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**ON TRANSIENT HEATING AND DYNAMIC RESPONSE TO ACOUSTIC  
OSCILLATION OF A VAPORIZING DROPLET**

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**Summary**

*This paper deals with the analysis of an evaporating droplet. First classical vaporization theory along with the hypotheses are recalled. A brief summary of the state of the art on droplet evaporation including heat transfer inside the liquid is given. As far as heat transfer inside the droplet is considered, a new model is proposed (the two-layer model). It is an alternative to the most simple (uniform temperature) or more complex models such that developed by Abramzon and Sirignano.*

*The second part of the paper deals with droplet dynamics. Following Heidmann and Wieber analysis, droplet response submitted to a pressure fluctuation is studied. The transfer function of the droplet is determined in two cases: a uniform droplet temperature and non uniform droplet temperature (two-layer model). The results presented and discussed in terms of response factor and cut-off frequency.*

**1. INTRODUCTION**

Droplet evaporation and combustion has been recognized by several investigators [17,44] as a possible mechanism for driving combustion instability in liquid propellant rocket engines.

The combustion process depends on the chemical composition of the injected propellants, on the injector design, and on the injection conditions. The initial diameter distribution of droplets is an important parameter of the problem. Interactions between droplets play a role (Borghini & Lacas, 1995 [5]; Dietrich et al., 1999 [12]; Rangel & Sirignano, 1989 [27]; Villepinte et al., 1999 [38]) and may not be neglected for dense clouds. Droplets are often considered as spherical. But this hypothesis becomes no more valid, in particular for a small gas-liquid surface tension in the presence of a non zero relative velocity.

Liquid droplets are generally injected in a gaseous environment with different temperature, pressure and velocity. A relaxation time is then necessary for a droplet to reach the equilibrium values. At the end of this relaxation period, the velocities of gas and liquid are equal, and droplet temperature reaches a uniform temperature equal to the temperature of saturated vapor at the liquid surface.

Sometimes, the droplet life-time is too short for reaching these equilibrium conditions. It is then possible to be always in a transient situation for temperature or/and for velocity. In the opposite case, equilibrium temperature and velocity are reached during the droplet life time.

A lot of papers have been published about combustion instabilities in liquid propellant rocket engines (Bhatia & Sirignano, 1991 [4]; Delplanque & Sirignano, 1994 [9], 1996 [10]; DiCicco & Buckmaster, 1994 [11]; Dubois et al., 1995 [13]; Duvur et al., 1996 [14]; Fachini, 1998 [15]; Heidmann & Wieber 1966 [18]; Heidmann 1972 [19]; Laroche et al., 2000 [20]; Mauriot, 1992 [23]; Schmitt, 1986 [31]; Sirignano et al., 1994 [32]; Tong & Sirignano, 1989 [37]; Weber & Mickelsen, 1960 [39]; Williams, 2000 [41])

High frequency combustion instability in liquid rocket engines is a result of coupling between combustion processes and the chamber acoustics. Droplet evaporation is one possible driving

mechanism of combustion instability. In order to investigate this possibility, we consider an evaporating droplet submitted to an acoustic field. The objective of the study is to determine droplet response. For this purpose we calculate the response factor giving the mass flow rate perturbation resulting from a given pressure or velocity perturbation. It is then necessary to consider transient regimes.

We will consider here a spherical droplet in an infinite atmosphere.

Many researchers have studied transient regimes of spherical droplets, assuming a quasi-steady regime behavior for the gaseous phase (Crespo & Liñan, 1975 [8]; Prud'homme & Habiballah, 2001 [26]; Strahle, 1963 [36]). This suppose that the gaseous phase has a very small response time. For the liquid phase, unsteady evolutions must be considered.

## 2. TRANSIENT REGIMES AND THE EXTENDED QUASI-STEADY HYPOTHESES

The quasi-steady hypothesis can be extended to droplet vaporization in the presence of an external flow, with exchanges of mass, momentum and energy between the droplet and the external flow (Abramzon, Sirignano, 1988 [1]. The equations of the gaseous phase are modified to account for these exchanges. We suppose an ideal mixture of perfect gases. For high compressibility effects, see Arias-Zugasti et al., 1999 [3]; Delplanque & Sirignano, 1994 [9]; Nieto de Castro, 1991 [24]; Okay et al., 2000 [25]; Rosner & Chang, 1973 [29]; Sohn et al., 1998 [33]; Spalding, 1953 [34]; Wieber, 1963 [40]; Yang et al., 1992 [42], 1995 [43].

### 2.1. Evaporation with temperature jump but without external flow

This is nearly the classical problem (Godsave, 1953 [16]; Spalding, 1953 [34]) but with heat exchange between gas and liquid droplet. The equations of the quasi-steady gas flow are the following:

$$\dot{M} = 4\pi r^2 \dot{m}, \quad \dot{m} = \rho u \tag{1}$$

$$\dot{M} \frac{dY_j}{dr} - 4\pi \rho D r \frac{d^2(rY_j)}{dr^2} = 0 \tag{2}$$

$$\dot{M} c_p \frac{dT}{dr} - 4\pi k r \frac{d^2(rT)}{dr^2} = 0 \tag{3}$$

with the boundary conditions

$$\text{- at infinity } (r \rightarrow \infty) : Y_j = Y_{j\infty}, T = T_\infty \tag{4}$$

$$\text{- and at the droplet surface } (r_s) :$$

$$\left| \begin{array}{l} Y_{js}, \quad 4\pi \rho D r_s^2 \frac{dY_j}{dr} \Big|_s = -\dot{M} (Y_{j\infty} - Y_{js}) \end{array} \right. \tag{5}$$

$$\left. \begin{array}{l} T_s, \quad 4\pi \frac{k}{c_p} r_s^2 \frac{dT}{dr} \Big|_s = \dot{M} \frac{\ell}{c_p} \end{array} \right. \tag{6}$$

Then solution of the diffusion equation writes

$$\dot{M} = 4\pi \rho D r_s \ln(1 + B_M), \quad \text{with } B_M = \frac{Y_{FS} - Y_{F\infty}}{1 - Y_{FS}} \tag{7}$$

On the other hand, solution of the conduction equation is modified because of the change of the condition at the droplet surface.

One introduces the heat flux  $Q_L$  which characterizes the heat brought to the drop in addition to that necessary for evaporation. The mass flow rate becomes

$$\dot{M} = 4\pi \frac{k}{c_p} r_s \ln(1 + B_T), \text{ with this time } B_T = \frac{c_p(T_\infty - T_s)}{\ell + Q_L/\dot{M}} \quad (8)$$

To continue the resolution, it is also necessary to have an expression for  $Q_L$ . For that we need to study heat exchanges inside the droplet.

