

# ON TRANSIENT HEATING AND DYNAMIC RESPONSE TO ACOUSTIC OSCILLATION OF A VAPORIZING DROPLET

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# 1. Introduction

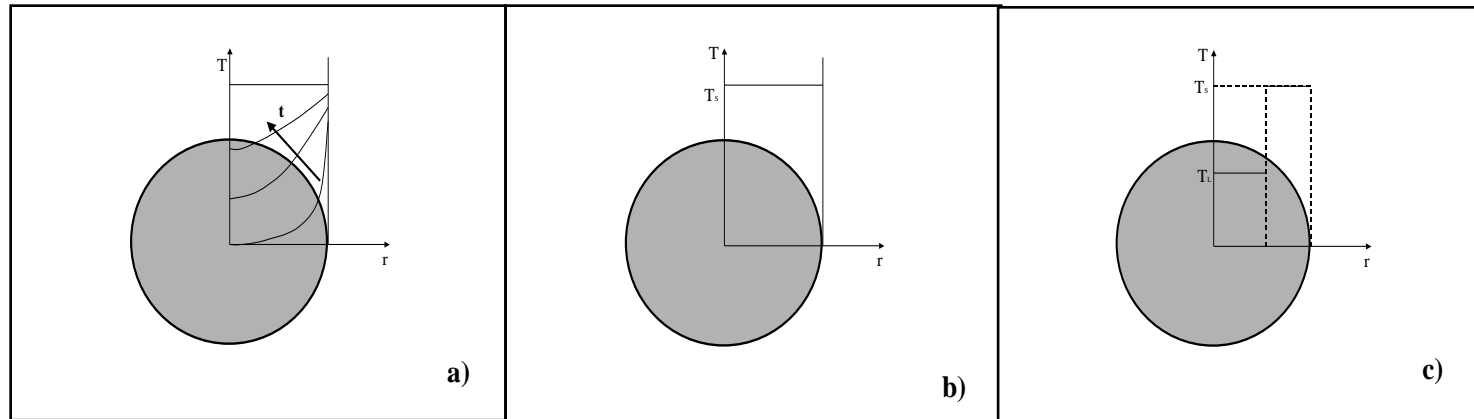
Basic considerations on transient heating and dynamic response of an evaporating droplet are discussed.

It seems that thermal exchange inside the droplet has an important effect on transient behavior and on the vaporization stability. A two-layer model is proposed here to take into account heat transfer inside the droplet (assuming a finite heat exchange coefficient between both layers).

High frequency combustion instability in liquid rocket engines is a result of coupling between combustion processes and the chamber acoustics. Droplet evaporation is one possible driving mechanism of combustion instability.

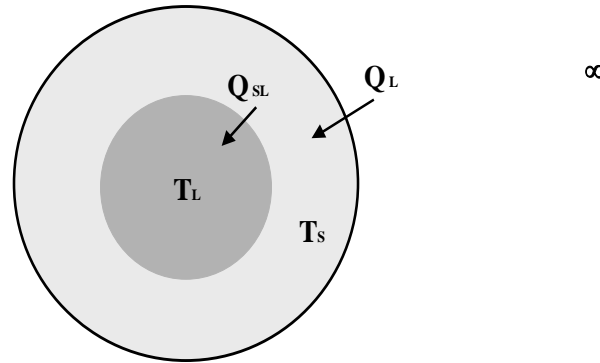
In order to investigate this possibility, we consider an evaporating droplet submitted to an acoustic field. The objective of the study is to determine droplet response.

