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# Sustainable management of natural resource using bipolar analysis. Application to water resource sharing.

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## Abstract:

This communication considers a problem of sharing a continuous resource (a duty taken on by a central authority) among a certain number of users in order to satisfy as much as possible some interests declined in terms of objectives by stakeholders (that constitute a community). Such problems are encountered in many socioeconomic domains and mainly in situations of sharing natural resources such as water resource for different socioeconomic activities such as drinking, irrigation, breeding, industrial used, etc.. Formerly speaking, these problems constitute multi-objectives optimization problem solved in the literature by a plethora of approaches. In this communication, we choose to highlight bipolar analysis approach that permit to introduce some flexibility (a necessity for such problems) during the recommendation phase. Bipolar analysis allow to aggregate separately incentives of the same nature (positive incentives separated from negative ones) so that the trade-off process will be reduced to balancing positive and negative aspects.

*Keywords:* Bipolar Analysis, Multiple Objectives Optimization, Continuous Resource Sharing, Satisficing Game Theory, Group Decision Making.

## 1. INTRODUCTION AND PROBLEM STATEMENT

Natural resources sharing is a big issue nowadays because of the multiplicity of parties interested in such process with potentially antagonist stakes and the ultimate necessity to ensure sustainability in the usage of these resources. One such natural resource that can be almost qualified as critical or strategic resource is water so that researchers are devoting their effort to develop methods and procedures to manage efficiently and sustainably this resource in the interest of humanity. For instance, in Davijani *et al.* (2016), the problem of optimizing water resource allocation to maximize socioeconomic efficiency has been considered. As main objective functions, authors of this study consider the maximization of economic profit in various sectors including agriculture sector, industry sector, municipality (in terms of water for drinking, sanitation and recreation) and the maximization of employment as a social performance indicator in these sectors mainly in that of agriculture and industry. A multi-objectives optimization model is developed and solved using soft-computing techniques such as evolutionary computation algorithms Zitzler (1999). A distributed constraint optimization problem (DCOP) have been used in Amigoni *et al.* (2015) for modeling the management of water resource systems consisting of farmers, a dam, economic operators, and a city. The problem here consists in supplying these different actors in water withdrawn from the dam when meeting some requirements of minimum inflow to some systems and maximum withdraw from the dam. In

Singh *et al.* (2015), authors use multiple criteria decision making approach to evaluate alternative management options in dairy farming with water resource limitation as a constraint to satisfy. Many other applications related to water resource management and/or sharing with sustainability stakes can be found in references cited in the above analyzed publications. The main characteristics that one can draw from previous mentioned studies are that, water resource sharing and/or management problems consist in optimizing many objectives, which objectives depend on multiple attributes of alternative options, and that many actors that we refer to as stakeholders' concerns or preferences must be taken into account when deriving the resolution procedure. These problems therefore fall into a large framework known as multi-objectives / multi-attributes or criteria and group decision making problems. Most of the approaches (that in general consider only one aspect of that mentioned here) used to solve such problems up to now in the literature, consist in transforming the "multi" into mono using some aggregation approach in order to use the well established "mono" optimization's algorithms that abound in optimization literature (Luenberger (1984); Moré and Wirright (1993)) or in searching for Pareto (see Pareto (1896)) set and then choosing an "appropriate" alternative within this set when using some additional information or constraints. These approaches do not distinguish, in earlier stages of decision process, the difference that may exist between positive incentives of an alternative and its negative impact with regard to a given

objective. In this communication we will highlight the necessity to consider separately the positive and negative aspects during evaluation process; indeed, any beneficiary or user of such natural resource as water will generate positive incentives for some aspects of the humanity or community but also negative impact on other aspects.

The remainder of this communication is organized around following items: in the second section the considered problem will be formally defined; a third section is devoted to recalling quickly main approaches used in the literature to solve similar problems with most of the time restricting the approach to one aspect (for instance without taking into account the inherent interaction between components of the problem for instance); in section four the main contribution of this communication is presented, continuous resource sharing integrating many ignored aspects of the literature are presented with the concept of bipolarity as the underlying modeling notion; section five considers using the developed approach to solve an illustrative application problem; and section six concludes the communication.