For example, if they are infinitely fast, droplet temperature remains uniform at any moment but varies with time; thus we have

$$M c_L \frac{dT_s}{dt} = Q_L \quad (9)$$

where  $C_L$  is the specific heat of the liquid. Temperature of the droplet surface is no more fixed, contrarily to the classical theory. It varies during the heating period. There is always evaporation at equilibrium at the surface, but the temperature being variable, the partial pressure of species F varies with time and thus the concentration  $Y_{FS}$  adapts itself to temperature variations.

In non-stabilized regimes, characteristic time for liquid heating tends to be higher than the time for boundary-layer heating.

Heating time  $\Delta t_{hu}$  and droplet lifetime  $\Delta t_{ex}$  are of the same order of magnitude for kerosene droplets, but for LOX droplets  $\Delta t_{hu} \gg \Delta t_{ex}$ .

In the case of infinitely fast heat exchanges in the liquid phase, we need to solve the following system of equations:

$$\frac{dT_s}{dt} = \frac{3k\ell}{\rho_L c_L c_p r_s^2} \left( \frac{B'_T}{B_T} - 1 \right) \ln(1 + B_T) \quad (10a)$$

$$r_s \frac{dr_s}{dt} = \frac{k}{\rho_L c_p} \ln(1 + B_T) \quad (10b)$$

where  $B_T$  is defined by (8) and where

$$B'_T = \frac{c_p(T_\infty - T_s)}{\ell} \quad (11)$$

Equation (10a) is simply deduced from (8) and (9), and equation (10b) derives from (8) and  $\dot{M} = -dM/dt$  (12)

The Spalding parameter  $B'_T$  depends on the droplet temperature  $T_s$ . The Spalding parameter for heat exchange  $B_T$ , defined by (8), is connected to the Spalding parameter for mass exchange  $B_M$ , defined by (7), by the following equation:

$$\dot{M} = 4\pi \frac{k}{c_p} r_s \ln(1 + B_T) = 4\pi \rho D r_s \ln(1 + B_M) \quad (13)$$

which is derived from (7) and (8). So  $B_T$  is a function of  $B_M$ , which depends on gaseous fuel mass fraction at the droplet surface (if the Lewis number  $Le$  is equal to unity, we have  $B_T = B_M$ ). This mass fraction is connected to temperature  $T_S$  by the equilibrium relation:

$$\mu_L = \mu_F \quad (14)$$

When  $\mu_L$  is function of  $T$  only, and for an ideal gas mixture, this lead to:

$$p X_{FS} = p_{sat}(T_S) \quad (15)$$

The equilibrium law writes [7] for instance:

$$p_{sat}(T_S) = \exp\left(a - \frac{b}{T_S - c}\right) \quad (16)$$

where  $a$ ,  $b$  and  $c$  are constant coefficients.

Then, we can relate  $Y_{FS}$  to the surface molar fraction  $X_{FS}$ , thus the surface mass fraction  $Y_{FS}$  appears as a function of surface temperature  $T_S$  and total pressure  $p$ , which is assumed uniform and constant:  $Y_{FS} = f(T_S, p)$ . Thus  $B_T$  and  $B'_T$  are functions of  $T_S$  only.

The latent heat itself is function of temperature and concentration at the surface. Indeed, from the relation of Clapeyron, one has per mole of pure substance:  $L = (RT^2/p)(dp/dT)_{sat}$ . This relation, applied here to unit mass, by replacing the pressure by the partial pressure and using (15) and (16), gives:

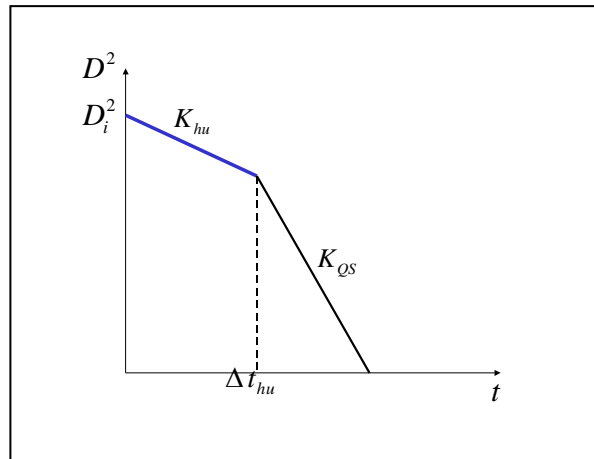
$$\ell = \frac{(M_A - M_F)Y_{FS} + M_F}{M_A M_F} \frac{b R T_S^2}{(T_S - c)^2} \quad (17)$$

As temperature and concentrations are not constant in the droplet surrounding, the averaged properties  $c_p$ ,  $k$  are evaluated at the reference temperature and composition

$$\bar{T} = T_S + A_r (T_\infty - T_S), \quad \bar{Y}_F = Y_{FS} + A_r (Y_{F\infty} - Y_{FS}) \quad (18)$$

$A_r = 1/3$  is generally chosen.

Thus the coefficients appearing in (10a) and (10b) are functions of  $r_s$  and  $T_S$  only. They form a non linear system of two differential equations. Solution of this system gives both functions  $T_S(t)$ ,  $r_s(t)$  and consequently  $Y_{FS}(t)$ .



**Figure 1.** Chin et Lefebvre approximation (Chin et Lefebvre, 1985 [7]).  
In the QS zone, droplet temperature does not vary anymore.

*Simplification (Chin & Lefebvre)*

Simplified hypotheses are used by Chin & Lefebvre (Chin et Lefebvre, 1985 [7]; Lefebvre, 1989 [22]) to solve system (10). Two  $D^2$  laws can be considered, one for the heat-up period  $\Delta t_{hu}$ , the other for the steady-state phase  $\Delta t_{QS}$  (Figure 1).

**2.2. Evaporation with an external flow**

Abramzon and Sirignano (1988, 1989) [1, 2] studied the case of an external flow by assuming a quasi-steady mode (QS) in gas phase.

The transfer coefficients are replaced by averages using the « 1/3 rule » (see Equation 18). They corrected the previous results by introducing modified Nusselt and Sherwood numbers  $Nu^*$  et  $Sh^*$  (to account for blowing caused by evaporation).

In the presence of external flow, there are movements inside the drop. These movements can be modeled by Hill vortices or, which is simpler, one can introduce an effective coefficient of conduction

$$k_{eff} = \chi k_L \tag{19}$$

It is then necessary to ensure the coupling between these phenomena and global droplet movement. The equation which describes this movement is that of Stokes in the turbulent case (the other forces acting on the droplet are supposed to be negligible).

