The closed-loop system corresponding to the force equations (2) is

$$M_i\dot{\tilde{\eta}}_i + C_i(\eta_i)\tilde{\eta}_i + k_{di}\tilde{\eta}_i = \Phi_i(t, \dot{\eta}_i^*, \eta_i^*, \tilde{\eta}_i)\tilde{\Theta}_i \quad (26a)$$

$$\dot{\tilde{\Theta}}_i = -\gamma\Phi_i(t, \dot{\eta}_i^*, \eta_i^*, \tilde{\eta}_i)^\top\tilde{\eta}_i. \quad (26b)$$

In view of (the proof of) Proposition 4, uniform global asymptotic stability of the origin $(\tilde{\eta}_i, \tilde{\Theta}_i)$ for (26) follows directly from [22, Theorem 3], provided that $\Phi_i(\cdot, \dot{\eta}_i^*(\cdot), \eta_i^*(\cdot), 0)$ is persistently exciting. Now, for $i = 1$, this means that $\Phi_1(\cdot, \dot{\eta}_r(\cdot), \eta_r(\cdot), 0)$ must be persistently exciting, which holds by assumption. We conclude that $\eta_1 \to \eta_r$ and $\dot{\eta}_1 \to \dot{\eta}_r$ hence, $\Phi_2(\cdot, \dot{\eta}_1(\cdot), \eta_1(\cdot), 0)$ is also persistently exciting. The result follows by induction. ■

V. Simulation results

For the sake of illustration, we have performed some numerical simulations using Simulink of Matlab. We consider a group of four mobile robots required to follow a virtual leader while assuming a diamond-shape formation, which is designed by imposing desired distances between the robots as follows: $[d_{x_{r,1}}, d_{y_{r,1}}] = [0, 0]$, $[d_{x_{1,2}}, d_{y_{1,2}}] = [-1, 0]$ and $[d_{x_{2,3}}, d_{y_{2,3}}] = [1/2, -1/2]$ and $[d_{x_{3,4}}, d_{y_{3,4}}] = [0, 1]$. Each vehicle is considered to be modelled by (1)–(3) with $m_{i1} = 0.6227$, $m_{i2} = -0.2577$, and $c_i = 0.2025$. The control gains are set to $k_{x_i} = k_{y_i} = k_{\theta_i} = 1$, and $k_d = 15$ while $p(t) := 20\sin(0.5t)$, which has a persistently exciting time-derivative. The results of two simulation tests are presented. In the first case, the reference vehicle trajectories are generated by (4) with $v_r(t)$ and $\omega_r(t)$ such that $|\eta_r(t)|$ is persistently exciting —see Figure 1. That is, in the first simulation test, the reference trajectories satisfy Inequality (9a) and so does the function

$$F(\eta_r) := \begin{cases} 0 & \text{if } \eta_r \in (0, 0.1] \\ |\eta_r| & \text{otherwise.} \end{cases}$$

The position and velocity tracking errors, in norm, are shown in Figures 2 and 3 respectively.

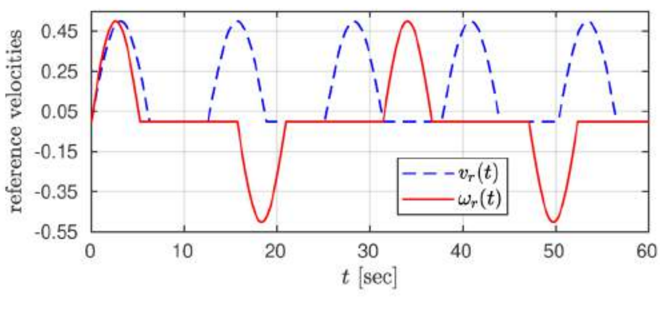

Fig. 1. Reference velocity trajectories $v_r(t)$ and $\omega_r(t)$

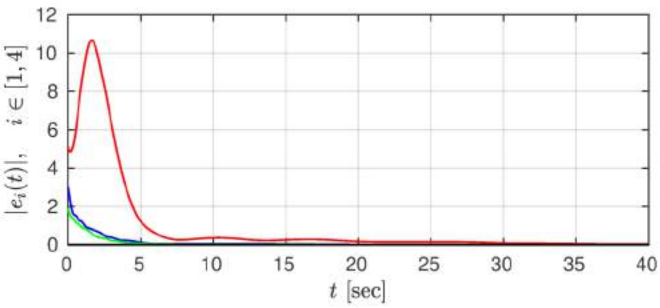

Fig. 2. Normed relative errors for each pair leader-follower, with persistently exciting reference trajectories

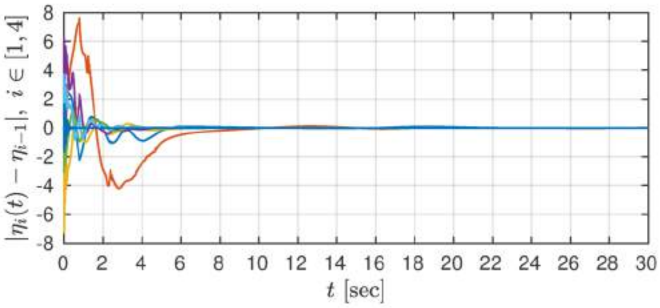

Fig. 3. Normed relative velocities for each pair leader-follower, with persistently exciting reference trajectories

The path followed by the vehicles in formation is depicted in Figure 4.