## 2 Two-layer model for droplet transient heating



### Temperature profiles inside the liquid droplet for three models

- a) Conductive heat exchange (Law & Sirignano, 1977 [21])
- b) Infinite thermal conductivity (cf. Chin & Lefebvre, 1985 [7])
- Two-layer model (Present work)



$$M_L c_L \frac{dT_L}{dt} = Q_{SL}, \quad M_S c_L \frac{dT_S}{dt} = -Q_{SL} + Q_L$$

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$$Q_{SL} = 2\pi k_L r_L Nu_L (T_S - T_L), \quad Q_L = \dot{M} \ell \left( \frac{B'_T}{B_T} - 1 \right)$$

$$M = \frac{4}{3} \pi r_S^3 \rho_L, \quad M_L = \frac{4}{3} \pi r_L^3 \rho_L, \quad M_S = \frac{4}{3} \pi (r_S^3 - r_L^3) \rho_L$$

$$\dot{M} = 2\pi \frac{k}{c_p} r_S Nu \ln(1 + B_T)$$

$$\beta = \frac{3k Nu}{2\rho_L c_L r_S^2}, \quad \beta_S = \frac{\beta}{1-\varepsilon}, \quad \beta_L = \frac{3k_L Nu_L}{2\rho_L c_L r_L^2}, \quad \beta' = \frac{\varepsilon \beta_L}{1-\varepsilon}, \quad \varepsilon = \left(\frac{r_L}{r_S}\right)^3$$

$$\begin{cases} \frac{dT_L}{dt} = \beta_L (T_S - T_L) \\ \frac{dT_S}{dt} = -\beta' (T_S - T_L) + \beta_S \frac{\ln(1+B_T)}{B_T} (T_\infty - T_S) - \beta_S \frac{\ell}{c_P} \ln(1+B_T) \end{cases}$$

- With no evaporation, we have  $B_T \rightarrow 0$  and we find

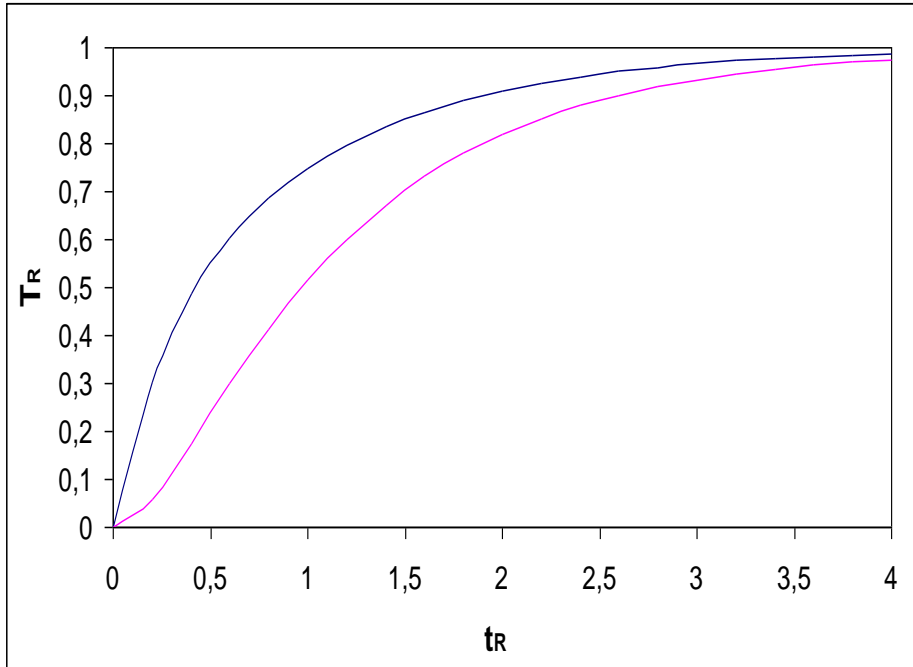
$$\frac{dT_L}{dt} = \beta_L (T_S - T_L), \quad \frac{dT_S}{dt} = -\beta' (T_S - T_L) + \beta_S (T_\infty - T_S)$$

- For one layer  $\alpha=0, \beta'=0, \beta_S=\beta, T_S=T_L$ , and we find evaporation with a droplet at uniform temperature:

$$\frac{dT_S}{dt} = \beta \frac{\ln(1 + B_T)}{B_T} (T_\infty - T_S) - \beta \frac{\ell}{c_p} \ln(1 + B_T)$$