## 2. FORMAL SPECIFICATION OF THE PROBLEM

In this communication we consider a problem where a central decision maker (a central government, a regional authority, a municipality, etc.) must decide which quantity of a certain resource (water for instance) it must give to each user in order to satisfy some collective objectives (considered as a social choice) as well as individual or local objectives such as getting a minimum of the shared resource; more precisely we consider a water management problem that consists in deciding which quantity of water withdrawn from a common infrastructure such as a river or a dam an authority must assign to each potential user for its activity. Formerly we consider a certain continuous quantity  $Q$  (without loss of generality we consider that  $Q = 1$ ) of a natural resource (quantity of water that can be withdrawn from a natural or man made infrastructure for some period) must be shared among  $n$  users for their activities. The activities of users generate positive outputs as well as negative outcomes with regards to a broad panel of objectives that stakeholders engaged in the decision process want to achieve. The assignment problem is therefore organized around following elements.

- A discrete set  $\mathcal{O} = \{o_1, o_2, \dots, o_l\}$  of objectives that constitute a social choice of the community of stakeholders (a stakeholder is an individual, a group of individuals, an organization, etc. that are interested in or are impacted by the outcome of decision being made) that the central decision maker must satisfy as much as possible is identified; meanwhile community may express its preference by ranking objectives by assigning them some weight where for instance  $\omega_j$  will be the global weight assigned to objective  $j$ . Determination of such weights may not be easy, but some recent research in the domain of voting, measuring and ranking show that Condorcet technique that is rank compatible, see Balinski and Laraki (2007), can be used for this purpose. Indeed, Balinski and Laraki (2007) proves that Condorcet's method is appropriate for ranking, so by using this approach where each stakeholder will supply a rank profile of objectives,

Condorcet count will constitute a sort of brut measure of the weight to assign to each objective. Alternatively each stakeholder may use pair wise comparison such as AHP (see Saaty (1980)) to assign a weight to each objective so that actual weight will be commensurate to total weights from stakeholders.

- $0 \leq x_i \leq 1$  is the proportion of quantity  $Q$  given to stakeholder  $i$  and  $x = [x_1, x_2, \dots, x_n]^T$  is the assigned proportion vector.

Let us designate by  $\mathcal{X}$  the compact set

$$\mathcal{X} = \left\{ x : x_{i,\min} \leq x_i \leq x_{i,\max}, \sum_{i=1}^n x_i \leq 1 \right\}; \quad (1)$$

where  $x_{i,\min}, x_{i,\max} \in [0, 1]$  are minimum and maximum proportion of resource allowable to stakeholder  $i$ ; the assignment problem can be therefore stated as: find the vector  $x \in \mathcal{X}$  such that following objectives are satisfied

$$\begin{cases} \max_x \{f_k^+(x)\}, \forall k = 1, \dots, l_p; \\ \min_x \{f_k^-(x)\}, \forall k = 1, \dots, l_m; \end{cases} \quad (2)$$

where  $f_k^+$  (positive incentive),  $k = 1, \dots, l_p$  and  $f_k^-$  (negative impact),  $k = 1, \dots, l_m$ , are some objective functions to optimize ( $l_p$  is the number of objective functions that must be maximized and  $l_m$  those that are to be minimized); this is a multi-objectives decision making or optimization problem.

Besides of continuous resource (water) allocation problem being considered in this communication, many real-world problems are often formulated in terms of multiple objectives optimization problems, see for instance Coello Coello (1998); Steuer (1986); Zitzler (1999) and references therein for some practical cases.

The broad and practical nature of multi-objectives optimization problem attracted many research approaches and methods to solve these problems in the best ways as much as possible; these methods that can be referred to as classical methods are reviewed in the following section.

## 3. CLASSICAL APPROACHES FOR SOLVING MULTI-OBJECTIVES OPTIMIZATION PROBLEMS

Some textual materials of this section come from Tchangani (2010). Many classical approaches for solving multiple objectives decision problems rely on the notion of the so-called Pareto dominance Zitzler (1999) and Pareto-optimal set and the resolution is organized around two processes: search and decision making. Depending on how search (finding a sample of Pareto-optimal set) and decision process are combined, multiple objectives optimization methods can be classified in three categories Zitzler (1999).

