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1 Numerical simulation of a turbulent channel flow with
2 an acoustic liner

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6 **Abstract**

Numerical simulations of a compressible turbulent channel flow with an acoustic impedance boundary condition are performed to assess how the flow is modified compared with a channel flow with rigid walls. When the liner resonance frequency is not too large and the resistance sufficiently small, turbulent statistics deviate from those obtained with rigid walls and surface waves are found traveling along the liner surface. For small resonance frequencies these waves are two-dimensional, they have a large wavelength compared to the turbulent structures and modulate these structures. As a result, they transport momentum toward the impedance wall, causing a drag increase. When the resonance frequency increases, the waves along the liner surface progressively lose their spanwise coherence while their streamwise wavelength decreases to get close to the flow typical length scales, which may also result in a drag increase when the resistance is sufficiently small. In the cases in which the surface waves are two-dimensional, a connection is established between them and the unstable modes computed by using a linear stability analysis. Given the streamwise periodicity of the channel, a temporal stability analysis is performed rather than a spatial analysis, the latter being more frequently encountered in acoustic mode computations. This temporal analysis shows that the unstable mode in the vicinity of an acoustic liner arises from the A-branch of wall modes.

7 *Keywords:* duct acoustics, acoustic liner, numerical simulation, instability,
8 turbulence, channel flow

9 *PACS:* 43.28.Py, 43.20.Mv, 47.27.nd

10 **1. Introduction**

11 Perforate acoustic liners are an important technology to absorb sound in
12 ducts such as turbofan engines or to suppress combustion instabilities. In
13 many practical situations, the liners are subject to high velocity flows and
14 turbulence, and much research has been devoted to the effect of a grazing flow
15 on the liner impedance. It is for example well known that the resistance tends
16 to increase linearly with the grazing flow speed whereas the reactance tends to
17 decrease [1, 2, 3] as a result of the interaction between the acoustic and vortical

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18 modes in the holes of the perforated face sheet. Conversely, the liner may
19 modify the flow and turbulence in its vicinity, compared with a rigid wall. An
20 effect of this is a drag increase [4], especially for large liner porosity. Another
21 effect is the flow instability observed in the vicinity of a low resistance liner [5].
22

23 Several numerical simulations in flow ducts with liners have been performed
24 in connection with this topic. The objective of many simulations has been
25 to study sound propagation in lined ducts with a known base flow using
26 the linearized Euler or Navier-Stokes equations. A difficulty is then to
27 impose a well-posed impedance boundary condition, especially in time-domain
28 solvers [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. These simulations neglect
29 both the effect of the grazing flow on the impedance and the effect of the
30 impedance on the flow. Other simulations are based on the full nonlinear
31 Navier-Stokes equations and the flow is computed together with the acoustic
32 field [17, 18, 19, 20, 21, 22]. Among these simulations, some include the liner
33 back cavity and the face sheet perforations [17, 18, 19, 20] so as to include
34 all possible flow-acoustics interactions. Others use an impedance boundary
35 condition with a given impedance [21, 22, 23], which means that the effect
36 of the flow on the impedance is no part of the computation. Jimenez et
37 al (2001) compute a turbulent incompressible channel flow above a purely
38 resistive porous surface [23]. They observed modified turbulent statistics over
39 the surface compared with a rigid wall, and part of this change was associated
40 with spanwise coherent structures moving along the wall. This observation was
41 partly confirmed by a linear stability analysis, unstable modes being present for
42 low values of the resistance. The porous surface being a purely resistive one,
43 resonance mechanisms typically encountered in acoustic liners were not present
44 in this investigation. Olivetti et al. (2015) [21] compute the sound propagation
45 in a lined pipe, a simple model for a nozzle, in order to suppress resonant modes
46 in the duct which have a strong impact on the noise produced by the jet out of
47 the nozzle. Scalo et al. (2015) [22] study the turbulent flow in a compressible
48 periodic channel flow with an impedance boundary condition and describe how
49 the structure of turbulence is modified as the liner resistance decreases. They
50 set the resonance frequency of the liner so that it corresponds to some typical
51 time scale of the flow. As a result the liner resonance frequency is rather high,
52 and larger than typical frequencies encountered in aeroacoustic applications.
53 Compared with Scalo et al. (2015) smaller resonance frequencies will be
54 considered in the present work. More specifically the resonance frequency and
55 other liner parameters are close to those used in the experiments in Marx et
56 al (2010) in which an instability was observed, and for which related linear
57 stability analyses were performed in Marx and Aurégan (2010,2013) [24, 25].
58 Apart from sound damping in ducts, there is a growing interest in passive
59 methods for aeroacoustics and flow control [26], and a better knowledge of the
60 behavior of the flow in the vicinity of non-rigid wall is useful in general.
61

62 This paper makes a new contribution to the investigation of turbulent chan-
63 nel flows with acoustic liners (limited to locally reacting, perforate-like liners).

64 Numerical Implicit Large Eddy Simulations (ILES) of compressible turbulent
65 channel flows are performed to study what changes in the flow may result from
66 using an impedance boundary condition, in comparison with a rigid wall bound-
67 ary condition. For some liner parameters, surface waves are computed and their
68 effect on drag is evidenced. A temporal stability analysis complements the simu-
69 lations and is a useful counterpart to the more classical spatial stability analysis
70 for these waves. The numerical model is presented in Section 2, including the
71 impedance boundary condition, taken to be of the mass-spring-damper type.
72 The different configurations and the corresponding flow statistics are presented
73 in section 3. In particular, the effect of the impedance resonance frequency and
74 resistance on the structures of turbulence is investigated. The surface waves
75 are described in Section 4. In Section 5 it is shown how they can increase the
76 drag on the liner surface. The temporal linear stability analysis is conducted in
77 Section 6. Finally conclusions are given in Section 7.

78 2. Numerical model

79 In this section, we introduce the equations of our problem, then present
80 quickly the numerical methods to solve these equations, and finally introduce
81 the model for the impedance boundary condition as well as its numerical imple-
82 mentation.

83 2.1. The equations

84 Simulations of channel flows that are periodic in the stream-wise and span-
85 wise directions are performed. The bottom and upper wall are either rigid or
86 modeled as an impedance, with a fixed wall temperature (see Section 2.3 for
87 the boundary conditions). The compressible Navier-Stokes equations are imple-
88 mented in the particular characteristics-like form presented by Sesterhenn [27]
89 which has also been used in [28] for the direct numerical simulation of com-
90 pressible channel flows. The Cartesian coordinates are denoted by x, y, z (or
91 by x_1, x_2, x_3) for the stream-wise, wall-normal and span-wise directions. The
92 non-dimensional equations are given by:

$$\frac{\partial u}{\partial t} = - \left(\frac{1}{2}(X^+ - X^-) + Y^u + Z^u \right) + \frac{1}{\text{Re}} \frac{1}{\rho} \frac{\partial \tau_{1j}}{\partial x_j} + F \quad (1)$$

$$\frac{\partial v}{\partial t} = - \left(X^v + \frac{1}{2}(Y^+ - Y^-) + Z^v \right) + \frac{1}{\text{Re}} \frac{1}{\rho} \frac{\partial \tau_{2j}}{\partial x_j} \quad (2)$$

$$\frac{\partial w}{\partial t} = - \left(X^w + Y^w + \frac{1}{2}(Z^+ - Z^-) \right) + \frac{1}{\text{Re}} \frac{1}{\rho} \frac{\partial \tau_{3j}}{\partial x_j} \quad (3)$$

$$\frac{\partial s}{\partial t} = -(X^s + Y^s + Z^s) + \frac{(\gamma - 1)}{\gamma} \frac{1}{\text{Re}} \frac{1}{p} \left(\Phi + \frac{1}{\text{Pr}} \nabla \cdot (K_t \nabla T) \right) \quad (4)$$

$$\begin{aligned} \frac{\partial p}{\partial t} = & -\frac{\rho c}{2} [(X^+ + X^-) + (Y^+ + Y^-) + (Z^+ + Z^-)] \\ & + \frac{1}{\text{Re}} (\gamma - 1) \left[\Phi + \frac{1}{\text{Pr}} \nabla \cdot (K_t \nabla T) \right] \end{aligned} \quad (5)$$

93 where the velocity components along the x , y , and z -directions are denoted by
 94 u , v , w (or u_1 , u_2 , u_3), p is the pressure, ρ the density, s the entropy. c is the
 95 sound speed given by $c^2 = \gamma p / \rho$, where $\gamma=1.4$ is the ratio of specific heats. The
 96 quantities X^\pm , Y^\pm , Z^\pm are the rates of change of the amplitude of the acoustic
 97 waves ; X^s , Y^s , Z^s are related to the entropy wave, and Y^u , Z^u , X^v , Z^v , X^w ,
 98 Y^w are related to the vorticity wave. They are given by the following relations:

$$X^\pm = (u \pm c) \left(\frac{1}{\rho c} \frac{\partial p}{\partial x} \pm \frac{\partial u}{\partial x} \right) \quad (6)$$

$$Y^\pm = (v \pm c) \left(\frac{1}{\rho c} \frac{\partial p}{\partial y} \pm \frac{\partial v}{\partial y} \right) \quad (7)$$

$$Z^\pm = (w \pm c) \left(\frac{1}{\rho c} \frac{\partial p}{\partial z} \pm \frac{\partial w}{\partial z} \right) \quad (8)$$

$$Y^u = v \frac{\partial u}{\partial y} \quad Z^u = w \frac{\partial u}{\partial z} \quad X^v = u \frac{\partial v}{\partial x} \quad (9)$$

$$Z^v = w \frac{\partial v}{\partial z} \quad X^w = u \frac{\partial w}{\partial x} \quad Y^w = v \frac{\partial w}{\partial y} \quad (10)$$

$$X^s = u \frac{\partial s}{\partial x} \quad Y^s = v \frac{\partial s}{\partial y} \quad Z^s = w \frac{\partial s}{\partial z} \quad (11)$$

The viscous stress tensor is:

$$\tau_{ij} = \mu (\partial u_i / \partial x_j + \partial u_j / \partial x_i) + \left(\kappa - \frac{2}{3} \mu \right) (\partial u_k / \partial x_k) \delta_{ij}$$

104 and $\Phi = \tau_{ij} (\partial u_i / \partial x_j)$ is the viscous dissipation. In addition the state equation
 105 is $p = ((\gamma - 1) / \gamma) \rho T$ and the thermodynamic relation reads $\rho = p^{1/\gamma} e^{-s}$. The
 106 dependence of μ on T is given by the power law: $\mu = T^{0.7}$.

107
 108 The equations above are normalized using the following reference scales
 109 for a channel flow (in the following the tilde $\tilde{\cdot}$ indicates a dimensional quantity):
 110 the speed of sound \tilde{c}_w at wall temperature \tilde{T}_w (imposed since an
 111 isothermal wall boundary conditions is used), the viscosity $\tilde{\mu}_w$ and thermal
 112 conductivity $\tilde{K}_{t,w}$ at the wall temperature, the half-height of the channel
 113 \tilde{H} , the bulk density $\tilde{\rho}_b$ to be defined below, and the adiabatic specific heat
 114 $\tilde{c}_p = \gamma \tilde{r} / (\gamma - 1)$ with \tilde{r} the gas constant. The thermal conductivity is related to
 115 the viscosity through the Prandtl number, $\text{Pr} = \tilde{\mu}_w \tilde{c}_p / \tilde{K}_{t,w}$, and $\text{Pr}=0.7$. The
 116 Reynolds number resulting from the previous normalization is $\text{Re} = \tilde{H} \tilde{\rho}_b \tilde{c}_w / \tilde{\mu}_w$.

The bulk density used for normalization is defined by $\tilde{\rho}_b = \langle \tilde{\rho} \rangle_{xyz}$, where $\langle \cdot \rangle_{xyz}$ denotes the mean over all spatial directions. The bulk velocity is defined by $\tilde{u}_b = \langle \tilde{\rho} \tilde{u} \rangle_{xyz} / \tilde{\rho}_b$. As no mass can escape the channel $\tilde{\rho}_b$ is constant, and in normalized form $\rho_b=1$. The normalized velocity u_b is also forced to keep a constant value, which is ensured by changing dynamically the uniform force F appearing in Eq. (1), as done by [29]. Note that due to normalization, $u_b = \mathcal{M}$ where $\mathcal{M} = \tilde{u}_b / \tilde{c}_w$ is the Mach number based on the bulk velocity and the sound speed at the wall. A non-dimensional number commonly used to specify the operating point of a turbulent channel flow is the bulk Reynolds number defined by:

$$\text{Re}_b = \frac{\tilde{\rho}_b \tilde{H} \tilde{u}_b}{\tilde{\mu}_w} \quad (12)$$

118 In the following we will prescribe Re_b and \mathcal{M} , from which the Reynolds number
 119 appearing in the equations can be derived using $\text{Re} = \text{Re}_b / \mathcal{M}$.

Another Reynolds number, to which Re_b may be related by an empirical relationship, is the friction Reynolds number defined by:

$$\text{Re}_\tau = \tilde{\rho}_w \tilde{H} \tilde{u}_\tau / \tilde{\mu}_w \quad (13)$$

120 where $\tilde{\rho}_w$ is the mean density at the wall, and the friction velocity defined as
 121 $\tilde{u}_\tau = \sqrt{\tilde{\tau}_w / \tilde{\rho}_w}$ is computed from the mean wall shear stress $\tilde{\tau}_w = \tilde{\mu}_w (\partial \tilde{U} / \partial \tilde{y})_w$,
 122 where \tilde{U} represents the time-averaged streamwise velocity. Classically the vis-
 123 cous length scale is defined by $\tilde{l}_\tau = \tilde{\mu}_w / (\tilde{\rho}_w \tilde{u}_\tau)$. Quantities scaled with \tilde{l}_τ and
 124 \tilde{u}_τ are indicated with a superscript + in the following.

125 2.2. Numerical schemes

126 With the characteristic formulation (1-5) it would be possible to use upwind
 127 schemes [30] in the characteristic directions to introduce some dissipation and
 128 stabilize the computation as in [27] and this is indeed what used to be done
 129 in this code. In the present work, centered schemes have been preferred as the
 130 dissipation is tuned more easily by managing it with the second order derivative.
 131 The equations are discretized in a collocated manner and the first derivatives
 132 are computed with a 6th order compact scheme (scheme (2.1.7) in [31]) for
 133 the central points. For the grid point next to the boundary a centered 4th
 134 order compact scheme is used (scheme (2.1.6) in [31] with $\alpha=1/4$), while at
 135 the boundary a 3rd order compact upwind scheme is used (scheme (4.1.3)
 136 in [31] with $\alpha=2$). Since the centered scheme is non-dissipative, the extra
 137 dissipation needed to stabilize the computations or serve for ILES is introduced
 138 through the diffusive terms with the second-order derivative [32, 33, 34] (rather
 139 than with the convective terms using the first-order derivative if upwind
 140 schemes were used). Specifically the second derivative is a 6th order compact
 141 scheme with a 3-9 stencil (Eq. (8) in [34]) for which some coefficients can be
 142 freely adjusted to impose the level of dissipation. More details can be found
 143 in Sebastian et al. [35]. Finally, the time-advancement relies on a classical
 144 fourth-order four-step Runge-Kutta method.

