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# Period-doubling bifurcation tracking for the Harmonic Balance Method: application to a Duffing oscillator with asymmetric clearances

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**Abstract.** This work presents a contribution to numerical bifurcation analysis of nonlinear vibrations via techniques based on the Harmonic Balance Method (HBM) and pseudo-arclength continuation. In particular, an original procedure for the detection, computation and tracking of period-doubling bifurcations -as well as branch switching- within said formalism is proposed. This approach is then used to explore the effects of asymmetry in a simple piecewise-continuous test case, where it successfully predicted phenomena such as 2T-periodic isola formation and subharmonic-response branches.

## Introduction

Strong nonlinearities, such as impacts or friction, give rise to complexity and a multiple possible dynamic regimes, which correspond to specific zones in parameter space delimited by *stability boundaries*. A small variation in a given parameter may thus entail significantly different responses from an otherwise unchanged system. For instance, heat exchanger supports with clearances that are initially symmetric may develop offsets due to wear, leading to irregular tube motion under excitation by crossflow.

An efficient way of directly determining stability boundaries is to employ continuation algorithms. In [1], Xie et al. characterized and tracked bifurcations of limit cycles from frequency-response curves by using pseudo-arclength continuation on extended systems in the frequency domain. Specifically, they treated Limit Point (LP), Branch Point (BP) and Neimark-Sacker (NS) bifurcations, but not Period Doubling (PD), which is nonetheless a generic scenario that is common, among others, in systems with clearances or in 2:1 internal resonance. In this work, we propose an original extended system for calculating PD bifurcations in the framework of the multi-harmonic balance method (HBM). In addition, we present a test case to show how a profound understanding of global dynamics can be gained by bifurcation tracking, i.e. a series of local problems.

## PD bifurcation in HBM formulation

Hill's method provides the following quadratic eigenvalue problem for stability evaluation [2], whose solutions include the Floquet Exponents (FE) of the system:

$$(\mathbf{R}_Q + \Lambda \mathbf{D}_1 + \Lambda^2 \mathbf{D}_2) \phi = \mathbf{0} \quad (1)$$

where  $\mathbf{R}_Q$  is the Jacobian matrix of the system with respect to Fourier coefficients  $\mathbf{Q}$ ,  $\mathbf{D}_1$  and  $\mathbf{D}_2$  are constant terms depending on its linear part. Period doubling corresponds to the existence of two purely imaginary FEs,  $\Lambda_{1,2} = \pm i\frac{\omega}{2}$ , while all the remaining ones have negative real parts. Inserting this condition into (1), separating real and imaginary parts and appending a normalization equation for the eigenvector  $\phi = \phi_R + i\phi_I$ , we obtain the following original augmented system defining PD bifurcations:

$$\mathbf{Y}_{PD}(\mathbf{Q}, \phi, \omega) = \begin{pmatrix} \mathbf{R}(\mathbf{Q}, \omega) \\ \left( \mathbf{R}_Q - \left(\frac{\omega}{2}\right)^2 \mathbf{D}_2 \right) \phi_R + \frac{\omega}{2} \mathbf{D}_1 \phi_I \\ \left( \mathbf{R}_Q - \left(\frac{\omega}{2}\right)^2 \mathbf{D}_2 \right) \phi_I - \frac{\omega}{2} \mathbf{D}_1 \phi_R \\ \mathbf{p}^T \cdot \phi_I - b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

Here,  $\mathbf{p}$  is a vector with non-zero projection over  $\text{Span}\{\phi_R, \phi_I\}$  and  $b \in \mathbb{R}^*$ ,  $b < 1$ . While calculating frequency-responses, a PD bifurcation is detected between two steps by monitoring the sign of a specific test function. (2) is then solved by Newton-like iterations, providing an initial point for tracking with respect to a parameter.

## 2T-periodic resonances: branching, subharmonic cascades, isolas

Period doubling corresponds to the phase-locking of an invariant torus and thus, in a sense, to a special case of NS bifurcation with commensurate frequencies. Another interesting and more computationally useful way to look at this phenomenon, which we adopt herein, is as a symmetry breaking of the second-iterate application of the system. Thus, by performing a change of variables  $\omega \rightarrow \frac{\omega}{2}$  and then solving (1), PD points appear as BP's, and 2T-periodic solution branches emanating from such points can then be calculated in the usual way, see [3].