- Decision making before search: the objective functions are aggregated into a single objective by using some preference of the decision maker.
- Search before decision making: here a sample (or totality) of Pareto-optimal set is obtained first and then a choice is made by a decision maker.
- Decision making during search: an interactive sequential optimization is performed where after each search step, the decision maker is presented with a number of alternatives and decide to stop or continuous the search.

The first approach to deal with multiple objectives decision making problems has been the aggregation of objectives into a single objective in different ways leading to weighting methods, constraint methods and goal programming methods. The advantage of these methods is that efficient and broad algorithms developed for single objective optimization problems (Luenberger (1984); Moré and Wiright (1993) and references therein) can be used to solve the resulting problems. The drawback of these techniques is that the subjective intervention of the user is needed to fix weights for factors and it is known, see for instance Zitzler (1999), that these methods are most of the time not able to finding Pareto-optimal solutions in the case of non convex feasible space. To overcome these drawbacks, new methods have been designed based on evolutionary algorithms, mainly genetic algorithms that are able to generating efficiently Pareto-optimal solutions.

Classical basic methods that are used to solve these problems are:

- weighting method, Steuer (1986), where original multiple objectives optimization problem is converted to a single objective;
- constraint method, Steuer (1986): here  $l - 1$  objective functions are transformed into constraints and the remaining objective is optimized under these constraints.
- goal programming, Charnes *et al.* (1955): some goals are defined for each objective function as: less-than-equal-to,  $f_j(x) \leq t_j$ ; greater-than-equal-to,  $f_j(x) \geq t_j$ ; equal-to,  $f_j(x) = t_j$ ; and within a range,  $f_j(x) \in [t_j^l, t_j^u]$ ; and the problem now consists in finding an assignment that ensures these goals; many variants of this approach exist in the literature.
- evolutionary algorithm, see Zitzler (1999).

Rather than proposing a revolutionary (if any) approach for solving problems presented so far, this communication is dedicated to deriving a practical oriented framework for efficiently constructing objective functions based on decisions makers preferences, the problem at hand, and most importantly, the underlying concept (or ultimate goal) that the final decision must follows or satisfies. Given, economic, social, and environmental concerns manifested by citizens nowadays, the concept of sustainability that relies on these three objectives can be the underlying ultimate goal of natural resource allocation problem such as that considered in this communication. In the following section, resource allocation among a certain number of activities that generate positive incentives as well as negative impacts on the well being of humans will be formulated when taking into account many issues ignored in classical approaches such as: interaction between attributes in the contribution or opposition in the realization of an objective, interaction between objectives in the realization of ultimate goal, or the relative importance of each activity, etc..

#### 4. PROPOSED APPROACH

Most difficulties in modelling decision making problems such as that of resource sharing presented in the previous sections are related to formulating functions to optimize; constraints are in general straightforward as they are re-

lated to available resource limitation. In this communication we rely on the notions of positive (or supporting) incentives and negative (or rejecting) incentives an activity may convey with regards to community's objectives to satisfy to design an assignment procedure to balance these aspects, an approach referred to as bipolar analysis, see for instance Tchangani (2015, 2014); Tchangani *et al.* (2012); Tchangani (2006), for some developments related to these issues. Indeed, we define  $\nu_i^+(o_k, x)$  (respectively  $\nu_i^-(o_k, x)$ ) to be the positive outcome (respect. negative outcome) generated by the activity of stakeholder  $i$  on the community objective  $k$  for a particular assignment  $x$ . Performance indicators  $\nu_i^+(o_k, x)$  (respectively  $\nu_i^-(o_k, x)$ ) can be obtained by combining (in some sense) positive and negative function defined in equation (2) respectively or can simply be obtained using AHP (see Saaty (1980)) by answering question like "how positively (respect. negatively) does stakeholder  $i$  contribute to the objective  $k$  compared to stakeholder  $j$ ".