145

146 *2.3. Impedance boundary condition*

147 *2.3.1. Rigid wall boundary condition*

For a rigid isothermal wall with $u=v=w=0$ and $T=T_w$ the boundary conditions to apply have been given by Lechner et al. [28]. At the bottom wall for example, Y^- is a known characteristic flux coming from the interior of the domain, and the reflected characteristic flux Y^+ should be calculated. The situation is reversed at the top wall. Lechner et al. give [28] (adapted to the present notation/normalization):

$$Y^+ = Y^- + 2 \frac{1}{\text{Re}} \frac{1}{\rho} \frac{\partial \tau_{2j}}{\partial x_j} \quad (\text{bottom wall}) \quad Y^- = Y^+ - 2 \frac{1}{\text{Re}} \frac{1}{\rho} \frac{\partial \tau_{2j}}{\partial x_j} \quad (\text{top wall}) \quad (14)$$

The isothermal boundary condition is obtained by replacing the pressure and entropy equations at the wall with:

$$\frac{\partial p}{\partial t} = -\frac{p}{2c}(Y^+ + Y^-) \quad (15)$$

$$\frac{\partial s}{\partial t} = \frac{\gamma - 1}{\gamma} \frac{1}{2c}(Y^+ + Y^-) \quad (16)$$

148 This now needs to be extended to non-rigid walls.

149

150 *2.3.2. Mass Spring Damper boundary condition*

151 Generally speaking, an impedance boundary condition is a relation between
 152 p and v at a given position at the wall. The impedance (or its inverse, the admit-
 153 tance) is a quantity defined in the frequency domain. Its use in a time-domain
 154 solver requires that the impedance be transformed into an impulse response in
 155 the time-domain, which is done by an inverse Laplace transform. This is not
 156 straightforward, since impedance laws that would appear to be satisfying in the
 157 frequency domain may actually not satisfy certain desirable physical properties,
 158 which can show up in the time domain in the form of instabilities [36]. These
 159 properties are: passivity, causality, and reality, as explained by Rienstra (2006)
 160 [10]. In particular, when an impedance has been measured at some given real
 161 frequencies, a special care should be exercised when performing the complex
 162 continuation of the data so that all the above properties are verified.

163 Several methods have been proposed to comply with these requirements. Tam
 164 and Auriault (1996) [6] have introduced a mass-spring-damper (MSD) equation
 165 as a simple liner model that accounts for one resonance frequency. It is directly
 166 formulated as a second order equation in v in the time domain, with the p
 167 derivative acting as a source (see Eq. (17) below). This equation is solved with
 168 the same integration method as the other governing equations, that is, with
 169 the same time accuracy. The Laplace transform of this equation provides the
 170 admittance of the system, which is also of second order. This admittance has
 171 been shown to be causal, passive, and real by Rienstra [10] as long as physical

172 (positive) values of the parameters are used. Fung and Ju (2000) have advo-
 173 cated the use of the reflection coefficient in place of the impedance/admittance
 174 as well as the use of a convolution integral instead of a differential equation
 175 [37]. Hence, they also consider the MSD surface, but compute the impulse re-
 176 sponse corresponding to the reflection coefficient of this type of surface. The
 177 cost of computing the integral is reduced by the use of recursive convolution [36].
 178 Recursive convolution has also been used in conjunction with impedance or ad-
 179 mittance by Reymen et al [38]. Finally, broadband models of the impedance
 180 often rely on partial fraction expansion of either admittance/impedance [40, 39]
 181 or reflection coefficient[36] into first and second order systems, each of which
 182 satisfies the required properties. The impulse response of these systems can
 183 then be plugged into a convolution integral, computed recursively. Recursive
 184 convolution involves an integral that is often discretized with a second order
 185 method, which reduces the order in time of the code. However, by differentiat-
 186 ing this integral it is possible to obtain auxiliary differential equations (ADE),
 187 the convolution integral being then obtained from the solutions to the ADE.
 188 The advantage is that the ADE are integrated with the same method as the
 189 governing equations, and Dragna et al (2015) have shown that the order in time
 190 of the code is then maintained [41].

In the present study, the characteristics of the liner are inspired from refer-
 ence [5] in which an instability occurs around the resonance frequency. Hence,
 the physics of the interaction of the turbulent flow with a single resonance
 frequency (corresponding normally to absorption of sound) is of interest here,
 which justifies the choice of a simple mass-spring-damper model. As discussed
 above, several implementations are possible. Scalo et al (2015) used the reflec-
 tion coefficient of the MSD wall together with a convolution integral [22], as
 recommended by Fung and Ju [36]. Their method was extended in [44, 42],
 also for use with the Navier-Stokes equations. An inconvenient is that the order
 of the time integration method decreases (even if it could possibly be fixed by
 using ADE). In the present work, in order to retain the order of time integra-
 tion, the implementation of the differential equation has been privileged. At a
 bottom wall the MSD model in its differential equation form reads:

$$M \frac{d^2 v}{dt^2} + K v + R \frac{dv}{dt} = - \frac{dp}{dt} \quad (\text{bottom wall}) \quad (17)$$

191 where R is the resistance, M is the mass, and K is the spring constant. These
 192 quantities are normalized with $\tilde{\rho}_b \tilde{c}_w$, $\tilde{\rho}_b \tilde{H}$, and $\tilde{\rho}_b \tilde{c}_w^2 / \tilde{H}$, respectively . This

193 equation can be recast into a first-order system¹:

$$\frac{dv}{dt} = Q \quad (18)$$

$$\frac{dQ}{dt} = \frac{1}{M} \left[-\frac{dp}{dt} - RQ - Kv \right] \quad (19)$$

194 This system was first solved by Tam and Auriault for use with the linearized
 195 Euler equations [6], and the resolution involved a ghost point for pressure. A
 196 direct implementation has apparently been used by Olivetti et al [21] for solving
 197 the Navier-Stokes in a turbulent channel flow. By direct implementation it is
 198 meant that Eq. (18) replaces the momentum equation in the direction perpen-
 199 dicular to the wall, at the grid point located on the wall. Olivetti et al mention
 200 a stability issue in some cases, but it is not clear whether it is a numerical or
 201 a physical one. We have also used successfully such a direct implementation
 202 before in a linearized Euler code (see Marx (2015) [14]), and in the linearized
 203 Navier-Stokes equations or the nonlinear disturbance equations. However, one
 204 of the boundary scheme was only second order to improve stability. Gabard
 205 and Brambley [13], and Brambley and Gabard [15] had a stability issue when
 206 using a direct implementation in the linearized Euler equations and used a
 207 characteristic formulation to stabilize their scheme. In this formulation, the
 208 ingoing acoustic wave is written as a function of the incoming one, which in
 209 spirit is similar to dealing with the reflection coefficient, as in Fung and Ju [36].
 210 Here, since our solver is written in characteristic form, and motivated by the
 211 observation of Gabard and Brambley [13], the method of characteristics will be
 212 used to impose the MSD boundary condition for the Navier-Stokes equations.
 213 With this formulation we have not encountered stability issues.
 214 In addition, at the wall one still has to enforce the non-slip boundary conditions
 215 $u=0$ and $w=0$, as well as the isothermal wall condition $T=T_w$. All together,
 216 these are four conditions that need to be imposed in the characteristic formula-
 217 tion. Moreover, four characteristic quantities need to be computed at the wall:
 218 Y^+ , Y^- , Y^u , Y^s .

219

First note that due to the boundary condition $u=0$, one has: $X^+ = \frac{1}{\rho} \frac{\partial p}{\partial x}$ and
 $X^- = -\frac{1}{\rho} \frac{\partial p}{\partial x}$, leading to $X^+ + X^- = 0$ at the wall. One has also $X^s = X^v = X^w = 0$
 at the wall. Due to the boundary condition $w=0$, one has similarly: $Z^+ +$
 $Z^- = Z^s = Z^v = Z^w = 0$ at the wall. Accounting for these relations, and since one
 also has to satisfy $\partial u / \partial t = 0$ at the wall, Y^u can be fixed from Eq. (1). In the same
 fashion Y^w will be fixed in Eq. (3) to satisfy $\partial w / \partial t = 0$. One has:

$$Y^u = -\frac{1}{2}(X^+ - X^-) + \frac{1}{\text{Re}} \frac{1}{\rho} \frac{\partial \tau_{1j}}{\partial x_j}$$

¹Referring to the discussion above, it is obvious that this set of equations is connected to the two ADE which would be used for computing the convolution integral for a second order system, although it is outside our scope to establish precisely this connection.

$$Y^w = -\frac{1}{2}(Z^+ - Z^-) + \frac{1}{\text{Re } \rho} \frac{\partial \tau_{3j}}{\partial x_j}$$

To obtain the reflected wave Y^+ at the bottom wall from the incident wave Y^- one injects Eq. (18) into Eq. (2) to obtain:

$$Y^+ = Y^- + 2 \left(\frac{1}{\text{Re } \rho} \frac{\partial \tau_{2j}}{\partial x_j} - Q \right) \quad (\text{bottom wall}) \quad (20)$$

220 For a rigid wall, the same relation holds with $Q=0$, this is Eq. (14). Equa-
 221 tion (20) is the Navier-Stokes equivalent of the equation preceding Eq. (2.8) in
 222 Gabard and Brambley [13].

223

Finally, the isothermal character of the wall needs to be enforced, which imposes:

$$\frac{\partial s}{\partial t} = -\frac{(\gamma - 1)}{\gamma} \frac{1}{p} \frac{\partial p}{\partial t} \quad (\text{isothermal wall}) \quad (21)$$

Knowing Y^\pm it is easy to compute $\partial p/\partial t$ and to deduce directly $\partial s/\partial t$ from this relation. Alternatively, in order to draw a parallel with the rigid wall case presented by Lechner et al. [28] we may further inject Eq. (5) and Eq. (4) in the latter equation to obtain:

$$Y^s = \frac{-1}{2c}(\gamma - 1)(Y^+ + Y^-) + \frac{\gamma - 1}{p} \frac{1}{\text{Re}} \left[\Phi + \frac{1}{\text{Pr}} \nabla \cdot (K \nabla T) \right] \quad (22)$$

$$\frac{\partial p}{\partial t} = -\frac{p}{2c}(Y^+ + Y^-) + pY^s \quad (23)$$

$$\frac{\partial s}{\partial t} = \frac{\gamma - 1}{\gamma} \frac{1}{2c}(Y^+ + Y^-) - \frac{\gamma - 1}{\gamma} Y^s \quad (24)$$

224 where $Y^s \neq 0$ arises from $v \neq 0$ for an impedance wall. For a rigid wall, one has
 225 $Y^s=0$ and these equations become equivalent to Eqs (15-16).

226

2.3.3. Numerical validation

228 The implementation of the MSD boundary condition is validated against
 229 the reference solution of Zheng and Zhuang [9] for the reflection of an initial
 230 Gaussian pressure pulse by a plane MSD wall. To remain in the linear inviscid
 231 regime in which the analytical solution has been derived, the amplitude of the
 232 pulse is small, and the thermo-viscous terms are all neglected (the solver is
 233 then a Euler equations solver).

234 The computational domain is a square box with $(x, y, z) \in [-50 \ 50] \times [0 \ 100] \times$
 235 $[-50 \ 50]$. The bottom MSD wall is at $y=0$, and on the other boundaries non-
 236 reflecting boundary conditions are used. The MSD characteristics, pulse size,
 237 and pulse-wall distance are the same as those used by Zheng and Zhuang [9].
 238 Hence, we have: $R=0.2$; $M=2.0938$; $K=0.4758$, and the initial pressure (mean
 239 + pulse) is given by: $p(x, y, z, t=0) = \frac{1}{\gamma} + p_a e^{-\ln(2)/25(x^2 + (y-30)^2 + z^2)}$, where

240 the amplitude $p_a=1e-8$ is small. A regular mesh size is used in all directions,
 241 with $\Delta x=\Delta y=\Delta z$. Equal numbers of grid points are used in all directions,
 242 $N_x=N_y=N_z$. Figure 1 shows a comparison between the computed pressure
 243 and the analytical solution along the x -axis at $t=30$, obtained for $\Delta t=0.5$ and
 $N_x=101$. A very good agreement is observed. A convergence study has been

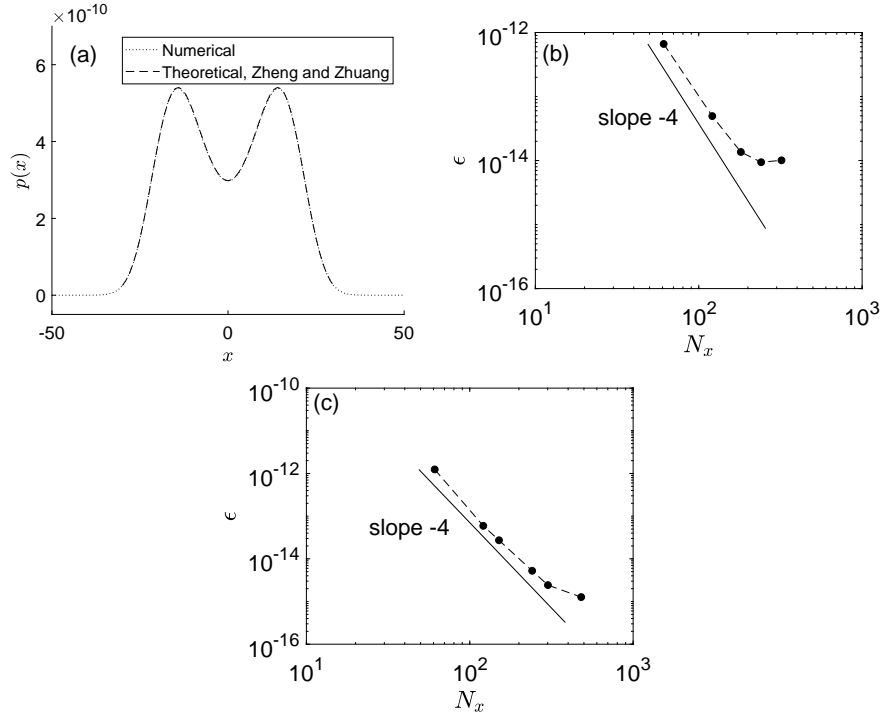


Figure 1: (a) Numerical vs theoretical pressure on the x -axis at time $t=30$ during Gaussian pulse reflection at an impedance wall. (b) Error (symbols) vs number of points in one direction for a fixed value of $\Delta t=0.01$. (c) Error (symbols) vs number of points in one direction for a fixed value of $CFL=0.5$.

