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Modeling, Measuring, and using BRDFs: Significant French Contributions

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The scattering of light by a surface is described by the Bidirectional Reflectance Distribution Function (BRDF). Unfortunately, this function cannot be straightforwardly acquired nor modeled. French researchers have proposed interesting contributions in the field, with several models and accurate experimental systems. For instance, the National Metrological Institute (LNE-CNAM) has implemented the best angular resolution goniospectrophotometer (0.015°). Modeling the BRDF has also been deeply studied in France, especially with the microfacet theory these recent years, a better understanding of the shadowing/masking function, new general distribution functions, visible normals, interfaced Lambertian microfacets and analysis concerning light multiple reflections. This paper proposes a state of the art concerning some significant French contributions in these fields. © 2019 Optical Society of America

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1. INTRODUCTION

The French language does not distinguish between radiometric and photometric quantities. The French term *luminance* refers to both radiance and luminance while *éclairage* refers to both irradiance and illuminance. The slight advantage of this ambiguity is that the Bidirectional Reflectance Distribution Function (BRDF) can be defined without specifying whether radiometry or photometry is considered. The BRDF of a surface is the ratio between the *luminance* scattered by this surface in one direction and the *éclairage* it receives from a given direction. The French scientist Pierre Bouguer is considered as the first author who laid the foundations of photometry [1–3] in the 18th century. In particular, he has designed the first photometers, correctly using Kepler's law (decrease of the illuminance in the inverse square of the distance between the light source and the illuminated surface). He established the principle of equalizing the brightness of two light sources, including one reference source, observed simultaneously. These photometers were used until the beginning of the 20th century and the rise of photoelectric detectors.

The other major contribution to the birth of photometry in

the 18th century was that of Johannes Heinrich Lambert. He is not strictly speaking French: he was born in the city of Mulhouse, now in France, but at the time linked to Switzerland. He published many of his writings in German but also in French and Latin, including *Photometria* [4]. If Bouguer was above all an experimenter, Lambert was more interesting in the formalization of laws describing general phenomena. In particular, he describes the ideal photometric signal of a scattering surface by the cosine law. This ideal surface is nowadays called a Lambertian surface. It reflects a constant luminance in all directions and, in the case of a flat surface, the intensity complies with the cosine law. This model gives a constant BRDF whatever the observation direction. It is a very effective tool, still widely used today to simulate the response of matte objects. It allows very fast calculations. However, it must be understood that this is an ideal response: there is no Lambertian surface in reality [5].

The opposite photometric response is that of an ideal reflector. Such surfaces reflect light exclusively in the specular direction. Photometry is not sufficient to describe the amount of light reflected specularly. It took a few decades and the birth of wave optics to understand the reflection coefficients associated with a specular surface depending on the incident light direction and state of polarization. These latter have been introduced by the French scientist Augustin Fresnel, in the early 19th century [6].

Most surfaces of the real world are actually more complex than the ideal Lambertian surface or the ideal reflector. The general optical property is described by the BRDF, expressed in the inverse of the steradian. It depends on the nature of the incident light (wavelength, polarization state) and the directions of both the incident light and the scattered light. Studies on the BRDF focus mainly on angular variations of the scattered flux. The nomenclature was defined by researchers of the U.S. National Bureau of standards (now NIST) [7] in 1977. The measurement of the BRDF requires sophisticated equipment developed mainly by metrology or instrumentation laboratories that are called *goniospectrophotometers*. The challenge for these devices is to measure quantities of light scattered in all directions, while illuminating the sample from controlled directions. For colored or even goniochromatic materials, the spectral bandwidth has to be narrow. For glossy materials, a very good angular resolution is required to accurately describe a sharp specular peak, that imposes the use of very small collecting diaphragms. In both cases,

the signal measured is always very small and the challenge is to get detectors sensitive enough to end up with a correct signal to noise ratio. Since the 2000s, improvements have been made in these directions, combined with significant gains in acquisition times. European metrological research projects have been run to improve the references and measurement techniques [8]. More compact devices have begun to be marketed. France has been an active player in bringing innovation and performance. Innovation by developing devices using a Fourier transform optical approach. Performance by pushing this technique at a level allowing to achieve the best angular resolution in BRDF measurement ever reached by a goniospectrophotometer. This story on BRDF metrology is outlined in Section 2.

The field of computer graphics has importantly contributed to the representation of reflectance since the 1970s. The Lambertian model was employed in the early years for rendering matte surfaces. The specular lobe necessary to give a glossy aspect to a surface was first described by Phong's empirical model [9], improved by Blinn [10]. The microfacet models were introduced a few years later, by the pioneering work of Torrance *et al.* [11, 12]. They consider physically-based parameters for representing a rough reflector with a statistical distribution of purely specular microfacets. The model accounts for the microfacets BRDF, the distribution of their orientations, and shadowing / masking effects. Since then, many questions and improvements have been introduced by researchers around the world [13], including several in France [14]. These French contributions to more physically based BRDF models are presented in Section 3. Finally, in Section 4, we present the uses of BRDF for physically-based rendering and for shape from shading workflow. We once again insist on the contributions of French researchers.