##### 4.1 Interaction between activities

In order to build an integrated framework for resource allocation that takes into account, as much as possible, the concerns of all stakeholders, one must consider possible interaction between activities of users. Indeed, in some cases, it may happen that the outcome or output of the activity of a user  $i$  affect the realization of activity of user  $j$  in the benefit of the community. To take this issue into account in our model, we introduce interaction indices  $a_{ij}^+(o_k)/a_{ij}^-(o_k)$  that measure the positive (respect. negative) contribution degree of the activity of user  $i$  in the realization of that of user  $j$  contributing to objective  $o_k$ . Building on Saaty and Vargas (1982), we propose a procedure in two steps to determine these degrees by experts for instance.

*Step 1: relative contribution of users to the realization of objectives* One determine a relative importance degrees  $0 < c_i^{r+}(o_k) < 1$  (respect.  $0 < c_i^{r-}(o_k) < 1$ ) that represents the relative contribution of user  $i$  in the satisfaction (respect. dissatisfaction) of objective  $o_k$  by using an AHP analysis for instance.

*Step 2: interdependency between users' activities* In a second step, degrees  $d_{ij}^+(o_k)/d_{ij}^-(o_k)$  are determined to represent the relative importance of the activity of user  $i$  to that of user  $j$  toward positive (respect. negative) incentives of objective  $o_k$ . To do so, for each user  $i$ , one can do a pair wise comparison of other  $n - 1$  users by answering question like "how important is the activity of user  $i$  to the realization of that of user  $j$  compared to that of user  $l$  in the achievement of positive (respect. negative) incentive of objective  $o_k$ ?" using AHP approach or any other procedure.

Finally the degree  $a_{ij}^+(o_k)/a_{ij}^-(o_k)$  can be obtained as given by equation (3)

$$a_{ij}^+(o_k) = c_i^{r+}(o_k)d_{ij}^+(o_k) ; a_{ij}^-(o_k) = c_i^{r-}(o_k)d_{ij}^-(o_k) \quad (3)$$

with the conditions of equation (4)

$$\sum_{i=1}^n c_i^{r+}(o_k) = 1 \text{ and } \sum_{j=1, j \neq i}^{n-1} d_{ij}^+(o_k) = 1 \quad (4)$$

where  $\times$  stands for  $+$  or  $-$ . Finally for each objective  $o_k$  users will interact through the interaction matrices  $A^+(o_k) = [a_{ij}^+(o_k)]$  and  $A^-(o_k) = [a_{ij}^-(o_k)]$ ; but this is the immediate interaction effects; in real world situation influence may circulate within the interaction network infinitely so that the overall or total interaction or influence matrices  $T^\times(o_k)$  will be given by equation (5)

$$T^\times(o_k) = \sum_{l=1}^{\infty} (A^\times(o_k))^l = A^\times(o_k) (I - A^\times(o_k))^{-1} \quad (5)$$

that always exists thanks to classical Perron-Frobenius theorem, see Meyer (2000). Indeed, by construction the sum of the rows as well as the sum of the columns of matrix  $A^\times(o_k)$  is less than 1 that ensures that the matrix  $I - A^\times(o_k)$  is invertible and all coefficients of its inverse are non negative.

#### 4.2 Bipolar allocation procedure

From the total interaction matrix  $T^\times = [t_{ij}^\times(o_k)]$ , users can be classified according to their global contribution to the objective  $o_k$ ; indeed one can calculate the global contribution  $D_i^+(o_k)$  (respect.  $D_i^-(o_k)$ ) of user  $i$  to the activities of other users as given by the following equation (6)

$$D_i^\times(o_k) = \sum_{j=1}^n t_{ij}^\times(o_k) \quad (6)$$

and the global contribution of other users  $R_i^\times(o_k)$  to the realization of activity of user  $i$  with regard to objective  $o_k$  given by equation (7)