The equations of this problem were solved numerically by Abramzon and Sirignano [1, 2]. Chiang, Raju & Sirignano [6] developed an advanced droplet vaporization model with few assumptions. They considered the unsteady equations of an evaporating droplet transported by a flow. Their model makes it possible to treat the evolution of a droplet starting from its injection location in the combustor by taking into account, in particular, evolution of relative velocity, transient heating, recirculation zone downstream of the droplet, movement inside the droplet and blowing due to evaporation. These "complete" models are very useful insofar as they can be used as references to calibrate more simplified models. The results obtained by the authors highlights the importance of transient regimes. They indicate that both transient droplet heating and reduction in Reynolds number (largely due to droplet deceleration) are major source of droplet unsteady behavior and persist during most of the droplet’s lifetime.

In the following section an alternative to the Abramzon and Sirignano model is proposed. The model allows to treat transient heat up of a droplet when the thermal conductivity is finite. In order to simplify the derivation and to get an analytical solution, some simplifications are made.

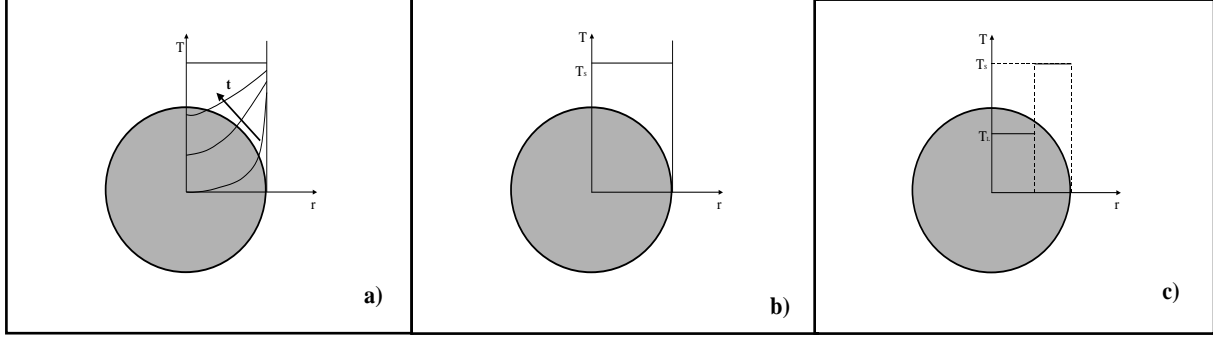
**2.3. Two-layer model for droplet transient heating**

Temperature profiles in a spherical drop are represented on Figure 2a. The most extreme simplification consists in assuming an infinite thermal conductivity, which leads to a uniform droplet temperature (Figure 2b). The two-layer model (Figure 2c) consists of two concentric spheres. In the first one, of radius  $r_L$ , the temperature is supposed to be uniform and equal to  $T_L$ . Between the spheres of radii  $r_L$  and  $r_S$  the temperature is also uniform and equal to  $T_S$ . Temperatures  $T_L$  and  $T_S$  depend on time.

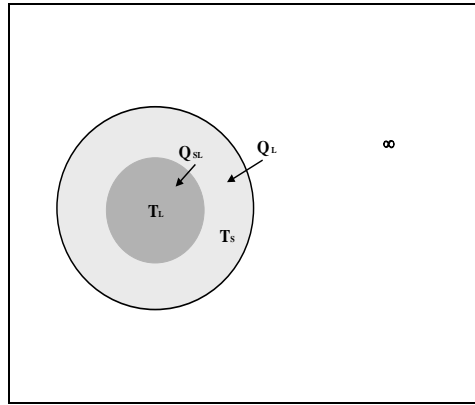
Figure 3 highlights the various heat exchanges.

The equations of the problem are:

$$M_L c_L \frac{dT_L}{dt} = Q_{SL}, M_S c_L \frac{dT_S}{dt} = -Q_{SL} + Q_L \tag{20}$$



**Figure 2.** Temperature profiles inside the liquid droplet for three models  
a) Conductive heat exchange (Law & Sirignano, 1977 [21])  
b) Infinite thermal conductivity (cf. Chin & Lefebvre, 1985 [7])  
c) Two-layer model (Present work)



**Figure 3.** Heat exchange in the two-layer model

with

$$Q_{SL} = 2\pi k_L r_L Nu_L (T_s - T_L), \quad Q_L = \dot{M} \ell \left( \frac{B'_T}{B_T} - 1 \right)$$

$$M = \frac{4}{3}\pi r_S^3 \rho_L, \quad M_L = \frac{4}{3}\pi r_L^3 \rho_L, \quad M_S = \frac{4}{3}\pi (r_S^3 - r_L^3) \rho_L \quad (21)$$

$$\dot{M} = 2\pi \frac{k}{c_p} r_S Nu \ln(1 + B_T)$$

Introducing reduced values for the heat exchange coefficients

$$\beta = \frac{3k Nu}{2\rho_L c_L r_S^2}, \quad \beta_S = \frac{\beta}{1 - \varepsilon}, \quad \beta_L = \frac{3k_L Nu_L}{2\rho_L c_L r_L^2}, \quad \beta' = \frac{\varepsilon \beta_L}{1 - \varepsilon}, \quad \varepsilon = \left( \frac{r_L}{r_S} \right)^3 \quad (22)$$

one obtains

$$\begin{cases} \frac{dT_L}{dt} = \beta_L(T_S - T_L) \\ \frac{dT_S}{dt} = -\beta'(T_S - T_L) + \beta_S \frac{\ln(1+B_T)}{B_T}(T_\infty - T_S) - \beta_S \frac{\ell}{c_p} \ln(1+B_T) \end{cases} \quad (23)$$

*Remark:*

Limiting cases can be obtained from (23).

- With no evaporation, we have  $B_T \rightarrow 0$ , and we find

$$\frac{dT_L}{dt} = \beta_L(T_S - T_L), \quad \frac{dT_S}{dt} = -\beta'(T_S - T_L) + \beta_S(T_\infty - T_S) \quad (24)$$

- For one layer:  $\varepsilon=0, \beta'=0, \beta_S=\beta, T_S=T_L$ , and we find again the analysis of section 2.1 for the heat up and evaporation with a droplet at uniform temperature:

$$\frac{dT_S}{dt} = \beta \frac{\ln(1+B_T)}{B_T}(T_\infty - T_S) - \beta \frac{\ell}{c_p} \ln(1+B_T) \quad (25)$$

- In the case of one layer without vaporization,  $B_T \rightarrow 0$ , (25) becomes the simple relaxation equation:

$$\frac{dT_S}{dt} = \beta(T_\infty - T_S) \quad (26)$$

The system (23) can be solved directly in simplified situations. We assume coefficients  $\beta', \beta_L, \beta_S$  and the Spalding parameter  $B_T$  to be constant. The parameter  $B_T$  associated with heat exchange can be deduced from the mass Spalding parameter  $B_M$ , using (12).