- If we assume coefficients  $\beta', \beta_L, \beta_S$  and the Spalding parameter  $B_T$  to be constant. The parameter  $B_T$  associated with heat exchange can be deduced from the mass Spalding parameter  $B_M$ . In the particular case  $Le=1$ ,  $B_T = B_M$ , and we obtain

$$\frac{d^2 T_L}{dt^2} + \left( \beta_L + \beta' + \beta_S \frac{\ln(1 + B_T)}{B_T} \right) \frac{dT_L}{dt} + \beta_S \beta_L \frac{\ln(1 + B_T)}{B_T} \left( T_L - T_\infty + \frac{\ell}{c_p} B_T \right) = 0$$



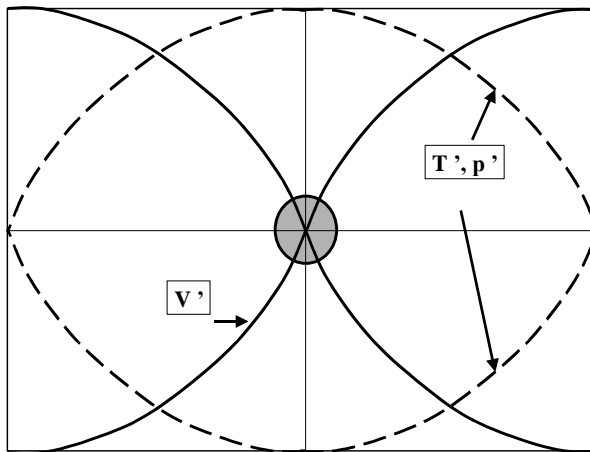
$$T_{RL} = (T_L - T_i) / (T_\infty - T_i - \ell B_T / c_p)$$

$$T_{SL} = (T_S - T_i) / (T_\infty - T_i - \ell B_T / c_p)$$

Evolutions of reduced temperatures  $T_{RL}$  of the inner sphere and  $T_{RS}$  of the outer sphere (surface) of an evaporating droplet, as functions of reduced time  $t_R = \beta' t$ , with the two-layer model for heat exchange



### 3. Analysis of frequency response characteristics

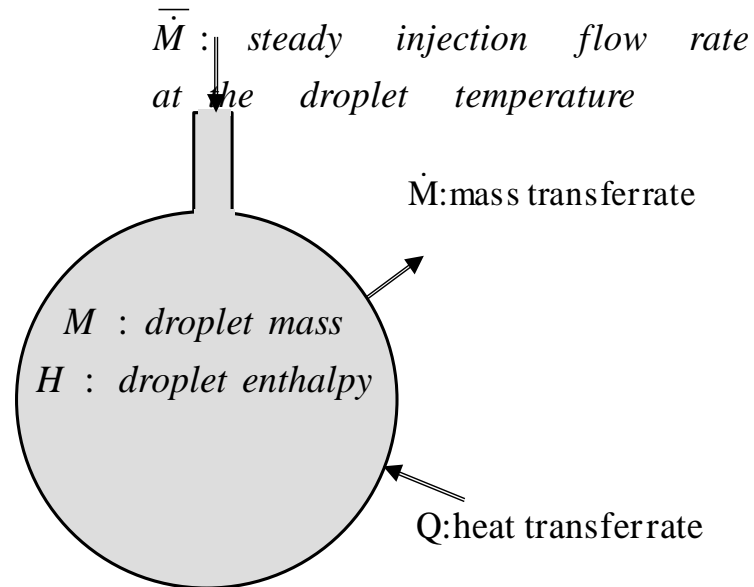


**Droplet at a velocity node  
in a closed cavity**

**Response factor:**

$$N = \frac{\iint_{V,t} q'(V,t) p'(V,t) dt dV}{\iint_{V,t} (p'(V,t))^2 dt dV}$$

For sinusoidal oscillations with the same period  
 $N = \left( \frac{|\hat{q}|}{|\hat{p}|} \right) \cos \theta$ , where  $|\hat{q}|$ ,  $|\hat{p}|$  are modulus and  
 $\theta$  the phase difference between  $q'$  and  $p'$ .