244 performed by varying N_x (with $N_y=N_z=N_x$) for a small $\Delta t=0.01$ and is shown
 245 in Fig. 1(b). The order observed is about 4. It is not unusual to observe for
 246 the global order the order of the boundary scheme plus 1 [43]. As discussed
 247 earlier, one advantage of implementing directly Eq. (18-19) is that the order of
 248 the integration scheme should be preserved. This is now verified. The order
 249 of the Runge-Kutta method is 4. Since the spatial order has been found to
 250 be about 4, the truncation error is expected to be $O(\Delta t^4, \Delta x^4)$. Hence, by
 251 keeping $CFL=\Delta t/\Delta x$ constant, we should observe a global order $O(\Delta x^4)$. This
 252 is indeed verified to be the case in Fig. 1(c).
 253
 254

255 **3. Mean flow statistics above the liner**

256 In this section the statistics of a compressible turbulent flow in a channel
 257 having an impedance boundary condition at the bottom wall and a rigid upper
 258 wall are considered. The different test cases for which simulations (ILES)
 have been carried out are summarized in Table 1. Of particular interest is

Table 1: Impedance wall simulation test cases

Case	\mathcal{M}	R	ω_{res}	ξ	ω_{flow}	$\text{Re}_\tau^{\text{bot}}$	$\text{Re}_\tau^{\text{top}}$	$\Delta c_f^{\text{bot}}\%$	$\Delta c_f^{\text{top}}\%$
AC01	0.3	0.23	0.185	0.109	1.84	1031	476	575	44
AC02	0.3	0.23	0.367	0.109	1.84	642	434	162	20
AC03	0.3	0.23	0.738	0.109	1.84	523	411	74	7
AC04	0.3	0.23	1.479	0.109	1.84	453	399	31	1
AC05	0.3	0.23	2.960	0.109	1.84	397	392	0	-3
AC06	0.3	0.23	5.887	0.109	1.84	390	386	-3	-5
AC09	0.4	0.10	2.961	0.045	2.51	514	413	66	7
AC11	0.3	0.60	0.185	0.284	1.84	396	390	1	-2
AC12	0.3	1.00	0.185	0.474	1.84	393	391	-1	-2
AC13	0.3	0.3	0.185	0.142	1.84	854	461	367	36
AC14	0.3	0.4	0.185	0.190	1.84	626	434	151	21
AC15	0.3	0.5	0.185	0.237	1.84	440	399	24	1

259 the dependence of the flow on the resonance frequency of the liner and on its
 260 resistance. Several Mach numbers have been used, but all of them remain small.
 261 The resonance angular frequency of the material is defined as $\omega_{\text{res}} = \sqrt{K/M}$
 262 (with a slight abuse of language, since strictly speaking this should be called
 263 the natural frequency). The liner may be characterized alternatively by the
 264 values of R , M , and K , or by the values of R , ω_{res} , and the damping ratio
 265 $\xi = R/(2\omega_{\text{res}}M)$. The latter quantities are provided in the table.
 266

267
 268 Recently, Scalo et al. [22] performed channel flow simulations with impedance
 269 walls, with Mach numbers up to $\mathcal{M}=0.5$ and resistance ranging from $R=0.01-1$.
 270 In addition, they tuned the resonance frequency of the liner so that it matches
 271 the typical angular frequency of the flow, defined to be: $\omega_{\text{flow}}=2\pi\mathcal{M}$. This
 272 frequency is typically high and much larger than the acoustic frequency that
 273 would be found in aeroengines. In the present work ω_{res} is first taken to be
 274 smaller than ω_{flow} for case AC01 and is progressively increased in the cases
 275 AC02-AC06, while keeping Re_b , \mathcal{M} , and R constant. The resistance is increased
 276 in cases AC11-AC12. Case AC09 corresponds to the smallest resistance used in
 277 this work and to a high resonance frequency.

278 The baseline frequency and resistance for case AC01 correspond to that
 279 for which an instability has been measured experimentally by Marx et al. [5],
 280 and the MSD characteristics are chosen to fit the impedance of the liner in
 281 the vicinity of the resonance frequency. The fit is the same as the one used

282 in Marx [45]. In dimensional units, this fit provides: $\tilde{R}=94.4 \text{ kg m}^{-2}\text{s}^{-1}$;
 283 $\tilde{M}=0.0685 \text{ kg m}^{-2}$; and $\tilde{K}=2.71 \cdot 10^6 \text{ kg m}^{-2} \text{ s}^{-2}$. The resonance frequency
 284 is thus $\tilde{f}_{\text{res}} = \tilde{\omega}_{\text{res}}/(2\pi) \sim 1 \text{ kHz}$, which corresponds to a realistic value for a
 285 liner. The normalized resistance $R \sim 0.23$ has a rather low value (lower than
 286 what would be found in practice in most aero-engines). In the experiments of
 287 Marx et al. [5], the half-height of the rectangular channel was $\tilde{H}=0.01\text{m}$ and
 288 this value is used here to compute normalized numbers. In the experiment
 289 the friction Reynolds number upstream of the liner was $\text{Re}_\tau \sim 3000$, which is
 290 costly to compute at the moment, even with a ILES. The simulations are thus
 291 performed at $\text{Re}_b=6900$, which corresponds to a smaller value of the friction
 292 Reynolds number, $\text{Re}_\tau=395$. In the following we will compare the results of the
 293 present simulations with the direct numerical simulations of an incompressible
 294 channel flow with rigid walls at $\text{Re}_\tau=395$ made by Moser et al. [46] (hereafter
 295 referred to as M395).

296
 297 Case AC01 was performed in a computational domain of size $L_x \times L_y \times L_z =$
 298 $6\pi H \times 2H \times \pi H$ (using a number of grid points $N_x \times N_y \times N_z = 351 \times 201 \times 125$).
 299 The domain size was chosen by running simulations on small domains to obtain
 300 the approximate wavelength of the structures in the direction of the flow.
 301 Then the domain length was increased to about two and then about three
 302 wavelengths to verify that the wavelength and flow statistics were stable. The
 303 independence of the statistics on the spanwise direction was then verified.
 304 The same procedure was followed for case AC02, leading to a domain size of
 305 $3\pi H \times 2H \times \pi H$, and this grid was then used for other cases for which the
 306 wavelength is smaller. For all the cases, we choose the grid resolution $\Delta x^+=20$
 307 and $\Delta z^+=10$, with wall units based on the conventional channel at $\text{Re}_\tau=395$.
 308 In the wall-normal direction, for case AC01, Δy^+ varies between 0.25 at the
 309 walls and 10 in the channel center, for case AC02 it varies between 0.5 and 10,
 310 whereas for the other cases Δy^+ varies between 1 and 10. A grid convergence
 311 study is provided in Appendix A.

313 3.1. Effect of the liner resistance R

314 The effect of liner resistance is first investigated. Olivetti et al. (2015)
 315 performed the simulation of a pipe flow with a liner having a resistance larger
 316 than 1 and reported that the turbulence statistics are not much modified by
 317 the liner, compared with a rigid wall [21]. Scalo et al. performed a series
 318 of simulations for resistance varying between 0.01 and 1 [22]. They observed
 319 important changes in the flow statistics for a low resistance value. The effect
 320 of resistance is presently investigated for 6 different values of the resistance:
 321 $R = 0.23$ (case AC01); $R = 0.3$ (case AC13); $R = 0.4$ (case AC14); $R = 0.5$
 322 (case AC15); $R=0.6$ (case AC11); and $R=1$ (case AC12). The mean velocity
 323 profile and the rms of the axial velocity are plotted in Fig. 2. Also plotted is
 324 the M395 case for rigid walls. In this figure, and at several occasions in the
 325 following, with a slight but unambiguous abuse of notation, y represents the
 326 distance to the wall. For example, y is used rather than $y + 1$ for the bottom

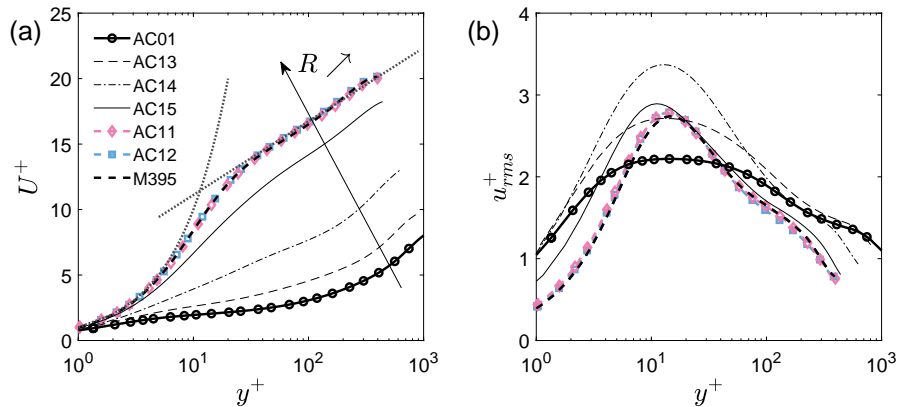


Figure 2: (a) Mean velocity profile and (b) rms of the streamwise velocity for varying resistance of the liner. Values of the resistance are $R = 0.23$ (AC01); $R = 0.3$ (AC13); $R = 0.4$ (AC14); $R = 0.5$ (AC15); $R=0.6$ (AC11); $R=1$ (AC12). In subplot (a), the straight dotted line indicates the log law, and the other dotted line the viscous sublayer (law of the wall).

327 lined wall located at $y=-1$. It is clear that for the lower resistance, $R \leq 0.5$,
 328 there are important changes in the flow statistics compared with the rigid wall,
 329 with at the lowest resistance a lower and broader peak of u_{rms} and a quasi-
 330 disappearance of the mean flow logarithmic region (it would be interesting to
 331 know if this remains true at higher values of the Reynolds number). The law
 332 of the wall is not followed either. For $R = 0.6$ and 1, the statistics are very
 333 close to the rigid wall channel statistics. This is in line with the findings in
 334 references [21, 22]. Hence, in the following we will focus on the configurations
 335 corresponding to a small resistance.

336 3.2. Effect of the resonance frequency ω_{res}

337 In this section we analyze the influence of the resonance frequency of the
 338 liner on wall turbulence in comparison with rigid wall turbulence. Cases
 339 AC01-AC06 are considered, which correspond to a constant value of Re_b and
 340 \mathcal{M} , and a given low resistance value $R=0.23$. The resonance frequency varies
 341 between $\omega_{res}=0.184$ and $\omega_{res}=5.887$. The typical frequency of the flow is
 342 $\omega_{flow}=2\pi\mathcal{M}=1.84$.

344 The mean velocity profile for the different cases is shown in Fig. 3. For
 345 low resonance frequency (cases AC01-AC04), the mean profile departs from
 346 the reference profile M395 and the law-of-the-wall is not valid anymore. The
 347 flow speed is smaller in the vicinity of the bottom impedance wall and due to
 348 flow-rate conservation and the top wall being rigid, the flow speed is larger in
 349 the upper half of the channel. For the high resonance frequency cases, AC05
 350 and AC06, the mean velocity profile follows the reference curve M395 for the
 351 rigid channel. Hence, for resonance frequencies somewhat higher than the flow

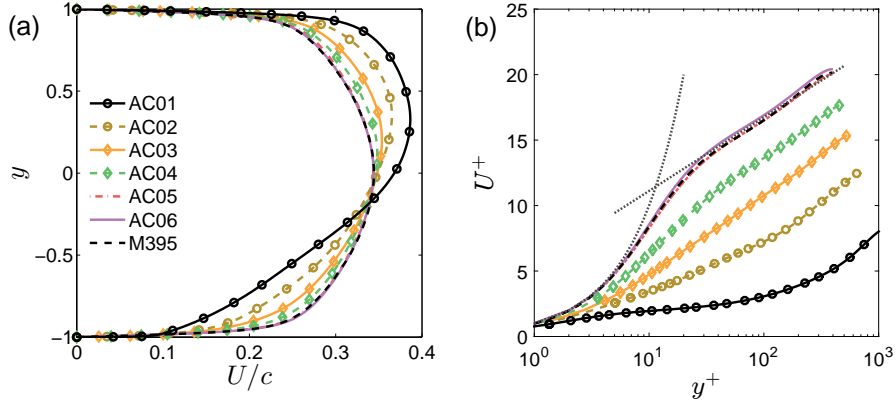


Figure 3: Mean velocity profile for varying resonance frequency of the liner as a function of the distance to the lined wall, scaled with (a) outer units c and H ; (b) wall units u_τ and l_τ . In (b) the straight dotted line indicates the log law, and the other dotted line the viscous sublayer (law of the wall).

352 frequency, the MSD wall behaves as a rigid wall, even at low resistance.

353

354 Figure 4 presents the rms of stream-wise velocity, rms of wall normal velocity,
 355 Reynolds stress, and rms of streamwise vorticity. Unless specified otherwise,
 356 all quantities are non-dimensionalised using the wall-variables from the bottom
 357 impedance wall. For higher resonance frequencies (AC05-AC06) the turbulent
 358 statistics do not differ significantly from the ones for the rigid channel M395,
 359 which is the same behavior as for the mean velocity profiles. For the low fre-
 360 quency cases (AC01-AC04) noticeable differences are seen in the profiles of all
 361 quantities compared to the rigid wall channel M395. A broader peak in u_{rms}
 362 is seen in the buffer layer (as has been already noticed in Fig. 2(b) for AC01),
 363 and it occurs at a different location. This peak results from turbulence produc-
 364 tion, meaning the turbulence production is strongly affected by a low resonance
 365 frequency acoustic liner. Figure 4(c) shows that large values of the Reynolds
 366 stress are found close to wall for cases AC01-AC04, which indicates an increased
 367 momentum transfer in the turbulent flow throughout the channel. This transfer
 368 increases the drag as we will see below (see Section 5). In addition, a drop in
 369 $\omega_{x rms}$ is observed in Fig. 4(d) for these cases, and a similar drop of the other
 370 components of the vorticity (not shown here) is observed. It is well known that
 371 there exists a near-wall turbulence regeneration mechanism involving streaks
 372 and stream-wise vortices. This classical mechanism is highly disturbed for cases
 373 AC01 and AC02, the flow dynamics being strongly affected by the acoustic
 374 liner and the nonvanishing wall normal velocity at its surface. The value v_{rms}^w
 375 of v_{rms} at the wall does not vanish for a liner and decreases when ω_{res} increases,
 376 as seen in Fig. 4(b) (see also Table 2). We find that v_{rms}^w is roughly inversely
 377 proportional to the resonance frequency. For large resonance frequencies (cases
 378 AC05-AC06), $v_{rms}^w \rightarrow 0$, the liner behaves nearly as a rigid wall, and the statis-

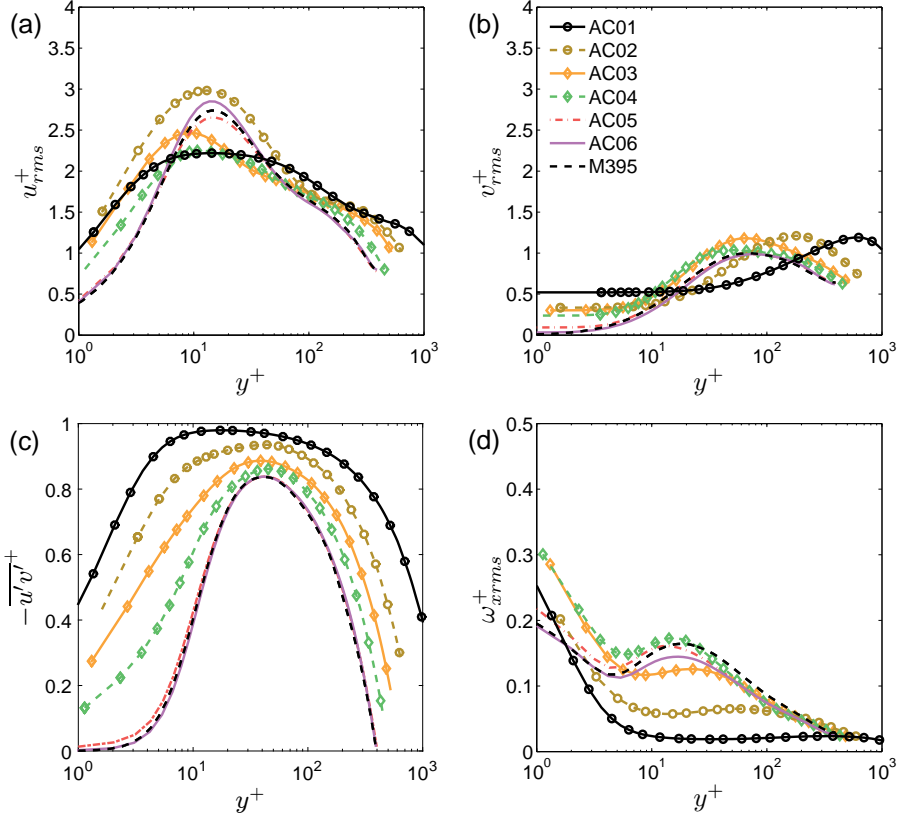


Figure 4: Profiles of (a) rms streamwise velocity; (b) rms wall-normal velocity; (c) Reynolds stress and (d) rms stream-wise vorticity for varying resonance frequency of impedance boundary condition.

379 tics of turbulence are close to those for a rigid wall, even for low value of the
 380 liner resistance.