In the particular case  $Le=1$ , we obtain  $B_T = B_M$ .

Eliminating  $T_S$  from the system (23), leads to:

$$\frac{d^2 T_L}{dt^2} + \left( \beta_L + \beta' + \beta_S \frac{\ln(1+B_T)}{B_T} \right) \frac{dT_L}{dt} + \beta_S \beta_L \frac{\ln(1+B_T)}{B_T} \left( T_L - T_\infty + \frac{\ell}{c_p} B_T \right) = 0 \quad (27)$$

It can be seen (system (23)), that, if we assume  $T_S = T_L = T_i$  at time  $t=0$ , the first time derivative of  $T_L$  vanishes, but not the time derivative of  $T_S$  which is positive.

Characteristic equation associated with (27) has generally two negative roots and the solution is the sum of two exponential functions.

At the end of the process, and if the droplet is not completely consumed, the final temperature  $T_L = T_S = T_\infty - \frac{\ell}{c_p} B_T$  is reached. This final temperature differs from  $T_\infty$  which was obtained for a sphere without evaporation.

Equation (27) can be solved with the following assumptions. We suppose small values for  $B_T$ , and for  $\beta_L = \beta_S = 2\beta'$ ; in this case the characteristic equation gives the roots:

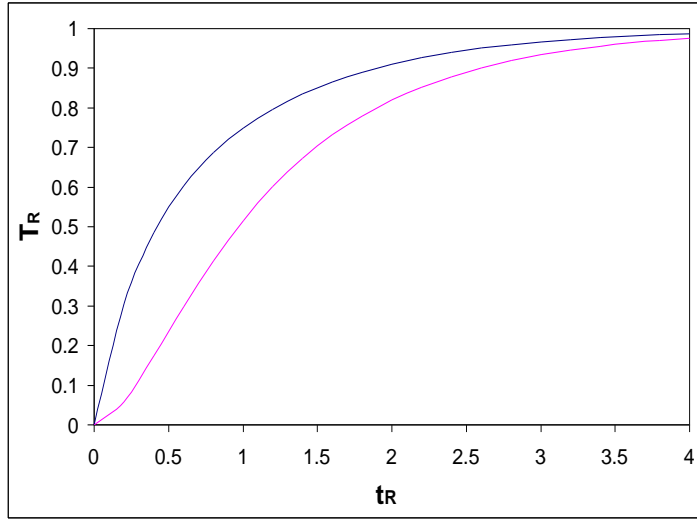
$$r' = -\beta' \left( \frac{1}{3} + \frac{2}{3} \frac{\ln(1+B_T)}{B_T} \right), r'' = -4\beta' \left( \frac{2}{3} + \frac{1}{3} \frac{\ln(1+B_T)}{B_T} \right) \quad (28)$$

Solution of eq. (27) leads to:

$$T_{RL} = (T_L - T_i) / (T_\infty - T_i - \ell B_T / c_p) = \frac{r''}{r' - r''} e^{r't} - \frac{r'}{r' - r''} e^{r''t} \quad (29)$$

$$T_{SL} = (T_S - T_i) / (T_\infty - T_i - \ell B_T / c_p) = 1 + \frac{(2\beta' + r')r''}{2\beta'(r' - r'')} e^{r't} - \frac{(2\beta' + r'')r'}{2\beta'(r' - r'')} e^{r''t}$$

The resulting curves have the shape given in Figure 4. This result is in accordance with results of more complex calculations (Scherrer, 1986 [30]). For  $B_T = 0$ , we find again the result of a non evaporating sphere:  $r' = -\beta'$ ,  $r'' = -4\beta'$ .



**Figure 4.** Evolutions of reduced temperatures  $T_{RL}$  — of the inner sphere and  $T_{RS}$  — of the outer sphere (surface) of an evaporating droplet, as functions of reduced time  $t_R = \beta' t$ , with the two-layer model for heat exchange

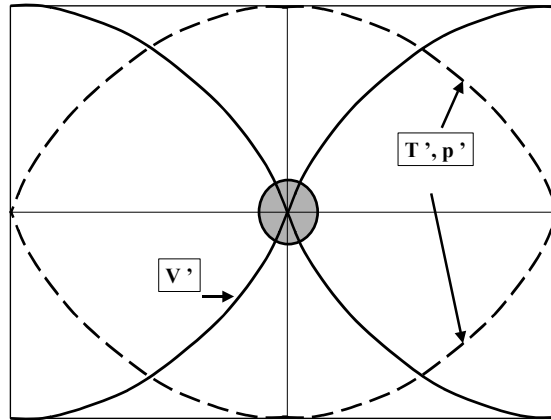
### 3. ANALYSIS OF FREQUENCY RESPONSE CHARACTERISTICS

Section 2 dealt with transient heating up of a droplet, which could be an important process with respect to combustion instability. In the section 3, we will investigate droplet dynamics of an evaporating droplet. That is the frequency response of a droplet submitted to an acoustic field. Indeed, following the well known Rayleigh criterion [28], droplet evaporation and burning could be one possible driving mechanism of instability.

An example of a droplet submitted to periodic oscillations is given in Figure 5. The drop is located at the center of a closed cavity, where there is a standing wave with a velocity node and pressure and temperature anti-node. In this case there is no external flow near the droplet. But the case of a velocity anti-node may also be considered and corresponds to a possible situation.

### 3.1. Response factor according to Heidmann and Wieber [18]

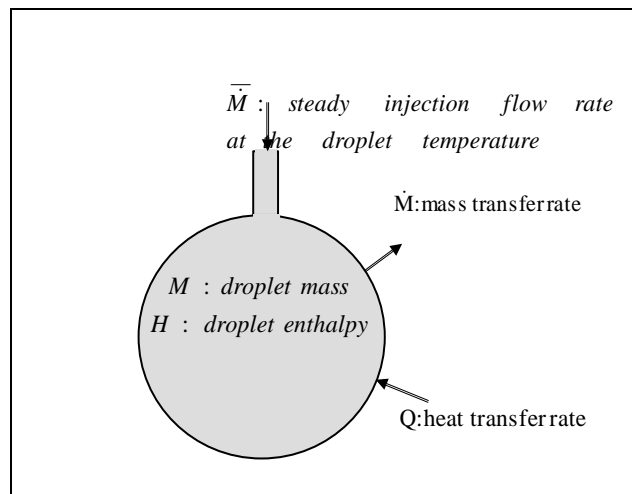
Heidmann and Wieber considered a droplet submitted to a pressure perturbation. The response is then a heat or mass resulting perturbation. Reduced pressure perturbation is defined as  $p'=(p-\bar{p})/\bar{p}$  and reduced heat or mass resulting perturbation is  $q'=(q-\bar{q})/\bar{q}$ .



