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## ► To cite this version:

Pierre Dublanquet. Fluid driven shear cracks on a strengthening rate-and-state frictional fault. Journal of the Mechanics and Physics of Solids, 2019, 132, pp.103672. 10.1016/j.jmps.2019.07.015 . hal-02367634

**HAL Id: hal-02367634**

**<https://hal.science/hal-02367634>**

Submitted on 20 Dec 2021

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# Fluid driven shear cracks on a strengthening rate-and-state frictional fault

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## Abstract

This study is dedicated to the dynamics of slow slip (or creep) reactivation on faults stimulated by a fluid injection. Fluid driven slow slip events involving slip rates too small to generate detectable elastic waves (typically less than  $1 \text{ cm.s}^{-1}$ ) are commonly observed on natural faults either in tectonic active areas or in the framework of deep energy exploitation. We model the fault as a planar 2D velocity strengthening rate-and-state frictional interface embedded in an elastic medium. The fluid is injected at a constant rate and spreads diffusively along the fault reducing the effective normal stress. We show that the fluid injection initiates a shear crack on the fault. In a first phase, the shear crack remains confined to the pressurized zone, and slip-rate increases exponentially with time. A second phase starts at the onset of a rapid crack expansion. The shear crack during phase two expands faster than the pressurized zone. Depending on the prestress conditions, the shear crack evolves towards two different regimes. If the initial shear stress  $\tau_0$  is larger than the steady-state residual frictional stress  $\tau_r$  within the crack, the slip rate and the crack expansion rate blow up in a finite time. This accelerating crack regime is similar to the nucleation of a dynamic rupture on the fault. On the other hand, if  $\tau_0 < \tau_r$ , the accelerated expansion progressively slows down so that the crack enters a steady regime, characterized by a constant expansion speed and a logarithmic increase of slip-rate. However, a large majority of fault scenarios ultimately lead to this steady expansion regime, which is also the most probable scenario on real tectonic faults. In addition to the numerical results, we develop asymptotic expressions for the the maximum

slip rate history on the fault and the crack length history, showing how initial prestress  $\tau_0$ , frictional conditions (ratio  $a/b$ ), hydraulic properties and injection history control the dynamics of fluid induced aseismic slip events.

*Keywords:* Strengthening and mechanisms (A), Friction (B), Geological materials (B), Crack mechanics (B)

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## 1. Introduction

Many observations suggest that fluid flow at depth can reactivate slip on preexisting crustal faults. Fluids may have a natural origin (rainfall events (Hainzl et al., 2006), deep source (Duverger et al., 2015), volcanic activity (Ágústsson et al., 2016)) or can be related to geo-resource exploitation (geothermal operations (Deichmann and Giardini, 2009), waste water injection (Ellsworth, 2013), reservoir lake impoundment (Gupta et al., 1969)). If rapid enough slip is reactivated (typically at slip rates greater than  $1 \text{ cm.s}^{-1}$  which allows the radiation of detectable elastic waves), the fault ruptures in an earthquake, such as the commonly observed induced earthquakes (Deichmann and Giardini, 2009; Ellsworth, 2013). However, the fluid-induced reactivation of faults can lead to much slower slip speeds, leading to the occurrence of a slow slip event (Cornet et al., 1997; Cornet, 2016; Guglielmi et al., 2015). Such slow slip events are also referred as accelerated creep, or aseismic events. Aseismic slip in turn may trigger earthquake sequences (Schaff et al., 1998; Bourouis and Bernard, 2007; Wei et al., 2015; Lengliné et al., 2017). Aseismic slip is even suspected to be one of the dominant mechanisms releasing injected energy in the context of fluid operations at depth (Goodfellow et al., 2015; Duboeuf et al., 2017). Understanding the physics controlling the dynamics of aseismic slip events triggered by fluid injections is therefore crucial to better constrain the energy partitioning in the subsurface or in a geological reservoir, and to better assess the associated seismic hazard.

Recent advances in the study of the mechanics of fluid-fault interaction have essentially focused on the stability of frictional slip when a fluid is lo-

25 cally injected and diffuses within a fault (Garagash and Germanovich, 2012;  
 Cappa et al., 2018; Bhattacharya and Viesca, 2019), or when a hydraulic frac-  
 ture propagates along a frictional fault (Azad et al., 2017). All these studies  
 have shown how the pore pressure related reduction in effective normal stress  
 triggers the reactivation of a slow aseismic slip, that eventually degenerates into  
 30 a dynamic rupture. In most of the fault scenarios investigated, the aseismic slip  
 (and the dynamic slip if initiated) is excited well beyond the pressurized region of  
 the fault. This strong aseismic response (stronger than the pore pressure pertur-  
 bation) is even more pronounced when the fault experiences significant frictional  
 weakening (Garagash and Germanovich, 2012; Azad et al., 2017; Cappa et al.,  
 35 2018), or if slip-induced permeability enhancement takes place (Cappa et al.,  
 2018; Bhattacharya and Viesca, 2019). The transition to dynamic rupture is  
 furthermore facilitated by initial stress conditions closer to failure, and by an  
 injection scenario leading to an abrupt increase of over-pressure within the fault  
 zone (Garagash and Germanovich, 2012; Azad et al., 2017).

40 All these studies rely on a frictional description involving either a constant  
 friction coefficient or a slip weakening friction coefficient. Such descriptions  
 impose a minimum over-pressure to trigger slow slip. Below this reactivation  
 threshold, no slip occurs. Furthermore, the slip-rate dependence of friction  
 needed to explain the dynamics of aseismic slip on tectonic faults (Marone et al.,  
 45 1991; Perfettini and Avouac, 2004; Helmstetter and Shaw, 2009; Jolivet et al.,  
 2013) is not accounted for by such frictional models. A long history of rock fric-  
 tion experiments has demonstrated that the rate-and-state friction (Dieterich,  
 1979; Ruina, 1983) captures both the slip-rate and the slip history dependence  
 of friction. Contrary to the slip weakening description, the rate-and-state law  
 50 does not impose a stress threshold to activate slip, which may lead to a fairly  
 different behavior than what has been obtained under slip weakening or con-  
 stant friction. This point still needs to be investigated. A rate-and-state friction  
 coefficient has however been tested by Cappa et al. (2018) to model the in-situ  
 fault reactivation experiment by Guglielmi et al. (2015), leading to a somewhat  
 55 larger slow slip response than predicted by the classical constant friction coeffi-

cient. However, more efforts still need to be done to understand the effects of the rate-and-state frictional rheology on fluid induced aseismic slip.

Here we propose to study the fluid induced reactivation of aseismic slip on a Dieterich-Ruina rate-and-state frictional fault. We will extend the previous  
60 studies on fluid fault interaction by studying, for a prescribed injection and fluid flow scenario, the effect of the rate-and-state parameters and the initial stress conditions on the induced aseismic slip. In particular, we will focus on the evolution of maximum slip rate and the size of the perturbed slip zone as the pore pressure perturbation proceeds. As far as possible we will derive closed  
65 form approximate solutions for the maximum slip rate history and for the size evolution of the aseismic slow slip events.

## 2. Fault model

We consider the anti-plane fault model depicted in figure 1, consisting in a linear 1D interface separating two 2D semi-infinite elastic media. The fault  
70 is loaded by a constant normal stress  $\sigma$ , and a remote shear stress  $\tau_0$ . Anti-plane slip -  $\delta(x, y, t) = w(x, 0^+, t) - w(x, 0^-, t)$ ,  $w$  being the  $z$  component of the displacement, and  $t$  the elapsed time - is resisted by friction within a finite length crack of size  $2L_0$ . Outside the crack, the slip-rate is imposed at a constant rate  $v^*$ . Here we consider rate-and-state friction within the crack,  
75 which accounts for the slip rate and slip history dependence of friction usually observed in laboratory experiments (Dieterich, 1979; Marone, 1998). The fault is furthermore permeated by a fluid injected at  $x = 0$ , and diffusing in the  $\pm x$  directions with pore pressure  $p$ . In this framework, the frictional stress along the  $x$  direction  $\tau$  is given by :

$$\tau(x, t) = f(x, t) [\sigma - p(x, t)], \quad (1)$$

80  $f$  being the rate-and-state friction coefficient defined as :

$$f(x, t) = f_0 + a \ln \frac{v(x, t)}{v^*} + b \ln \frac{\theta(x, t)v^*}{d_c}, \quad (2)$$

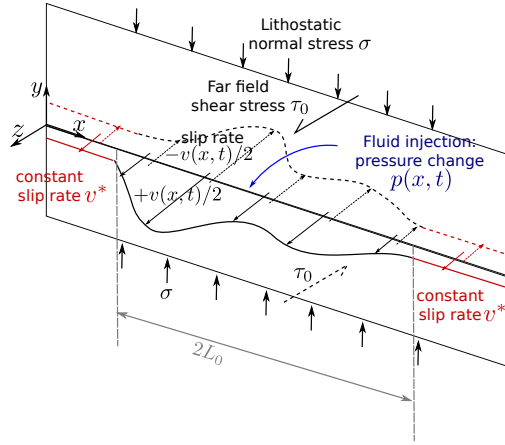


Figure 1: 2D mode III fault model considered in this study.

where  $f_0$  is a constant friction coefficient,  $a$  and  $b$  are rate-and-state parameters,  $d_c$  the critical slip of rate-and-state friction needed to renew a population of microscopic contacts.  $v$  is the slip rate defined as  $v = \dot{\delta}$ . The state variable  $\theta$  incorporates the slip history dependence of the friction coefficient. Here it is  
85 assumed to evolve with time and slip rate on the fault according to the ageing law (Ruina, 1983; Marone, 1998) :

$$\dot{\theta}(x, t) = 1 - \frac{v(x, t)\theta(x, t)}{d_c}. \quad (3)$$

The steady-state friction coefficient  $f_{ss}$  at the slip rate  $v$  is obtained when  $\dot{\theta} = 0$ , so that  $\theta = d_c/v$  from equation (3). We get:

$$f_{ss} = f_0 + (a - b) \ln \frac{v}{v_*}. \quad (4)$$

The steady-state frictional stress  $\tau_{ss}$  is then defined as  $\tau_{ss} = f_{ss}(\sigma - p)$ . Since  
90 we are interested in the dynamics of slow aseismic slip, rate strengthening properties are considered so that  $a > b$  and the steady state shear stress is an increasing function of slip rate. Assuming velocity strengthening properties prevents the development of spontaneous slip instabilities (Ruina, 1983; Rubin and Ampuero, 2005), or stick slip oscillations on the fault, and rather al-

95 lows the fault to undergo stable creep at  $v = v^*$  in the absence of pore pressure perturbation. Following Rubin and Ampuero (2005), we note  $\Omega = v\theta/d_c$  the distance to steady state, since  $f - f_{ss} = \ln \Omega$ .

We further assume that the slip distribution on the fault results from a balance between the frictional stress  $\tau$  (equation 1) and the  $z$  component of the  
 100 elastostatic stress  $\tau_{el} = \tau_{yz}(x, 0, t) = \mu \partial w / \partial y(x, 0, t)$ .  $\tau_{el}$  could be written as :

$$\tau_{el}(x, t) = \tau_0(x) - \frac{\mu}{2} \mathcal{H}[\delta'](x, t), \quad (5)$$

where  $\tau_0$  corresponds to the initial prestress, that is the stress prevailing on the fault before the onset of slip. The second term on the right-hand side is the static shear stress generated by the slip distribution  $\delta$ . The operator  $\mathcal{H}$  is the Hilbert transform, and the prime denotes a derivative with respect to the  
 105 spatial coordinate  $x$ .

The fluid injection is modeled as a prescribed pore pressure history resulting from a constant injection rate imposed at  $x = 0$ , and a diffusion along the infinite  $x$  axis with a constant diffusivity  $D$ . Under such conditions, the pore pressure history  $p$  is given by (see Turcotte and Schubert (2014) for details of  
 110 the derivation):

$$p(x, t) = 2q\sqrt{Dt} \left[ |\eta|(\operatorname{erf}(|\eta|) - 1) + \frac{e^{-\eta^2}}{\sqrt{\pi}} \right], \quad (6)$$

where the similarity variable  $\eta$  is defined as  $\eta = x/2\sqrt{Dt}$  and  $q$  is the change in pressure gradient (directly related to the fluid flux according to the Darcy's law) at the origin.