$$R_i^\times(o_k) = \sum_{j=1}^n t_{ji}^\times(o_k). \quad (7)$$

One can therefore consider ranking users using the index  $D_i^\times(o_k) - R_i^\times(o_k)$ : users for which  $D_i^\times(o_k) - R_i^\times(o_k) > 0$  constitute global active or dispatching users of incentive  $\times$  in the realization of objective  $o_k$  whereas those for which  $D_i^\times(o_k) - R_i^\times(o_k) < 0$ , are receiver or passive users for the corresponding incentive  $\times$ . This remark may be a basis for resource allocation procedure by choosing for instance the appropriate multi-objectives optimization scheme. Indeed, one can consider that the activity of a user is most important that it influences activities of many other users that is ranking with regards to the parameter  $D_i^\times(o_k)$  or that it is more an active user than a passive user that is using parameter  $D_i^\times(o_k) - R_i^\times(o_k)$  to rank; this will constitute a basis to determine positive and negative relative importance degrees  $\varpi_i^+(o_k)$  (respect.  $\varpi_i^-(o_k)$ ) of the activity of each user  $i$  with regard to objective  $o_k$ . The global positive  $\Psi_+^k(x)$  (respect. negative  $\Psi_-^k(x)$ ) contribution of a particular allocation  $x$  for the objective  $o_k$  is therefore given by equation (8)

$$\Psi_\times^k(x) = \sum_{i=1}^n \varpi_i^\times(o_k) \nu_i^\times(o_k, x). \quad (8)$$

From materials defined above and building on satisficing game theory (Stirling (2003)), we define the overall selectability measure  $\mu_S(x)$  and the overall rejectability measure  $\mu_R(x)$  for a particular assignment  $x$  by equation (9)

$$\mu_S(x) = \sum_{k=1}^l \omega_k \Psi_+^k(x) \text{ and } \mu_R(x) = \sum_{k=1}^l \omega_k \Psi_-^k(x) \quad (9)$$

where  $\omega_k$  is the relative importance degree of objective  $o_k$  with regards to the overall allocation goal.

The following paragraph presents approaches to select the best alternative  $x$  arguable to be satisficing or good enough allocation.

#### 4.3 Satisficing or good enough allocation

The satisficing or good enough allocations are those for which selectability degree  $\mu_S(x)$  exceeds a non decreasing function  $q$  of the rejectability degree  $\mu_R(x)$ . The set  $\Sigma_q$  of satisficing or good enough allocations is therefore defined by equation (10)

$$\Sigma_q = \{x \in \mathcal{X} : \mu_S(x) \geq q(\mu_R(x))\}; \quad (10)$$

the shape of non decreasing function  $q$  can be used to manage the attitude of decision maker; a typical shape of this function is given by the following Figure 1 that enhances the fact that many decision makers exhibits a risk taking attitude for low negative impact (rejectability degree) and risk aversion attitude for high negative impact; risk neutral attitude correspond to the case  $q(\mu_R(x)) = \mu_R(x)$ .

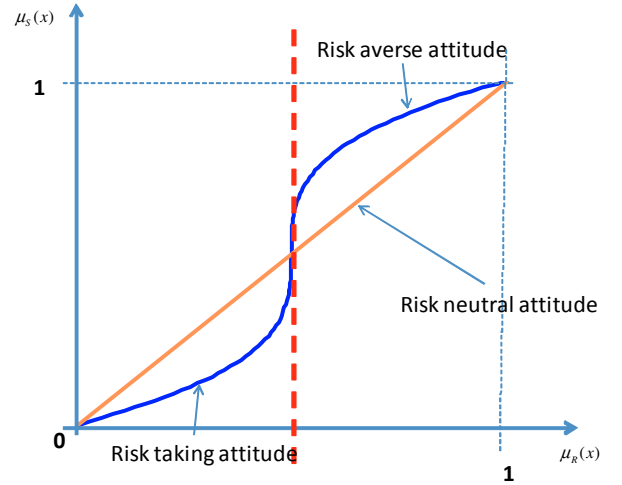


Fig. 1. Shape of boldness function  $q$  taking into account risk attitude

A particular satisficing alternative  $x \in \Sigma_q$  can be calculated by solving the following nonlinear programming problem (11)

$$\min_x (0) \text{ s.t. } -\mu_S(x) + q(\mu_R(x)) \leq 0, x \in \mathcal{X} \quad (11)$$

that can be solved using a general purpose software such as Matlab<sup>TM</sup> with Optimization Toolbox or writing one's own code. The final best alternative can then be selected using different criteria such as most selectable alternative, least rejectable alternative, maximal discriminant alternative, maximum boldness alternative, or other.

The framework established so far which is a particular case of BOCR analysis, see for instance Tchangani (2015), can be considered as a dimensionality reduction approach, see Giuliani *et al* (2014), based on practical considerations.