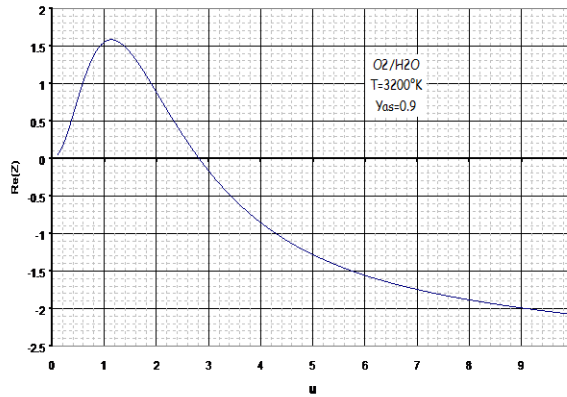
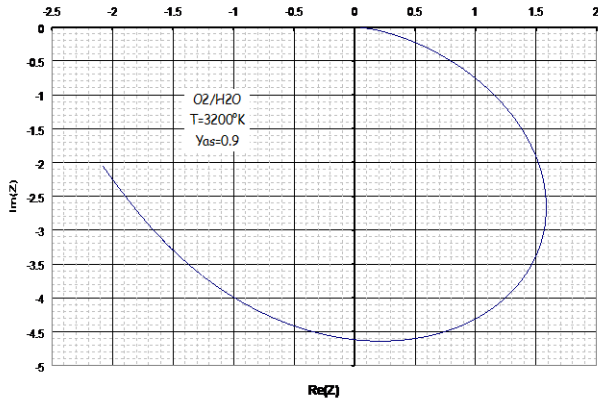


## The mean droplet of Heidmann and Wieber

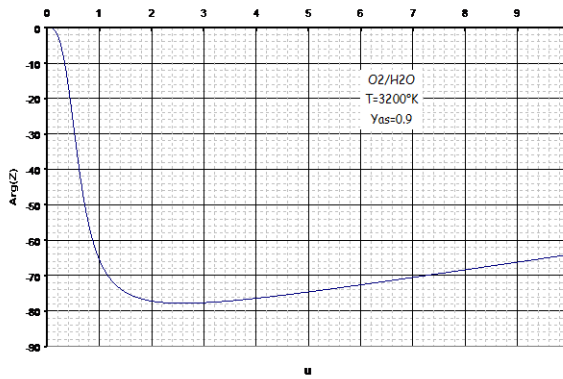
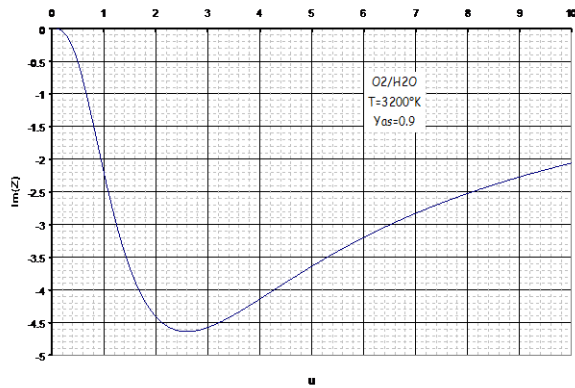
A vaporizing drop of radius  $r$ , continuously supplied by a steady flow rate  $\dot{M}$ .