The time derivative of the quasi-static stress balance  $\tau = \tau_{el}$ , along with  
 115 the state evolution law (3) leads to the following differential equations for the evolution of slip rate  $v$  and state variable  $\theta$  under prescribed pore pressure

history  $p$ :

$$\begin{cases} (\sigma - p) \left[ a \frac{\dot{v}}{v} + b \frac{\dot{\theta}}{\theta} \right] = f(v, \theta) \dot{p} - \frac{\mu}{2} \mathcal{H}[v'] \\ \dot{\theta} = 1 - \frac{v\theta}{d_c}. \end{cases} \quad (7)$$

Considering a characteristic slip rate  $v_c = v^*$ , a characteristic time  $t_c = d_c/v^*$ , a characteristic length  $x_c = \mu d_c/b\sigma$ , and a characteristic pore pressure  $p_c = \sigma$ , we can make the substitution:

$$v \Rightarrow v/v_c, \quad t \Rightarrow t/t_c, \quad x \Rightarrow x/x_c, \quad p \Rightarrow p/p_c, \quad (8)$$

so that the system (7) becomes in non-dimensional form:

$$\begin{cases} (1 - p) \left[ \alpha \frac{\dot{v}}{v} + \frac{\dot{\theta}}{\theta} \right] = \bar{f}(v, \theta) \dot{p} - \frac{1}{2} \mathcal{H}[v'] \\ \dot{\theta} = 1 - v\theta, \end{cases} \quad (9)$$

where  $\alpha = a/b$  and  $\bar{f} = \bar{f}_0 + \alpha \ln v + \ln \theta$ ,  $\bar{f}_0$  corresponding to  $f_0/b$ . Similarly, the frictional stress  $\tau$  and the steady-state frictional stress  $\tau_{ss}$  could be made non dimensional and become:

$$\begin{cases} \tau = [\bar{f}_0 + \alpha \ln v + \ln \theta] (1 - p) \\ \tau_{ss} = [\bar{f}_0 + (\alpha - 1) \ln v] (1 - p) \end{cases} \quad (10)$$

Finally, the pore pressure history (6) could be made non-dimensional. From the characteristic length  $x_c$  the characteristic time  $t_c$ , and the characteristic pore pressure  $p_c$ , we construct the characteristic diffusivity  $D_c = x_c^2/t_c = \mu^2 d_c v^* / b^2 \sigma^2$ , and the characteristic pore pressure gradient  $q_c = p_c/x_c = b\sigma^2/\mu d_c$ . Assuming the substitution

$$D \Rightarrow D/D_c, \quad q \Rightarrow q/q_c, \quad (11)$$

along with (8), the non-dimensional pore pressure history keeps the form (6).

In order to simplify the developments in the main text, we will in the following make only use of non-dimensional quantities removing the over-bar on



$f$  and  $f_0$ . However, the figures will be labeled with dimensional quantities, so that the relevant physical parameters appear more explicitly. A length  $L$  in the  
135 main text will therefore correspond to the non-dimensional  $\bar{L} = Lb\sigma/\mu d_c$ , but not in the figure label.

For a specific choice of the parameters  $\alpha$ ,  $q$ ,  $D$ , and initial conditions  $v(x, 0) = v_0(x)$ ,  $\theta(x, 0) = \theta_0(x)$  the system (9) is solved numerically using a standard Runge-Kutta Fehlberg scheme (Fehlberg, 1969) with adaptative time stepping  
140 as detailed by Dublanchet (2017). This requires to evaluate the Hilbert transform of the slip rate gradient at each time step. For that we follow the method proposed by Cochard and Rice (1997), on a grid of  $n = 8192$  identical computational cells of normalized size  $h = 0.03$  much smaller than the typical normalized process zone size  $L_b = 1$  (Perfettini and Ampuero, 2008; Rubin and Ampuero,  
145 2005) in order to ensure continuity. This algorithm results in the slip rate and state variable history  $v(x, t)$  and  $\theta(x, t)$ , in response to a fluid injection.

### 3. Results

We performed several fluid injection tests at constant rate in our fault model. In each scenario, we considered a normalized diffusivity  $D = 4.369$ , which cor-  
150 responds to hydraulic diffusivities ranging between  $10^{-6} \text{ m}^2.\text{s}^{-1}$  and  $0.1 \text{ m}^2.\text{s}^{-1}$  assuming standard values for the shear modulus  $\mu \sim 3.10^{10} \text{ Pa}$ , for the rate-and-state parameter  $b \sim 10^{-3} - 10^{-2}$ , for the reference creep rate  $v^* = 10^{-9} - 10^{-10} \text{ m.s}^{-1}$  (representative of creeping faults), for the lithostatic stress  $\sigma \sim 100 \text{ MPa}$  (representative of approximately 3 km depth) and the critical slip distance  $d_c$  be-  
155 tween 1 mm and 1  $\mu\text{m}$  (Marone, 1998). Typical diffusivities within fault gouge are expected to vary within this range (Rice, 2006; Jaeger et al., 2009). We considered a constant normalized pore pressure gradient at the origin  $q = 0.01$ , which corresponds to a pressure gradient of the order of  $300 \text{ Pa.m}^{-1}$  to  $100 \text{ MPa.m}^{-1}$  with the same reference parameters. We assumed a normalized refer-  
160 ence friction coefficient  $f_0 = 600$ , which corresponds to a non normalized value of 0.6 with  $b = 10^{-3}$  in agreement with laboratory experiments (Marone, 1998).

We tested 8 values of the velocity strengthening frictional parameter  $\alpha$  between 1.1 (weakly strengthening behavior) to 2.5 (strongly strengthening behavior).

All the injection scenarios were performed on a fault initially slipping at the uniform background slip rate  $v_0 = 1$ . For all the frictional parameters, we considered three different uniform initial state variable  $\theta_0 = 1, e^2$  and  $e^{-2}$ , so that the initial stress  $\tau_0 = f_0 + \alpha \ln v_0 + \ln \theta_0$  on the fault is respectively equal to, slightly above or slightly below steady state at  $v = 1$  from equations (10). The three initial stress conditions indeed correspond to  $\tau_0 - \tau_{ss}^0 = 0, 2$  or  $-2$ ,  $\tau_{ss}^0$  being the steady state frictional stress at the reference slip rate  $v = 1$ , and with  $p = 0$ . Although we expect stable creeping faults to be at steady-state, they may undergo stress perturbations associated with the occurrence of earthquakes in their vicinity, especially in tectonic active environments. Stress perturbations lead to transient deviations from the steady-state stress that can survive for a few hours to a few years, as suggested by the observations of post-seismic accelerated creep (Helmstetter and Shaw, 2009). Creeping faults may therefore be not exactly at steady-state at the time of an injection. For that reason we not only consider initial conditions at steady state.

All the simulations were stopped before the maximum pore pressure exceeds the lithostatic normal stress  $\sigma = 1$ , so that no mode I hydraulic fracture is created (which implies a loss of frictional contact, so that the model used here is no longer valid).

Two characteristic slip responses to a fluid injection are illustrated in figure 2: as the pore pressure perturbation develops, slip accelerates on a patch centered on the injection point (figures 2(a) and (b)). Within the slipping patch, the initial stress is reduced, and redistributed on the non perturbed remote portions of the fault (figures 2(c) and (d)). The fluid injection therefore initiates the development of a shear crack on the fault. Note that in both examples, the maximum slip rate is amplified by up to a factor  $10^5$ , which is typically in the range of slow aseismic slip (or creep) if we assume a reference slip rate of  $10^{-9}$  m.s $^{-1}$ . As this will be discussed later, two mechanisms drive the development of this crack: the reduction of effective normal stress due to the presence of fluid,

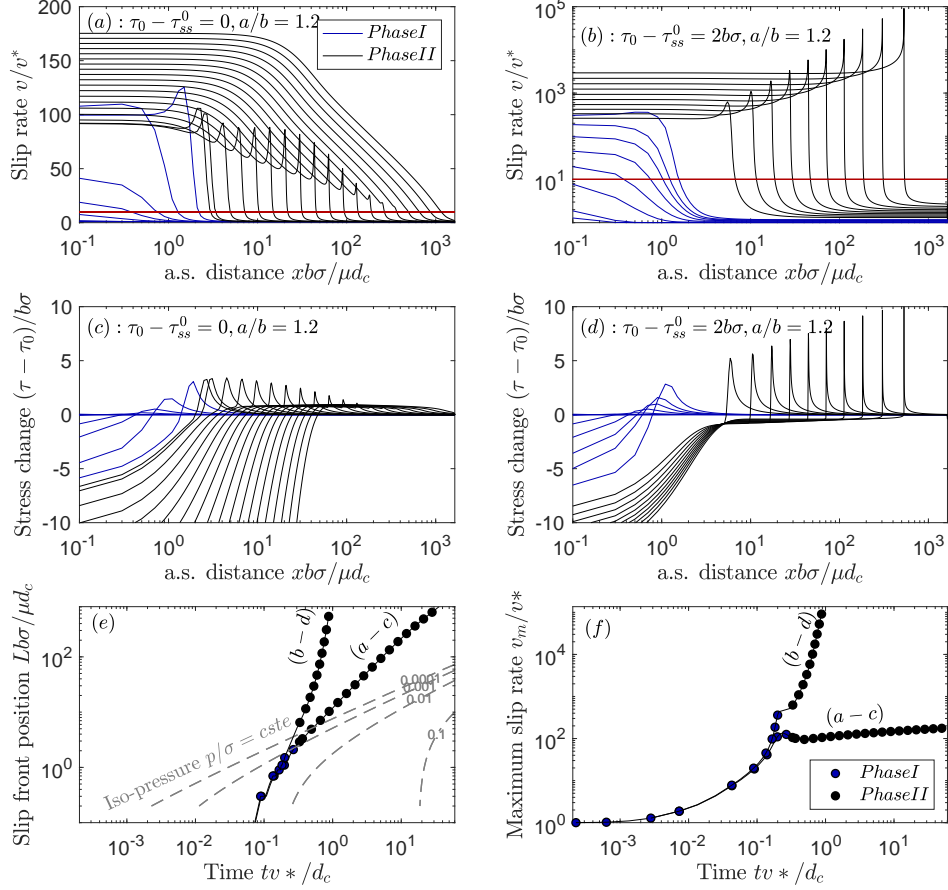


Figure 2: Two examples of fluid driven fault slip reactivation: fault initially at steady state (a,c), and initially above steady state (b,d). (a,b): Normalized slip rate. (c,d): Normalized shear stress change.  $\tau_0$  is the initial stress on the fault before the injection, and  $\tau_{ss}^0$  the steady-state frictional stress at the reference slip rate  $v^*$  and pore pressure  $p = 0$ . The profiles being symmetric around  $x = 0$ , only the right half profiles are represented. a.s. refers to along strike. (e): Slipping patch normalized half-length (or slip front position), defined as the size of the patch where slip rate exceeds the red line in figures (a,b). Gray dotted lines indicate iso-pressure levels. The grey numbers are levels of normalized pore pressure. (f): Maximum normalized slip rate. Each dot in (e,f) corresponds to a profile in figures (a,b,c,d). Blue lines and symbols correspond to phase I of slip reactivation, black lines and symbols correspond to the second phase of slip reactivation (see main text for details).

or the initial prestress.

We observe two successive phases in the evolution of such a pressurized crack: at the very beginning of the injection (phase I), the crack is confined to the central region of the fault, but rapidly, it starts to expand along strike (phase II). During phase I, the maximum slip rate on the crack increases exponentially with time from injection start. As illustrated in figure 2, we get a different behavior during phase II for different initial stress conditions: for a fault initially at steady state, the crack expansion occurs approximately at a constant rupture speed (figure 2(a,c,e)), and the maximum slip rate increases logarithmically in time (figure 2(f)). For a fault initially above steady state, both the crack length and the maximum slip rate blow up in a finite time (figure 2(b,d,e,f)). In the following we will refer to these two modes as the steady crack (constant rupture speed), or the accelerating crack (diverging slip rate).

The half crack length  $L(t)$  and the maximum slip rate  $v_m(t)$  histories for all the fault scenarios investigated are shown in figure 3. Here again, maximum slip rates remain within the range of what is typically called aseismic slow slip. As illustrated in figure 2,  $L(t)$  is defined as the half length of the patch experiencing more than a 10 times increase in slip rate. From figure 3,  $L(t)$  and  $v_m(t)$  are both influenced by the frictional conditions  $\alpha$  and by the initial state of stress. Interestingly, neither the steady nor the accelerating crack follows an iso-pressure path: the crack dynamics is not directly controlled by the pore pressure diffusion. In all the scenarios the crack propagates faster than the pressurized zone does. However, all the results lead either to the steady crack, or to the accelerating crack. The latter fault response is typically obtained for a  $\alpha$  parameter close to the weakening transition (i.e. for  $\alpha$  close to 1), and for a fault initially stressed above steady state.