Furthermore some parameters of the framework such as boldness function can be used to address real world applications' concerns such as that raised in Farina and Amato (2002).

## 5. APPLICATION

In this section we will illustrate the approach established so far by considering a small practical case study. Authorities of a rural area are confronted to a problem of efficiently managing water coming from a lake of this area. There are four main users (that we refer to as stakeholders) that utilize water from the lake for their activities, namely:

- Municipality that has to withdraw water from the lake for domestic use, that we refer to as stakeholder  $S_1$ ;
- Farmers that need water from the lake for irrigation purpose, known as stakeholder  $S_2$ ;
- Besides agriculture activity of previous point, there is a great breeding activity in the area that necessitates water from the lake, stakeholder  $S_3$ ;
- There is an industrial plant installed in the area that needs to use water from the lake for its activity, we call it stakeholder  $S_4$ .

Here  $x_i$  is the quantity of water stakeholder  $i$  is allowed to withdraw from the lake. The problem for these authorities is therefore to determine which quantity of water each stakeholder is allowed to withdraw from the lake each year in order to ensure sustainable development of the area; so that, as it is well accepted from the sustainability concept, there are three main objectives to satisfy when deciding how to share water from the lake, namely economic objective ( $o_1$ ), social objective ( $o_2$ ), and environmental objective ( $o_3$ ).

For this illustrative application (and because of the limitation of the communication length) we will ignore potential interaction; meanwhile we derive directly relative importance degrees  $\varpi_i^+(o_k)$  and  $\varpi_i^-(o_k)$  by using the following procedure. For each objective  $o_k$ , two comparison matrices  $\Omega_{o_k}^+$  and  $\Omega_{o_k}^-$  of stakeholders are built using classical AHP method to represent positive comparison score and negative comparison score respectively. A simple way to do this is to choose a pivot stakeholder  $S_p$  and compare other stakeholders against it by answering question like "how important is the positive (respectively negative) contribution of stakeholder  $S_i$  to the objective  $o_k$  in comparison of the contribution of pivot stakeholder?" to obtain notes  $v^+(i, p)/v^-(i, p)$  from the classical AHP table. Consistent matrices  $\Omega_{o_k}^+$  and  $\Omega_{o_k}^-$  are then obtained by equations (12)-(13)

$$\Omega_{o_k}^+(p, i) = \frac{1}{v^+(i, p)}; \quad \Omega_{o_k}^+(i, j) = \frac{v^+(i, p)}{v^+(j, p)} \quad (12)$$

$$\Omega_{o_k}^-(p, i) = \frac{1}{v^-(i, p)}; \quad \Omega_{o_k}^-(i, j) = \frac{v^-(i, p)}{v^-(j, p)} \quad (13)$$

Final positive/negative relative importance degrees  $\varpi_i^+(o_k)$  (respect.  $\varpi_i^-(o_k)$ ) of stakeholder  $S_i$  for objective  $o_k$  are determined from comparison matrices by equations (14)-(15)

$$\varpi_i^+(o_k) = \frac{1}{n} \left\{ \sum_{j=1}^n \left\{ \frac{\Omega_{o_k}^+(i, j)}{\sum_{l=1}^n \Omega_{o_k}^+(l, j)} \right\} \right\}; \quad (14)$$

$$\varpi_i^-(o_k) = \frac{1}{n} \left\{ \sum_{j=1}^n \left\{ \frac{\Omega_{o_k}^-(i, j)}{\sum_{l=1}^n \Omega_{o_k}^-(l, j)} \right\} \right\}. \quad (15)$$