**The case of a droplet at uniform temperature without external flow**

$$Z_0 = \frac{1}{\alpha} \frac{\hat{M}}{\hat{p}_C} = \frac{iu}{1+iu} \frac{A-iu}{B+iu}$$



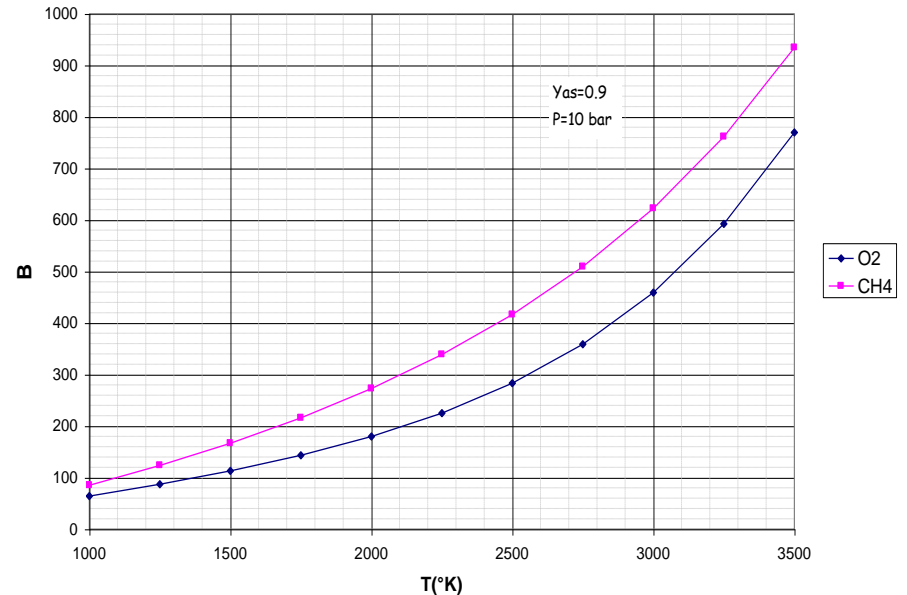
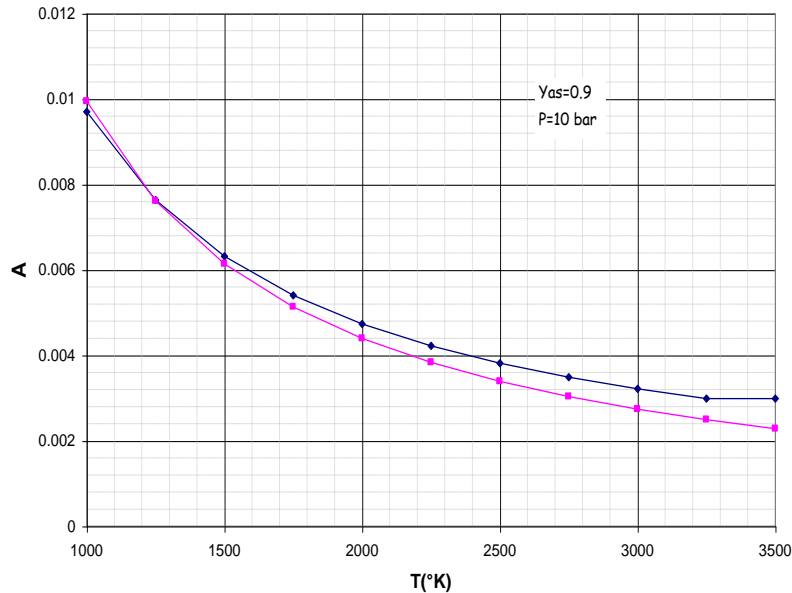
**Oxygen in water vapor**