In the following, we will study the details of slip rate acceleration during phase I, the transition to phase II, and the crack expansion during phase II.

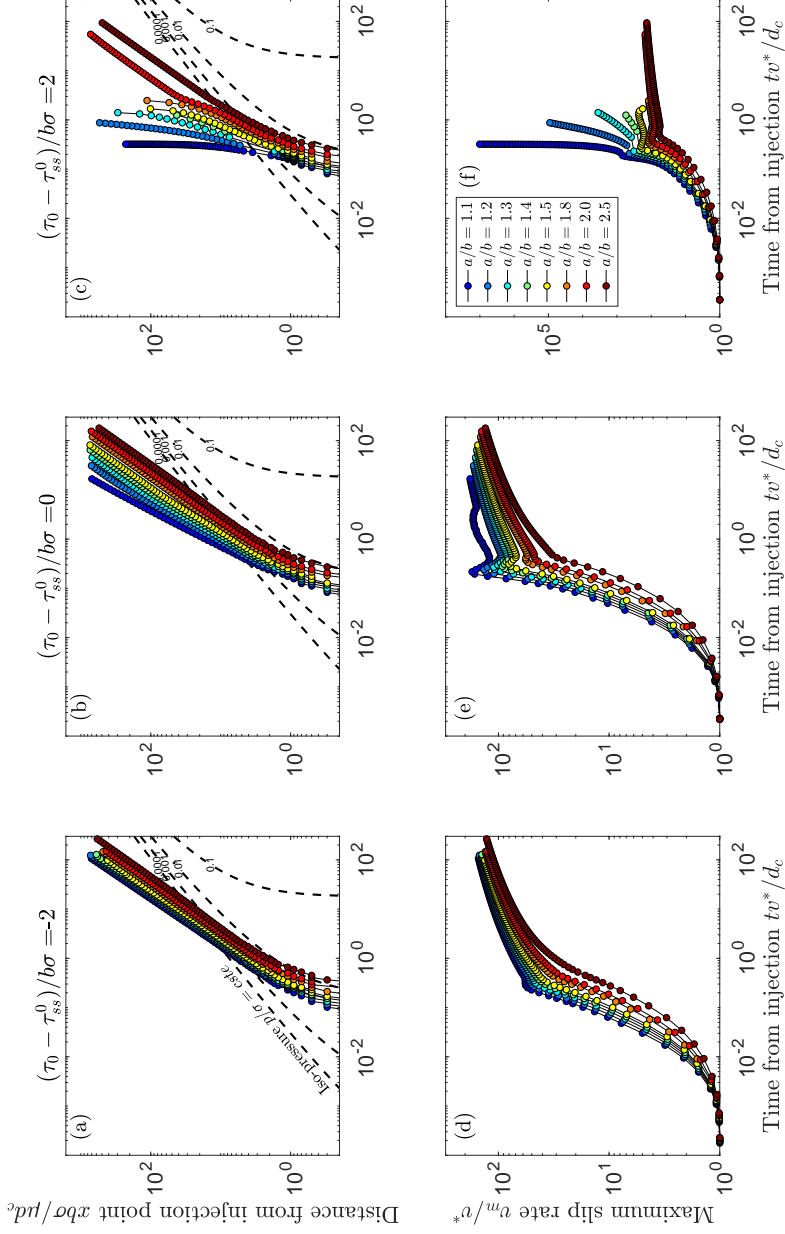


Figure 3: Shear crack half length (a,b,c) and maximum slip rate (d,e,f) as a function of time from the start of injection for all the fault scenarios investigated here. (a,d): Fault initially slightly below steady state ( $\tau_0 - \tau_{ss}^0 = -2b\sigma$ ). (b,e): Fault initially at steady state ( $\tau_0 - \tau_{ss}^0 = 0$ ). (c,f): Fault initially slightly above steady state ( $\tau_0 - \tau_{ss}^0 = 2b\sigma$ ). The color scale refers to the rate-and-state frictional parameter  $\alpha = a/b$ . Black dashed lines in figures (a,b,c) indicate iso-pressure levels. See figure 2 for details.

### 3.1. Phase I: initial slip rate acceleration

As shown in appendix Appendix A.1, the initial localized slip rate acceleration could be understood to the first order as a balance between the direct effect term  $\alpha \dot{v}/v$  and the approximate rate of change of effective normal stress  $f_0 \dot{p}$  in the central region of the fault. The other terms appearing in the first equation of (9) being negligible. In this framework, the maximum slip rate  $v_m$  is reached where the pore pressure is maximum, that is at the injection point. It is shown in appendix Appendix A.1 that  $v_m$  evolves approximately as:

$$v_m(t) = \exp(\sqrt{t/t_a}), \quad (12)$$

where  $t_a$  is a characteristic time scale given by:

$$t_a = \frac{\pi}{4} \frac{\alpha^2}{D f_0^2 q^2}. \quad (13)$$

This first phase ends when steady state is reached at the fault center (see figure A.8), marking the onset of significant crack expansion. It is shown in appendix Appendix A.1, that the onset of phase II occurs at time  $t_I$  given by:

$$t_I \simeq t_a (\ln 2t_a)^2. \quad (14)$$

As illustrated in figure 4, equation (12) provides a good estimates of the exponential increase of the maximum slip rate on the fault. Furthermore, the expression (14) for  $t_I$  provides the correct order of magnitude for the transition time to phase II, in particular for small values of  $\alpha$ . For larger values, it anticipates the transition by a factor of 2. This is certainly due to all the approximations leading to equation (14).

During phase I, we do not detect any significant increase of the length of the crack (no points for phase I in figures 3(a), (b) and (c)). This does not mean that the accelerating patch does not slightly expands: initially the slip rate is below the threshold used to track the crack size ( $v = 10$ ). Because of the balance between the direct effect and the pore pressure rate, the size of the

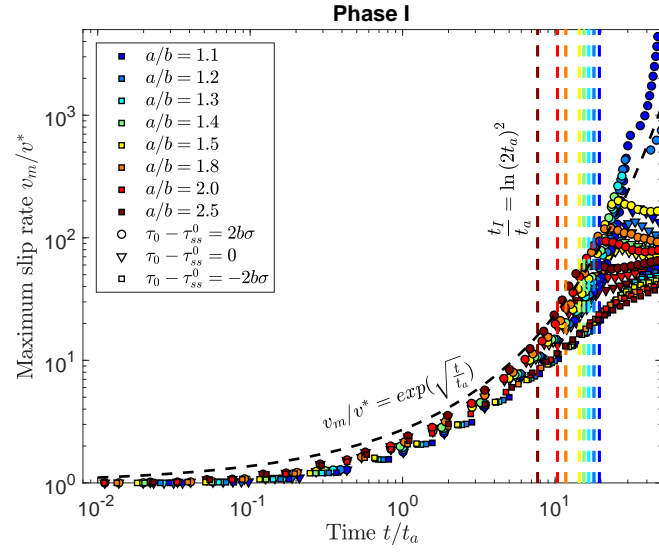


Figure 4: Normalized maximum slip rate increase during phase I. Colored symbols correspond to the different fault scenarios (different rate-and-state frictional parameter  $\alpha = a/b$  and different initial stress). The black dashed line is the theoretical prediction of equation (12). The colored dashed lines are the theoretical estimates for the end of phase I ( $t_I$ ) provided by equation (14).

accelerating patch is directly controlled by the diffusion length  $\sqrt{Dt}$ , as shown  
 245 in figures A.9(a) and A.10(a).

### 3.2. Phase II: slip rate increase and crack expansion

The onset of phase II is characterized by a steepening of the slip rate profile at the crack tip (last blue profiles and first black profiles in figure 2(a) and (b)). Simultaneously, the position of the slip rate maximum  $v_m$  is shifted from the  
 250 fault center to the crack front. The initiation of such a sharp slip front was observed for all the fault scenarios considered here. As noted earlier, the slip front then evolves in two different ways: either the peak slip rate increases, and the front accelerates in the case of an accelerating crack (figure 2(b-d)), or the peak slip rate decreases so that the sharp front dies away (figure 2(a-c)),  
 255 leading to a smoother front propagating at a constant speed (steady crack). In this latter case, the slip rate is maximum at the crack center. In the following, we will make the distinction between the maximum slip rate  $v_m$  and the peak slip rate  $v_p$ , that is the slip rate at the tip of sharp propagating fronts. In the case of an accelerating crack (figure 2(b)),  $v_m = v_p$ .

#### 3.2.1. Accelerating crack

In order to study in detail the dynamics of the accelerating crack, we concentrate on the fault scenarios characterized by  $\alpha = 1.1$  or  $1.2$  and an initial state of stress above steady state ( $\tau_0 - \tau_{ss}^0 > 0$ ), which provide the most characteristic examples of such a dynamics (figure 3). In appendix Appendix A.2.1,  
 265 it is shown that the accelerating crack is characterized by a square-root shaped displacement profile at the crack tip, and a positive stress drop  $\Delta\tau_{tip}$  near the tip of the crack (i.e. the difference between the initial stress and the residual stress left by the process zone at the crack tip). A quasi-static energy balance at the tip indicates that the crack half-length is approximately given by the  
 270 characteristic length  $L_c$  defined by Rubin and Ampuero (2005):

$$L \sim L_c = \frac{1}{\pi} \left[ \frac{\ln v_m \theta_0}{\ln \theta_0 - (\alpha - 1) \ln v_m} \right]^2. \quad (15)$$



Furthermore, the crack expansion rate  $\dot{L}$  is approximately given by the characteristic rupture speed  $v_c$  defined by Ampuero and Rubin (2008) as a function of the maximum (peak) slip rate  $v_m$ . From Ampuero and Rubin (2008), we thus have:

$$\dot{L} \sim v_c = 0.75 \frac{v_m}{\ln v_m \theta_0}. \quad (16)$$

275 As shown in figure 5(a) and (b), the results of the numerical simulations converge to the predictions of equations (15) and (16), for the two examples of accelerating cracks detailed here.

The simple quasi-static crack model defined by equations (15) and (16) provides an explanation to the observed diverging crack length and maximum (or  
280 peak) slip rate. Under velocity strengthening rheology ( $\alpha > 1$ ) and an initial prestress above steady state ( $\tau_0 - \tau_{ss}^0 = \ln \theta_0 > 0$ ), the two equations constrain  $L$ ,  $\dot{L}$ ,  $v_m$  (or  $v_p$ ) and  $\dot{v}_m$  to be increasing functions of time. In particular, according to (15), as  $v_m$  increases,  $L$  blows up and diverges at a finite slip rate  $v_l$ , approximately given by:

$$v_l = \theta_0^{1/(\alpha-1)}. \quad (17)$$

285 However, we did not try to find a closed form solution to equations (15) and (16) and we did not solve this system numerically. The solutions  $L(t)$  and  $v_m(t)$  would indeed be strongly influenced by our choice of initial conditions (poorly defined at the end of phase I), and by the fact that equation (15) is approached relatively slowly (see figure 5(a)). Note that in this case, since the pore pressure  
290 term does not enter into equation (15), the propagation is exclusively driven by the release of initial prestress. Beyond the limit speed  $v_l$ , the quasi-static crack model fails, since the stress drop at the crack tip (denominator in equation (15), see appendix Appendix A.2.1 for details) becomes negative.