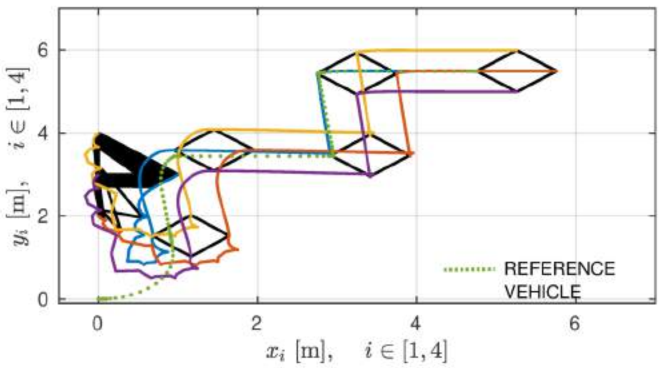

Fig. 4. Path followed by the formation with (truncated) persistently exciting trajectories.

Then, we use the adaptive velocity-tracking controller (24) with $\Theta := [m_1 \; m_2 \; c]^\top$ with adaptation gain $\gamma = 10$. The initial values for $\hat{\Theta}$ are set to zero. In Figure 5 are depicted the norms of the parameter-estimation errors for each of the four vehicles, which converge to zero since the regressor evaluated along the reference trajectories,

$$\Phi_1(t, \dot{\eta}_r, \eta_r, 0) := \begin{bmatrix} \dot{v}_r & \dot{\omega}_r & \omega_r^2 \\ \dot{\omega}_r & \dot{v}_r & -v_r\omega_r \end{bmatrix},$$

is persistently exciting.

In a second simulations test, the robots are required to follow a fictitious vehicle that slowly comes to a full-stop, that is, the reference velocities converge to zero with a "slow" convergence rate near zero; these are determined by

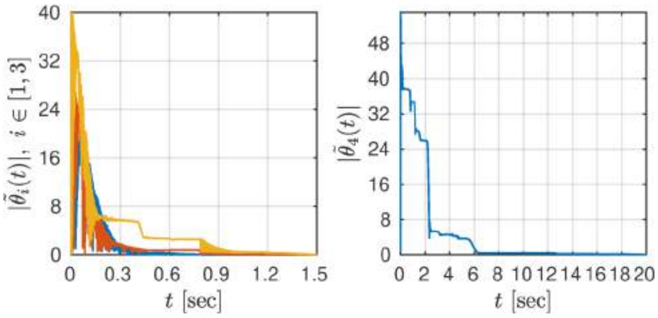

Fig. 5. Normed parameter estimation errors for each of the four vehicles

the solutions of $\dot{v}_r = -50v_r^3$ and $\dot{\omega}_r = -100\omega_r^3$ with initial conditions $v_r(0) = \omega_r(0) = 1$. The corresponding position and velocity tracking errors, in norm, are showed in Figures 6 and 7 below.

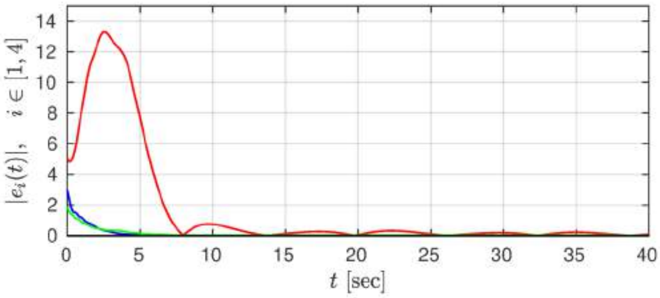

Fig. 6. Normed relative errors for each pair leader-follower, with vanishing reference trajectories

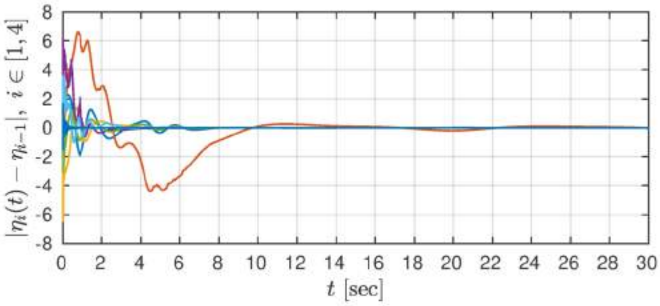

Fig. 7. Normed relative velocities for each pair leader-follower, with vanishing reference trajectories

VI. Conclusion

We presented an adaptive controller for groups of autonomous vehicles following in formation persistently-exciting, as well as converging reference trajectories. Our controller guarantees uniform asymptotic stabilisation and, in the case of persistently-exciting references, it also ensures the uniform convergence of the parameter estimation errors. Future work involves the formation control problem studied here with obstacle avoidance.

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