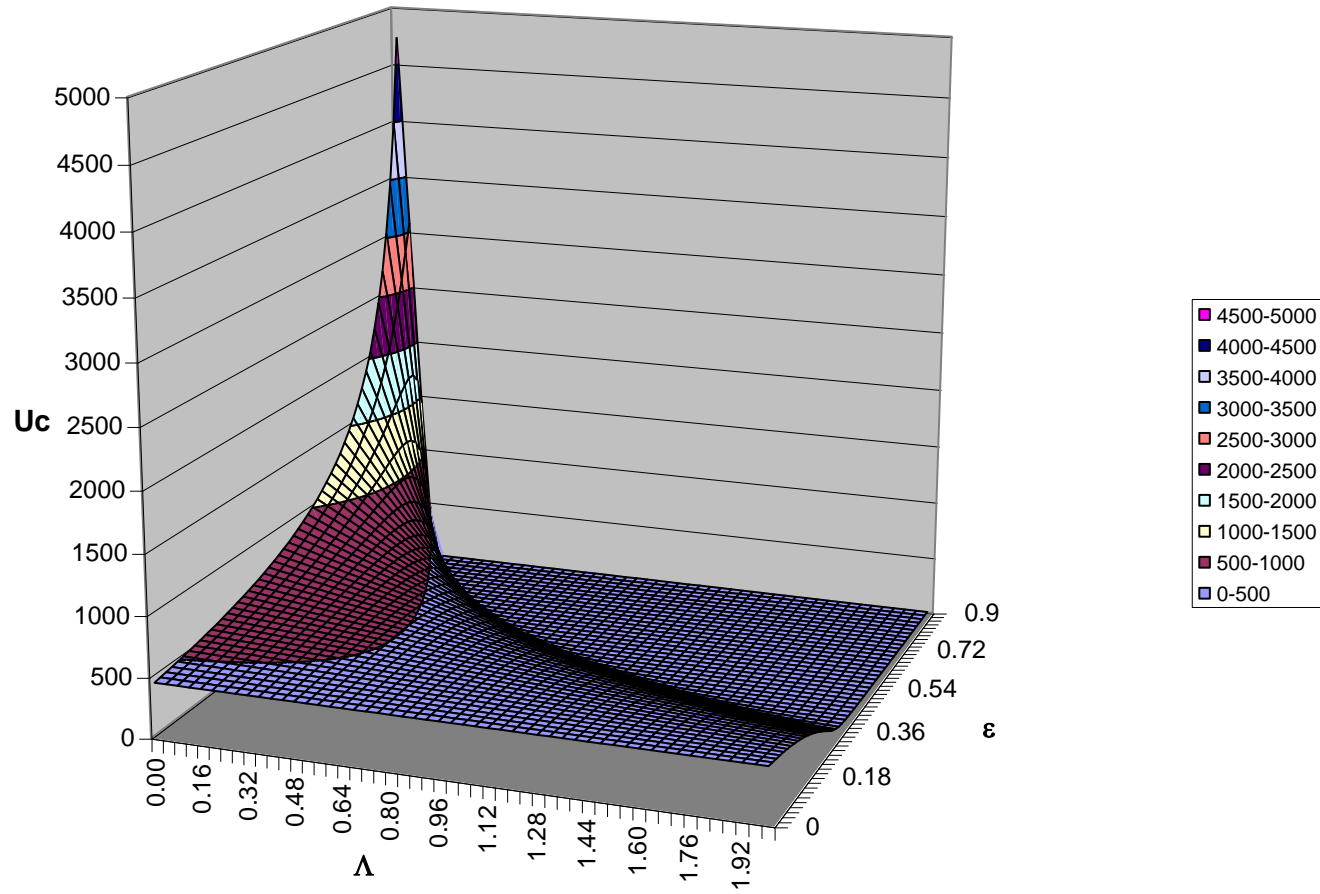
## Droplet with a temperature field (without convection): two-layer model

$$Z = \frac{1}{\alpha} \frac{\hat{M}}{\hat{p}_c} = \frac{i u}{1 + i u} \frac{(\Lambda + i u)(A - i u) - \varepsilon u^2}{(\Lambda + i u)(B + i u) + \varepsilon u^2}$$

$$u = 3 \omega \bar{\tau}_v, \quad \Lambda = 3 \beta_L \bar{\tau}_v = \bar{\tau}_v / \bar{\tau}_T, \quad \varepsilon = (r_L / r_S)^3$$