2. BRDF MEASUREMENT

As mentioned in the section above, the BRDF is defined by International Commission on Illumination (CIE) as the function describing the change according to the directions of irradiation $\mathbf{i}(\theta_i, \varphi_i)$ and observation $\mathbf{o}(\theta_o, \varphi_o)$ of the radiance reflected by a surface element centered on the position (x_o, y_o) , at a given wavelength λ , $L_o(\theta_i, \theta_o, \varphi_i, \varphi_o, x_o, y_o, \lambda)$, with respect to the irradiance incident on the surface from the given direction of irradiation at the given wavelength, $E_i(\theta_i, \varphi_i, \lambda)$ [15]

$$f(\theta_i, \theta_o, \varphi_i, \varphi_o, x_o, y_o, \lambda) = \frac{dL_o(\theta_i, \theta_o, \varphi_i, \varphi_o, x_o, y_o, \lambda)}{dE_i(\theta_i, \varphi_i, \lambda)}$$

In this equation, radiance and irradiance are given throughout infinitesimal surface and solid angles, that is obviously not implementable in practice. In order to collect light, a given surface measurement A is defined as well as the irradiation and observation solid angles Ω_i and Ω_o . They all must be considered as small according to the BRDF variation on the surface, in order to provide a valid measurement (see Figure 1). Assuming that the illuminated area and the measurement area are equal, the BRDF $f(\mathbf{i}, \mathbf{o}, A, \lambda)$ can be expressed according to the incident and reflected radiative fluxes by

$$f(\theta_i, \theta_o, \varphi_i, \varphi_o, A, \lambda) = \frac{\phi_o(\theta_i, \theta_o, \varphi_i, \varphi_o, A, \lambda)}{\phi_i(\theta_i, \varphi_i, A, \lambda) \cdot \cos \theta_o \cdot \Omega_o'}$$

with ϕ_i the incident flux that hits the surface A and ϕ_o the reflected flux in the geometrical extent defined by A and Ω_o .

To provide a correct measurement of the BRDF, the goniospectrophotometer must therefore illuminate the sample from a

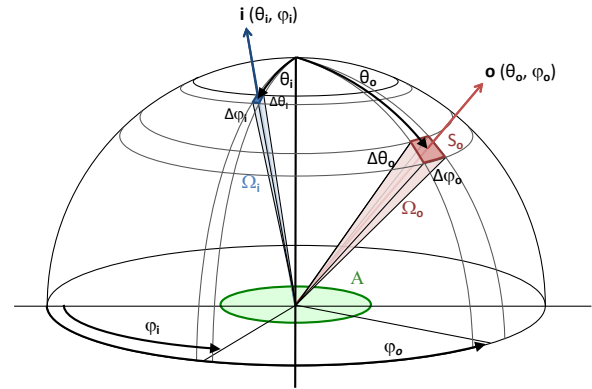


Fig. 1. Angular notations used for the description of the BRDF.

given direction with a known radiant flux, and measure the flux reflected in a given direction, within an aperture S_o that defines Ω_o . In consequence, classical goniospectrophotometers usually present four independent rotations to control $\theta_i, \theta_o, \varphi_i, \varphi_o$ [16–18]. In order to reduce uncertainties, fluxes ϕ_i and ϕ_o are collected thanks to the same detection system, with a single photodiode or an array spectroradiometer according to the type of equipment. The angular resolution of the facility is defined by $\max[\Omega_i, \Omega_o]$. The resolution has to be small enough according to the variation of the BRDF, this is why dimensions of goniospectrophotometers are usually large.

The acquisition of one BRDF value requests several steps: (a) move the sample out of the light, (b) move the detection in front of the light source, (c) measure the flux ϕ_i , (d) block the incident beam, (e) measure the straylight ϕ_{i0} , (f) unblock the incident beam, (g) place the sample in the light area, (h) move the detector to the desired angular position, (i) move the light source to the desired angular position, (j) measure the flux ϕ_o , (k) block the incident beam, (l) measure the straylight ϕ_{o0} , (m) unblock the incident beam. This defines a minimal sequence of 13 steps to collect only one BRDF measurement point. In most of the metrological setup, the measurement of ϕ_i and ϕ_{i0} will be repeated at the end, to cancel uncertainties due to the light source time drift. Besides, the acquisition of ϕ_o and ϕ_{o0} is done at several angular positions between the two acquisitions of ϕ_i and ϕ_{i0} for saving time.

Even when equipped with fast rotation stage or robot-based mechanical designs, the acquisition of the BRDF with goniospectrophotometers remains long. The measurement of a single BRDF point requires about 1 minute, leading to a full measurement time of about 10 hours for covering the half hemisphere with an angular step of 5° , for one wavelength and one direction of illumination. The same operation with an angular step of 1° would require about 10 days, which is practically untractable. Unfortunately, such a sampling (with a step of 1°) is sometimes needed for rendering applications based on real measurements.