More generally, such a mode of propagation is only possible if the stress drop  
295 at the crack tip is positive. In all the simulations this kind of crack front was observed, we computed the stress drop evolution  $\Delta\tau_{tip}$ . The results are reported

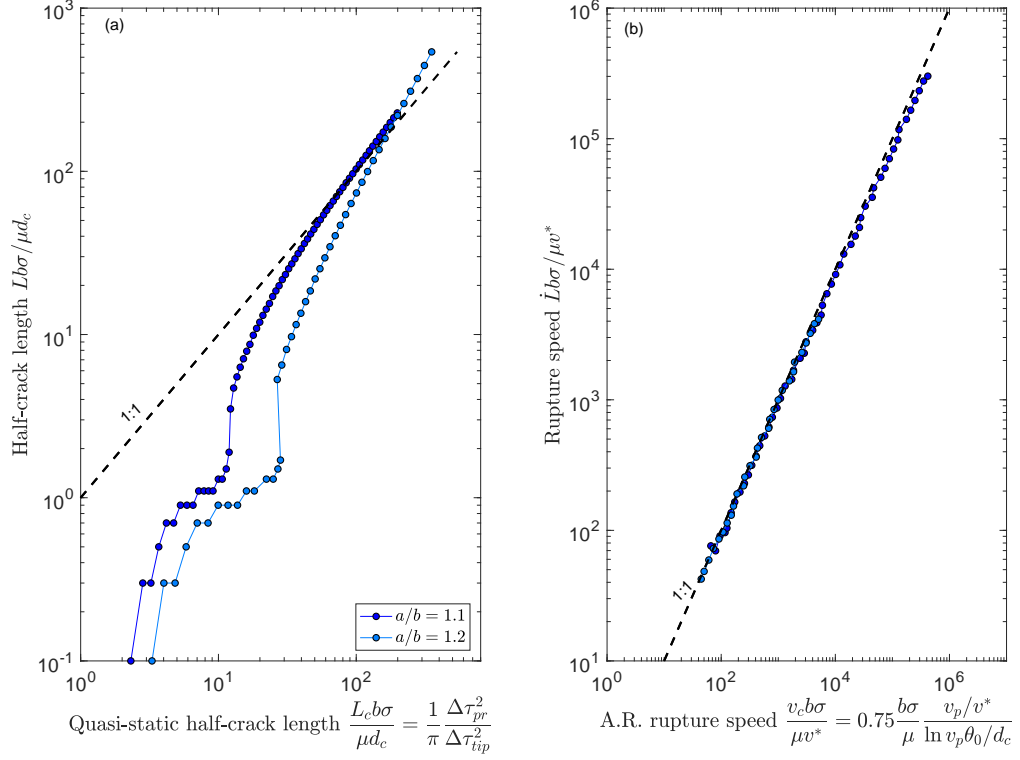


Figure 5: Numerical vs. theoretical dynamics of accelerating cracks, obtained with initial stress above steady state ( $\tau_0 - \tau_{ss0} = 2b\sigma$ ). (a): crack half-length as a function of the theoretical length  $L_c$  (expected from a quasi-static energy balance at the crack tip, equation (15)).  $\Delta\tau_{pr}$  is the peak to residual stress drop,  $\Delta\tau_{tip}$  is the stress drop at the crack tip (see main text for details). (b): rupture speed (crack expansion speed), as a function of the theoretical prediction by (Ampuero and Rubin, 2008) (A.R. rupture speed  $v_c$ , equation (16)).  $v_m$  is the maximum slip rate. Colored dots indicate the numerical solution. Black dashed lines indicate the perfect match between numerical solution and theoretical estimates  $L_c$  and  $v_c$ .

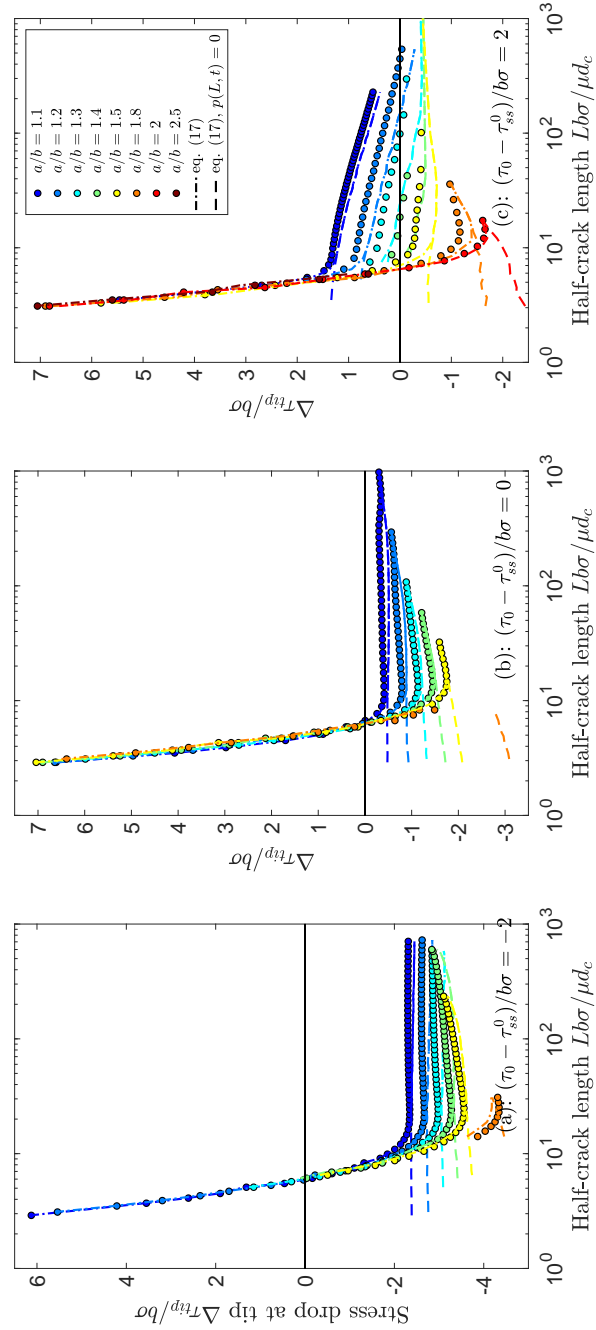


Figure 6: Stress drop at the crack tip  $\Delta\tau_{tip}$  during crack expansion (phase II) for all the scenarios an accelerating crack was enough developed to be observed. (a): Fault initially slightly below steady state ( $\tau_0 - \tau_{ss}^0 = -2b\sigma$ ). (b): Fault initially at steady state ( $\tau_0 - \tau_{ss}^0 = 0$ ). (c): Fault initially slightly above steady state ( $\tau_0 - \tau_{ss}^0 = 2b\sigma$ ). Color scale corresponds to the rate-and-state friction parameter  $\alpha = 1.8$ . Dots are the numerical results. Dashed and dashed dotted lines are the approximations of equation (18). The black horizontal line indicates  $\Delta\tau_{tip} = 0$ . The fault scenarios not represented (or partially represented) correspond to situations where the accelerating crack was too short lived.

in figure 6 as a function of the (increasing) crack length.  $\Delta\tau_{tip}$  is always positive at the onset of the propagation, then it strongly decreases as crack propagation proceeds and eventually becomes negative. At larger crack length,  $\Delta\tau_{tip}$  either  
300 increases smoothly if negative, or decreases smoothly if positive. As shown in figure 6, the evolution of  $\Delta\tau_{tip}$  is well captured by the approximate expression of  $\Delta\tau_{tip}$  provided in appendix Appendix A.2.1:

$$\Delta\tau_{tip}(t) = \ln \theta_0 - (\alpha - 1) \ln v_p(t) + f_0 p(L, t), \quad (18)$$

where  $p(L, t)$  is the pressure at the crack front  $x = L$  and time  $t$ . Initially, when the crack process zone is within the pressurized region, the third term  
305 dominates, so that  $\Delta\tau_{tip}$  is positive. The initiation of an accelerating crack could therefore be attributed to the decrease of effective normal stress within the pressurized region. As the crack expands faster than the pressurized region, this term decreases, and the first two terms become important, until they dominate (figure 6). When the pore pressure term becomes negligible, the evolution of  
310  $\Delta\tau_{tip}$  becomes smoother.

The evolution of  $\Delta\tau_{tip}$  shows that if in many cases, an accelerating crack could be initiated at early times, this kind of solution can not be sustained during the whole simulation, because  $\Delta\tau_{tip}$  quickly becomes negative. This is typically what happens in figure 2(a,c). The only way of maintaining this accelerating  
315 crack beyond the pressurized zone is to keep the first two terms of equation (18) positive, that is if  $v_p < v_l$ . From equation (17), we see that if the initial stress is below steady state ( $\theta_0 \leq 1$ ), then  $v_l$  is smaller than the initial slip rate  $v = 1$ , and the accelerating crack can not develop towards instability. For faults initially above steady state ( $\theta_0 > 1$ ), we obtain  $v_l = 4.85 \times 10^8$  for  $\alpha = 1.1$ ,  $v_l = 2.2 \times 10^4$   
320 for  $\alpha = 1.2$ , which is near or above the observed maximum slip rates (figure 3(f)). Under stronger velocity strengthening properties ( $\alpha > 1.2$ ), we get  $v_l$  between 3 and 800, which is of the order of (or below) the observed maximum slip rates. Equation (17) therefore provides a first order estimate of the range of slip rates that could be obtained on a prestressed velocity strengthening fault.

325 As the accelerating crack dies away, the simulations evolve towards a steady propagation, that is detailed in the next section.

### 3.2.2. Steady crack

It is shown in appendix Appendix A.2.2 that this second mode of crack propagation is to the first order controlled by a balance between the elasticity  
 330 (i.e. the elastic stress transfers) and the approximate rate of effective normal stress change  $f_0\dot{p}$ . In other words, the crack evolves as if it were governed by a constant friction coefficient within the slipping zone. Here the pore pressure increase drives the crack expansion (and not the initial prestress). When assuming this simplified stress balance within the crack along with the conservation of  
 335 total stress along the fault, it could be shown (appendix Appendix A.2.2) that the crack half size grows linearly in time as:

$$L(t) = v_r t = \frac{f_0 q D}{\lambda(\alpha - 1) - \ln \theta_0} t, \quad (19)$$

where  $v_r$  is a constant rupture speed. As during phase I, the maximum slip rate  $v_m$  is reached at the injection point, and approximately increases as the logarithm of time:

$$v_m(t) = c \left( 1 + \ln \sqrt{\frac{t}{t_s}} \right), \quad c = \frac{4f_0 q D}{\pi}, \quad t_s = 4 \frac{\lambda^2(\alpha - 1)^2 - \ln \theta_0^2}{f_0^2 q^2 D}. \quad (20)$$

340  $c$  and  $t_s$  are a typical slip rate and a characteristic time scale respectively. In both equations (19) and (20),  $\lambda$  corresponds to the logarithm of maximum slip rate on the fault, which has been assumed constant in the derivations of appendix Appendix A.2.2. Since the maximum slip rate approximately increases logarithmically in time for the steady crack, the changes in  $\lambda = \ln v_m$  are negligible compared to the linear increase of crack length  $L$ , and the variations of  
 345  $v_m$ . If we consider  $\lambda$  as the average value of  $\ln v_m$  during phase II for the steady crack, we get numerically  $4.6 < \lambda < 5.2$ . In the following we therefore assume  $\lambda = 5$ .

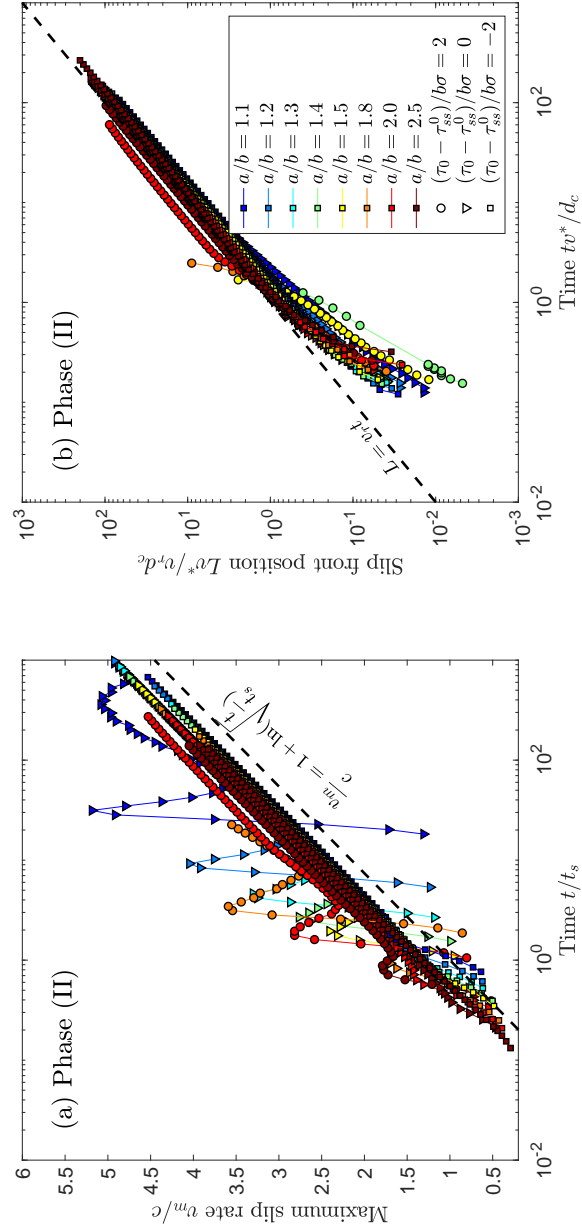


Figure 7: Normalized maximum slip rate (a) and normalized slip front position (b) for the simulations leading to a steady crack expansion during phase II. Only times  $t > t_I$  are represented (phase II). Colored dots are the numerical results. The black dashed lines are the approximations of equations (19) and (20) the characteristic slip rate  $c$ , rupture speed  $v_r$  and time  $t_s$  are defined in the main text: equations (19) and (20).