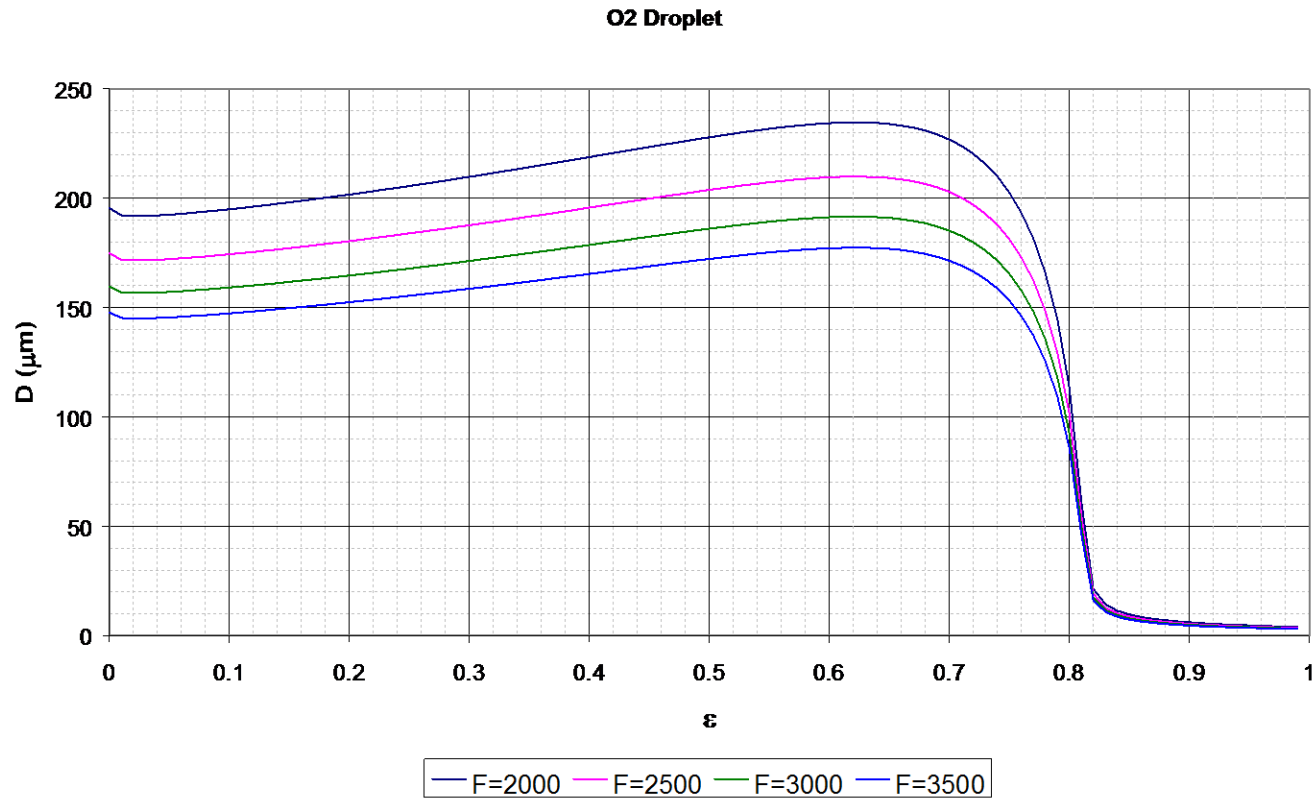
Evolution of coefficients A and B versus temperature