In 1993, T. Leroux puts the first stone of a new approach for the measurement of directional optical properties of materials, based on the combination of a Fourier optics and a CCD matrix sensor [19]. At this early stage, Leroux is developing this concept for the characterization of angular dependency of luminance and chromaticity of LCD panels [20]. He founded the company Eldim, which markets the device EZ-Contrast [21]. Few years later, with a beamsplitter introduced in the optical design, Eldim proposes an innovative and fast solution to measure the full

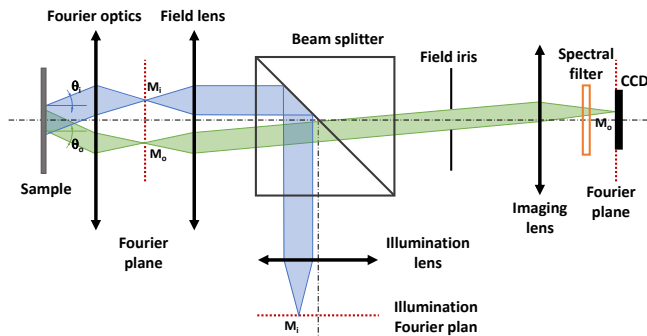


Fig. 2. Principle scheme of EZ-Contrast (from [22]).

hemispherical BRDF of a surface in few seconds, with an angular resolution of 0.4° .

The proposed system is described in Figure 2, where azimuthal angles are not shown in order to avoid complex 3D representation. An optical system provides a Fourier transform image of the material surface. Every light beam reflected from the test area along a direction $\mathbf{o}(\theta_o, \varphi_o)$ is focused on the Fourier transform plane at a location $M_o(x_o, y_o)$, with $x_o = f_o \tan \theta_o \cos \varphi_o$ and $y_o = f_o \tan \theta_o \sin \varphi_o$, f_o being the focal of the Fourier optics. A relay optical system allows imaging the Fourier plane on a cooled CCD sensor. Using a selection of spectral filters placed before the CCD, it is therefore possible to record the luminance, the chromaticity or even the spectrum at each pixel location $M_o(x_o, y_o)$, that corresponds to a given direction of reflection $\mathbf{o}(\theta_o, \varphi_o)$.

The illumination of the sample along the incident direction $\mathbf{i}(\theta_i, \varphi_i)$ is obtained by inverting this principle. When a point light source is introduced in the illumination Fourier plane of the device in $M_i(x_i, y_i)$, the sample is lit by a parallel light beam at the corresponding angle of incidence $\mathbf{i}(\theta_i, \varphi_i)$.

The angular coverage of the device depends upon the first lens of the Fourier optics. It can go up to 88° in extreme cases. The counterpart is that the measurement area cannot be larger than 2 mm. The angular resolution defined by Ω_o depends on the quality of the Fourier optics and size of the light source. The last version of this device is designed for a resolution below 0.2° .

Thanks to this optical approach, it is possible to record the BRDF of a sample at one given direction of illumination $\mathbf{i}(\theta_i, \varphi_i)$ with $\mathbf{o}(\theta_o, \varphi_o)$ such that $0^\circ \leq \theta_o \leq 88^\circ$ and $0^\circ \leq \varphi_o \leq 360^\circ$ with an angular sampling of 0.2° in few seconds instead of the 10 days described above. Using different incident angles and interference filters to select the wavelength, a full BRDF can be practically acquired for the first time. An example of the obtained results of a white sample is plotted in the Fourier plane using sRGB images in Figure 3.

Based on this optical principle, the LNE-Cnam (French National Metrological Institute for radiometric and photometric quantities) started in 2008 a program dedicated to the measurement of the BRDF specular peak. The goal is to study new approaches for measuring gloss attributes. Previous studies had shown that the shape of the specular peak could have an effect in the perception of gloss [23] and that the specular peak for glossy surfaces, i.e. with a specular gloss measured at 60° higher than

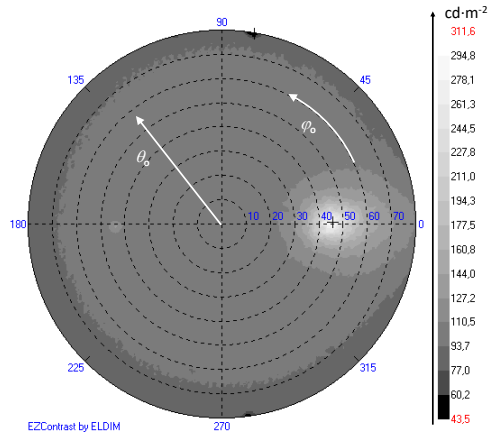


Fig. 3. Example of a BRDF measurement provided by EZ-Contrast, for a white semiglossy sample. Illumination along $\mathbf{i}(45, 180)$. Luminance is recorded for $0^\circ \leq \theta_o \leq 80^\circ$ and $0^\circ \leq \varphi_o \leq 360^\circ$ with a resolution of 0.4° .

80 GU, have a full width at half maximum (FWHM) $\leq 1^\circ$ [24]. In order to study the shape of a 1° width peak, the targeted angular resolution of the setup had been set to 0.03° , that corresponds to the acuity of the human visual system. The facility has been named *Conoscopic Device for Optic Reflectometry* (ConDOR).