**Figure 5.** Droplet at a velocity node in a closed cavity

The response factor N is defined as:

$$N = \frac{\iint_{V_t} q'(v_t) p'(v_t) dt dV}{\iint_{V_t} (p'(v_t))^2 dt dV} \quad (30)$$



**Figure 6.** A vaporizing drop of radius  $r$  is continuously supplied by a steady flow rate  $\bar{M}$ .

For sinusoidal oscillations with the same period  $N = (|\hat{q}|/|\hat{p}|) \cos \theta$ , where  $|\hat{q}|$ ,  $|\hat{p}|$  are modulus and  $\theta$  the phase difference between  $q'$  and  $p'$ .

Heidmann (1972, [19]) considered a mean droplet of vaporizing propellant inside an engine (Figure 6). The heat transfer rate  $Q$  to the drop causes a vaporization rate  $\dot{M}$ . The total heat and mass transferred establishes the instantaneous droplet mass  $M$  and droplet enthalpy  $H$ .

Some critical remarks can be formulated, not to the method of Heidmann and Wieber, but to a part of their equations. Then, we do not give their results. We prefer to perform the linear analysis of Heidmann and Wieber, starting with well established formula and for situations of increasing complexity.

### 3.2. Droplet dynamics investigations

#### 3.2.1. The case of a droplet at uniform temperature without external flow

This is the case of section 2.1. Changing the notations: use of index  $(C)$  instead of  $(\infty)$  (Equ. 7, 8, 11), and calling  $p_L(T_S)$  the vapor pressure at propellant surface instead of  $p_{sat}(T_S)$  (Equ. 16). After

linearization of (7), (12), (10a), (13), (15), with  $M = \frac{4}{3}\pi r_s^3 \rho_L$  and  $Y_F = \frac{M_F X_F}{M_F X_F + M_A(1-X_F)}$  we

obtain

$$\left\{ \begin{array}{l} \bar{\tau}_v \frac{dM'}{dt} = -\dot{M}', \quad \text{with} \quad \bar{\tau}_v = \bar{M}/\bar{M}', \quad \dot{M}' = \frac{1}{3}M' + \frac{\bar{B}_M}{(1+\bar{B}_M)\ln(1+\bar{B}_M)}B'_M, \quad \text{with} \\ B'_M = \frac{\bar{Y}_{AC}\bar{Y}_{FS}}{\bar{Y}_{AS}(\bar{Y}_{FS}-\bar{Y}_{FC})}Y'_{FS}, \quad Y'_{FS} = \frac{M_A}{M_F\bar{X}_{FS}+M_A\bar{X}_{AS}}X'_{FS}, \quad X'_{FS} = p'_L - p'_C \\ p'_L = \bar{b}T'_S, \quad -\frac{c_L\bar{T}_S}{\ell}\bar{\tau}_v \frac{dT'_S}{dt} = \frac{\bar{T}_S}{T_C-\bar{T}_S}T'_S - \frac{\bar{T}_C}{T_C-\bar{T}_S}T'_C + B'_T + \ell', \quad \text{with} \\ B'_T = \frac{\bar{B}_M}{Le} \frac{(1+\bar{B}_M)^{\frac{1}{Le}-1}}{(1+\bar{B}_M)^{\frac{1}{Le}-1}}B'_M, \quad \ell' = \frac{(M_A-M_F)\bar{Y}_{FS}}{(M_A-M_F)\bar{Y}_{FS}+M_F}Y'_{FS} - \frac{2c}{\bar{T}_S-c}T'_S \end{array} \right. \quad (31)$$

If formula (16) is used, we have  $\bar{b} = b\bar{T}_S / (\bar{T}_S - c)^2$ .

System (31) leads to the four equations

$$\begin{aligned} \bar{\tau}_v \frac{dM'}{dt} &= -\dot{M}', \quad \dot{M}' = \frac{1}{3}M' + \alpha (p'_L - p'_C), \quad p'_L = \bar{b}T'_S, \\ \alpha &= \frac{\bar{B}_M}{(1+\bar{B}_M)\ln(1+\bar{B}_M)} \frac{\bar{Y}_{AC}\bar{Y}_{FS}}{\bar{Y}_{AS}(\bar{Y}_{FS}-\bar{Y}_{FC})} \frac{M_A}{M_F\bar{X}_{FS}+M_A\bar{X}_{AS}} \\ \frac{c_L\bar{\tau}_v}{\ell} \frac{dT'_S}{dt} &= \frac{\bar{T}_C}{T_C-\bar{T}_S} \frac{\gamma-1}{\gamma} p'_C - \frac{\bar{T}_S}{T_C-\bar{T}_S} T'_S - \ell' - \frac{\bar{B}_M}{Le} \frac{(1+\bar{B}_M)^{\frac{1}{Le}-1}}{(1+\bar{B}_M)^{\frac{1}{Le}-1}} \frac{\bar{Y}_{AC}\bar{Y}_{FS}}{\bar{Y}_{AS}(\bar{Y}_{FS}-\bar{Y}_{FC})} \\ &\quad - \frac{M_A}{M_F\bar{X}_{FS}+M_A\bar{X}_{AS}} (p'_L - p'_C) \end{aligned} \quad (32)$$

Replacing  $p'_L$  and  $\ell'$  by their values given in (31), the last equation of system (32) can be written

$$\left\{ \begin{array}{l} \lambda \bar{\tau}_v \frac{dT'_s}{dt} + \mu T'_s - \bar{a} p'_c = 0 \quad \text{with} \quad \lambda = \frac{c_L \bar{T}_s}{\ell}, \quad \mu = \frac{\bar{T}_s}{T_c - \bar{T}_s} - \frac{2c}{T_s - c} + \bar{b} \varphi, \quad \bar{a} = \frac{\bar{T}_c}{T_c - \bar{T}_s} \frac{\gamma - 1}{\gamma} + \varphi \\ \text{and} \quad \varphi = \left( \frac{\bar{B}_M (1 + \bar{B}_M)^{\frac{1}{Le} - 1}}{Le (1 + \bar{B}_M)^{\frac{1}{Le} - 1}} \frac{\bar{Y}_{AC} \bar{Y}_{FS}}{\bar{Y}_{AS} (\bar{Y}_{FS} - \bar{Y}_{FC})} + \frac{(M_A - M_F) \bar{Y}_{FS}}{M_F \bar{Y}_{AS} + M_A \bar{Y}_{FS}} \right) \frac{M_A}{M_F \bar{X}_{FS} + M_A \bar{X}_{AS}} \end{array} \right. \quad (33)$$