Numerical simulations leading to a steady crack approximately converge to  
 350 the predictions of equations (19) and (20), as illustrated in figure 7. For  $t > 10t_s$ ,  
 the maximum slip rate and the crack half size follow a logarithmic and linear  
 increase respectively. The approach to the asymptotic solution for  $v_m$  is in some  
 cases preceded by a quick oscillation, in particular for  $\tau_0 - \tau_{ss}^0 = 2b\sigma$  and for  
 small values of  $\alpha$ . This corresponds to the transient phase (between phase I  
 355 and II) where an accelerating crack is initiated, so that the maximum slip rate  
 is the peak slip rate at the tip  $v_m = v_p$ . However,  $v_p$  quickly decreases as the  
 accelerating crack can not propagate any further.

#### 4. Discussion

We have shown that a fluid injection into a velocity strengthening rate-and-  
 360 state fault initiates a slow (aseismic) slip event in the form of a shear crack  
 expanding faster than pore pressure diffuses along the fault (the crack front  
 neither follows an iso-pressure, nor expands diffusively as  $\sqrt{t}$ ). Aseismic slip  
 outpaces the pore pressure diffusion for the range of initial stress and frictional  
 conditions analyzed here. This dynamics is similar to the results obtained by  
 365 Cappa et al. (2018); Bhattacharya and Viesca (2019) in the case of significant  
 permeability enhancement. We did not consider permeability enhancement, so  
 that the strong slow slip response is essentially the result of the rate-and-state  
 rheology.

We were able to highlight many new features about the dynamics of fluid  
 370 driven slow slip under constant injection rate. Our results show two successive  
 phases of evolution: in a first phase, slip abruptly accelerates around the in-  
 jection point, and the crack remains within the pressurized zone. In a second  
 phase, the crack expands along strike while the slip rate keeps increasing. Two  
 modes of crack expansion were observed: an accelerated expansion character-  
 375 ized by diverging slip rate and rupture speed, or a steady crack expanding at  
 a constant speed with a slip rate increasing as the logarithm of time. The ac-  
 celerating crack could eventually lead to radiative slip rates ( $v_{sis}$  of the order

of  $1\text{cm.s}^{-1}$ ): assuming a reference slip rate  $v^* = 10^{-9} \text{ m.s}^{-1}$ , this would correspond in our simulations to a slip rate increase of 8 orders of magnitude. The  
380 accelerating crack could therefore be considered as the nucleation phase of a dynamic event, that would eventually stop when the slip rate reaches the limiting speed  $v_l = \theta_0^{1/(\alpha-1)}$  defined in equation (17). The steady crack on the other hand only involves slow slip rates, so that it could be considered as ultimately stable. These two crack modes could be compared to the stability analysis  
385 by Garagash and Germanovich (2012); Azad et al. (2017), where a fluid driven aseismic crack either nucleates a dynamic instability if the initial stress  $\tau_0$  is larger than the residual stress  $\tau_r$  of the slip-weakening friction law, or remains stable and aseismic if  $\tau_0 < \tau_r$ . Here we see that under rate-and-state friction, unstable cracks can develop up to seismic speeds if  $v_l > v_{sis}$ , which leads to  
390 (from the definition of  $v_l$ ) :

$$f_0 + \ln \theta_0 > f_0 + (\alpha - 1) \ln v_{sis}, \quad (21)$$

which is nothing else than the initial stress  $\tau_0$  being larger than a residual (steady state) stress  $\tau_r^a$  taken at the seismic slip rate  $v_{sis}$  according to the second equation of (10). Note that this result does not necessarily imply important initial stress that would be non-physical: as  $\alpha$  goes to one, the initial deviation from  
395  $f_0$  leading to slip rate of seismic magnitude  $v_{sis}$  could be arbitrarily small. For  $\alpha = 1.1$  and assuming  $v_{sis} \simeq 10^9$ , it is only necessary to perturb  $f_0$  by 0.35 percent to promote dynamic failure. Considering typical values of friction parameters, along with a lithostatic normal stress at approximately 3 km depth, this implies an increase in shear stress of approximately 0.21 MPa from a refer-  
400 ence value of 60 MPa. This order of magnitude is below the typical stress drop of earthquakes (Abercrombie, 1995; Allmann and Shearer, 2009). It is thus within the range of stress variations expected on tectonic faults. Under constant slip rate  $v_{sis}$ , rate-and-state friction indeed behaves as a slip weakening law, the steady state value of stress being the residual stress. This is typi-  
405 cally what happens at the crack tip: slip rate instantaneously increases to  $v_{sis}$ ,



which remains approximately constant as the crack propagates further. The conditions leading to an unstable crack propagation are therefore equivalent to the conditions needed to nucleate a dynamic rupture under slip weakening law (Garagash and Germanovich, 2012; Azad et al., 2017).

410 An important difference between the rate-and-state response and the slip weakening response is the absence of a minimum stress (or pore pressure increase) to trigger a slip transient. This feature is similar to the spontaneous nucleation process on a velocity weakening rate-and-state fault (Rubin and Ampuero, 2005; Ampuero and Rubin, 2008).

415 We have shown that during phase I, the crack expansion occurs diffusively, following the pore pressure diffusion. A spontaneous expansion starts at the beginning of phase II. We defined the characteristic time  $t_I$  for the transition to phase II that could also be used to define a characteristic crack length  $L_I$  at the transition to phase II. Since the initial crack expansion is diffusive, we  
 420 have  $L_I \simeq \sqrt{Dt_I}$ .  $t_I$  and  $L_I$  are the time needed to start crack expansion, and the critical length allowing crack expansion. If an accelerating crack develops,  $t_I$  and  $L_I$  could therefore be used as estimates for the time to instability and the critical crack length for instability. Assuming that variations in  $\ln t_a$  are negligible compared to changes in  $t_a$ , we can assume from equation (14) that  
 425  $t_I \propto t_a$ , and from the definition of  $t_a$  (13) we get:

$$\begin{cases} t_I \propto \frac{\alpha^2}{Df_0^2q^2} \\ L_I \propto \frac{\alpha}{f_0q}. \end{cases} \quad (22)$$

This simple scaling indicates first that the injection scenario controls the transition to instability:  $t_I$  and  $L_I$  decrease as the injection rate increases. A decrease of time to instability with increasing injection rate was already obtained by Garagash and Germanovich (2012); Azad et al. (2017), in the framework of slip-  
 430 weakening friction. However, the authors did not observe a significant change in the critical length with the injection scenario. Under rate-and-state friction, the critical length and time to instability also depend on the parameter  $\alpha$ : stronger

strengthening behavior (large  $\alpha$ ) leads to a delayed  $t_I$  and a larger  $L_I$ .

Note that for the range of initial stress investigated (in the vicinity of steady state at the reference speed), the dynamics of phase  $I$  does not significantly  
435 depend on the initial state of stress, as observed by Garagash and Germanovich (2012); Azad et al. (2017). As shown in appendix A, phase I is essentially controlled by a balance between the direct effect of the friction law and the rate of pore pressure change, so that only the initial slip rate matters and the state  
440 evolution could be neglected. Since we have considered situations where initial stress is controlled by the initial value of the state variable, the initial stress has no real influence on the dynamics of phase I. Considering a much higher or smaller value of the initial state variable may break the balance between the direct effect and the pore pressure change, so that the initial stress would  
445 influence the dynamics of phase I (in particular  $t_I$  and  $L_I$ ). Alternatively, considering a higher initial stress by imposing a higher initial slip rate would make the dynamics of phase I more sensitive to the initial stress (in particular  $t_I$  and  $L_I$ ). Therefore, we do not rule out the possibility that initial stress influences the transition to the (eventually accelerated) crack expansion.

450 Here we have decided not to investigate extreme values of the initial state variable in order to remain under initial stress conditions close to steady-state friction. Such close to steady state conditions are expected to prevail on aseismic faults that have not experienced significant stress perturbations for a long time (at least longer than the characteristic time for slip relaxation, that could  
455 be estimated from the observed duration of afterslip following large earthquakes, or from the characteristic duration of aftershock sequences driven by frictional afterslip. This characteristic time is of the order of hours to years (Helmstetter and Shaw, 2009; Schaff et al., 1998)). Even following a stress perturbation, a velocity strengthening fault remains close to steady-state (Perfettini and Ampuero,  
460 2008; Dublanchet et al., 2013). We therefore expect the probability of a fault to be far from steady-state to be very small. The simulations performed from initial steady-state conditions and the steady-crack solution are therefore the most relevant results of this study concerning the understanding of tectonic

fault behavior.

465 The crack expansion characterizing the second phase occurs either at an increasing rupture speed (accelerating crack) or at a constant rupture speed (steady crack). The accelerating crack being a transient state (until a limit speed is reached), we expect it to evolve towards a dynamic rupture, or to the steady crack (as observed for faults initially above steady state and characterized by intermediate frictional parameter  $\alpha$ ) . A dynamic rupture is controlled by inertia, 470 so that rupture speed is a fraction of the shear wave speed of the elastic medium. If slowing down to the steady crack, the expansion will ultimately proceed at constant rupture speed  $v_r$  provided by equation (19). As shown in appendix Appendix A.2.2, the constant rupture speed is a consequence of the linear increase of average pore pressure under constant injection rate. Considering a 475 different injection scenario could lead to a different dynamics of crack expansion. Many seismological observations show a migration of micro-earthquakes and tremor activity along faults at a constant speed (Duverger et al., 2015; Roland and McGuire, 2009; Ghosh et al., 2010; Houston et al., 2011) along with 480 clear evidences that such activity is triggered by a slow aseismic slip event. The constant migration speed of seismic activity could therefore be related to an aseismic slip event propagating at a constant speed. Our result suggest that such phenomena are compatible with a triggering mechanism involving a fluid over-pressure emanating from a local and constant injection rate.

485 We show in appendix Appendix A.2.2 that the steady crack rupture speed  $v_r$  results from a stress balance at the scale of the entire fault. The reduction of frictional stress (related to the effective normal stress decrease) is balanced by the velocity strengthening rheology (shear stress increase caused by slip rate acceleration) on the cracked region. The reduction of effective stress is essentially 490 accommodated by an expansion of the high slip rate region, rather by a stronger increase of slip rate. Assuming that the parameter  $\lambda$  in equation (19)

is a measure of the log of the maximum slip rate,  $v_r$  could be rewritten as:

$$v_r = \frac{f_0 q D}{\tau_r^s - \tau_0}, \quad (23)$$

where  $\tau_r^s = f_0 + (\alpha - 1)\lambda$  could be understood as a typical (steady-state) residual stress within the crack from equation (10), that is larger than the initial stress  $\tau_0$  for the steady propagating crack. Here again, the dynamics of the steady crack strongly depends on the difference between the initial stress and a residual stress of the friction law.  $v_r$  is expected to diverge as the initial stress gets closer to  $\tau_r^s$ . If  $\tau_0 > \tau_r^s$  we enter the accelerated crack solution. The importance of  $\tau_0 - \tau_r^s$  in the steady crack dynamics is here again analog to the slip weakening case (Garagash and Germanovich, 2012; Azad et al., 2017). Hydraulic properties also contribute to the crack expansion speed: a higher injection rate or a stronger diffusivity leads to an increase of  $v_r$ .

An important result to note is that both  $\tau_r^a$  controlling the possibility of dynamic slip rates, and  $\tau_r^s$  allowing the steady crack propagation correspond to a residual steady-state stress within the expanding crack at zero pore pressure, whatever crack regime prevails.  $\tau_r^s$  and  $\tau_r^a$  could therefore be unified so that the relevant parameter incorporating the effect of prestress conditions and the parameter  $\alpha$  is the (slip-rate dependent) residual stress within the crack  $\tau_r$  at zero pore pressure. The different crack propagation regimes are therefore controlled by  $\tau_0 - \tau_r$ .