We will illustrate this approach for objective  $o_1$  (economic objective) of our application to get comparison matrices of equations (16) and (17)

$$\Omega_{o_1}^+ = \begin{pmatrix} o_1 & S_1 & S_2 & S_3 & S_4 \\ S_1 & 1 & 1/5 & 1/3 & 1/9 \\ S_2 & 5 & 1 & 5/3 & 5/9 \\ S_3 & 3 & 3/5 & 1 & 3/9 \\ S_4 & 9 & 9/5 & 9/3 & 1 \end{pmatrix}; \quad (16)$$

$$\Omega_{o_1}^- = \begin{pmatrix} o_1 & S_1 & S_2 & S_3 & S_4 \\ S_1 & 1 & 1/2 & 1/2 & 1/2 \\ S_2 & 2 & 1 & 1 & 1 \\ S_3 & 2 & 1 & 1 & 1 \\ S_4 & 2 & 1 & 1 & 1 \end{pmatrix}; \quad (17)$$

it means that in terms of economic objective ( $o_1$ ), experts consider that farmers ( $S_2$ ) contribute 5 times, agriculture ( $S_3$ ) 3 times, and industry ( $S_4$ ) 9 times positively than municipality ( $S_1$ ); and these three activities contribute each 2 times negatively to economic objective than municipality; other relative proportions are straightforward to interpret. Similar considerations for other objectives lead to results of following Table 1 in terms of relative importance degrees.

Stakeholders	$\varpi_i^+(o_k)$			$\varpi_i^-(o_k)$		
	$o_1$	$o_2$	$o_3$	$o_1$	$o_2$	$o_3$
$S_1$	0.05	0.17	0.62	0.14	0.08	0.04
$S_2$	0.28	0.33	0.16	0.29	0.23	0.23
$S_3$	0.17	0.33	0.12	0.29	0.31	0.31
$S_4$	0.50	0.17	0.10	0.29	0.38	0.41

Table 1: Evaluation of each user with regard to pursued objectives

If we consider the three objectives to be with equal importance and that the contribution of each user is commensurate to the proportion it receives with the same proportional coefficient, that is  $\nu_i^x(o_k, x) = x_i \forall k$ , one can consider the selectability measure and the rejectability measure to be given by equations (18) and (19) respectively

$$\mu_S(x) = 0.84x_1 + 0.77x_2 + 0.62x_3 + 0.77x_4; \quad (18)$$

$$\mu_R(x) = 0.26x_1 + 0.75x_2 + 0.91x_3 + 1.08x_4. \quad (19)$$

Let us suppose that it is admitted that none of the user should receive less than 1% of the allocation meaning that  $x_{i, \min} = 0.01 \forall i$ ; then the satisficing allocation set at boldness index of  $q = 1$  is given by equation (20)

$$\Sigma_1 = \left\{ x : \begin{array}{l} -0.58x_1 - 0.02x_2 + 0.29x_3 + 0.31x_4 \leq 0 \\ x_1 + x_2 + x_3 + x_4 \leq 1 \\ 0.01 \leq x_i \leq 1, \forall i \end{array} \right\} \quad (20)$$

which is, mathematically speaking, a constraint satisfaction problem; an example of an allocation obtained by optimizing a null function subjected to constraints of (20) using Matlab function *linprog* is

$$x = [0.1923 \ 0.0590 \ 0.0127 \ 0.0102]^T$$

meaning that a total of 27.41% of the capacity is utilized and allocated according to following scheme: 19.23% for the municipality, 5.90% for the farmers, 1.27% for breeding, and 1.02% for the industry. Now let us suppose that we want to maximize the total allocation that is maximizing the function  $x_1 + x_2 + x_3 + x_4$ , in this case the "optimal" allocation vector is given by

$$x = [0.3901 \ 0.2361 \ 0.0896 \ 0.2842]^T$$

that is the total available quantity is shared according to the following scheme: 39.01% for the municipality, 23.61% for the farmers, 8.96% for breeding, and 28.42% for the industry

## 6. CONCLUSION

The problem of allocating continuous resource such as water to different users for their activities when meeting some societal requirements in terms of sustainable development is considered in this communication. Because any socioeconomic activity convey positive outputs as well as negative ones, bipolar analysis that relies on aggregating positive and negative features of an alternative decision (here an allocation) separately is used as the underlying concept to formulate the allocation problem. The contribution consists in proposing a framework within which one can analyze an allocation problem when taking into account inevitable interaction between actors engaged in the decision process. From the so called interaction matrix, a sort of preemptivity analysis can be done to find which user's activity is susceptible to influence other activities in the spirit to privilege those activities that have more impact on the overall decision goal such as the sustainability of allocation. A small real world application in the domain of water sharing between four activities with the sustainability as the main objective shows the feasibility of the proposed approach.

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