381 4. Existence of a wave along the liner

382 In the previous section, it has been shown that for a liner with a resistance
 383 sufficiently small and a resonance frequency not too large, the statistics of turbu-
 384 lence differ from that of a turbulent channel flow with rigid walls. It is shown
 385 in this section that this is due to (or accompanied with) the presence of waves
 386 along the impedance surface. To evidence these waves the velocity spectra in
 387 the stream-wise direction are considered in Fig. 5. The axial wavenumber is
 388 denoted by k_x . The spectra are obtained at a position $y=0.015$ close to the
 389 impedance wall. The effect of the acoustic liner on the stream-wise spectra is
 390 clearly observed, as energy piles up at the resonance frequency, which leads to
 391 a partial modification of the turbulent energy cascade. For cases AC01-AC05

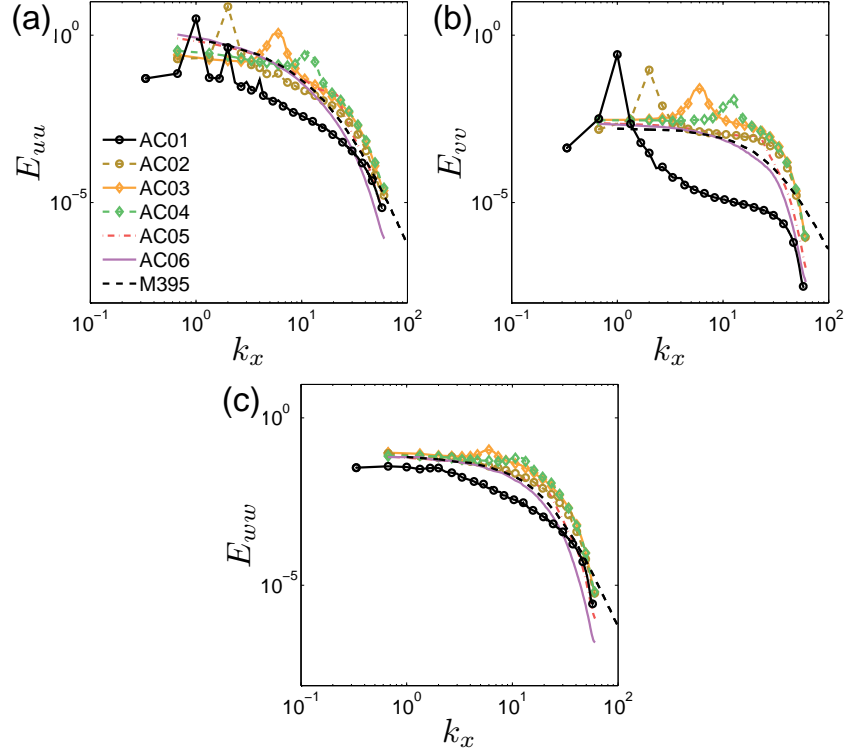


Figure 5: Velocity spectra of (a) streamwise velocity; (b) wall normal velocity; (c) spanwise velocity, versus the wavenumber k_x , at a location close to the wall ($y=0.015$), for several resonance frequencies (cases AC01-AC06).

392 we observe spikes in E_{uu} and E_{vv} , whose wave-number corresponds to the resonance
393 frequency of the material (see Fig. 5a). We also find harmonics for
394 case AC01. For cases AC03-AC04, we observe spikes in E_{ww} . Spectra for case
395 AC06 are in good agreement with the spectra for a rigid wall (case M395). The
396 angular frequency of the waves, ω_{wave} , has been computed from the peak of the
397 Fourier transform of the wall-normal velocity measured at a point belonging to
398 the liner surface. It is reported in Table 2. Overall, the observed frequency
399 corresponds to the resonance frequency of the liner. For low values of ω_{res} , the
400 convection speed for the surface wave, c_{wave} , is such that $c_{\text{wave}}/u_b \approx 0.6$. In
401 order to assess if the vertical movement at the lined wall can perturb wall tur-
402 bulence, the vertical displacement amplitude at the lined surface is estimated
403 from $d_a = v_{rms}^w / \omega_{\text{wave}}$. In wall units it becomes $d_a^+ = d_a \text{Re}_\tau$, where the fric-
404 tion Reynolds number for a rigid wall is used here ($\text{Re}_\tau=395$). For case AC01,
405 d_a^+ is more than 50, which means that the vertical displacement from the wall
406 goes well beyond the turbulence production region well known to be located at
407 $y^+=15$. For case AC02, d_a^+ is also large. This explains why turbulence is so
408 affected by the liner in these cases. As the resonance frequency increases the

Table 2: Characteristics of the wave along the liner: ω_{wave} , k_x , and c_{wave} are the measured angular frequency, axial wavenumber, and phase speed of the wave along the impedance wall, while v_{rms}^w is the rms of the wall-normal velocity computed at the impedance wall. The quantity d_a^+ is the estimated vertical amplitude of the displacement of a point on the lined surface, in wall units.

	AC01	AC02	AC03	AC04	AC05	AC06
ω_{res}	0.185	0.370	0.739	1.48	2.96	5.92
ω_{wave}	0.185	0.380	0.78	1.5	3.0	5.1
k_x	1	2	3	12.3	20.3	20.3
c_{wave}/u_b	0.61	0.62	0.42	0.41	0.49	0.87
v_{rms}^w	0.024	0.0094	0.0068	0.0047	0.0016	0.0005
d_a^+	51	10	3.5	1.2	0.21	0.039

409 value of the vertical displacement decreases. For cases AC05 and AC06, the
 410 displacement is less than $d_a^+=1$, and $y^+ = 1$ is often taken to be the first grid
 411 point position off the wall in large eddy simulations. Hence, for these two cases
 412 the wall displacement is too small to modify turbulence and turbulent statistics
 413 are similar to the rigid wall channel (M395), as observed above.

414 Some flow visualizations are now presented to illustrate the presence of
 415 the wave and show its effect on the flow. Instantaneous visualizations of the
 416 turbulent structures for the bottom half of the channel is shown in Fig. 6, where
 417 flow direction is from left to right. Among the displayed quantities is a slice
 418 of the wall-normal velocity component close to the wall (red color for blowing
 419 with v out of wall, and blue color for suction with v into wall). Also shown
 420 are iso-contours of Q2 events (ejections) and Q4 events (sweeps), colored with
 421 wall-normal location (darker shade close to wall and lighter away from the wall,
 422 blue (red) shade is used for Q2 (Q4) events). Iso-contours of the Q-criterion
 423 are finally colored in yellow to show the near-wall turbulent structures.

424 For the low resonance frequency case AC01, a large scale two-dimensional
 425 (2D) wave is clearly visible which propagates in the streamwise direction,
 426 with alternating blowing and suction regions. This wave strongly modifies
 427 and modulates the flow: structures (in yellow) are mainly present in the
 428 blowing regions and are absent in suction regions, leading to an inhomogeneous
 429 distribution of turbulent structures. In addition we do not see the near-wall
 430 streaks which are characteristic of the rigid wall turbulence, being an essential
 431 ingredient of its regeneration cycle. Q2 ejections are logically found in the
 432 blowing region (v out of wall), since the fluid is pushed away from the wall
 433 there, and Q4 sweeps are found in the suction regions (v into wall).

434 For the higher resonance frequency case AC04, waves of v at the impedance
 435 wall are observed, which are not 2D anymore and undulations are present in
 436 the spanwise direction. The Q2 and Q4 events are rooted in the impedance
 437 wall but tend to merge farther away from the wall where their scale become
 438 larger than the wavelength. This indicates an interaction between the flow

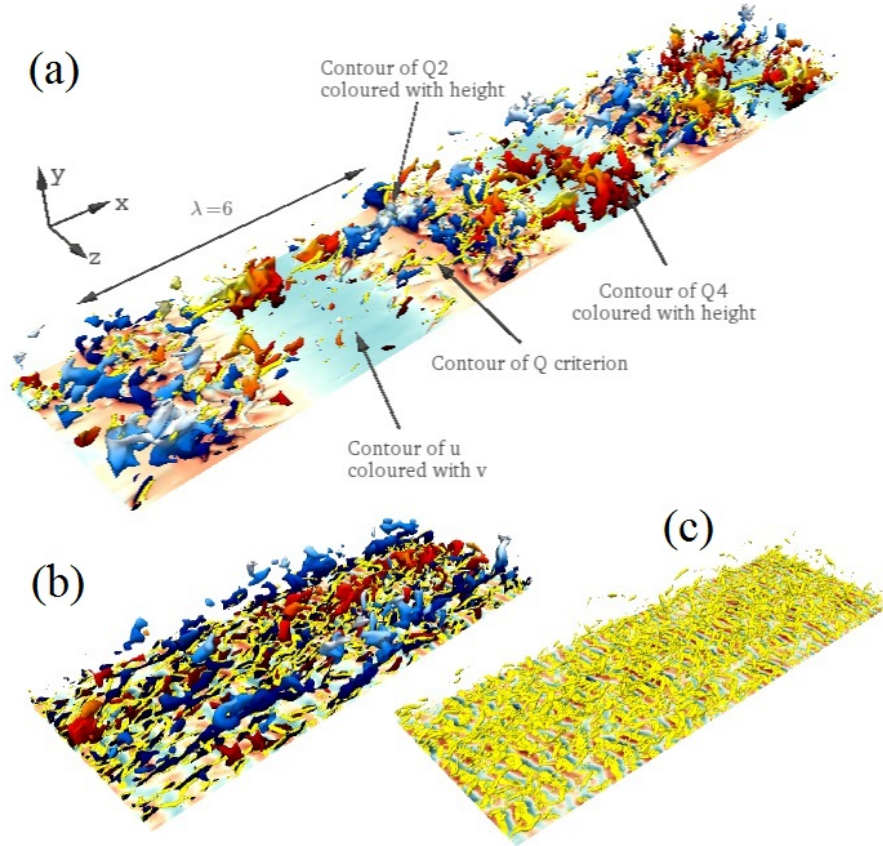


Figure 6: Instantaneous visualization of turbulent structures for the cases: (a) AC01, (b) AC04 and (c) AC09. Several quantities are shown: a colormap of v at the wall (light blue is for negative v corresponding to flow into the wall, light red is for v positive corresponding to flow out of the wall); iso-contours of $Q2^{+/-4}$, shown in blue and corresponding to ejection events; isocontours $Q4^{+/-4}$ shown in red and corresponding to sweep events; isocontours of Q-criterion $Q^{+/-3}$ in yellow color.

439 and wave when $\omega_{\text{flow}} \approx \omega_{\text{res}}$. Unlike the flow for case AC01, the flow for case
 440 AC04 is densely populated with near wall turbulent structures. For cases with
 441 higher $\omega_{\text{res}} > \omega_{\text{flow}}$ (not shown here) we observe near-wall turbulent structures
 442 similar to rigid wall turbulence. This may not be the case if the resistance is
 443 smaller than for our baseline configuration AC01-AC06. For example, Fig. 6(c)
 444 corresponds to case AC09, where ω_{res} is large (but not much larger than ω_{flow})
 445 and where the resistance is very small ($R=0.1$). In that case there is a large
 446 interaction between the small scales in the flow and the liner, leading to small
 447 spanwise rollers. Parameters for case AC09 are close to those used by Scalo et
 448 al. [22], and these authors observed very similar patterns along their impedance
 449 surface.

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A slice of the instantaneous fluctuations of stream-wise velocity at wall distance $y=0.03$, and slices of wall-normal velocity and pressure at the wall are shown in Fig. 7. The computational domain being larger for case AC01, only a half of it is shown in the streamwise direction. For the low resonance frequency

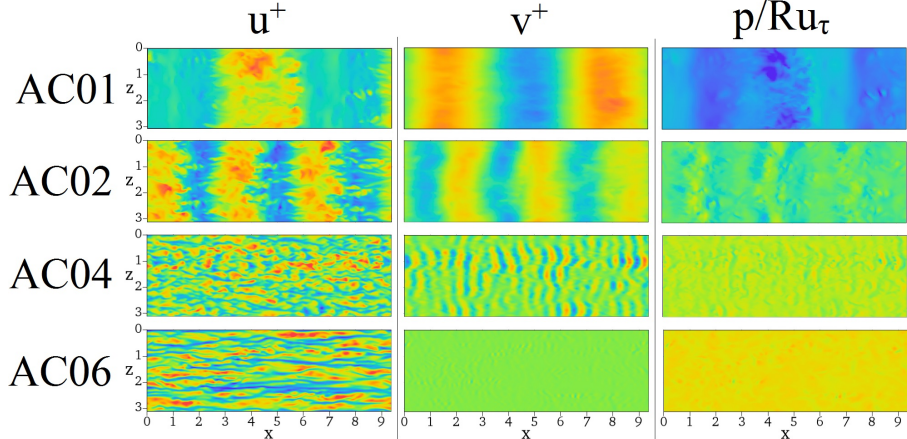


Figure 7: Slices of instantaneous fluctuations of u^+ (left) at wall distance $y=0.03$, and of v^+ (center) and p/Ru_τ (right) at the impedance wall for the cases AC01, AC02, AC04, and AC06. Blue (red) color corresponds to negative (positive) values, and the considered range is $-6 \leq u^+ \leq 6$, $-1 \leq v^+ \leq 1$ and $-6 \leq p/Ru_\tau \leq 6$ for all cases.

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cases AC01 and AC02, waves are seen for all three variables. The waves are 2D with a phase difference of π between stream-wise and wall-normal velocity. As ω_{res} increases the wave progressively becomes less 2D. This is clearly seen in case AC04 (v component). For the high resonance frequency case AC06, elongated streaks are found close to the wall (left plot for u^+), which is the feature of rigid wall flows. No pressure wave is observed.