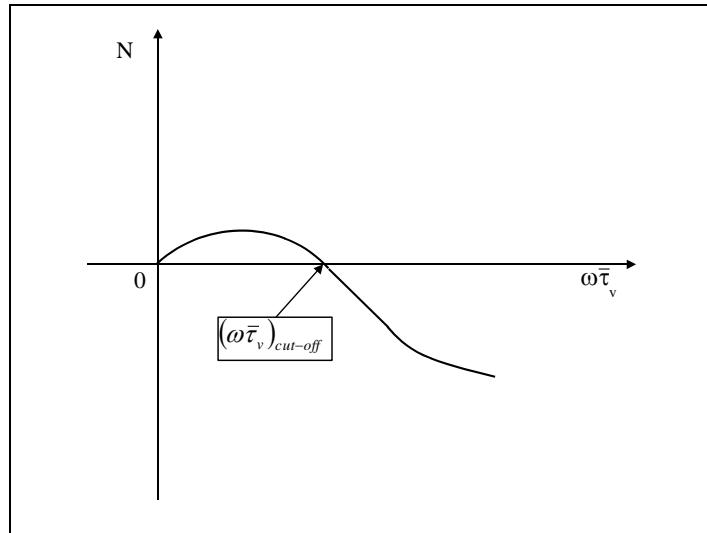
Putting  $f = \hat{f} e^{i\omega t}$  (with  $\hat{f}$  complex and  $\omega$  real) for any parameter, one obtains

$$\frac{\hat{M}}{\hat{p}_c} = \frac{(\bar{a}\bar{b} - \mu - \lambda i \omega \bar{\tau}_v) 3\alpha i \omega \bar{\tau}_v}{(\mu + \lambda i \omega \bar{\tau}_v) (1 + 3i \omega \bar{\tau}_v)}, \quad (34)$$

$$N_0 = \text{Re} \left( \frac{\hat{M}}{\hat{p}_c} \right) = \frac{3\alpha \omega^2 \bar{\tau}_v^2 (\bar{a}\bar{b} \lambda + 3\mu(\bar{a}\bar{b} - \mu) - 3\lambda^2 \omega^2 \bar{\tau}_v^2)}{(\lambda^2 \omega^2 \bar{\tau}_v^2 + \mu^2)(1 + 9\omega^2 \bar{\tau}_v^2)}$$

We suppose that  $\bar{a}\bar{b} \lambda + 3\mu(\bar{a}\bar{b} - \mu) > 0$ . In such a case, the response factor  $N$  vanishes for  $\omega^2 \bar{\tau}_v^2 = 0$  and for  $\omega^2 \bar{\tau}_v^2 = \bar{a}\bar{b} \lambda + 3\mu(\bar{a}\bar{b} - \mu) / 3\lambda^2$  and the curve  $N(\omega \bar{\tau}_v)$  has the shape of Fig. 7

with a cut-off reduced frequency given by:  $(\omega \bar{\tau}_v)_c = \sqrt{\frac{\bar{a}\bar{b} \lambda + 3\mu(\bar{a}\bar{b} - \mu)}{3\lambda^2}}$ .



**Figure 7.** The response factor in the case of an evaporating drop with uniform internal temperature

Let us consider the response factor expression (34) and let us set:

$$Z_0 = \frac{1}{\alpha} \frac{\hat{M}}{\hat{p}_c} = \frac{i u}{1 + i u} \frac{A - i u}{B + i u}, \quad (35)$$

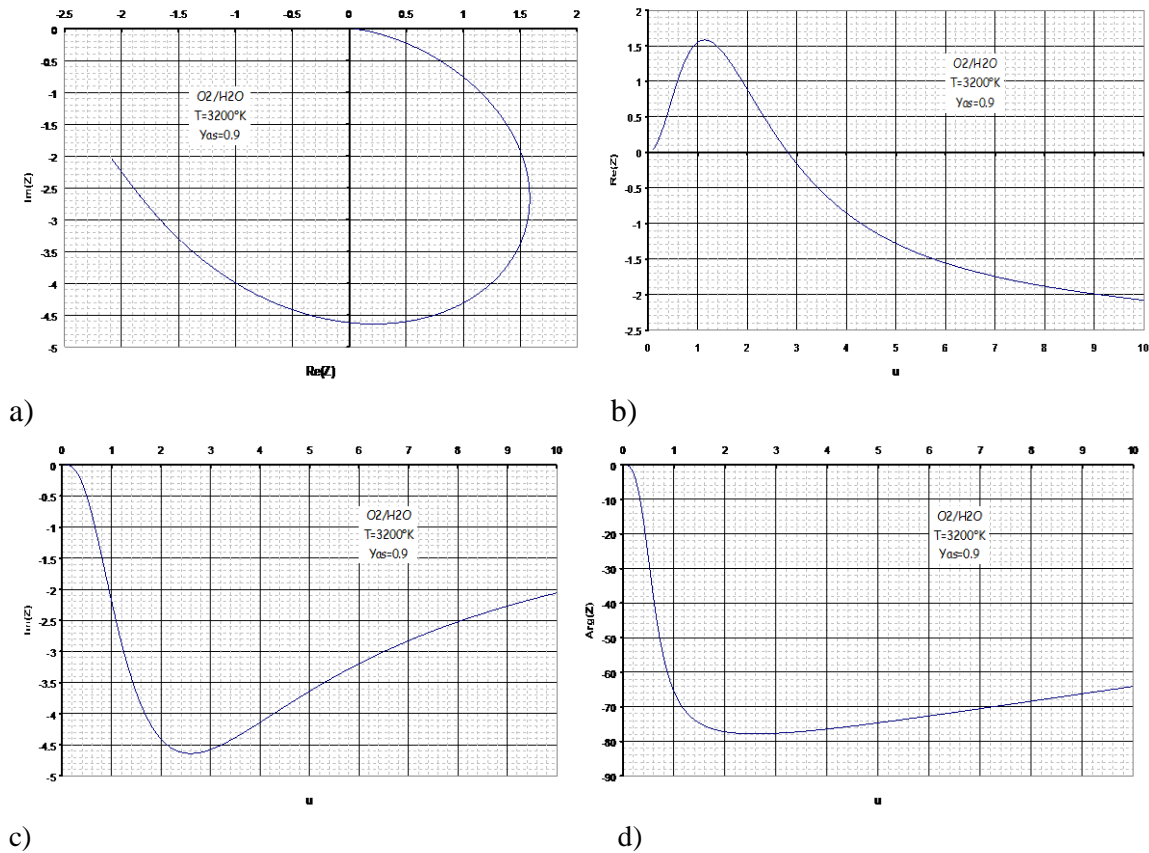
with  $u = 3\omega\bar{\tau}_v$ ,  $A = 3(\bar{a}\bar{b} - \mu)/\lambda$ ,  $B = 3\mu/\lambda$ . The real part of  $Z_0$  becomes:

$$\frac{N_0}{\alpha} = \frac{u^2 (AB + A + B - u^2)}{(1 + u^2)(B^2 + u^2)} \quad (36)$$

and its phase  $\theta_0$  can be written:  $\theta_0 = \frac{\pi}{2} + \text{Arctg}\left(\frac{-u}{A}\right) - \text{Arctg}(u) - \text{Arctg}\left(\frac{u}{B}\right)$

The cut-off value is  $u_{c0}^2 = AB + A + B$  (37)

Figure 8a, b, c and d shows respectively  $Z_0$ ,  $\text{Re}(Z_0)$ ,  $\text{Arg}(Z_0)$  for oxygen in water vapor (see section 3.3.2).



**Figure 8.** Oxygen in water vapor :

- a) Function  $Z_0 = \frac{i u}{1 + i u} \frac{A - i u}{B + i u}$ , b) Real part of  $Z_0$  as a function of  $u$ ,  
c) Imaginary part of  $Z_0$  as a function of  $u$ , d) Phase of  $Z_0$  as a function of  $u$ .

**Remark :** In their analysis Heidmann & Wieber [18] suppose that variations in the evaporation heat  $\ell$  with drop temperature perturbations are negligible ( $c=0$ ). They take a Sherwood number

Sh proportional to  $(r_s p_C)^{1/2}$  because they consider an external flow (in the present case we have  $Sh=2$ ). Their mass flow rate formula is obtained for equal molar masses  $M_A = M_F$  and for a zero concentration of fuel at infinity,  $Y_{FC} = X_{FC} = 0$ . Lewis number can be taken equal to unity. In the present case and with these hypotheses, equations (32)-(34) apply by replacing  $1+B_M = p_L/(p_C - p_L)$ ,  $\alpha = [p_L/(p_C - p_L)]/\ln[p_C/(p_C - p_L)]$ ,  $\varphi = 1 + \bar{B}_M = \bar{p}_L/(\bar{p}_C - \bar{p}_L)$ , to recover Heidmann and Wieber results.

### 3.3. Droplet with a temperature field without convection

The case of a droplet with a temperature field is more complex. As a first step, we propose to solve the problem of droplet dynamics by using the two-layer model developed in section 2.3.

#### 3.3.1 Case $T=T_L(t)$ for $0 < r < r_L$ and $T=T_S(t)$ for $r_L < r < r_s$ (two-layer model)

With the two-layer model, the heat exchange equation is replaced by the two equations of system (20). The corresponding perturbation equations are

$$\begin{cases} \frac{dT'_L}{dt} = \beta_L (T'_S - T'_L) \\ (1 - \varepsilon) \frac{dT'_S}{dt} = -\varepsilon \beta_L (T'_S - T'_L) - \frac{\bar{\ell}}{c_L \bar{T}_S \bar{\tau}_v} \left( \frac{\bar{T}_S}{\bar{T}_C - \bar{T}_S} T'_S - \frac{\bar{T}_C}{\bar{T}_C - \bar{T}_S} T'_C + B'_T + \ell' \right) \end{cases} \quad (38)$$

System (38) can be rearranged to give

$$\begin{cases} \frac{dT'_L}{dt} = \beta_L (T'_S - T'_L) \\ \varepsilon \frac{dT'_L}{dt} + (1 - \varepsilon) \frac{dT'_S}{dt} = -\frac{\bar{\ell}}{c_L \bar{T}_S \bar{\tau}_v} \left( \frac{\bar{T}_S}{\bar{T}_C - \bar{T}_S} T'_S - \frac{\bar{T}_C}{\bar{T}_C - \bar{T}_S} T'_C + B'_T + \ell' \right) \end{cases} \quad (39)$$

Then, we obtain instead of equation (33), the new system

$$\begin{cases} \frac{dT'_L}{dt} = \beta_L (T'_S - T'_L), \quad \lambda \bar{\tau}_v \left[ \varepsilon \frac{dT'_L}{dt} + (1 - \varepsilon) \frac{dT'_S}{dt} \right] + \mu T'_S - \bar{a} p'_C = 0 \\ \text{with } \lambda = \frac{c_L \bar{T}_S}{\bar{\ell}}, \quad \mu = \frac{\bar{T}_S}{\bar{T}_C - \bar{T}_S} - \frac{2c}{\bar{T}_S - c} + \bar{b} \varphi, \quad \bar{a} = \frac{\bar{T}_C}{\bar{T}_C - \bar{T}_S} \frac{\gamma - 1}{\gamma} + \varphi, \\ \varphi = \left( \frac{\bar{B}_M}{Le} \frac{(1 + \bar{B}_M)^{1/Le - 1}}{(1 + \bar{B}_M)^{1/Le - 1}} - \frac{\bar{Y}_{AC} \bar{Y}_{FS}}{\bar{Y}_{AS} (\bar{Y}_{FS} - \bar{Y}_{FC})} + \frac{(M_A - M_F) \bar{Y}_{FS}}{M_F \bar{Y}_{AS} + M_A \bar{Y}_{FS}} \right) \frac{M_A}{M_F \bar{X}_{FS} + M_A \bar{X}_{AS}} \end{cases} \quad (40)$$

The other equations remain valid

$$\bar{\tau}_v \frac{dM'}{dt} = -M', \quad \dot{M}' = \frac{1}{3} M' + \alpha (p'_L - p'_C), \quad p'_L = \bar{b} T'_S \quad (41)$$

Setting  $f' = \hat{f} e^{i\omega t}$  for any parameter leads to

$$\frac{\hat{M}}{\hat{p}_C} = \frac{\left[ (\bar{a}\bar{b} - \mu) \beta_L + (1 - \varepsilon) \lambda \omega^2 \bar{\tau}_v - i \omega (\lambda \tau_v \beta_L - \bar{a}\bar{b} + \mu) \right] 3\alpha i \omega \bar{\tau}_v}{\left[ \mu \beta_L - (1 - \varepsilon) \lambda \omega^2 \bar{\tau}_v + i \omega (\mu + \beta_L \lambda \bar{\tau}_v) \right] (1 + 3i \omega \bar{\tau}_v)} \quad (42)$$