To keep the targeted resolution below 0.03° , the illumination part of ConDOR is made of a 200 W lamp placed behind a $V(\lambda)$ filter that illuminates a pinhole of 100 μm diameter. The pinhole is set at the focal plane of a doublet with a focal length of $f = 400$ mm, that gives a divergence of the collimated beam in the image space below 0.015° . The shape of the beam is circular and can be adjusted between 1 mm up to 15 mm of diameter according to the sample under test.

The sample holder is mounted at the extremity of a six axes robot arm that can undergo up to 10 kg. The robot performs three rotations around the axes α , β and γ (see Figure 4). The fourth rotation is obtained with the 2 m diameter ring bearing the illumination. The three remaining degrees of freedom are used for translations along the three axis x , y and z (see Figure 4) that allow the alignment of samples at the center of the goniospectrophotometer. With this mechanical design, ConDOR is able to reach all the angular configurations defined by a given direction of illumination and a given direction of observation belonging to the half sphere above the sample's surface. Measurements in transmission are also possible but limited in term of angular configurations range.

The detection system uses the principle of Fourier optics and was developed in collaboration with Eldim. The system gathers light in a field of $\pm 1^\circ$, located at a distance of 1700 mm from the center of the sample (see Figure 4). Due to the distance, the first lens of this optical system has a diameter of 110 mm. As ConDOR has been developed to study achromatic gloss of surfaces, a $V(\lambda)$ filter is used to restrict the spectral domain to photometry. The Fourier plane is imaged on a high sensitivity cooled CCD camera (Hamamatsu ORCA II), the sensor is a matrix of 512 by 512 pixels. The combination of both water and Peltier cooling maintains a working temperature of -70°C . Using the association of multi-exposure time, extreme care of straylight and a 16 bits A/D converter, we generate images with a dynamic of 7 decades. With this system, the half angle that describes the solid angle Ω_o is 0.004° , and the angular resolution

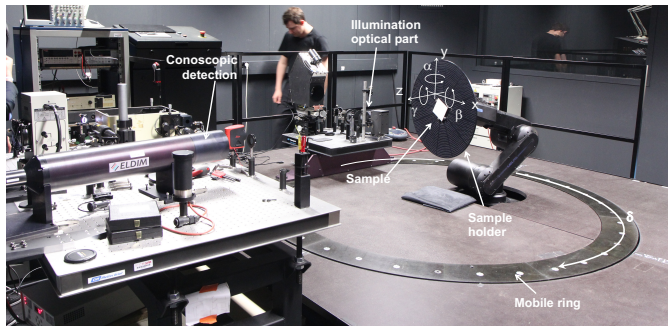


Fig. 4. Picture of the ConDOR BRDF acquisition system.

of ConDOR is defined by $\max[\Omega_i; \Omega_o] = 0.015^\circ$.

The full metrological characterization of ConDOR's detection includes corrections of wavefront across the different optics, of the flux non-linearity (number of counts per second at a CCD pixel is not a linear function of the luminous flux hitting the pixel), of the exposure time non-linearity (real exposure time of a pixel is not equal to the addressed exposure time), and of the optical and thermal noise. Corrections are made pixel per pixel. A validation of the setup has been made by comparison of ConDOR measurement to the primary goniospectrophotometer of LNE-CNAM using a spectralon sample. The deviation of 5% observed with the reference measurement is acceptable, knowing that ConDOR has not been made to measure matte samples. Works are in progress for reducing this deviation.

Thanks to this association of a classical mechanical design and a Fourier optics based detection, ConDOR is today the goniospectrophotometer that has the highest angular resolution ever reached. It can, for the first time, measure the size and the shape of the specular area of the BRDF of glossy surfaces [25], at the same level of information as a human observer [26].

ConDOR brings new and unique metrological knowledge about the optical properties of materials that is of interest for the community working on the measurement of appearance. Since 2012, LNE-CNAM coordinates the European efforts devoted to the development of new primary standards, measurement protocols and transfer artefacts for the control and measurement of appearance. This research is carried on in the frame of EMRP (European Metrology Research Project), a program co-funded by the European Union and EURAMET. The first project called "Multidimensional Reflectometry for Industry" (xDReflect), has tackled metrological issues on goniochromatism, gloss, sparkle and fluorescence between 2013 and 2016 [27]. The second project, called "Bi-directional Reflectance Definitions" (BiRD, 2017-2020), focuses on the definitions and recommendations for BRDF, gloss and sparkle measurements under the frame of CIE [28]. A new project, called "New quantities for the measurement of Appearance" (BxDiff), has been started in May 2019 for establishing primary standards for BTDF and BSSRDF. It will also address issues with BRDF measurement, such as the effect of polarization, speckle and small size samples.