Hence, for a low resonance frequency, a wave is present along the liner surface. This wave has a rather two dimensional character which is lost as the resonance frequency increases and the wave length approaches the typical size of the structures in the flow. Phase averaging is now performed in order to obtain the spatial distribution of the wave. Phase averaging allows distinguishing between the effect of the wave and that of random turbulence. Phase averaging relies on the following triple decomposition [47] for any quantity a :

$$a = \bar{a} + a' = \bar{a} + \tilde{a} + a'' \quad (25)$$

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where \bar{a} is the Reynolds average, \tilde{a} is the contribution from the wave, and a'' is the contribution from turbulence (note that depending on the context a tilde $\tilde{\cdot}$ is used either for a dimensional quantity or for a wave component). The extraction of the wave is based on phase averaging: $\tilde{a}(\phi) = \langle a \rangle_\phi$ where $\langle \cdot \rangle_\phi$ is the

466 phase average, that is, the average over all the available samples corresponding
 467 to the same phase ϕ , with $0 \leq \phi < 2\pi$. When the wave results from some
 468 periodic external triggering, the phase reference for phase-averaging is provided
 469 by this external trigger. Here the wave is self excited and we cannot rely on
 470 such external reference. However, in cases where the wave is two-dimensional
 471 its normal component v at the impedance wall is not noisy (see Figs. 6 and 7)
 472 and sinusoidal; the phase of v at the wall is thus taken as the phase reference.
 473 The procedure used to extract the wave (amplitude and phase angle) is thus
 474 the following:

475 1) For any flow field, average v at the wall in the span-wise direction to obtain
 476 a periodic 1D wave of v in the flow direction. This is legitimate as long as the
 477 wave is 2D; 2) Define several phases within a period (in the present case we use
 478 12 bins); 3) For any component (u, v, \dots) assign a phase bin to any streamwise
 479 position. This phase bin is simply taken to be that of the 1D wave of v which
 480 serves as a phase reference; 4) Compute the phase averaged components at each
 481 phase, by averaging the samples corresponding to the same phase bin; 5) Sub-
 482 tract the global mean to obtain wave profiles at each phase; 6) Compute the
 483 amplitude and phase angle of the waves using the wave profiles at each phase.
 484 Several instantaneous flow fields are used for this process. The random compo-
 485 nents a'' can then be obtained by subtracting the global mean \bar{a} and the phase
 486 average \tilde{a} from instantaneous fields. 2D surface waves which go all the way up to
 487 the channel center are obtained at low values of ω_{res} . Hence, we limit ourselves
 488 to cases AC01-AC03 for the phase averaging. Satisfactory 2D surface waves are
 489 observed for cases AC01 and AC02. However we do not expect entirely trust-
 490 worthy results from AC03 where the 2D character is partially lost.
 491 The amplitude of the phase-averaged stream-wise and wall-normal velocity, deno-
 492 ted respectively by $|\hat{u}(y)|$ and $|\hat{v}(y)|$, are shown in Fig. 8. The stream-wise

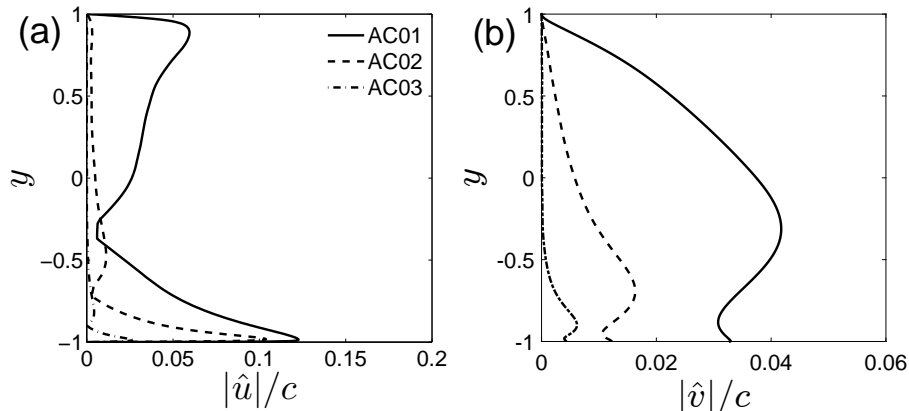


Figure 8: Amplitude of the wave along the liner: (a) stream-wise velocity component; (b) wall-normal velocity component.

492
 493 velocity component of the wave is larger close to the wall, and the maximum

494 decreases as the resonance frequency increases. For the wall-normal velocity
 495 component, the peak amplitude is obtained away from the wall, and also de-
 496 creases when ω_{res} increases. Some characteristics of the wave (wavelength, phase
 497 speed) were given in Table 2. The shapes in Fig. 8 are reminiscent of the wave
 498 measured and modeled by Marx and Aurégan [25] (see their Fig. 14 where nor-
 499 malized eigenfunctions for u and v are given). However, in the present case
 500 the wave is not in its linear regime. More details are given in Section 6. The
 501 eigenfunctions given here also resemble those obtained by Jimenez et al [23]
 502 for a pure resistance (actually these authors replace their resistive wall by a
 503 controlled wall actuation to perform a phase average process similar to the one
 504 performed here).

505 5. Drag increase

506 In this section it is shown that the modifications in the flow induced by the
 507 liner are associated with an increase in the drag, compared with the channel
 508 with rigid walls. The friction at the wall is classically measured by Re_τ defined
 509 in Eq. (13). This is given in Table 1 for both the bottom impedance wall and the
 510 top rigid wall. Other useful quantities are the drag coefficient and the change
 511 in drag coefficient in percent computed as follows:

$$c_f = \frac{2\tau_w}{\rho_w u_b^2} \quad (26)$$

$$\Delta c_f \% = \frac{c_f - c_f^{\text{ref}}}{c_f^{\text{ref}}} \times 100 \quad (27)$$

512 The change in drag is computed with respect to the conventional channel
 513 flow M395, c_f^{ref} being the drag coefficient for case M395. Thus, a positive
 514 value indicates a drag larger than the drag in a channel flow with rigid walls.
 515 $\Delta c_f \%$ is indicated in Table 1 for both walls. Table 1 shows that the friction
 516 at the impedance wall is more important than for the rigid channel in the
 517 cases for which the flow modifications are important. This is the case for low
 518 resistance and a not too large resonance frequency, that is for cases AC01-AC03,
 519 AC13-AC15, and case AC09. For three of these cases a flow visualization has
 520 been shown in Fig. 6. For case AC01 the drag is increased by as much as 575%.

521
 522 The connection between the wave along the liner and the drag increase is
 523 now discussed. Using the phase-averaging process introduced in the previous
 524 section, it is possible to compute the drag increase at each phase of the wave.
 525 This is shown in Fig. 9(c).

526 The phase averaged wall-normal velocity at the impedance wall, which is
 527 used as reference for phase averaging, is also presented in the figure. Phases
 528 $\Phi=0$ (and 2π) correspond to blowing (v out of wall) and phase $\Phi=\pi$ to suction
 529 (v into wall). We see that during suction we have an important increase in
 530 drag, with $\Delta c_f \%$ multiplied by a factor 22 for case AC01 for $\Phi = \pi$, due to a
 531 strong impingement of the flow at the wall. Averaged over a period the drag

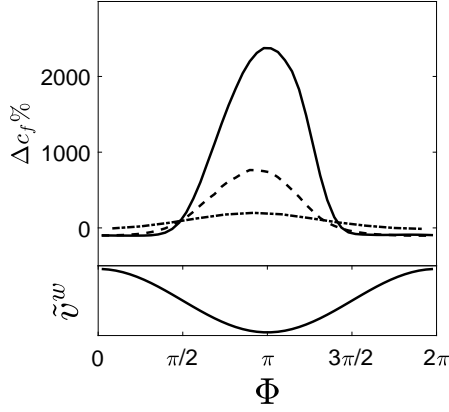


Figure 9: Phase-averaged drag increase in percent (for reference, the vertical component of the wave at the wall is also indicated in the bottom of the plot).

532 increase is 575% (Table 1). Remember from Fig. 6 that for case AC01 no
 533 turbulent structures are present in suction regions. Hence, the drag increase is
 534 not due to the effect of turbulence, but rather to the effect of the wave, which
 535 brings in high speed flow to the wall. Figure 9c also shows that there is a small
 536 drag reduction ($\Delta c_f\% < 0$) during blowing for cases AC01 and AC02.

537

538 We now consider the total Reynolds stress $\overline{u'v'}$. Using the triple decompo-
 539 sition, this can be decomposed into several contributions:

$$\begin{aligned} \overline{u'v'} &= \overline{(\tilde{u} + u'')(v + v'')} \\ &= \overline{\tilde{u}v} + \overline{u''v''} + \overline{\tilde{u}v''} + \overline{u''\tilde{v}} \end{aligned} \quad (28)$$

540 The 3 contributions are those from the wave, from the turbulence, and from
 541 cross terms. In the original triple decomposition [47] these cross terms are
 542 null because the wave and the turbulence are supposed to be uncorrelated,
 543 but such an assumption cannot be made a priori in the present case when the
 544 wave can modulate the turbulence. Nevertheless, it has been verified that the
 545 contribution of these terms is very small. The contributions for the other two
 546 terms are shown for case AC01 in Fig. 10. Wall units based on the bottom
 547 impedance wall are used for the scaling, therefore the peak values close to
 548 the top rigid wall have a small magnitude. Close to the impedance wall, the
 549 major contribution to the Reynolds stress comes from $\overline{\tilde{u}v}$, whereas away from
 550 the wall it comes from $\overline{u''v''}$. Hence, close to the wall an increased momentum
 551 transfer is due to the wave, and since $\overline{\tilde{u}v}$ is negative, momentum is transferred
 552 to the wall, which contributes to increasing the drag. This is in agreement with
 553 the drag increase being correlated with the wave, as observed above in Fig. 9(c).

554

555 It is interesting to draw a parallel between the wave created here spontane-
 556 ously in the vicinity of the liner and the waves that are sometimes imposed

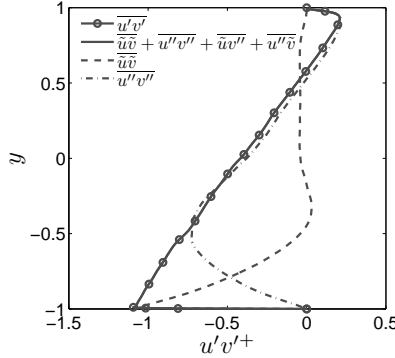


Figure 10: Contributions to the Reynolds stress $\overline{u'v'}$, scaled with u_τ and ρ_w from the bottom impedance wall. Case AC01.

557 using blowing and suction (or some related forcing) in flow control [48, 49, 50].
 558 In flow control it is well known that the surface waves resulting from blow-
 559 ing/suction should generally have a negative phase speed to obtain a drag
 560 reduction. For example Mamori and Fukagata [49] performed simulations of a
 561 channel flow with a wave-like wall-normal body-force. For upstream traveling
 562 waves (having a propagation speed smaller than the bulk velocity) they find a
 563 drag reduction of up to 40%. They reported the presence of span-wise rollers
 564 which produces positive $\overline{u'v'}$ close to wall and contribute to reducing the drag.
 565 Here in Fig. 10 we find the opposite behavior since the wave has a positive
 566 phase speed and $\overline{\tilde{u}\tilde{v}}$ is negative, leading to a drag increase. In our case the wave
 567 is formed spontaneously by the interaction between the flow and the boundary
 568 condition, thus it is not a controlled configuration. Höpfner and Fukagata [50]
 569 study wall actuation, such as wall deformation or wall blowing and suction.
 570 They argue that both actuation can be characterized as pumping and this
 571 pumping is strongly connected to drag reduction. Here the drag increases as a
 572 result of the phase between the \tilde{u} and \tilde{v} components of the wave, and this also
 573 corresponds to some pumping by the wave. The acoustic liners produce a wave
 574 with a phase speed in the flow direction and a magnitude of the order of u_b and
 575 it is unlikely that they can be used for passive drag reduction. Nevertheless, a
 576 slight drag decrease is reported for some cases in Table 1.

577

578 6. Stability analysis

579 In this section it is shown that the wave observed above the liner can be
 580 connected to an unstable surface wave. In the literature there has been several
 581 investigations of the surface wave modes in flow duct acoustics [51, 52, 53, 54,
 582 45, 25]. All these investigations have been done in a spatial frame of work, in
 583 which the (real) mode frequency is given and the (complex) mode wavenumber

584 is computed. This is the traditional way of computing a wavenumber spectrum
585 in acoustics. In the present case, since the computational domain is periodic in
586 the flow direction, a temporal analysis is more relevant: the (real) wavenumber
587 k_x is given, and the spectrum of the (complex) angular frequency $\omega = \omega_r + i\omega_i$
588 is computed. Temporal analyses have been performed by Jimenez et al [23] for
589 a purely resistive surface, by Tilton and Cortelezzi [55] for a model of porous
590 surface, and by Rahbari and Scalò [57], again for a purely resistive surface.
591 Also, it should be mentioned that global stability analyses have been performed
592 recently by Pascal et al [56] and Rahbari and Scalò [58].

593 Both the linearized Euler or Navier-Stokes equations, possibly comple-
594 mented with a turbulent eddy viscosity model, are encountered for modal
595 analysis. Normally, an unstable surface mode is found by using an inviscid
596 model, and including the dissipative phenomena provides a better estimation
597 of its characteristics [25]. In the present work, the two-dimensional linearized
598 Navier-Stokes equations for compressible perturbations are employed. Given
599 the low Mach number used in the numerical simulations, the shear base flow
600 $U_0(y)$ for the linearization is almost incompressible, and the mean density and
601 temperature are uniform. The equations are given in Eqs. (B.1-B.5) presented
602 in Appendix B, where the same normalization (see Section 2.1) as for the
603 numerical simulation is used. These equations are discretized in the wall
604 normal direction in the same way as in Ref. [25], which leads to the eigenvalue
605 matrix problem in Eq. (B.6). The MSD boundary condition at the bottom
606 wall at $y=-1$ is easily included in this eigenvalue problem, see Eq. (B.16-B.17).
607 The top wall at $y=1$ is rigid. The solution of Eq. (B.6) relies on standard
608 libraries [25], and provides for each value of the wavenumber k_x an eigenvalue
609 spectrum ω . The solver has been validated against spatial solvers that have
610 themselves been extensively validated [25].

611 In the following, the spectrum of standard canonical flows are first considered
612 briefly to show how a MSD wall can lead to instability before the method is
613 applied to the numerical simulation.

614

615 6.1. Instability due to MSD

A parabolic mean flow $U_0(y) = U_c(1 - y^2)$ for a channel with rigid walls
is first considered, where $U_c=0.1$ is the velocity at the center of the channel
(which is related to the Mach number $\mathcal{M} = 2U_c/3$ given the normalization
with the speed of sound). For a Reynolds number based on the center velocity
of $Re_c=2000$ ($Re_b=2/3Re_c$) and $k_x=1$ this flow is known to be stable in the
incompressible regime [59], which is confirmed by the spectrum of the phase
speed $C = \omega/k_x = C_r + iC_i$ presented in Fig. 11(a). Indeed, all eigenvalue have
 $C_i < 0$ (that is, also $\omega_i < 0$), meaning the flow is stable. Acoustic modes are
indicated by square symbols. The inviscid limit for these modes in a uniform
flow of Mach number \mathcal{M} is given by:

$$\omega = k_x \mathcal{M} \pm \sqrt{k_x^2 + \left(\frac{n\pi}{2}\right)^2} \quad \forall n = 0, 1, 2, \dots \quad (29)$$

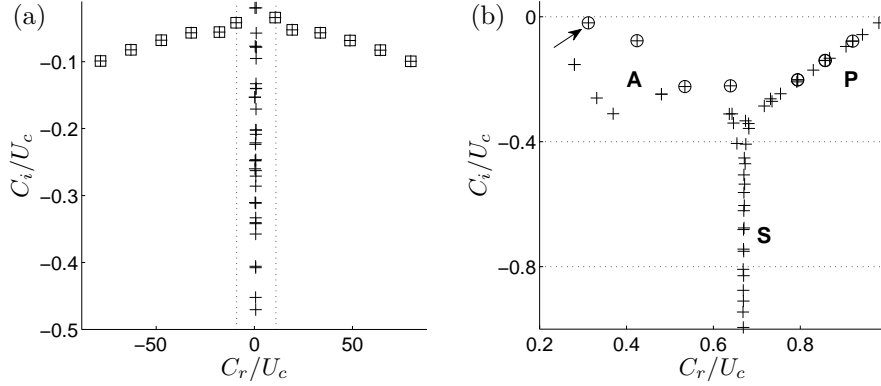


Figure 11: Complex phase speed spectrum (+) for a parabolic flow with rigid walls and $k_x=1$, $Re_c=2000$. (a) Full spectrum. Symbols (\square) indicate the acoustic modes; The vertical dotted lines are at $C_r/U_c = 1+1/U_c$ and $C_r/U_c = 1-1/U_c$. (b) Zoom in on the non-acoustic modes. Symbols (\circ) are some reference values obtained for an incompressible flow.

616 Two vertical lines indicate the speed $U_c - c_w = U_c - 1$ and $U_c + c_w = U_c + 1$,
 617 which are the propagation speeds of the upstream and downstream plane sound
 618 waves. The modes located outside the region comprised within the two vertical
 619 lines are all non-plane acoustic modes. Figure 11(b) presents a zoom in of the
 620 region within the two vertical lines. This region consists of non-acoustic modes.
 621 It classically displays a Y-shaped spectrum with 3 branches denoted A, P,
 622 and S. The A-branch modes are often designated as wall modes because their
 623 eigenfunctions are maximal close to the walls. The P modes are designated
 624 as center modes, since their eigenfunctions reach their maxima close to the
 625 channel center. Some reference values for an incompressible flow (given in
 626 the appendix A.7 of reference [59]) are added to the plot: for the low Mach
 627 number value $U_c=0.1$ taken here the agreement between the present results
 628 and the reference value is already quite good (the agreement can be reached
 629 at any order of accuracy by lowering compressibility by reducing the value of U_c).