So far we have only considered a single injection scenario consisting in a constant injection rate (or fixed pore pressure gradient at the origin). Other injection scenarios could be investigated, in particular involving a finite duration of injection. When the injection stops, pore pressure progressively decreases along the fault, and as the pore pressure rate becomes negligible, the fault is essentially controlled by a balance between elasticity and friction. At large time after injection (over timescales exceeding the diffusion timescale  $L_0^2/D$  on the entire fault) the system behaves as a velocity strengthening fault responding to an instantaneous stress perturbation. This problem has already been studied by

520 Perfettini and Ampuero (2008), showing that the maximum slip rate ultimately  
 relaxes back to the steady state reference velocity  $v^*$  following a  $1/t$  decay. How-  
 ever, the behavior at intermediate times needs to be further investigated. Differ-  
 ent scenarios could also be analyzed, involving a controlled overpressure at the  
 injection point (Garagash and Germanovich, 2012), or a propagating hydraulic  
 525 fracture (Azad et al., 2017). Slip and stress induced permeability enhancement  
 seems also to play an important role in fluid induced aseismic slip as shown  
 by the in-situ hydro-shearing experiments (Guglielmi et al., 2015; Cappa et al.,  
 2018; Bhattacharya and Viesca, 2019). If permeability enhancement increases  
 the aseismic slip response under slip-weakening friction, more efforts need to be  
 530 done to evaluate this effect under rate-and-state conditions.

Finally, we did not investigate here the whole rate-and-state spectrum: we  
 did not consider other state evolution laws, such as the slip law (Ruina, 1983).  
 We do not present any results about the velocity weakening behavior. Never-  
 theless, our results suggest that in the absence of strengthening, the accelerated  
 535 crack would be favored by velocity weakening rheology, leading to dynamic  
 ruptures if the fault is large enough for the nucleation to fully develop. If  
 nucleation on dry, velocity weakening, rate-and-state faults has been largely  
 explored (Rubin and Ampuero, 2005; Ampuero and Rubin, 2008; Dublanchet,  
 2017), fluid induced earthquake nucleation needs to be further investigated.

## 540 **5. Conclusion**

The effect of a local injection of fluid into a stable velocity weakening fault  
 could be summarized as follows: in a first step, the reduction in effective nor-  
 mal stress triggers a slip acceleration in the form of a shear crack confined to  
 the pressurized zone. During this first phase, slip rate increases exponentially  
 545 in time, and the crack expands diffusively along strike. In a second phase,  
 the shear crack expands much faster than the pore pressure diffuses, and the  
 regime of slip rate increase changes. The transition to the expansion phase  
 is facilitated by a larger injection rate, and frictional properties closer to the

weakening transition  $a/b = 1$ . During phase II, we observe either an acceler-  
550 ating crack regime, where slip rate and rupture speed blow up in a finite time,  
or a steady crack expansion characterized by a constant expansion speed and  
a logarithmic increase of maximum slip rate. The transition between the two  
regimes could be understood in terms of the difference between the initial stress  
along the fault  $\tau_0$ , and the residual stress left within the crack  $\tau_r$ . The accel-  
555 erating crack regime is obtained when  $\tau_0 > \tau_r$ , while the steady crack occurs if  
 $\tau_0 < \tau_r$ . The rupture speed of the steady crack decreases approximately as the  
inverse of  $\tau_r - \tau_0$ , and increases linearly with the injection rate. Considering  
the possible stress conditions on tectonic faults, the steady crack is the most  
probable reactivation scenario expected from these results. The slip response of  
560 a rate-and-state fault to a fluid injection is therefore controlled by the frictional  
properties, the injection history, and the initial stress conditions. The numerical  
and theoretical results presented here provide some new insights about the haz-  
ards associated with fluid operations at depth, by demonstrating quantitative  
relationships between injection parameters and fault slip response.

## 565 **Appendix A.**

In this section, we derive approximate expressions for the slip-rate profiles,  
the maximum slip rate  $v_m(t)$  and the half-crack size evolution  $L(t)$ . As men-  
tioned in the main text, the slip rate evolution during a fluid injection could be  
decomposed into two phases: in the early phase (phase I), slip rate increases  
570 exponentially on a small patch centered on the injection point. In the sec-  
ond phase, the slip rate increases further while the accelerating crack expands  
along strike. Recall that in all the following developments we make use of the  
non-dimensional notations defined in the second section.

### *Appendix A.1. Early slip acceleration*

575 As shown in figure A.8(a,d,g), the early phase is characterized to the first  
order by a balance between the shear stress rate related to the pore pressure

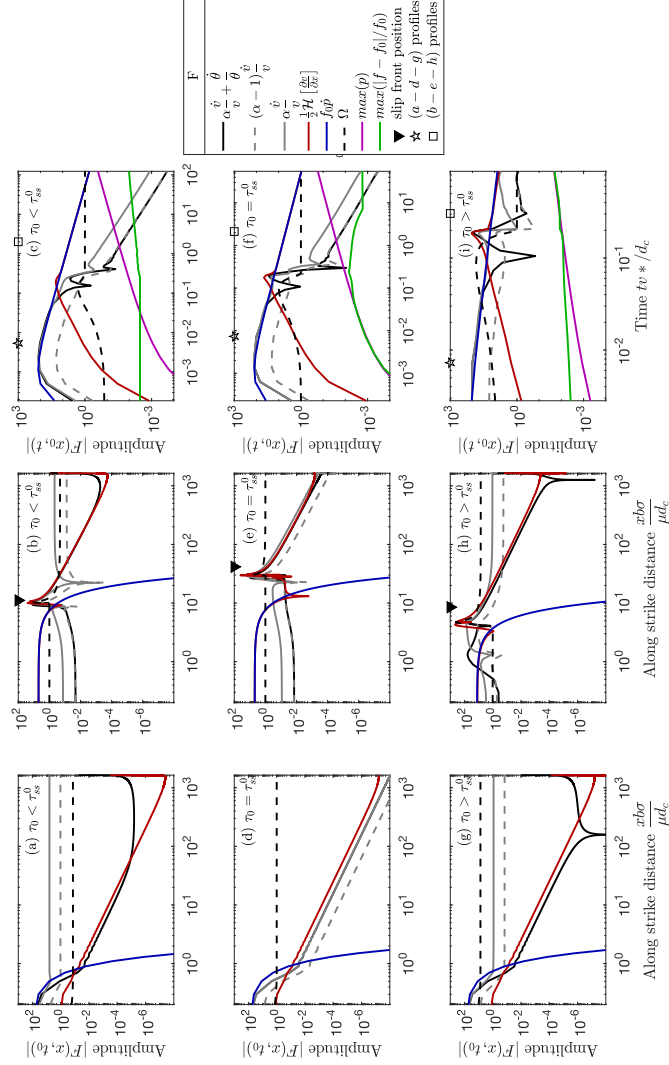


Figure A.8: Stressing rate contributions along the fault, for the fault scenarios with  $\alpha = a/b = 1.2$ . The different lines represent the different terms of the stressing rate balance (9), along with the distance to steady state  $\Omega = v\theta$  (dashed black line), the maximum value of pore pressure  $p$  (purple solid line), and the deviation of the friction coefficient  $f$  from its reference value  $f_0$  (green line). The two left columns represent the different contributions along strike (only the right half fault is represented) at two selected instants ( $t = t_0$ ): one during phase I (left column), the second during phase II (middle column). The black triangle marks the position of the crack front. The right column represents the different contributions at the fault center ( $x = x_0$ ) as a function of time from the start of injection. The black star and the black square indicate the time corresponding to the profiles of the left column and the middle column respectively. (a,b,c): Fault initially slightly below steady state ( $\tau_0 - \tau_{ss}^0 = -2b\sigma$ ). (d,e,f): Fault initially at steady state ( $\tau_0 - \tau_{ss}^0 = 0$ ). (g,h,i): Fault initially slightly above steady state ( $\tau_0 - \tau_{ss}^0 = 2b\sigma$ ). Note that non-dimensional notation is used in the legend.

rate ( $f\dot{p}$ ) and the changes in friction coefficient ( $\alpha\dot{v}/v + \dot{\theta}/\theta$ ), in the central region of the fault where slip accelerates ( $|x| < 1$  in figures A.8(a,d,g)). The elastic stress transfers (Hilbert transform term in the first equation of (9)) are negligible. Outside this central region, the elastic stress transfers dominate over the pore pressure variations. Note also that in figures A.8(a,d,g), the black solid curve overlaps the gray solid line within the pressurized zone, so that the state evolution term is negligible and the changes in the friction coefficient are dominated by the direct effect  $\alpha\dot{v}/v$ . This balance is maintained in particular at the crack center during the whole duration of phase I (see figures A.8(c,f,i)). Furthermore, during this early phase,  $f \simeq f_0$  and  $p \ll 1$  (figure A.8(c,f,i)) so that around the injection point, the system (9) approximately reduces to:

$$\alpha \frac{\dot{v}}{v} = f_0 \dot{p}, \quad (\text{A.1})$$

which leads to, after integrating from  $t = 0$  to  $t$  along with  $v(x, 0) = v_0 = 1$  and  $p(x, 0) = 0$ :

$$v = e^{f_0 p / \alpha}. \quad (\text{A.2})$$

The maximum slip rate  $v_m$  is thus expected to occur at the injection point  $x = 0$ , where the fluid pressure is maximum. Making use of equation (6) at  $x = 0$ , we get:

$$v_m(t) = e^{\sqrt{t/t_a}}, \quad t_a = \frac{\pi}{4} \frac{\alpha^2}{D f_0^2 q^2}. \quad (\text{A.3})$$

The state evolution term could also be considered in these developments, in particular if the distance to steady state  $\Omega = v\theta$  is significant. If  $\Omega \ll 1$  for instance, we have from the state evolution law (3)  $\theta \simeq \theta_0 + t$  (Helmstetter and Shaw, 2009), leading to a slip rate of the form:

$$v = \frac{e^{f_0 p / \alpha}}{(\theta_0 + t)^{1/\alpha}}. \quad (\text{A.4})$$



Alternatively, if  $\Omega \gg 1$ , the state evolution law leads to  $\dot{\theta}/\theta \simeq -v$  (Helmstetter and Shaw, 2009), so that the stress balance could be integrated as:

$$v = \frac{e^{f_0 p/\alpha}}{1 - \Pi(t)/\alpha}, \quad (\text{A.5})$$

where  $\Pi(t)$  is given by:

$$\Pi(t) = \int_0^t e^{f_0 p(t')/\alpha} dt'. \quad (\text{A.6})$$

600 It can be seen in figure A.9(a) and A.10(a), that approximations (A.4) and (A.2) are supported by our numerical solutions during the initial phase of slip rate evolution, for the two cases  $\tau_0 - \tau_{ss}^0 = -2b\sigma$  and  $\tau_0 - \tau_{ss}^0 = 0$  respectively, even at large distances from the pressurized zone. The maximum slip rate at the fault center  $v_m$  (figures A.9(d), A.10(d) and 4) is in any case close to the  
605 simple expression (A.3). Note that the grid used to compute the numerical solution does not allow to get the slip rate exactly at  $x = 0$ , and the maximum slip rate obtained numerically is instead the slip rate at  $x = \pm h/2$ ,  $h$  being the grid size, which underestimates the maximum slip rate. The agreement between numerical and analytical solution is improved when using expression (A.2), with  
610 the pressure computed at  $x = \pm h/2$ .