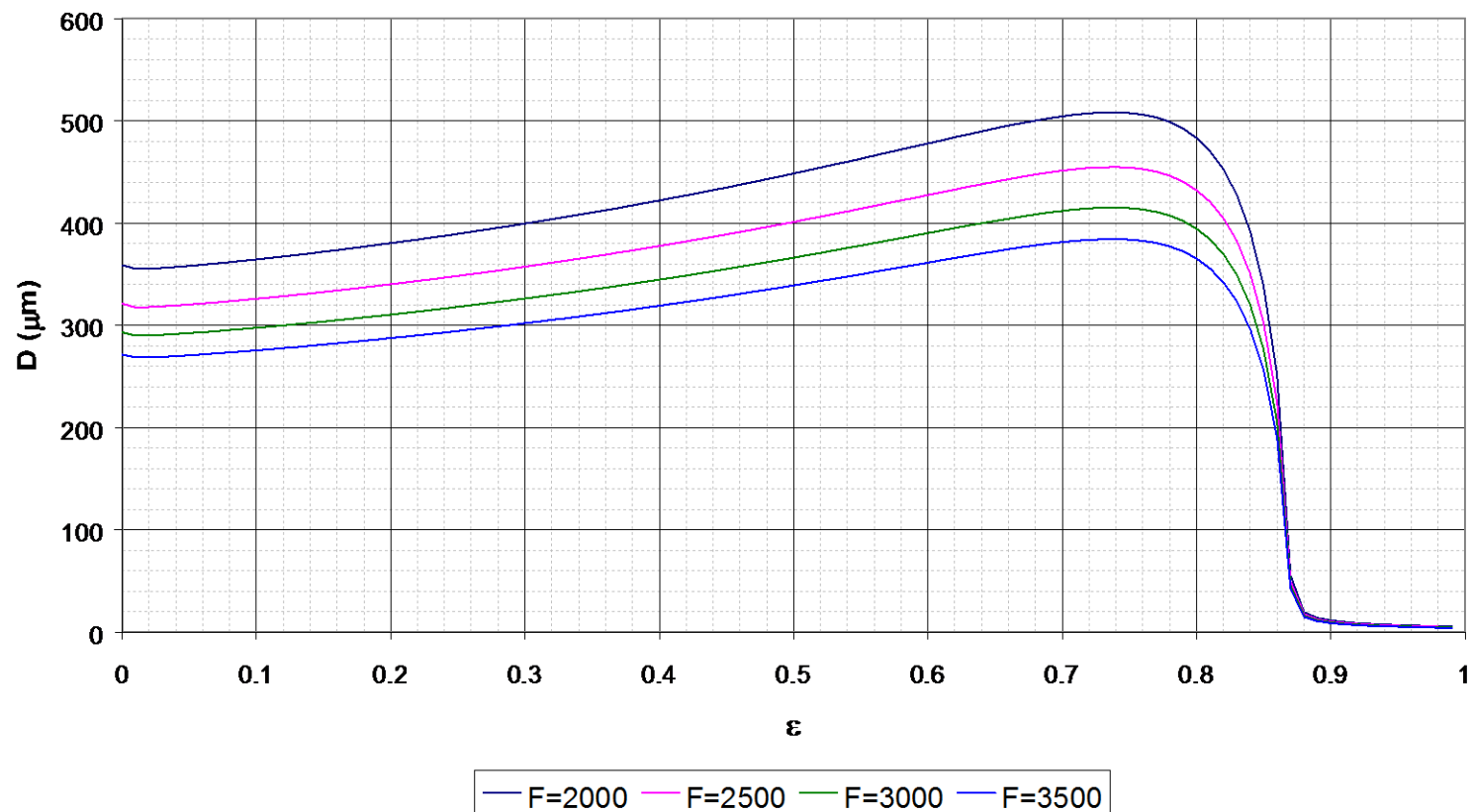
**Effects of  $\epsilon$  and  $\Lambda$  on the reduced cut-off frequency  $u_c$**

Influence of  $\varepsilon$  for liquid Nusselt number equal to 2:

$$\Lambda \varepsilon^{2/3} = 3k_L / c_L \bar{\rho} \bar{D} \ln(1 + \bar{B}_M)$$



## Méthane Droplet



#### **4. Conclusions and prospects:**

**A linearized model is developed on the basis of Heidmann and Wieber theory . A dynamic model is investigated on the basis of the two previous models, and is then used in the stability analysis to determine the complex transfer function and the response factor of a vaporizing droplet submitted to small periodic perturbations. Only the case of a pressure perturbation is considered.**

**In every case the results shows the important effect of thermal exchange inside the droplet on the vaporization stability.**

**To continue these investigations, it should be interesting:**

- To conceive a N-layer model to compare with the two-layer model and find the volume fraction**
- To study the effect of convection due to relative velocity gas/droplet**
- For the future, to validate numerically the results**

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