630
 631 The effect of the MSD boundary condition on stability is now evidenced.
 632 The particular mode shown by an arrow in Fig. 11(b) belongs to the A-branch
 633 for rigid walls and corresponds to $C/U_c \sim 0.31-0.020i$ and $\omega \sim 0.031-0.0020i$.
 634 The bottom wall of the channel is now changed from rigid to MSD, and the
 635 MSD resonance frequency is taken to match the frequency of that particular
 636 mode, with $\omega_{res} = \sqrt{K/M} \sim 0.031$. The resistance is given an arbitrary small
 637 value, $R=0.0001$, all other parameters being unchanged. The corresponding
 638 spectrum is shown in Fig. 12(a). A new mode, indicated by an arrow, stands
 639 just above the original particular mode chosen. This new mode has a small
 640 positive value $C_i \sim 0.00002$, indicating that the flow is now unstable, other
 641 modes remaining approximately the same. By modifying the resonance

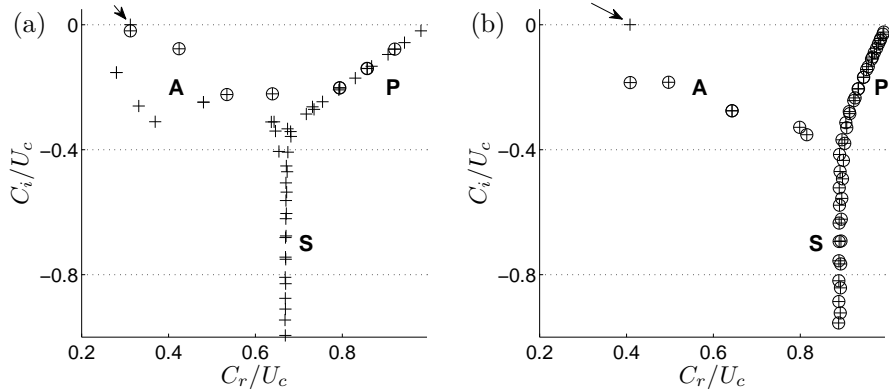


Figure 12: (a) Complex phase speed spectrum (+) for a parabolic flow with a top rigid wall and a MSD bottom wall tuned to resonance frequency 0.031. $k_x=1$, $Re_c=2000$. The reference values for the incompressible flows with rigid wall are still indicated (o). (b) Comparison of the spectra obtained for a $U_c(1-y^8)$ profile with a top rigid wall and: (o) a bottom rigid wall; (+) a bottom MSD wall tuned at the correct resonance frequency. $k_x=1$, $Re_c=2000$.

642 frequency of the MSD wall, we have observed that it is possible to render
 643 unstable any mode of the A-branch and many modes of the P-branch. However,
 644 the modes with the highest growth rate are those with a lower value of C_r .
 645 Hence, a MSD wall can prompt some modes to become unstable, particularly
 646 those of the A-branch. Unstable modes above liners are often called surface
 647 waves due their fast decrease away from the wall. This is in agreement with
 648 these modes arising from the A-branch of wall modes. Tilton and Cortelezzi
 649 [55] for a porous surface, and Rahbari and Scalo [57] for a purely resistive
 650 surface, observed that two modes may be unstable due to the channel walls not
 651 being rigid, one symmetric, and one anti-symmetric. The mode obtained here
 652 for a resonant material is very similar in nature to their symmetric mode (as
 653 a result of only the bottom wall being non-rigid in the present study). Hence,
 654 qualitatively, purely resistive, porous, and resonant surfaces all give birth to
 655 the same type of instability.

656
 657 A mean flow with a steeper profile $U_0(y) = U_c(1-y^8)$ (let us call it
 658 "turbulent like") is now considered, again with $k_x=1$, $Re_c=2000$. The spectrum
 659 for rigid walls is shown with o symbols in Fig. 12(b). Overall, compared with
 660 the parabolic profile, the spectrum is shifted toward higher phase velocities.
 661 All eigenvalues are found in the bottom half plane, indicating that the flow is
 662 stable. The spectrum obtained by replacing the bottom rigid wall by a MSD
 663 wall is shown with + symbols in Fig. 12(b). The resonance frequency is tuned
 664 to the frequency of the leftmost mode of the rigid wall case (corresponding to
 665 $\omega_r \sim 0.04$, or $C_r/U_c \sim 0.4$). With a MSD bottom wall, a slightly unstable mode
 666 (indicated with an arrow) is appearing on top of the leftmost stable mode. All

667 other modes are unchanged. This is the same behaviour as for the parabolic
 668 profile. Only the modes pertaining to the A-branch could be destabilized in
 669 that way. The normalized norm of the streamwise velocity eigenfunction is
 given in Fig. 13 for both the parabolic and eighth-power velocity profiles. For

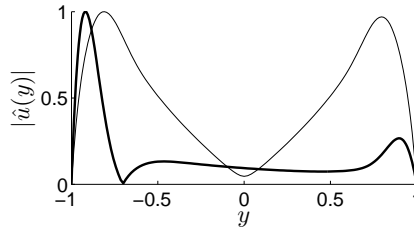


Figure 13: Streamwise velocity eigenfunction (normalized to have unit maximum) for the unstable mode for: — the parabolic mean velocity profile with $U_0(y) = U_c(1 - y^2)$; — the steeper profile $U_0(y) = U_c(1 - y^8)$. $k_x=1$, $Re_c=2000$. The bottom wall at $y=-1$ is of the MSD type.

670
 671 the parabolic flow, the maxima close to the rigid and MSD walls are about
 672 the same. For the steeper profile, the peakedness of the eigenfunction close
 673 to the MSD wall is more pronounced and resembles the shape of a surface mode.

674
 675 In this subsection, the major ingredients for the temporal stability of a
 676 channel flow with a bottom MSD wall have been given. If the MSD wall is
 677 tuned to the frequency of the modes of the A-branch, these modes can be
 678 destabilized. The destabilization is more effective for the modes located on
 679 the left of the A-branch. This is also where the classical Tollmien-Schlichting
 680 (TS) unstable wave can be found at higher Reynolds numbers. There is thus
 681 some similarity between this TS wave and the present liner-due surface mode,
 682 although the latter can exist even without viscosity. Note that Jimenez et
 683 al [23] argue that for a fully permeable surface (purely resistive with zero
 684 resistance), the mode is connected to a Kelvin-Helmholtz instability.
 685

686 6.2. Comparison with the numerical simulations

687 The waves observed in the numerical simulations and reported in Section 4
 688 for cases AC01-AC03 are 2D waves, and we now try to explain their presence by
 689 the same type of 2D stability analysis as in the previous subsection. These waves
 690 are non-linearly saturated waves not really prone to a linear stability analysis.
 691 To circumvent this limitation the following method that has been employed:
 692 the configurations AC01, AC02, and AC03 were run with a bottom *rigid* wall
 693 rather than a MSD wall until a statistically stationary turbulent channel flow is
 694 observed. Then at some instant chosen as the origin of time, $t = 0$, the bottom
 695 rigid wall is suddenly replaced by a MSD wall. Due to the flow being unstable,
 696 an instability develops in the numerical simulation in the vicinity of the bottom

697 wall and for some time it should be linear.
698 The base flow $U_0(y)$ for the stability analysis should be close to be the actual
699 mean velocity profile of the turbulent channel flow with rigid walls at $Re_b=6900$
700 ($Re_\tau=395$), which is the profile when the MSD wall is set up at $t=0$. The
701 numerically calculated profile could be used but it is more convenient to use
702 an analytical velocity profile that matches this mean flow. In particular this
703 avoids the need for interpolation, and facilitates grid convergence study. As in
704 reference [25] (see Eqs (3-4) therein), the Cess mean velocity profile is used for
705 that purpose. It is indeed verified in Fig. 14 that it is a very good approximation
706 to the computed mean flow. However, since the unstable mode is known to be
707 sensitive to the velocity profile, results obtained with the actual mean velocity
708 profile (interpolated on the Chebyshev grid) will also be presented in some places
(Table 3, Fig. 17(b)).

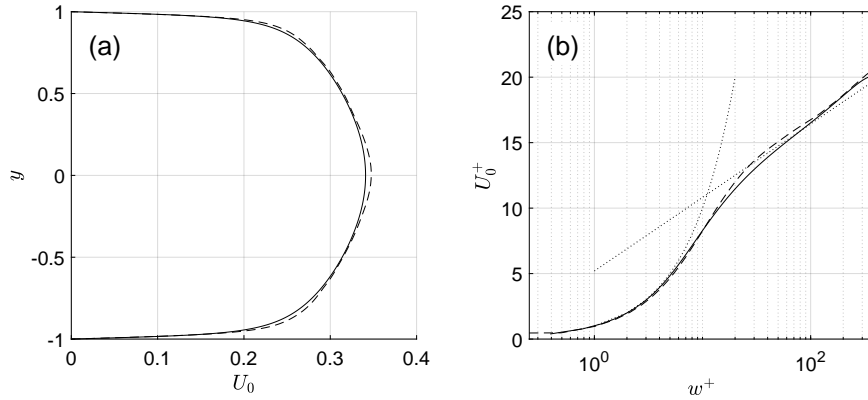


Figure 14: Comparison of the mean velocity profile obtained numerically (dashed line) with *rigid* walls and the analytical Cess profile (plain line). (a) in outer units; (b) in wall units ($w = y + 1$ represents the distance to the bottom wall).

709
710 The time evolution of the amplitude of the dominant spectral component
711 (spectral refers to Fourier transform in the x -direction) of the wall-normal ve-
712 locity v on the bottom MSD wall is shown in Fig. 15(a), for case AC02. The
713 value of the streamwise wavenumber k_x corresponding to this maximal spec-
714 tral amplitude is given in Fig. 15(b). Before saturation starts at time $t \sim 150$
715 the amplitude corresponds to a constant value of $k_x=3.55$ and exhibits an ex-
716 ponential growth typical of an instability. At later times, saturation leads to
717 a final state that is the same as described in Section 4, and the wavenumber
718 corresponding to the final state ($k_x \sim 2$ in Fig. 15(b), see also Table 2) differs
719 from the wavenumber for the initial instability ($k_x=3.55$). The characteristics
720 of this instability (wavenumber, angular frequency, growth rate) are computed
721 for $t < 150$ and compared to those predicted by the stability analysis. The char-
722 acteristics of the wave extracted from the numerical simulations is compared

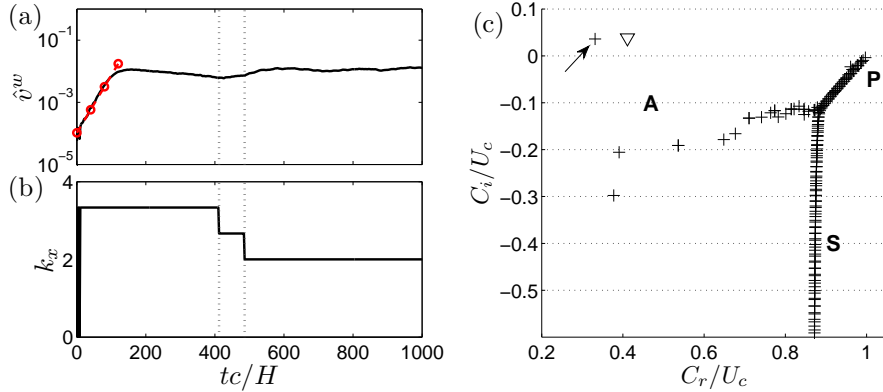


Figure 15: For case AC02: (a) Time evolution of the spectral component of the wall-normal velocity at the impedance wall having the largest amplitude (the straight dashed line with symbols is a fit to the initial exponential growth); (b) Streamwise wavenumber of the spectral component having the largest amplitude; (c) Normalized phase speed spectrum (+) resulting from a linear stability analysis performed for $k_x=3.55$. The triangle is the phase speed computed from the numerical simulation.

723 to that of the stability analysis in Table 3. For the numerical simulation the
724 characteristics are the one of the observed wave. For the stability analysis, a
725 search of the most unstable mode has been done, and the reported value of k_x
726 is the one for which the largest value of ω_i is obtained. The stability analysis
727 has been performed for the Cess mean velocity profile and the numerical mean
728 velocity profile. The latter is computed for rigid walls, just before the bottom
729 MSD wall is introduced at $t = 0$. The characteristics predicted by both profiles
730 are reasonably close, and if not perfect, the agreement with the numerical simu-
731 lation is satisfactory. For example, for case AC02 the wavenumber k_x and the
732 growth rate ω_i agree rather well. In any case the value of ω_r is slightly larger
733 than the angular resonance frequency of the liner, $\omega_{\text{res}} = \sqrt{K/M}$. Figure 15(b)
734 shows the spectrum predicted by the stability analysis, for case AC02. It bears
735 many similarities with that for the canonical eighth-power velocity flow consid-
736 ered above. In particular, the unstable mode indicated by an arrow stands on
737 the left side of the A-branch. The mode calculated from the numerical simula-
738 tion is indicated with a triangle symbol. The eigenfunctions for the streamwise
739 and wall normal velocities for case AC02 are compared in Figs 16(a) and 16(b),
740 respectively. Both are normalized with the value of the wall normal velocity
741 eigenfunction at the wall.

742 If the values are not exactly the same, especially for \hat{u} , the general trends
743 are similar. The eigenfunctions for the saturated state obtained at large times
744 (which were already included in Fig. 8) are also shown. Obviously the satu-
745 rated state will differ in many respects of the initial instability leading to this
746 state. Nevertheless, some similarities exist between the eigenfunction in the two

Table 3: Comparison between the characteristics of the instability wave observed in the numerical simulation during the growth period and the characteristics of the most amplified wave predicted by the linear stability analysis. For the stability analysis, either the Cess velocity profile or the numerical profile have been used.

Case	ω_{res} $= \sqrt{K/M}$	Numerical			Stability Analysis					
		Simulation			Cess profile			Numerical profile		
		k_x^{max}	ω_r	ω_i	k_x	ω_r	ω_i	k_x	ω_r	ω_i
AC01	0.185	2.3	0.22	0.028	1.65	0.210	0.0282	2.1	0.22	0.030
AC02	0.367	3.3	0.47	0.044	3.55	0.408	0.0445	4	0.41	0.051
AC03	0.738	6.7	0.79	0.073	7.05	0.787	0.0585	7.3	0.79	0.073

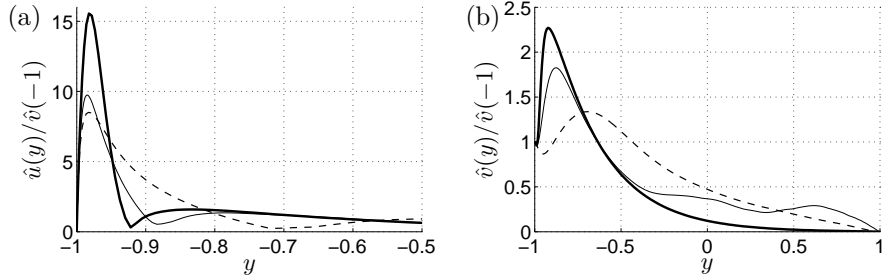


Figure 16: (a) Streamwise velocity eigenfunction. (b) Wall normal velocity eigenfunction. Case AC02. $k_x=3.55$. — Stability analysis; — Numerical computation, during the exponential growth of the instability; - - - Numerical computation, final saturated state.