As illustrated in figures A.8(c), (f) and (i), this balance between the direct effect and the pore pressure rate remains valid until the injection region reaches steady state. This could be seen with the evolution of the distance to steady-state  $\Omega = v\theta$  at the injection point (recall that  $\Omega = 1$  corresponds to steady-  
615 state). Afterwards, the elastic interaction term  $\mathcal{H}[v']$  becomes dominant over the pore pressure rate term, and equilibrates the friction coefficient variations  $(\alpha\dot{v}/v + \dot{\theta}/\theta)$  at the crack center. In order to get an approximate duration for phase I, we need an expression for the evolution of  $\Omega$  at the fault center. From the definition of  $\Omega$ , and the state evolution law (3), we have:

$$\frac{\dot{\Omega}}{\Omega} = \frac{\dot{v}}{v} + \frac{\dot{\theta}}{\theta} = \frac{\dot{v}}{v} + v \left( \frac{1}{\Omega} - 1 \right). \quad (\text{A.7})$$

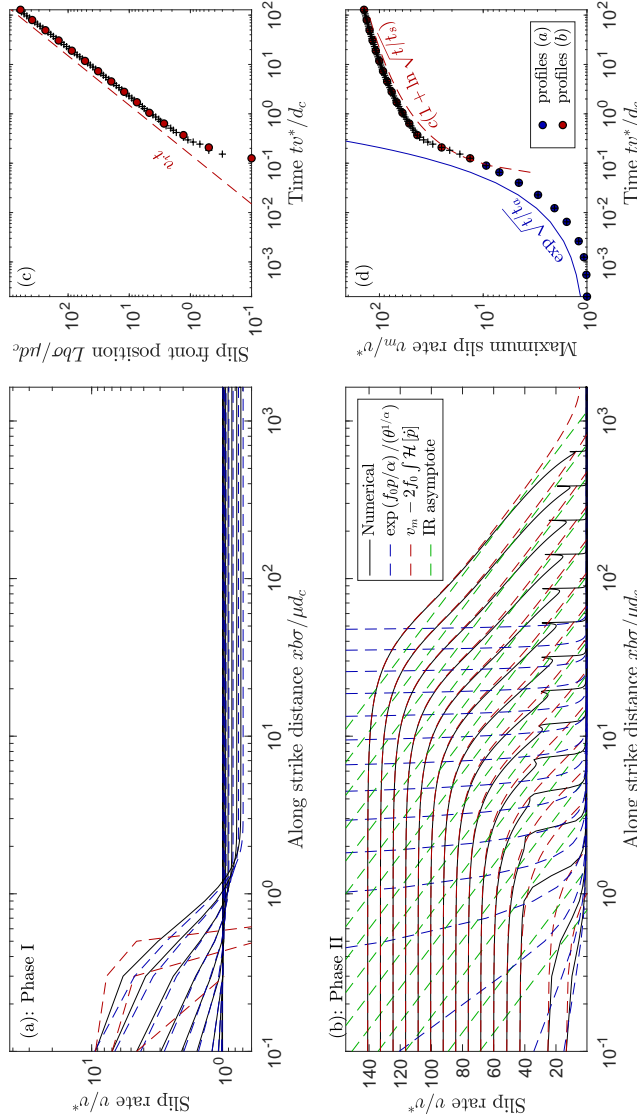


Figure A.9: Detail of slip rate evolution on a fault initially below steady state ( $\tau_0 - \tau_{ss}^0 = -2b\sigma$ ) and characterized by a rate-and-state parameter  $\alpha = a/b = 1.4$ , evolving towards the steady crack solution. (a): Slip rate profiles during phase I. (b): Slip rate profiles during phase II. Here again in (a,b) only the right half fault is represented (the solution is symmetric). Black profiles correspond to the numerical solution, and the colored ones to the different possible approximations mentioned in the legend. Here again, non dimensional notations are used in the legend. The blue dashed profiles correspond to the approximation (A.4) expected during phase I. The red dashed profiles correspond to the approximation (A.23) suggested for phase II. The green dashed line is the simplified form of equation (A.23) in the intermediate distance range (IR): equation (A.26). (c): Crack half-length evolution. (d): Maximum slip rate evolution. The blue dots in (d) indicate the time and the maximum slip rate of each profile of (a). The red dots in (c) and (d) correspond to the profiles shown in (b). The blue solid line in (d) is the approximation (A.3) for the evolution of maximum slip rate during phase I. The red dashed lines in (c) and (d) indicate the approximations (A.36) and (A.37) for the crack half-length and the maximum slip rate during phase II.

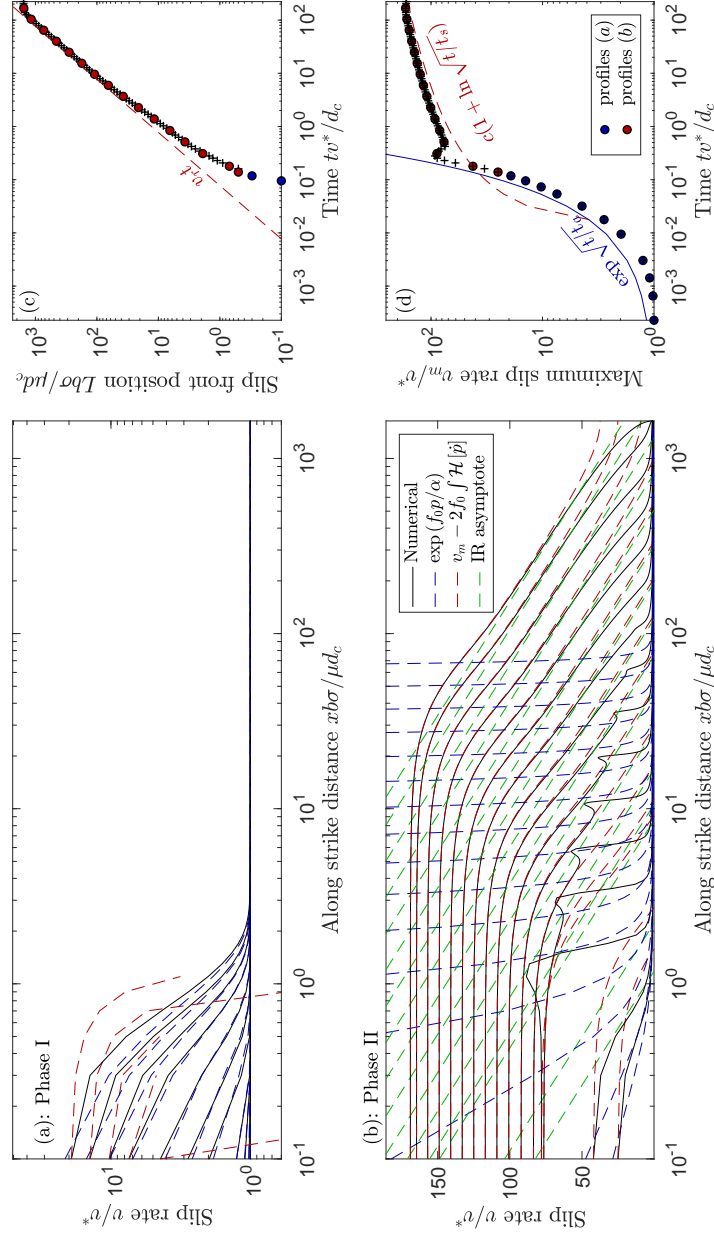


Figure A.10: Same as figure A.9 for a fault initially at steady state ( $\tau_0 - \tau_{ss}^0 = 0$ ) and characterized by a rate-and-state parameter  $\alpha = a/b = 1.4$ , evolving towards the steady crack solution. Note that the blue dashed profiles correspond here to the approximation (A.2).

620 As shown in figures A.8(f) and (i), if the fault is initially above steady state  $\Omega \geq 1$ , phase I is characterized by  $\Omega > 1$ . We thus assume  $\Omega \gg 1$ , so that equation (A.7) becomes:

$$\frac{\dot{\Omega}}{\Omega} = \frac{\dot{v}}{v} - v. \quad (\text{A.8})$$

At the injection point ( $x = 0$ ), this equation could be integrated for  $\Omega$  making use of the approximate expression (A.3), leading to:

$$\ln \frac{\Omega}{\Omega_0} = \sqrt{\frac{t}{t_a}} - 2t_a \left[ e^{\sqrt{t/t_a}} \left( \sqrt{\frac{t}{t_a}} - 1 \right) + 1 \right], \quad (\text{A.9})$$

625 where  $\Omega_0$  is the initial value of  $\Omega$ . This expression leads to first an increase followed by a decrease of  $\Omega$  as the slip rate blows up. The end of phase I occurs when  $\Omega = 1$ . In our simulations, this occurs at a time  $t_I \gg t_a$ , satisfying the last expression for  $\Omega = 1$ . For a  $|\ln \Omega_0| \ll \sqrt{t_I/t_a}$ , as this is approximately the case in our simulations, we end up with the following approximate expression  
630 for  $t_I$ :

$$t_I = t_a (\ln 2t_a)^2. \quad (\text{A.10})$$

If the fault is initially slightly below steady-state,  $\Omega$  increases during phase I, and quickly gets higher than 1 (figure A.8(c)). We will therefore use the same expression of  $t_I$  for all the initial stress conditions assumed here.

#### *Appendix A.2. Slip acceleration and crack expansion*

635 During the second phase (i.e. for  $t > t_I$ ), slip on the fault consists in an expanding and accelerating crack of half size  $L(t)$ . As shown in figures 2 and 3, two modes of crack expansion are observed: an accelerated crack propagation, where maximum slip rate and crack size blow up in a finite time, or a steady crack propagation, where expansion occurs at a constant rate, and slip rate  
640 increases smoothly. The accelerated crack propagation dominates at early times, but most of the simulations performed here ultimately lead to a steady crack expansion.

### Appendix A.2.1. Accelerated expansion

Here we study the accelerating crack dynamics. For that we focus on the fault scenario  $\tau_0 - \tau_{ss}^0 = 2$ ,  $\alpha = 1.1$ . The change of slip rate  $v$ , slip  $\delta$  and stress in the vicinity of the crack tip during the propagation are illustrated in figure A.11. Following the analysis by (Ampuero and Rubin, 2008; Rubin and Ampuero, 2009) for the nucleation pulses under velocity weakening conditions, the shape of the slip profiles (figure A.11) suggest a scaling of the form:

$$\delta(x, t) = \psi_0(t)f(X) \simeq 1.5\psi_0(t)\sqrt{X}, \quad (\text{A.11})$$

where  $\psi_0(t) = \ln v_m \theta_0$  is a scaling factor introduced by Rubin and Ampuero (2005); Ampuero and Rubin (2008); Rubin and Ampuero (2009), and  $X = x - L(t)$  is the distance to the crack tip.

The stress profiles show a progressive increase of stress from the initial stress (at large  $X$ ) to the peak stress at the crack tip ( $X = 0$ ), then a weakening over a process zone of size  $W$  until reaching a constant value of residual stress within the crack. The residual stress is the steady state frictional stress approximately at the maximum slip rate  $v_m$ . As shown by Rubin and Ampuero (2005) and in figure A.11(c), the process zone width under rate-and-state friction is of the order of  $W = 1$  when considering the ageing law. Note also that the stress outside the crack approximately decreases as the inverse of the square root distance from the tip (as expected when taking the Hilbert transform of the slip gradient, assuming the simple form (A.11)).

At the crack tip, the energy release rate  $G$  balances the fracture energy  $G_c$ . From (Rubin and Ampuero, 2005), the (non-dimensional) fracture energy for such a crack is given by:

$$G_c = \Delta\tau_{pr}^2, \quad (\text{A.12})$$

where  $\Delta\tau_{pr}$  is the difference between the peak  $\tau_p$  and the residual stress  $\tau_r$  at the tip. The (non-dimensional) energy release rate is approximately given by

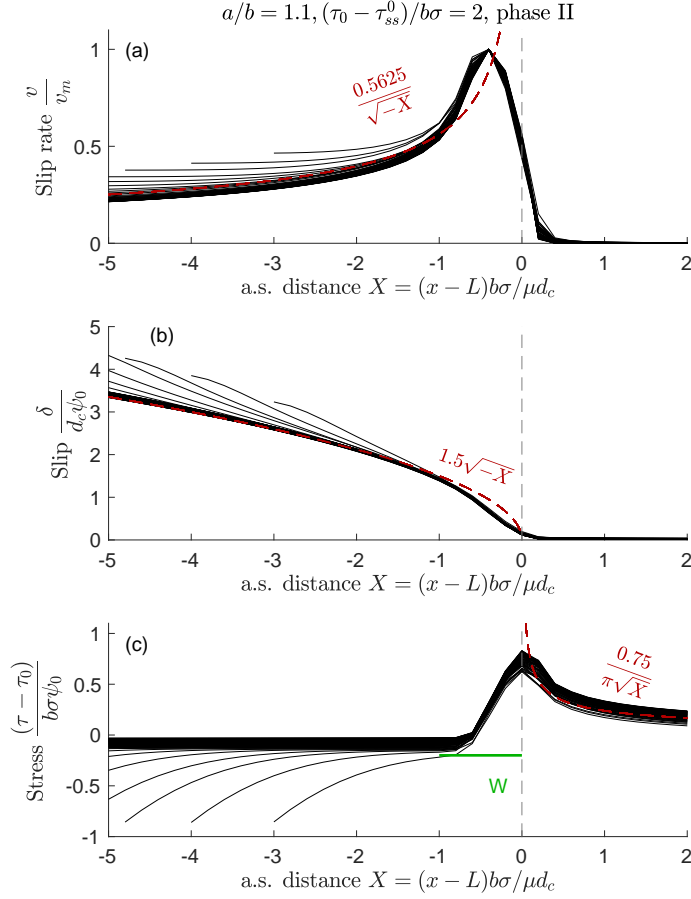


Figure A.11: Details of the slip rate (a), slip (b) and stress (c) changes in the vicinity of an accelerating crack tip, obtained on fault initially above steady state ( $\tau_0 - \tau_{ss}^0 > 0$ ) and characterized by a rate-and-state friction parameter  $\alpha = 1.1$ .  $X$  is the distance to the crack tip:  $x$  being the along strike distance and  $L$  the half length of the crack. Slip and stress profiles are normalized by  $\psi_0 = \ln(v_m\theta_0/d_c)$ ,  $v_m$  being the maximum (peak) slip rate. Black lines are the numerical solution, red dashed lines are the approximate profiles. See main text for details. The dashed vertical gray line indicates the crack tip (defined here as the position of the peak stress).