3. BRDF MODELS

The birth of global illumination in the 80s and its expansion since then have favored the development of physically-based models [29–38]. In this context, microfacet representations introduced by Torrance *et al.* [11, 12] have been favored by many authors because of their physical completeness. They corre-

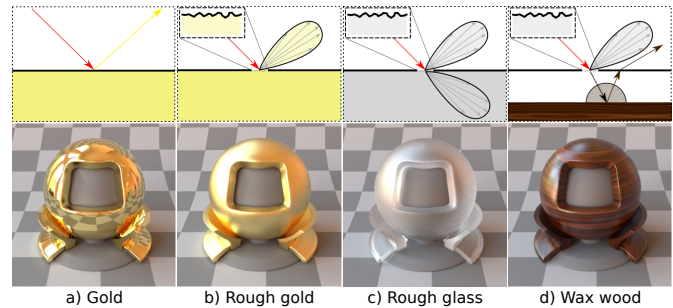


Fig. 5. Four computer-generated images with various materials: a) flat gold surface generated using spectral refraction indices; b) rough gold surface corresponding to Cook and Torrance model [12]; c) rough glass corresponding to the contribution of Walter *et al.* [41]; d) wax wood approximation resulting from a texture associated with a rough dielectric interface.

spond to a statistical distribution of purely specular microfacets. Figure 5 illustrates several computer-generated images of such materials. In France, several outstanding contributions took part in the progression of the domain.

L. Simonot and G. Obein studied the deformation of the reflection by a rough surface in oblique incidence [39]. The width of the specular peak perpendicular to the incidence plane for isotropic roughness decreases in cosine with the incidence angle; the diffusion plane for a highly anisotropic roughness is distorted. They proposed new representations of the BRDF to overcome these geometric deformations as alternatives to Rusinkiewicz's [40].

These properties are properly handled by the microfacet theory, and the remainder of this section is dedicated to this family of models since it has motivated an overwhelming majority of research work in this area.

A. Microfacet representations

Microfacet models consist in describing a rough surface by a set of microfacets defined by its normal distribution function. They are physically-based and the specular peak is consistent with elementary geometric properties. Given incidence and observation directions (resp. denoted by \mathbf{i} and \mathbf{o}), a BRDF based on a distribution of specular microfacets can be written as the product of the microfacets reflectance $F(\mathbf{i}, \mathbf{m})$ given by the Fresnel relations, each associated with a normal \mathbf{m} , the normal distribution function $D(\mathbf{m})$ and the shadowing-masking function $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$ that determines the visible portion of microfacets of orientation \mathbf{m} :

$$f(\mathbf{i}, \mathbf{o}, \mathbf{n}) = \frac{F(\mathbf{i}, \mathbf{m})D(\mathbf{m})G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4|\mathbf{i}, \mathbf{n}||\mathbf{o}, \mathbf{n}|}, \quad (1)$$

where $\mathbf{m} = \frac{\mathbf{i} + \mathbf{o}}{|\mathbf{i} + \mathbf{o}|}$ is the bisector angle between \mathbf{i} and \mathbf{o} and the operator $|\mathbf{u}, \mathbf{v}|$ corresponds to the absolute value of the scalar product between two directions \mathbf{u} and \mathbf{v} .

The initial model considers a Beckmann distribution of normals [42], which corresponds to a Gaussian distribution of slopes. Only one parameter is required to specify the surface roughness. The shadowing-masking function used assumes that the rough surface can be represented by a set of long V-shaped cavities randomly oriented on the surface. The opening of the cavities are given by the chosen distribution [11]. The most controversial point is the calculation of this latter term. Completely correct from a mathematical point of view, this function

is based on physically unrealistic assumptions. To overcome this defect, Smith's approach [43] was introduced in computer graphics by He *et al.* [44]. Smith assumes that the orientation of one facet is not correlated with other facets, even close ones, more in line with BRDF measurements. At that time, the French co-author F. Sillion was in Cornell University, later researcher at INRIA-Grenoble. Besides, C. Schlick *et al.* have proposed a methodology based on rational fraction approximations for reducing computation time of each term of these models [45–47]. Over the past fifteen years, computing performance increase has made it possible to more fundamentally question each factor of microfacet models. In 2007, Walter *et al.* [41] have revisited microfacet theory, starting from the general expression of the microfacet model whatever the BRDF of the microfacets and formally demonstrate the achievement of Equation 1 in the case of specular microfacets. For the anecdote, this work was presented at Eurographics Symposium on Rendering organized in Grenoble in 2007. For the past 10 years, the paper of Walter *et al.* has inspired several French research teams, which in turn have enriched microfacet models, as discussed in the following paragraphs for each factor.

Fresnel term and transparency P. Callet's work takes spectral variations into account in the reflectance of microfacets, calculated from the material's complex refractive index, itself measured by ellipsometry [48, 49].

Walter *et al.* have extended the microfacet theory to transparent surfaces [41] (see Figure 5.c). From this formalism, they define a BTDF (where T stands for Transmittance) model of a rough dielectric interface.

Normal Distribution Function The normal distribution function plays a very important role in material appearance. The GGX distribution (or Towbridge-Reitz [50]) introduced by Walter *et al.* [41] very quickly became an alternative to Beckmann distribution in computer graphics because, with its long tail, it often better approximates the measurements of materials. GGX distribution also comes with Smith shadowing-masking function, offering the advantage of a close-form expression.