Further PD or NS points can occur along these branches, hence leading to sub-harmonic cascades. In addition, PD bifurcation curves containing local extrema may indicate the formation of a 2T-periodic isola. By either tracking LP's on an isola or the extrema of PD curves, we can define existence boundaries for this attractor in parameter space. We note that the presence of such isolas is equivalently evidenced through Nonlinear Mode calculation, where they appear as Branch Points of the second-iterate backbone curve.

### Test case: Duffing oscillator with asymmetric clearances

The system under study is the same as that in [4], an experimental apparatus available at CEA Saclay. It is modeled by a forced, damped, Duffing oscillator plus a (regularized) piecewise-linear contact force. The non-dimensional symmetry factor  $\bar{j}$  is the ratio of clearances, and is thus equal to 1 for perfect symmetry, 0 for "full asymmetry" (one stop at the origin). Low values of  $\bar{j}$  lead to period doubling and a corresponding resonance of the main sub-harmonic ( $\frac{\omega}{2}$ ) along the frequency-response curve; tracking shows that this happens independently of forcing amplitude and contact stiffness. Besides, as seen in Figure 1 (left), there exists a local maximum of  $\bar{j}$ ,  $\bar{j}_{det}$ , which marks the detachment of an isola comprising a stable and an unstable branch delimited by two LP's, see Figure 1 (right). In turn, the isola LP curve shows an extremum,  $\bar{j}_{max}$ , across which the system is "asymmetric enough" for the ( $\frac{\omega}{2}$ ) subharmonic to respond, albeit in the form of an isola.

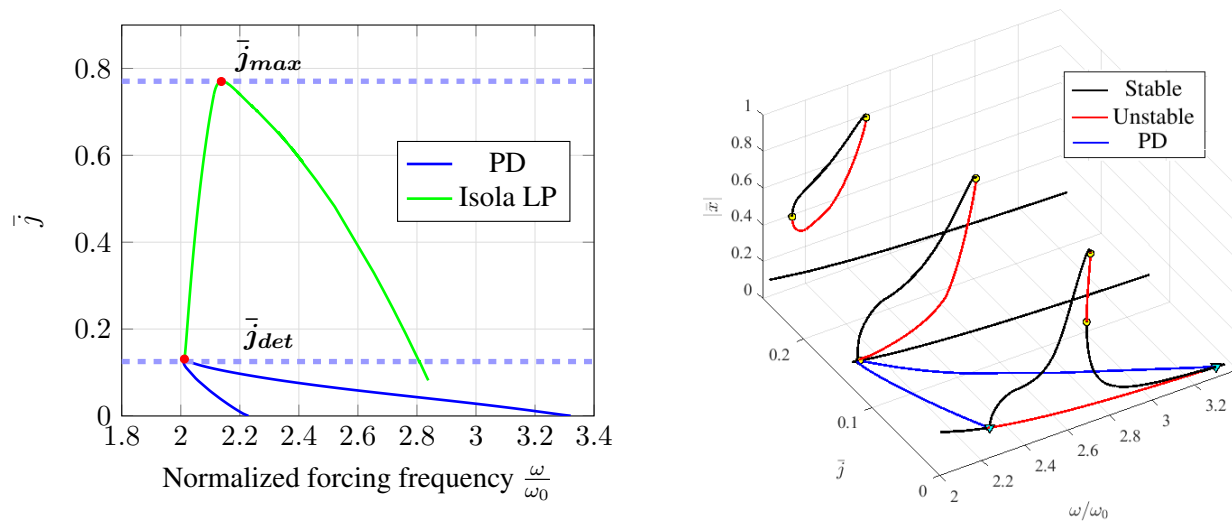


Figure 1: Analysis of main subharmonic resonance in an asymmetric Duffing oscillator. Left: Bifurcation tracking. Right: 2T-periodic isola formation.

### Conclusions

Isolated resonances, not previously reported in [4], have been predicted by the proposed method. Furthermore, the crucial role of symmetry regarding this phenomenon could be not only highlighted but also quantified and, even more, verified experimentally afterwards. Future work includes the integration of fluid-elastic coupling forces into HBM formalism, in order to study vibrating heat exchanger tubes under crossflow. In addition, the ideas presented here are to be generalized to detect and track phase-locking bifurcations of any order, an useful feature for nonlinear n-DOF systems where modal interactions are commonplace.

### References

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