747 regimes, which tends to indicate that the dynamics of the wave observed in the
748 vicinity of the lined wall in Section 4 is partially governed by a linear instability.
749

750 6.3. Link between resistance, growth rate, and observed drag increase

751 A critical value of the resistance is expected for a liner, above which there
752 should be no instability. Moreover, the drag increase observed in the numerical
753 simulations is due to the wave along the liner, and this wave is partly governed
754 by a linear instability. Hence, it is also expected that the predicted growth rate
755 of the instability and the observed drag increase should be correlated below the
756 critical resistance. This is now investigated.

757 A linear stability analysis is performed for case AC01 in table 1. The baseline
758 resistance for case AC01 is $R = 0.23$. However, the resistance is here allowed
759 to vary from $R = 0.01$ to $R = 0.8$ so that its effect can be investigated. The
760 growth rate as a function of the wavenumber is shown for the unstable mode

761 in Fig. 17(a), for several values of R . For each R the growth rate is maximal
762 for some given wavenumber, as indicated with a filled symbol. The maximal
763 growth rate is given as a function of resistance in Fig. 17(b). It is confirmed
764 that there exists a critical resistance $R = 0.7$ above which configuration AC01 is
765 stable (configuration AC02 is also shown, and corresponds to a critical resistance
766 $R = 0.6$). It is verified for case AC01 in Fig. 17(b) that using the numerical
767 velocity profile in the stability analysis rather than the Cess profile has almost
768 no effect on the critical resistance. In addition, the growth rate increases linearly
769 as R decreases and saturates as the resistance approaches a zero value. The
770 angular frequency of the instability obtained at the wavenumber corresponding
771 to maximal growth is given in Fig. 17(c) for both AC01 and AC02 cases. Also
772 indicated in this figure is the theoretical resonance frequency for a damped
773 oscillator [60], given by $\sqrt{K/M - (R/2M)^2}$ (a more precise definition compared
774 to $\omega_{\text{res}} = \sqrt{K/M}$ used everywhere else in this paper). The frequency of the
775 instability in the presence of flow is classically larger than this theoretical value,
776 but follows the same trend. In particular, the frequency of the most amplified
777 instability saturates to a value slightly larger than $\sqrt{K/M}$ when $R \rightarrow 0$. This
778 would not be the case for a purely resistive system (such as considered by
779 Jimenez et al [23] or Rahbari and Scalo [57]), in which the frequency of the
780 most amplified instability (as well as the corresponding wavenumber) would
781 increase importantly as $R \rightarrow 0$. Finally, the phase speed of the most amplified
782 instability is given as a function of resistance in 17(d) for case AC01. The phase
783 speed decreases with decreasing R . It would appear that this is opposite to
784 the observations by Rahbari and Scalo [57] (see their Fig. 6) that the phase
785 velocity increases when R decreases, at a fixed wavenumber. However, even
786 for a purely resistive liner, we have verified that the phase velocity decreases
787 when R decreases as long as the wavenumber is changed along with R to always
788 correspond to maximal growth (if α is fixed instead, we also find that the phase
789 speed increases with decreasing R). This means that the qualitative behavior
790 of the phase speed is the same for a purely resistive and a MSD liner. For the
791 original AC01 case (with $R = 0.23$), the phase speed in Fig. 17(d) is $c_{r,max}/u_b \sim$
792 0.42 . This is less than the value $c_{wave}/u_b = 0.61$ reported in Table 2. However,
793 this difference is logical, since the latter value corresponds to the saturated state,
794 whereas $c_{r,max}/u_b \sim 0.42$ corresponds to the initial linear instability. Since the
795 wavenumber decreases (as shown in Fig. 15(b) for AC02) as the instability
796 saturates, and meanwhile the frequency does not change much, the phase speed
797 increases during saturation.

798 The drag increase obtained in the numerical simulations (as reported in Table 1)
799 for cases AC01, AC13, AC14, AC15, AC11, and AC12 (all corresponding to
800 baseline configuration AC01 with different values of R) is plotted in Fig. 18.
801 The drag increase here corresponds to the saturated state. Below a critical
802 resistance, the value of which is between $R = 0.5$ and $R = 0.6$, the drag increase
803 decreases approximately linearly with increasing resistance, which is in line with
804 the predicted growth rate of the linear instability also decreasing linearly with
805 resistance (see Fig. 17(b)). Moreover, the critical resistance observed in the

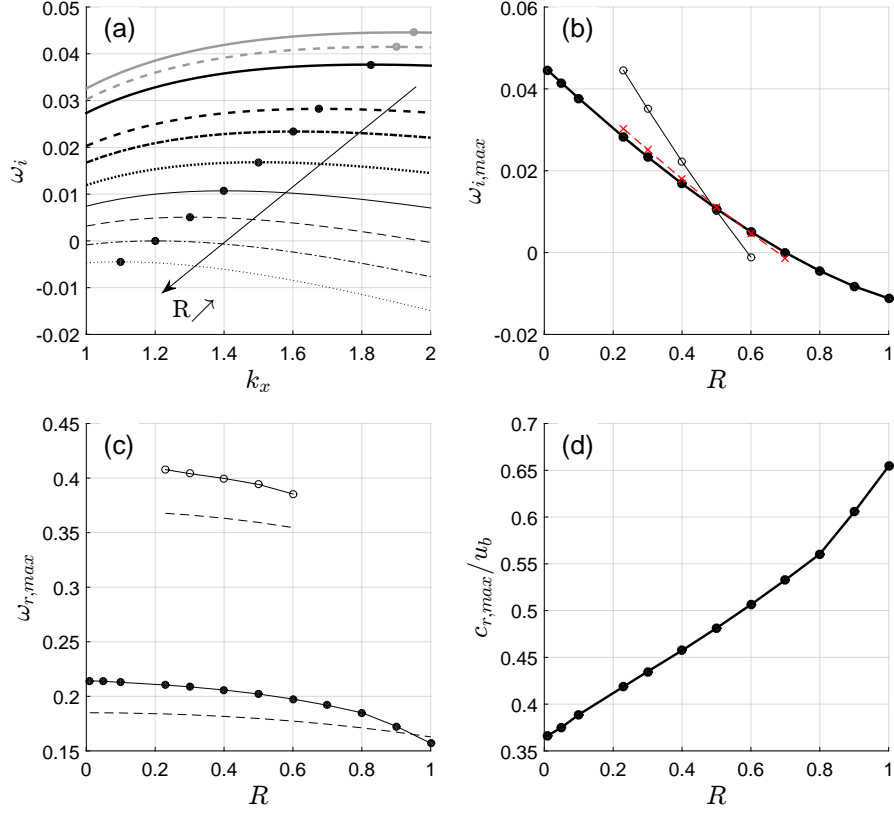


Figure 17: (a) Growth rate of the unstable mode versus wavenumber, given by a linear stability analysis of test case AC01 with several values of the resistance: — $R = 0.01$; - - - $R = 0.05$; — $R = 0.1$; - - - $R = 0.23$; - · - $R = 0.3$; · · · $R = 0.4$; — $R = 0.5$; - - - $R = 0.6$; - · - $R = 0.7$; · · · $R = 0.8$. The maximum growth rate is indicated by filled symbols. (b) Maximum growth rate of the unstable mode as a function of resistance: the plain line with filled circles corresponds to case AC01, and the one with open circles to case AC02, both are obtained with the Cess velocity profile. The dashed line with \times symbols corresponds to case AC01 when the stability analysis is based on the numerical velocity profile. (c) real angular frequency of the most amplified instability (filled circles for AC01; open circles for AC02; dashed lines for theoretical resonance frequencies of these MSD systems)(d) Phase speed normalized by bulk velocity as a function of resistance, for the instability having maximal growth, for baseline case AC01.

806 numerical simulations is not too different from the linear stability prediction,
 807 $R = 0.7$. This thus confirms the statements made in the beginning of this
 808 section.

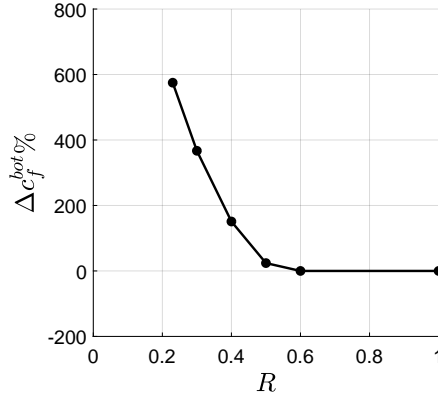


Figure 18: Drag increase at the bottom lined wall as a function of resistance, obtained in the numerical simulations by varying the resistance of baseline case AC01 (the corresponding cases are AC01, AC13, AC14, AC15, AC11, AC12).

809 7. Conclusion

810 Numerical simulations of a turbulent channel flow with an impedance
 811 boundary condition have been performed. When the liner resistance is small,
 812 and when its resonance frequency is not too large compared to a typical fre-
 813 quency of the flow, the turbulent statistics differ from those for a channel with
 814 rigid walls. Surface waves are also observed along the liner surface. The waves
 815 have a large wavelength compared to the turbulent structures. They modulate
 816 these structures and transport momentum toward the impedance wall, causing
 817 a drag increase. As the resonance frequency increases these waves progressively
 818 lose their spanwise coherence while their wavelength decreases to get close
 819 to the flow typical length scales, which may also results in a drag increase
 820 when the resistance is sufficiently small, as was also observed by Scalo et al. [22].

821
 822 At low resonance frequencies the two-dimensional waves have a spatial
 823 distribution which resembles the waveforms observed in former experiments [5],
 824 even if the conditions differ (the simulation are periodic in the streamwise
 825 direction, the Reynolds number in the simulation is lower).

826
 827 A linear two-dimensional temporal stability analysis has been performed.
 828 A temporal analysis suits the streamwise periodic configuration of the channel
 829 flow and offers a new perspective on unstable modes, compared with the more
 830 widespread spatial analysis of surface modes in acoustics. It has been shown
 831 that by tuning the resonance frequency of the liner to the frequency of a mode
 832 pertaining to the A-branch of wall modes, it is possible to destabilize this
 833 mode. Numerical simulations of a channel flow have been performed where the
 834 bottom rigid wall is suddenly replaced by an impedance wall. An instability

835 is then observed with characteristics similar to those obtained from a linear
836 stability analysis. This instability saturates and leads to a final state of the
837 flow which corresponds to a waveform different from the initial instability, but
838 not completely different.

839
840 The liner was modeled by an impedance boundary condition corresponding
841 to a mechanical oscillator. This has first been introduced by Tam and
842 Auriault [6] and has been recast here to match the characteristic form of
843 the equations used in the solver. It is a simple model that accounts for a
844 resonance of the liner. As virtually any other impedance model, it can be
845 derived by supposing that the incident acoustic wave has a long wavelength
846 compared to the perforations of the face sheet. This model has been used
847 as is in the turbulent channel numerical simulations, but some turbulent
848 scales may be so small that the model is not valid anymore. Also it is not
849 clear how the rugosity due to liner perforations can be accounted for by an
850 impedance boundary condition. As a result, the low frequency waves certainly
851 correspond to some reality (and have been observed experimentally) but
852 the presence of small spanwise rollers at high frequency probably needs to be
853 confirmed by experiments or simulations including the full geometry of the liner.

854
855 Ongoing work focus on spatial simulations, with a well defined inlet and
856 outlet and no periodicity assumption. This is closer to practical situations, and
857 also allows introducing a sound wave into the domain. The triggering of the
858 surface wave by an incoming wave is indeed important physically. In the present
859 paper, the periodic streamwise boundary conditions probably act as means to
860 sustain the wave as an incoming wave would do in a spatial simulation.

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864 Poitiers.

865 **Appendix A. Grid convergence study**

866 The mesh size requirement in the vicinity of an impedance wall is not as
867 well established as for a rigid wall. Hence, in this section we perform a grid
868 convergence study to determine the grid requirements for accurate numerical
869 simulations with impedance walls. The different configurations studied in this
870 appendix are summarized in Table A.4. Four test cases from Table 1 are consid-
871 ered (AC01,AC02,AC03,AC05), which are those for which the impedance affect
872 the more the flow dynamics. For AC01 and AC02, a 2D traveling surface wave
873 is observed in the domain, which strongly interacts with the flow due to blowing
874 and suction at the wall. Hence, we perform the grid convergence study mainly
875 focused on refinement in the wall-normal direction. For AC03 we observe quasi-
876 2D surface waves with ripples in the span-wise direction, therefore we study

Table A.4: Test cases for grid convergence analysis using impedance wall boundary condition.

	$L_x \times L_y \times L_z$	\mathcal{M}	R	ω_{res}	$N_x \times N_y \times N_z$	Δx^+	Δy_{min}^+	Δz^+	$\Delta c_f^{\text{bot}\%}$	$\Delta c_f^{\text{top}\%}$
AC01cc	$6\pi h \times 2h \times \pi h$	0.3	0.23	0.185	$351 \times 151 \times 125$	20	1	10	443	44
AC01c	$6\pi h \times 2h \times \pi h$	0.3	0.23	0.185	$351 \times 171 \times 125$	20	0.5	10	573	45
AC01	$6\pi h \times 2h \times \pi h$	0.3	0.23	0.185	$351 \times 201 \times 125$	20	0.25	10	575	44
AC02c	$3\pi h \times 2h \times \pi h$	0.3	0.23	0.367	$185 \times 151 \times 125$	20	1	10	162	20
AC02	$3\pi h \times 2h \times \pi h$	0.3	0.23	0.367	$185 \times 171 \times 125$	20	0.5	10	159	18
AC03c	$3\pi h \times 2h \times \pi h$	0.3	0.23	0.738	$185 \times 151 \times 125$	20	1	10	74	7
AC03	$3\pi h \times 2h \times \pi h$	0.3	0.23	0.738	$91 \times 151 \times 61$	40	1	20	78	-1
AC05	$3\pi h \times 2h \times \pi h$	0.3	0.23	2.960	$185 \times 151 \times 125$	20	1	10	0	-3
AC05w	$3\pi h \times 2h \times 2\pi h$	0.3	0.23	2.960	$185 \times 151 \times 251$	20	1	10	-2	-4
R0.1cc	$6h \times 2h \times 6h$	0.5	0.10	3.140	$115 \times 151 \times 251$	20	1	10	161	188
R0.1c	$6h \times 2h \times 6h$	0.5	0.10	3.140	$115 \times 171 \times 251$	20	0.5	10	176	176
R0.1	$6h \times 2h \times 6h$	0.5	0.10	3.140	$115 \times 201 \times 251$	20	0.25	10	181	181

877 the grid coarsening in the stream- and span-wise directions. With case AC05
878 we check the influence of domain size in the span-wise direction. In addition,
879 to validate our computations against the simulations of Scalo et al. [22] (to
880 our knowledge, the only published work for which turbulence with impedance
881 walls has been documented), a configuration taken from this reference has been
882 computed and is referred to as R0.1 in the table. In this configuration, the
883 resistance $R=0.1$ is smaller than generally used in this work, and both top
884 and bottom walls are impedance walls, whereas for cases AC01-AC05 only the
885 bottom wall is an impedance wall. Also, the Mach number value is larger,
886 $\mathcal{M}=0.5$. All in all, configuration R0.1 is not hugely different from the test case
887 AC09, except that for the latter only the bottom wall is lined. Our domain
888 size for test case R0.1 is the same as Scalo et al [22], and our finer grid verifies
889 $\Delta x^+ \times \Delta y^+ \times \Delta z^+ = 20 \times 0.25 \times 10$ (with + units based on $\text{Re}_\tau=395$). Scalo
890 et al grid resolution is $\Delta x^{++} \times \Delta y^{++} \times \Delta z^{++} = 27.7 \times 0.6 \times 21.6$ where ++
891 denotes normalization by wall units of the simulation. Converted into + units
892 based on $\text{Re}_\tau=395$, this gives $\Delta x^+ \times \Delta y^+ \times \Delta z^+ \sim 17.5 \times 0.37 \times 13.5$. Hence,
893 the resolutions are comparable.