Some other functions have emerged, leading to more general models, for two main reasons: (i) designers sometimes require more control on the material parameters; (ii) with measured materials, the existing distributions are not sufficient for accurately fitting data [51–53]. Deviations are sometimes due to non-centered orientations of microfacets around the macrosurface normal. This is why M. Bagher *et al.* propose a shifted Gaussian distribution [51] with one additional parameter for controlling roughness, that better fits many BRDF data, especially for manufactured materials. Unfortunately, the shadowing-masking function cannot be analytically expressed: the authors propose to pre-calculate and tabulate the values.

M. Ribardière *et al.* employ a *Student's T Distribution* (STD) [53, 54], that generalizes both Beckmann and GGX distributions, with only one additional parameter. The advantage is a better control of the distribution bell-shape and tail. Furthermore, the Smith's shadowing-masking function can be derived analytically.

Anisotropic distributions have also been proposed. For example, A. Comar measured BRDF from plant leaves, with an anisotropic Beckmann distribution to simulate the BRDF [55]. This latter representation requires one parameter in the tangent plane for each local coordinate system axis. A more general representation proposed by M. Ribardière *et al.* consists in defining

a piecewise representation dedicated to normal distributions, with the associate shadowing-masking function [56].

Shadowing-Masking Function B. Walter [41] formalized the calculation of the shadowing function according to Smith model based on the work of C. Bourlier, a French radar specialist [57]. In 2014, E. Heitz wrote a research report of more than 50 pages, entitled *Understanding the masking-shadowing function in microfacet-based BRDFs* [58]. He brought several significant contributions to the community: demonstration that Smith's model is equivalent to the tabulated approach proposed by M. Ashikhmin in 2000 [59]; concept of *the distribution of visible normals*, in order to improve importance sampling [60]; generalization of the microfacet approach to participating media containing microflakes [61, 62]; different joint forms of the shadowing and masking functions, including one of the most elaborate approaches with C. Bourlier [63].

Multiple Reflections By construction, microfacet models do not account for multiple light reflections between microfacets. They thus underestimate BRDF, and in practice they are limited to slightly rough surfaces. E. Heitz proposed an approach that would allow multiple reflections to be taken into account with Smith representation and by using a Monte Carlo integration method [64].

D. Saint-Pierre describes and quantifies the impact of multiple reflections in a single V-shaped cavity for Lambertian [65] or specular [66] surfaces, for directional or diffuse illumination.

M. Ribardière *et al.* [56] also propose a method for reconstructing a 3D mesh that corresponds to a given distribution. Path tracing methods are employed for estimating the contribution of light multiple reflections on the microfacets.

The results obtained by the above contributions show that the effects due to these reflections do not only depend on the normal distributions, additional assumptions are required on possible correlations between slopes and heights. These properties have not received much attention yet, to the best of our knowledge.

B. Extending Appearances with Microfacets

Microfacet models have mainly been employed for representing rough conductors and glossy objects. Along with the approach proposed by Walter for transparent objects, French authors have put forward new methods for handling various types of appearances.

Interfaced Lambertian Microfacets Microfacets are often considered ideal reflectors and a Lambertian term is used to model the diffuse response. The specular lobe and the diffuse part are thus considered as two independent components of the BRDF. This approach is not physically correct: modifying the roughness or the refractive index of a surface changes both the specular and the diffuse part of the reflected light. In 1994, Oren and Nayar generalized the model to a distribution of Lambertian microfacets [67], introducing a roughness parameter to describe the diffuse part of the BRDF. In 2001, M. Elias *et al.* developed a model of an *interfaced Lambertian surface* in order to describe the reflection of light by a gloss paint layer [68]. This model considers a flat interface, *i.e.* a change in refractive index, above a Lambertian material. L. Simonot then considered a distribution of interfaced Lambertian microfacets [69]. It includes the classic microfacet models: Cook-Torrance (no scattering), Oren-Nayar (refractive index equal to 1), interfaced Lambertian plane surface (no roughness) and Lambert model (no change of refractive in-

dex and no roughness). The model has recently been improved and brought to computer graphics by D. Meneveau *et al.* [70].

Multi-Layered Surfaces To describe more precisely the scattering of light within a laminated material, multilayer BRDF models have been developed, especially by W. Jakob [71, 72]. L. Belcour proposes an alternative framework [73]. The materials are considered as a succession of media defined by their absorption and scattering properties, and interfaces described for example by a microfacet model. Simonot *et al.* also propose a multilayer BRDF model where light scattering is approximated by a 4-flux model [74]. This article also demonstrates that a model of a rough interface above a Lambertian background is not equivalent to the model of interfaced Lambertian microfacets except when the surface is flat.

L. Belcour and P. Barla also extend the microfacet models to rough surfaces covered with an interference layer [75]. In particular, they propose a method that produces correct iridescent colors with using RGB components instead of a more costly spectral processing.

Diffraction Effects Microfacet models are based on geometric optics and neglect the wave properties of light. However, below a certain scale, the diffractive aspects due to roughness must be handled. The model proposed by N. Holzschuch and R. Pacanowski assumes that BRDF can be decomposed on two scales, one microscopic in the sense of microfacets and the other nanoscopic [76]. In short, micro facets are considered as diffracting elements and no longer as simple ideal reflectors that follow the law of geometric optics. There is therefore a convolution of a diffraction term on a microfacet distribution. By using some approximations, this convolution can be summarized in a semi-analytical and tabulated manner, which makes it possible to obtain a BRDF model as the sum of a lobe from microfacet model and a lobe from diffraction. Handling diffraction also improves the acquisition process [77, 78].