(Freund, 1998):

$$G = \pi \Delta \tau_{tip}^2 L, \quad (\text{A.13})$$

where  $\Delta \tau_{tip}$  is the difference between the initial stress  $\tau_0$  and the residual stress  $\tau_r$  left by the process zone. Following the developments of Rubin and Ampuero (2005) for this kind of crack front, we have:

$$\begin{cases} \tau_0 = f_0 + \ln \theta_0 \\ \tau_p \simeq (f_0 + \alpha \ln v_m + \ln \theta_0) [1 - p(L, t)] \\ \tau_r \simeq [f_0 + (\alpha - 1) \ln v_m] [1 - p(L, t)]. \end{cases} \quad (\text{A.14})$$

We end up with:

$$\begin{cases} \Delta \tau_{pr} = \tau_p - \tau_r \simeq \ln v_m \theta_0 [1 - p(L, t)] \\ \Delta \tau_{tip} = \tau_0 - \tau_r \simeq \ln \theta_0 - (\alpha - 1) \ln v_m + f_0 p(L, t), \end{cases} \quad (\text{A.15})$$

670 When the accelerating crack is well developed, the crack tip is far from the pressurized zone, so that  $p(L, t) \ll 1$  and can be neglected in the above expressions. The expression for  $\Delta \tau_{tip}$  has therefore been simplified assuming that  $p$  and the slip dependent component of the friction coefficient are small deviations from the reference normal stress 1 and the reference friction coefficient  
675  $f_0$  respectively. We have therefore neglected terms of second order. The last term in this expression quickly becomes also negligible as the crack expansion proceeds (see figure 6).

Neglecting the pressure at the crack tip, the energy balance ( $G = G_c$ ) leads, after making use of expressions (A.15), to the following expression for the crack  
680 half length  $L$ :

$$L = \frac{1}{\pi} \left[ \frac{\ln v_m \theta_0}{\ln \theta_0 - (\alpha - 1) \ln v_m} \right]^2. \quad (\text{A.16})$$

Another feature of such a propagating crack shown by (Ampuero and Rubin, 2008) is that the rupture speed (or expansion speed  $\dot{L}$ ) is controlled by the maximum slip rate  $v_m$ . Following (Ampuero and Rubin, 2008), we should ap-

proximately have:

$$\dot{L} = 0.75 \frac{v_m}{\ln v_m \theta_0}. \quad (\text{A.17})$$

685 The two equations (A.16) and (A.17) govern the coupled evolution of  $v_m$  and  $L$  during the crack expansion.

Note that the form (A.11) leads to a slip rate  $v$  that follows:

$$v = \dot{\psi}_0 f(X) - \psi_0 \dot{L} f'(X). \quad (\text{A.18})$$

If we neglect the first term, we get, after making use of expression (A.17):

$$v \simeq -0.75 v_m f'(X) \simeq 0.5625 \frac{v_m}{\sqrt{X}}, \quad (\text{A.19})$$

which provides a good approximation to the simulated slip-rate profiles within  
690 the crack (see figure A.11(a)).

#### *Appendix A.2.2. Steady expansion*

Here we concentrate on the steady expansion phase, typically illustrated in figures 2(a,c), A.9(b) and A.10(b). Within the crack ( $x < L(t)$ ), frictional changes become negligible compared to the elastic stressing rate and the stress-  
695 ing related to the pore pressure changes, as shown in figure A.8(b,c,e,f). Note that during this phase the inner part of the crack has evolved to the frictional steady state. The slip evolution is therefore controlled to the first order by a balance between the two terms on the right-hand side of the first equation of (9). Since during this phase as well  $f \simeq f_0$  (figure A.8(c,f)), the slip rate evolution  
700 within the crack is approximately given by :

$$\frac{1}{2} \mathcal{H}[v'] = f_0 \dot{p}. \quad (\text{A.20})$$

This last equation could be inverted, and making use of the inverse property of the Hilbert transform ( $\mathcal{H}[\mathcal{H}(f)] = -f$  for any functional  $f$  (King, 2009)), we



get:

$$v' = -2f_0\mathcal{H}[\dot{p}] = -2f_0q\sqrt{\frac{D}{\pi t}}\mathcal{H}\left[e^{-\eta^2}\right] = -\frac{4f_0q}{\pi}\sqrt{\frac{D}{t}}D_w(\eta), \quad (\text{A.21})$$

where  $D_w$  is the Dawson's function (Temme, 2010) defined as:

$$D_w(\eta) = e^{-\eta^2} \int_0^\eta e^{y^2} dy. \quad (\text{A.22})$$

705 In writing equation (A.21) we have used the time derivative of the pore pressure field (6), the Hilbert transform of a Gaussian function and the scaling property of the Hilbert transform (King, 2009). Equation (A.21) could be further integrated with respect to  $x$  leading to:

$$v(x, t) = v_m(t) - \frac{8f_0qD}{\pi} \int_0^\eta D_w(\xi) d\xi, \quad (\text{A.23})$$

where  $v_m(t) = v(0, t)$ . Equation (A.23) could be further simplified, considering  
710 the asymptotic expansion of  $D_w$  at small and large  $\eta$ . We have :

$$\begin{cases} D_w(\eta) = \eta + o(\eta^3), & \text{as } |\eta| \rightarrow 0 \\ D_w(\eta) = \frac{1}{2\eta} + o(\frac{1}{\eta^3}), & \text{as } |\eta| \rightarrow +\infty. \end{cases} \quad (\text{A.24})$$

In the following we will therefore make the assumption that:

$$\int_0^\eta D_w(\xi) d\xi \simeq \begin{cases} \int_0^\eta \xi d\xi = \frac{\eta^2}{2}, & \text{if } \eta < 1 \\ \int_0^1 \xi d\xi + \int_1^\eta \frac{d\xi}{2\xi} = \frac{1}{2}(1 + \ln \eta) & \text{if } \eta > 1 \end{cases} \quad (\text{A.25})$$

The slip rate inside the crack (A.23) could therefore be approximated as:

$$v(x, t) \simeq v_m(t) - \begin{cases} \frac{4f_0qD}{\pi}\eta^2, & \text{if } \eta < 1 \\ \frac{4f_0qD}{\pi}(1 + \ln \eta) & \text{if } \eta > 1 \end{cases} \quad (\text{A.26})$$

As shown in figures A.9(b) and A.10(b), the expression (A.23) provides a good approximation to the slip rate within the crack. These figures also show that  
715 the second expression of (A.26), labeled IR in the figures, could be used as an

approximation for the slip rate profiles at intermediate distances between the crack center and the crack tips. Note that the intermediate asymptote (IR, second expression in (A.26)) goes to minus infinity as  $x$  goes to infinity, at a given fixed time. This expression is therefore no longer valid at least when it  
720 vanishes. The zero of this expression could be used as an approximation of the crack tip position  $L(t)$  as illustrated in figures A.9(b) and A.10(b). We therefore have the following relationship between  $v_m(t)$  and  $L(t)$ :

$$v_m(t) \simeq \frac{4f_0qD}{\pi} \left( 1 + \ln \frac{L(t)}{2\sqrt{Dt}} \right). \quad (\text{A.27})$$

At this point we have one equation relating two unknowns: the crack tip (or slip front) position  $L(t)$ , and the maximum slip rate  $v_m(t)$ . A second expression  
725 could be obtained by considering the quasi-static shear stress balance on the entire fault. The global shear stress balance reads:

$$\int_{-\infty}^{+\infty} [\tau(x, t) - \tau_{el}(x, t)] dx = 0, \quad (\text{A.28})$$

which leads to (from equations (1) and (5)):

$$\int_{-\infty}^{+\infty} \left[ (f_0 + \alpha \ln v + \ln \theta)(1 - p) - \tau_0 + \frac{1}{2} \mathcal{H}[\delta'] \right] dx = 0, \quad (\text{A.29})$$

Since we consider here an initial slip rate uniform on the fault  $v = 1$  along with a uniform  $\theta = \theta_0$ , the local stress balance leads to  $\tau_0 = f_0 + \ln \theta_0$ . Furthermore,  
730 the integral of the stress transfers related to the slip distribution  $\delta$  on the real line vanishes. Finally, assuming that the pore pressure remains small compared to the lithostatic pressure  $p \ll 1$  (see figure A.8(c,f,i)), this latter expression could be simplified to:

$$\alpha \int_{-\infty}^{+\infty} \ln v dx + \int_{-\infty}^{+\infty} \ln \theta dx = 2L_0 \ln \theta_0 + f_0 \int_{-\infty}^{+\infty} p dx, \quad (\text{A.30})$$

where  $L_0$  is the half length of the fault initially stressed. Concerning the integral  
735 of  $p$  on the real line, let's consider the diffusion equation governing the evolution

of pore pressure:

$$\frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2} + 2qD\delta_D(x), \quad (\text{A.31})$$

where  $\delta_D$  is the Dirac delta function. Integrating the diffusion equation on the real line, leads to:

$$\frac{\partial}{\partial t} \int_{-\infty}^{+\infty} p dx = 2qD, \quad (\text{A.32})$$

since  $p$  and its space derivatives are supposed to vanish at infinity. We finally end-up with:

$$\int_{-\infty}^{+\infty} p dx = 2qDt. \quad (\text{A.33})$$

The right hand side of the stress balance could be further simplified, assuming that the frictional stress is at steady-state at  $v_m$  on the crack, close to the initial stress outside. Accounting for the shape of the slip-rate profiles represented in figures A.9(b) and A.10(b) we have, to the first order:

$$\begin{cases} \alpha \int_{-\infty}^{+\infty} \ln v dx + \int_{-\infty}^{+\infty} \ln \theta dx \\ \simeq (\alpha - 1) \int_{-L(t)}^{+L(t)} \ln v_m dx + \left( \int_{-L_0}^{-L} + \int_L^{L_0} \right) \ln \theta_0 dx \\ \simeq 2L(t) \ln v_m(t) + 2(L_0 - L(t)) \ln \theta_0. \end{cases} \quad (\text{A.34})$$

The stress balance therefore leads to a second equation relating  $L(t)$  and  $v_m(t)$ :

$$(\alpha - 1)L(t) \left[ \ln v_m(t) - \frac{\ln \theta_0}{\alpha - 1} \right] = f_0 q D t. \quad (\text{A.35})$$

Note that this stress balance and the following developments become valid when the left hand side is positive, that is when the maximum slip rate exceeds the limit slip rate  $v_l = \theta_0^{1/(\alpha-1)}$ .  $v_l$  has been defined in the previous sections (equation (17)) as the maximum possible slip rate for accelerated crack propagation.

During the expansion phase,  $v_m$  seems to increase as the logarithm of time, while  $L(t)$  increases much faster, approximately linearly in time, as shown in figures 2(e,f), 3, A.9(c-d) and A.10(c-d). The variation of  $\ln v_m$  in equation (A.35) could therefore be neglected, and  $\ln v_m$  could be replaced by a constant

$\lambda$  providing the order of magnitude of the maximum log slip rate. This way, the  
755 linear increase of  $L(t)$  with time could be retrieved from equation (A.35):

$$L(t) = v_r t = \frac{f_0 q D}{\lambda(\alpha - 1) - \ln \theta_0} t, \quad (\text{A.36})$$

where  $v_r$  is an (approximately) constant slip front (rupture) speed. Similarly, the  
logarithmic increase of  $v_m$  with time could be obtained by re-injecting equation  
(A.36) into expression (A.27), so that we get:

$$v_m(t) = c \left( 1 + \ln \sqrt{\frac{t}{t_s}} \right), \quad c = \frac{4f_0 q D}{\pi}, \quad t_s = 4 \frac{\lambda^2 (\alpha - 1)^2 - \ln \theta_0^2}{f_0^2 q^2 D}. \quad (\text{A.37})$$

As illustrated in figures A.9(c,d) and A.10(c,d), equations (A.36) and (A.37)  
760 provide good approximations to the slip front position and the maximum slip  
rate obtained numerically.

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