894 All simulations share the same value $\text{Re}_b=6900$. For an incompressible
895 channel flow with rigid walls (a configuration referred to as M395 in the paper)
896 at similar Re_b and \mathcal{M} one would have $\text{Re}_\tau=395$. For impedance walls it is not
897 possible to estimate the friction Reynolds number *a priori*. Hence, all the grid
898 resolution mentioned in Table A.4 are based on the value of Re_τ for a rigid wall
899 simulation. Similarly $\Delta c_f\%$ computed for the two walls are based on the velocity
900 and length scales computed for bottom impedance and top rigid wall separately.

901
902 Results of the grid convergence analysis are shown in Fig. A.19, where the
903 mean stream-wise velocity, as well as the rms of stream-wise and wall-normal

velocity are presented. Results in Fig. A.19 are non-dimensionalised with

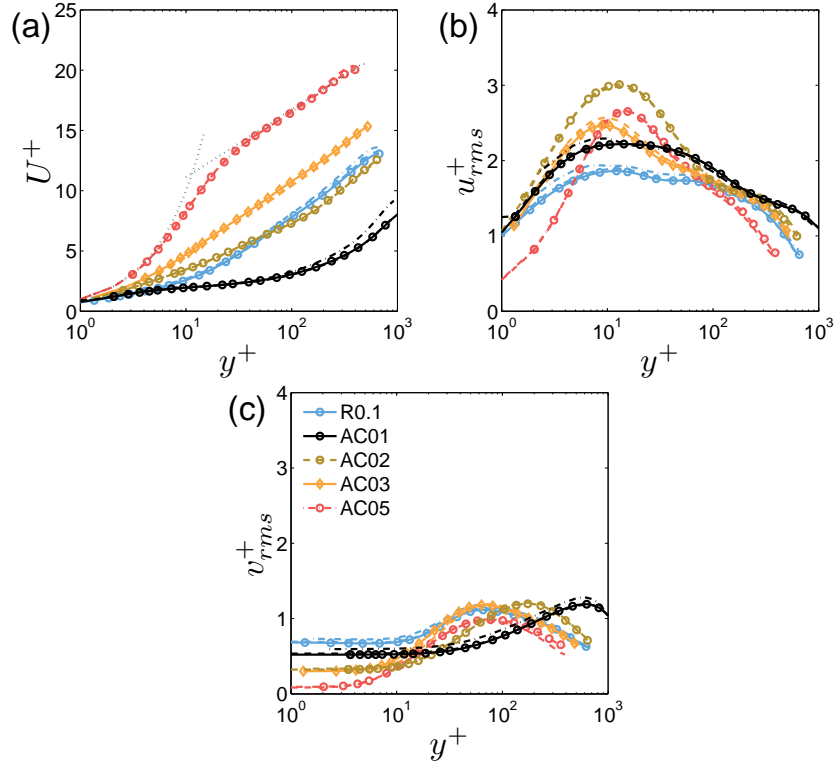


Figure A.19: Grid convergence for impedance wall test cases. In the legend only the the most well resolved cases (e. g. AC01) are indicated (line+symbols). Cases with medium resolution (e. g. AC01c) are shown by a dashed line, while cases with the worst resolution (e. g. AC01cc) are shown by a dash-dotted line, where lines are of the same color as the most well resolved case. Lines of medium and high resolution are superimposed, only the line corresponding to the worst resolution stands apart in some cases.

904

905 wall units of the bottom impedance wall. These results do not depend much
 906 on the grid resolution, which is therefore deemed to be sufficient in all test
 907 cases. From this study the grid resolution that is required in the directions
 908 parallel to the wall is $\Delta x^+=20$ and $\Delta z^+=10$ (computed with equivalent friction
 909 Reynolds number $Re_\tau=395$ in rigid-wall simulation). In the wall-normal
 910 direction, a minimal mesh size $y_{min}^+=0.25-1$ is necessary. These values are
 911 about the same, or slightly more stringent than required for a rigid wall. In
 912 particular the smaller the resistance of the MSD wall the smaller y_{min}^+ should be.
 913

914 The profiles for the mean flow, rms of streamwise velocity, and rms of span-
 915 wise velocity for case R0.1 are compared to the ones obtained by Scalo et al. [22]
 916 for the same configuration in Fig. A.20, where the rigid wall case M395 is also
 917 shown. Although an exact agreement with the results of Scalo et al. [22] is not

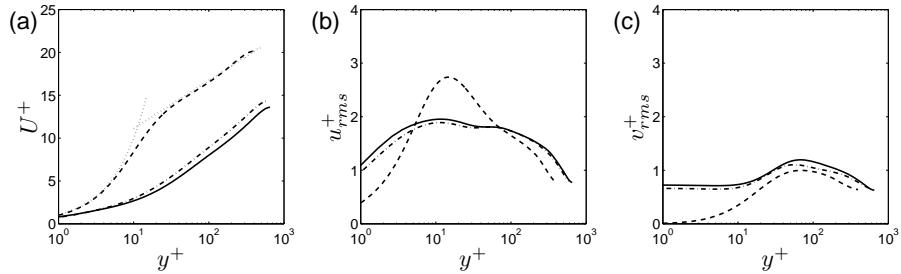


Figure A.20: Comparison between the present simulation and that from Scalo et al. [22] for case R0.1. (a) Mean velocity; (b) rms of streamwise velocity; (c) rms of wall-normal velocity. Lines are: — presents result; - - - Scalo et al. results; - - - rigid wall case M395.

918 obtained (their simulation is also a large eddy simulation), both simulations
 919 provide close results. The difference seen in the mean velocity profile also shows
 920 up in the drag increase: we computed a larger drag increase ($\approx 180\%$) for case
 921 R0.1, whereas Scalo et al. report a value of 148%. Nevertheless, despite these
 922 differences, it is clear that both simulations show similar changes compared to
 923 the rigid channel flow.

924 Appendix B. Linear stability analysis

To perform the stability analysis, the linearized two-dimensional Navier-Stokes equations are considered. For the linearization all variables are splitted as a sum of a base flow indicated by subscript 0, and a perturbation indicated with a tilde: $\rho(x, y) = \rho_0 + \tilde{\rho}(x, y, t)$; $u(x, y, t) = U_0(y) + \tilde{u}(x, y, t)$; $v(x, y, t) = \tilde{v}(x, y, t)$; $p(x, y, t) = p_0 + \tilde{p}(x, y, t)$; $T(x, y, t) = T_0 + \tilde{T}(x, y, t)$. The base flow is taken to be essentially a shear flow with a streamwise component $U_0(y)$ that depends on y . In our simulation, the Mach number is small, as a result no dependence of the base density ρ_0 and temperature T_0 upon y is considered. The linearized

equations for the perturbations are:

$$\frac{\partial \tilde{p}}{\partial t} + U_0 \frac{\partial \tilde{p}}{\partial x} + \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) = 0 \quad (\text{B.1})$$

$$\begin{aligned} \frac{\partial \tilde{u}}{\partial t} + U_0 \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{dU_0}{dy} + \frac{\partial \tilde{p}}{\partial x} = \frac{1}{\text{Re}} \mu_T \Delta \tilde{u} + \frac{1}{\text{Re}} \left(\frac{\mu_T}{3} + \frac{\kappa}{\mu} \right) \frac{\partial}{\partial x} \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) \dots \\ + \frac{1}{\text{Re}} \frac{d\mu_T}{dy} \left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x} \right) \end{aligned} \quad (\text{B.2})$$

$$\begin{aligned} \frac{\partial \tilde{v}}{\partial t} + U_0 \frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{p}}{\partial y} = \frac{1}{\text{Re}} \mu_T \Delta \tilde{v} + \frac{1}{\text{Re}} \left(\frac{\mu_T}{3} + \frac{\kappa}{\mu} \right) \frac{\partial}{\partial y} \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) \dots \\ - \frac{2}{3} \frac{1}{\text{Re}} \frac{d\mu_T}{dy} \frac{\partial \tilde{u}}{\partial x} + \frac{4}{3} \frac{1}{\text{Re}} \frac{d\mu_T}{dy} \frac{\partial \tilde{v}}{\partial y} \end{aligned} \quad (\text{B.3})$$

$$\begin{aligned} \frac{\partial \tilde{p}}{\partial t} + U_0 \frac{\partial \tilde{p}}{\partial x} + \left(\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} \right) = \frac{1}{\text{RePr}} \left(1 + \frac{\nu_t \text{Pr}}{\gamma \text{Pr}_t} \right) \Delta \tilde{T} + \frac{1}{\gamma \text{RePr}_t} \frac{d\nu_t}{dy} \frac{\partial \tilde{T}}{\partial y} \dots \\ + \frac{(\gamma - 1)}{\text{Re}} \frac{dU_0}{dy} \left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x} \right) \end{aligned} \quad (\text{B.4})$$

$$\tilde{p} = \frac{1}{\gamma} (\tilde{T} + \tilde{\rho}) \quad (\text{B.5})$$

925 These equations are normalized with the same reference quantities as used
 926 in the numerical solver (see Section 2.1). Since the temperature is uniform
 927 here, the speed of sound at the wall is simply the speed of sound anywhere.
 928 For completeness, a turbulent eddy viscosity $\mu_t(y)$ depending on y has been
 929 retained. The result obtained with this eddy viscosity were slightly different
 930 but neither better or worse than the ones presented in the paper which were
 931 obtained by accounting only for the molecular viscosity. The total viscosity
 932 (molecular + turbulent) is $\mu_T(y) = \mu + \mu_t(y)$. The corresponding dynamic
 933 viscosity (obtained after division by the uniform ρ_0) is $\nu_T(y) = \nu + \nu_t(y)$.
 934 For the results presented in the paper and obtained with the sole molecular
 935 viscosity, one has: $\mu_t(y)=0$, $\nu_t(y) = 0$, $\nu_T(y) = \nu$.

936

For a temporal stability analysis, modal solutions of the form $\tilde{u}(y) = \hat{u}(y)e^{i(k_x x - \omega t)}$ are searched for, where k_x is a real wavenumber and ω is the complex angular frequency. Injecting this solution into Eqs. (B.1-B.5), discretizing these equations on a y -grid and denoting by \mathbf{D} the corresponding derivation matrix [25] provides the generalized eigenvalue problem:

$$(\mathbf{A} + \omega \mathbf{B}) \Psi = 0 \quad (\text{B.6})$$

where:

$$\mathbf{A} = \begin{bmatrix} ik_x \mathbf{U}_0 & ik_x \mathbf{I} & \mathbf{D} & & \\ & ik_x \mathbf{U}_0 + \mathbf{A}_1 & \mathbf{U}'_0 + \mathbf{A}_2 & & \\ & & \mathbf{A}_3 & \mathbf{D} & \\ & ik_x \mathbf{I} + \mathbf{A}_5 & \mathbf{D} + \mathbf{A}_6 & ik_x \mathbf{U}_0 & \mathbf{A}_7 \\ -\frac{1}{\gamma} \mathbf{I} & & & \mathbf{I} & -\frac{1}{\gamma} \mathbf{I} \end{bmatrix} \quad (\text{B.7})$$

$$\mathbf{B} = \begin{bmatrix} -i\mathbf{I} & & & & \\ & -i\mathbf{I} & & & \\ & & -i\mathbf{I} & & \\ & & & -i\mathbf{I} & \\ & & & & \end{bmatrix} \quad (\text{B.8})$$

937

$$\mathbf{A}_1 = \frac{1}{\text{Re}} \left(\frac{4}{3} \mathbf{N}_T + \mathbf{K} \right) k_x^2 - \frac{1}{\text{Re}} (\mathbf{N}_T \mathbf{D}^2 + \mathbf{N}'_T \mathbf{D}) \quad (\text{B.9})$$

$$\mathbf{A}_2 = -\frac{1}{\text{Re}} \left(\frac{1}{3} \mathbf{N}_T + \mathbf{K} \right) i k_x \mathbf{D} - \frac{i k_x}{\text{Re}} \mathbf{N}'_T \quad (\text{B.10})$$

$$\mathbf{A}_3 = -\frac{1}{\text{Re}} \left(\frac{1}{3} \mathbf{N}_T + \mathbf{K} \right) i k_x \mathbf{D} + \frac{2}{3\text{Re}} i k_x \mathbf{N}'_T \quad (\text{B.11})$$

$$\mathbf{A}_4 = -\frac{1}{\text{Re}} \left(\frac{4}{3} \mathbf{N}_T + \mathbf{K} \right) \mathbf{D}^2 + \frac{1}{\text{Re}} \left(k_x^2 \mathbf{N}_T - \frac{4}{3} \mathbf{N}'_T \mathbf{D} \right) \quad (\text{B.12})$$

$$\mathbf{A}_5 = -\frac{2(\gamma-1)}{\text{Re}} \mathbf{U}'_0 \mathbf{D} \quad (\text{B.13})$$

$$\mathbf{A}_6 = -\frac{2(\gamma-1)}{\text{Re}} i k_x \mathbf{U}'_0 \quad (\text{B.14})$$

$$\mathbf{A}_7 = \frac{1}{\text{RePr}} \left(\mathbf{I} + \frac{\text{Pr}}{\gamma \text{Pr}_t} \mathbf{N}_t \right) (k_x^2 \mathbf{I} - \mathbf{D}^2) - \frac{1}{\gamma \text{RePr}_t} \mathbf{N}'_t \mathbf{D} \quad (\text{B.15})$$

938 The vector of unknown is $\Psi = [\mathbf{R} \ \mathbf{U} \ \mathbf{V} \ \mathbf{P} \ \mathbf{T}]^t$ where \mathbf{U} for example contains the
 939 values of \hat{u} at the grid points.

940 \mathbf{U}_0 and \mathbf{U}'_0 are matrices containing the velocity and velocity derivative at the grid
 941 points. \mathbf{N}_T is the square matrix containing the total viscosity (ν_T) at the grid points
 942 in its diagonal; \mathbf{N}'_T is the square matrix containing the y -derivative of the total
 943 viscosity ($d\nu_T/dy$) in its diagonal; \mathbf{N}_t is the square matrix with the turbulent eddy
 944 viscosity only (ν_t) in its diagonal; \mathbf{N}'_t is the square matrix with the y -derivative of
 945 the turbulent eddy viscosity only ($d\nu_t/dy$) in its diagonal.

946

947 Finally, the implementation of the impedance boundary condition is discussed.
 948 Contrary to the spatial stability case, in the general case when the impedance $Z(\omega)$ is
 949 a transcendental function of ω it would not be possible to easily insert the impedance
 950 boundary condition into Eq. (B.6). As the present MSD boundary condition leads to
 951 an algebraic function of ω , there is no such difficulty. Taking the Fourier transform of
 952 Eqs. (18-19), one obtains:

$$\tilde{q} + \omega (i\tilde{v}) = 0 \quad (\text{ at } y = -1) \quad (\text{B.16})$$

$$K\tilde{v} + \omega (-iM\tilde{q} - iR\tilde{v} - i\tilde{p}) = 0 \quad (\text{ at } y = -1) \quad (\text{B.17})$$

953 To introduce this into Eq. (B.6), one has to add one unknown \tilde{q} in the vector Ψ , and
 954 add an extra line in Eq. (B.6) that corresponds to the first equation, Eq. (B.16), in the
 955 former system. The second equation of the system, Eq. (B.17), replaces the equation
 956 for \tilde{v} at the bottom wall in Eq. (B.6). Overall the matrices \mathbf{A} and \mathbf{B} are square of size
 957 $(5N+6) \times (5N+6)$, where N is the number of discretization points in the y -direction.

958

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