C. Higher-level Material Representations

Even though BRDF representations have motivated a lot of interest, the appearance of materials is much more complex, with low- and high-frequency variations and/or surface weathering for instance. Higher-level models are mandatory for modeling real-world surfaces. We would like to mention several valuable French contributions in this area.

G. Gilet and his co-authors have proposed several methods for producing noise and control methods for generating procedural textures, notably with multi-scale approaches [79, 80]. In addition, a set of methods have been proposed for generating detailed textures [81, 82], including volumetric representations [83, 84].

Besides, weathering has been addressed for long by several authors, notably by S. Merillou *et al.* [85, 86]. Surfaces change over time, and their appearance may be altered with pollution or weathering. Good examples in this area concern the simulation of urban pollution for increasing the realism of 3D cities. Several French authors have also proposed methods for modeling and rendering surface scratches, that alter the surface geometry and roughness [87, 88].

4. APPLICATIONS

The physical description of material appearance has brought interesting improvements in various applications. For instance with computer graphics and global illumination, the use of BRDF

models requires additional technical constraints. First of all the BRDF have to be calculated given incident and observation directions. Furthermore, with ray-tracing and path tracing systems, importance sampling allows to efficiently gather incoming contributions in the specular lobe. With shape-from-shading applications, the BRDF formulation should be ideally invertible. Unfortunately, Lambert formulation only complies with this constraint, and complex representations practically require setting advanced resolution mechanisms for fitting data.

A. Physically-Based Rendering

The direct use of a reflectance model looks straightforward with computer-generated images. However, physically-based systems very often rely on lighting simulation techniques that are constrained by numerical integration. Basically, light transfer has been formalized by J. Kajiya in 1986 [89], who also proposed the use of Monte-Carlo integration, with a process called *path tracing*. These methods consist in: (i) determining the surface (or set of surfaces) visible through each pixel of an image, and (ii) estimating the radiance reflected toward the camera. The process requires integrating all incoming radiances from the hemisphere. The general idea consists in following light paths through space, while managing light contributions [89]. In practice many rays are traced through each pixel, each of them is the source of a path. When a ray hits a surface, it is reflected according to several parameters: direct illumination sampling with known light sources, and importance sampling according to BRDF for noise reduction (uniform sampling on the hemisphere is prone to noisy artifacts with glossy surfaces).

In this context, many studies have been proposed with various types of material representations. Important contributions have been proposed in the area, with several goals, described in the next paragraphs.

BRDF Parameterization and fitting Unfortunately, no single general model is adaptable to the very wide variety of materials existing in the real world. Fitting the parameters of existing models from measured data only rarely provide comparable predictive curves and appearances. Several authors have employed basis functions for exploiting BRDF data with global illumination algorithms. For instance, cosine lobes have been first employed by Phong [9], for empirically representing glossy materials. The idea has been extended by Lafortune [90] for managing several lobes.

In France, such representations have been used for fitting image-based data [91], and for interactive rendering [92] for instance. N. Noé and B. Péroche have employed a hierarchical approach and locally supported functions [93]. L. Claustres *et al.* have employed wavelets for representing and efficiently rendering materials [94, 95]. C. Soler *et al.* benefit from a Gaussian process for representing, managing and interpolating BRDF data [96, 97]. J. Dupuy *et al.* also propose a method for recovering microfacet distribution parameters and refraction indices using power iterations [98].

Besides, the choice of a BRDF correct parametrization is not straightforward because of high frequency effects and anisotropy. Several French authors have proposed studies for determining a hemispherical representation that relies on the reflection geometry [99–102].

Rendering and Global Illumination The representations and parameterizations described above have been often designed for improving both the model quality and the global illumination process. In this context, one of the most difficult challenge

concerns the estimation of the incoming radiance distribution, crucial for providing accurate results, especially with glossy surfaces. J. Zaninetti and B. Péroche have proposed a vector approach for approximating the incident light flux [103]. Generally speaking, most global illumination methods were first designed for Lambertian surfaces, and derived later for more general materials.

Importance sampling have been used with path tracing, so as to improve convergence [104]. The idea is to rely on the BRDF reciprocity for gathering incoming radiance from the gloss region of interest. While these methods are favored for their physical correctness, other rendering systems have been developed for providing visually correct results in lower computation time (for instance dedicated to real-time rendering used in video games).

Radiance caching methods consist in interpolating between values. The initial method proposed by Ward [105] was designed for Lambertian surfaces. It has been extended by P. Gautron *et al.* to glossy materials, using spherical harmonics, but their use remains limited to low-frequency BRDFs because of the number of coefficients [106, 107]. Highly glossy surfaces can be managed thanks to another method, proposed by M. Omidvar *et al.* [108].

Several French authors have also proposed approaches for managing multi-scale appearance of surfaces. Normal maps and displacement maps are often used for managing meso-scale surface variations. When the user moves away, these variations are likened to microgeometry, and the appearance can be managed using microfacet models. For instance, Dupuy *et al.* have proposed a method for properly handling the change of scale [109] in the context of interactive rendering. More recently, X. Chermain *et al.* propose to employ a mixture of anisotropic and noncentered Beckmann distributions [110].

Another interesting area concerns the user control of appearance. Artists often need to change the material according to their rendering needs. For instance, P. Barla *et al.* propose several methods for controlling highlights on glossy objects [111, 112].

B. Shape from Shading

Initially proposed by B.K.P. Horn in the 70s [113], shape from shading approaches consist in estimating the surface geometry from one photograph, based on a known illumination. Traditionally, the BRDF is considered as Lambertian and only one light source illuminates the surface. With the emergence of digital cameras, significant improvements have been proposed at international and French levels. E. Prados and O. Faugeras [114] have demonstrated the importance of the distance attenuation term of the illumination for obtaining a well-posed problem. F. Courteille *et al.* [115] aim at reducing the number of unknowns, using a B-spline to approximate the surface, with a stochastic optimization process. Even nowadays, shape from shading is considered as difficult and non-Lambertian surfaces lead to ill-posed problems.

Based on the same idea, R.J. Woodham [116] proposes to use more than one image, for making the problem well-posed. Under the Lambertian assumption, and for two directions of illumination, if the reflectance is uniform on the whole surface, the mathematical equation has only one solution. This technique is known as photometric stereo and two steps are generally required to estimate the surface geometry: according to the scattering of light by a surface, the first one estimates a gradient field and the second one consists of a 2D integration (see Figure 6). Because acquiring more than one image might be intractable for many applications, some authors have proposed to use a multispectral imaging systems where gray images used in case

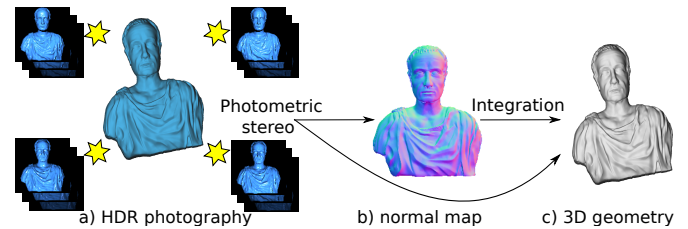


Fig. 6. Photometric stereo pipeline used for 3D geometry measurement; historically, a normal map (b) is computed from a series of images with fixed viewpoint and several fixed light sources (a) and a BRDF model, then an height map (c) is reconstructed. More recently, the height map (c) is estimated directly from the series of images (a) and a BRDF model.

of usual photometric stereo are replaced by the three sensor channels (red, green and blue). Based on the work proposed by C. Hernandez [117] or B. Bringier [118], E. Prados [119] tackles the challenge of textured surfaces in multispectral photometric stereo system. Unfortunately, this technique can again not be easily extended to non-Lambertian surfaces with only three images. S. Barsky and M. Petrou [120] propose to detect shadow or specular reflection to overcome this non-linear system, with four directions of illumination and the four corresponding images. They assume that light scattering can be approximated by the sum of a Lambertian and a specular component. An extension of this work for n -directions of illumination has been developed by B. Bringier *et al.* [121]. The concept of scattering separation has the advantage to be very fast compared with other methods that use a non-Lambertian BRDF model and a mathematical optimization algorithm. Based on R. Mecca research [122], Y. Quéau *et al.* [123] consider specular reflection as noise in case of many illumination directions and propose a new methodology that solves the photometric stereo challenge in a single step with a variational approach. As shown in the state of the art of photometric stereo, dealing with a complex BRDF model is complicated and an acceptable compromise has to be found between the amount of acquired data required, the computing time and the depth of scattering light analysis.

In the context of the computer vision, another popular method in French research teams for estimating the reflectance parameters of an object is the multi-view processing [91, 124, 125]. This approach is less direct than shading analysis to estimate all parameters of a surface (geometry, color and appearance) because geometry is measured from the position of an object in relation to the camera's direction.

5. CONCLUSION

Since the 2000s, a research community has been active in France around both the measurement and the modeling of BRDF. This multidisciplinary community is federated around the publication of a collective book in French (27 chapters, 40 authors) whose title can be translated by *When matter scatters light* [14]. This community takes part in the national research group on the material appearance (GDR APPAMAT CNRS 2044) created in January 2019. It brings together more than 160 researchers from 27 laboratories across the country and in a wide variety of disciplines, as well as people from the industrial sector.

As stated along this paper, several challenges remain to be overcome. For instance, glossy-colored material are difficult to represent because of the interdependence between the specular

lobe and the colored appearance. More generally, fitting data with predictive models is from our point of view still unresolved, partly because the models do never perfectly define the materials. One of the interesting issue concerns the management of the error (or difference between prediction and measurements). Another enthusiastic question is related to surface manufacturing. Many industries are interested in predicting surface appearance for specific needs with simulation tools so as to avoid a costly construction process for validating a chosen appearance. Unfortunately in many cases the virtual models do not reach the reality. Finally, physical descriptions are also mandatory for recovering and / or identifying information from photographs (or more generally acquisition systems). Therefore, appearance models are at the heart of many researches.

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