Let us define  $Z$  as follows:

$$Z = \frac{1}{\alpha} \frac{\hat{M}}{\hat{p}_C} = \frac{i u}{1 + i u} \frac{(\Lambda + i u)(A - i u) - \varepsilon u^2}{(\Lambda + i u)(B + i u) + \varepsilon u^2} \quad (43)$$

$$\text{with } u = 3\omega \bar{\tau}_v, \quad A = 3(\bar{a}\bar{b} - \mu)/\lambda, \quad \Lambda = 3\beta_L \bar{\tau}_v, \quad B = 3\mu/\lambda, \quad \varepsilon = (r_L/r_S)^3 \quad (44)$$

Coefficients  $\varepsilon$  and  $\Lambda$  are non-dimensional. As shown in (44),  $\varepsilon$  is a volume ratio. Coefficient  $\Lambda$  is a time ratio  $\Lambda = \bar{\tau}_v/\bar{\tau}_T$ , where  $\bar{\tau}_T$  can be deduced from (22) and is a heat transfer time

$$\bar{\tau}_T = \frac{2\rho_L c_L \bar{r}_L^2}{9k_L Nu_L} \quad (45)$$

- For  $\varepsilon=0$  (only one layer,  $r_L=0$ ), we obtain again  $Z=Z_0$  (Eq. 35). The cut-off frequency is thus  $u_{c0}$  (Eq.37).

- For  $\varepsilon \neq 0$  and  $\varepsilon \neq 1$ ,  $\Lambda = 0$ , in this case there are two layers without any heat exchange, then  $Z=Z_1 \neq Z_0$ :

$$Z_1 = \frac{i u}{1 + i u} \frac{A - i u (1 - \varepsilon)}{B + i u (1 - \varepsilon)} \quad (46)$$

$$\frac{N_1}{\alpha} = \frac{u^2 \left[ AB + A + B - \varepsilon(A + B) - u^2 (1 - \varepsilon)^2 \right]}{(1 + u^2) \left[ B^2 + u^2 (1 - \varepsilon)^2 \right]}$$

and the cut-off reduced frequency becomes

$$u_{c1}^2 = \frac{AB + A + B - \varepsilon(A + B)}{(1 - \varepsilon)^2} \geq u_{c0}^2 \quad (47)$$

Then, for a given droplet radius, the absence of heat exchange between the two layers leads to an extension of the instability domain. At the limiting, the case  $\varepsilon=1$ , corresponds to one layer without heat exchange. In this case, there is no cut-off frequency.

- In the general case of the two layer model (with  $0 < \varepsilon < 1$ )

$$\frac{N}{\alpha} = \frac{u^2 \left\{ \Lambda^2 (AB + A + B) + u^2 \left[ AB + A + B - \Lambda^2 - \varepsilon(A + B + \Lambda B - \Lambda A) \right] - u^4 (1 - \varepsilon)^2 \right\}}{(1 + u^2) \left[ (\Lambda B - (1 - \varepsilon)u^2)^2 + u^2 (\Lambda + B)^2 \right]} \quad (48)$$

The cut-off frequency is then given by

$$u_c^2 = \frac{AB + A + B - \Lambda^2 - \varepsilon(A + B + \Lambda B - \Lambda A) + \sqrt{\Delta}}{2(1 - \varepsilon)^2} \quad (49)$$

with  $\Delta = \left[ AB + A + B - \Lambda^2 - \varepsilon(A + B + \Lambda B - \Lambda A) \right]^2 + 4(1 - \varepsilon)^2 (AB + A + B) \Lambda^2$ .

In the following section, we investigate, by solving numerically equations (47) and (48), the influence of the volume ratio  $\varepsilon$  and the heat exchange coefficient between the two layers on the response factor  $N$  and the cut-off frequency  $u_c$ .

### 3.3.2. Coefficient A and B for oxygen and methane in water vapor

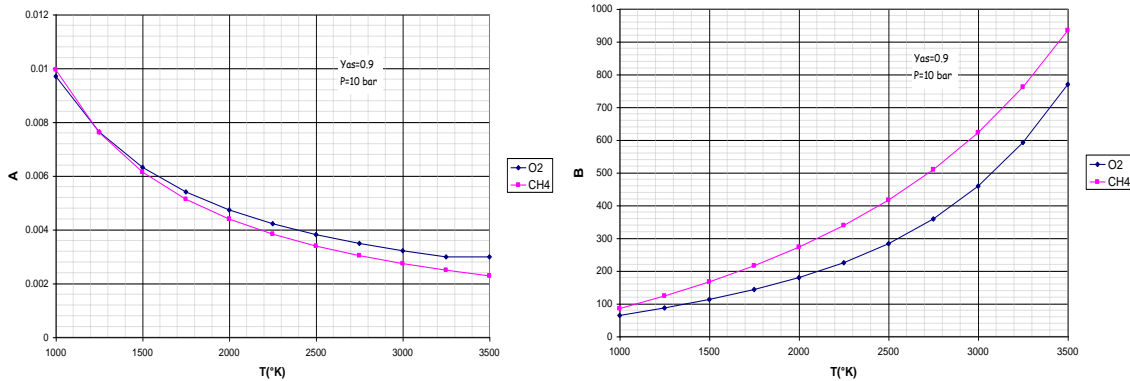
A program was written to solve equations (48) and (49). The user of the program should select the diluent and the droplet type then the pressure, the temperature and the mass fraction of the diluent in the combustion chamber .

The coefficient  $A$  and  $B$  are computed in these conditions. We consider the case of a chamber pressure of 10 bar, a mass fraction of the water in the combustion chamber of 0.9 and a temperature of 3000°K. One can see in Figure 9a and 9b that the coefficient  $A$  is very small compared to  $B$  in these conditions.

### 3.3.3. Influence of $\varepsilon$ and $\Lambda$ on the response factor $N$

We consider a liquid oxygen (LOX) droplet evaporating in a mixture of  $O_2$  and  $H_2O$  (the diluent  $A$  is water, which is the main combustion product) at  $T=3000K$ ,  $p=10bar$ ,  $Y_{AC}=0.9$ .

- For a fixed value of  $\varepsilon$ , chosen between 0 and 1, and  $\Lambda$  varying between 0 and infinity, the cut-off reduced frequency  $u_c$  decreases from  $u_{c1}$  to  $u_{c0}$  (the instability region is restricted) as shown on Figure 10 in the case  $\varepsilon=0.5$ .



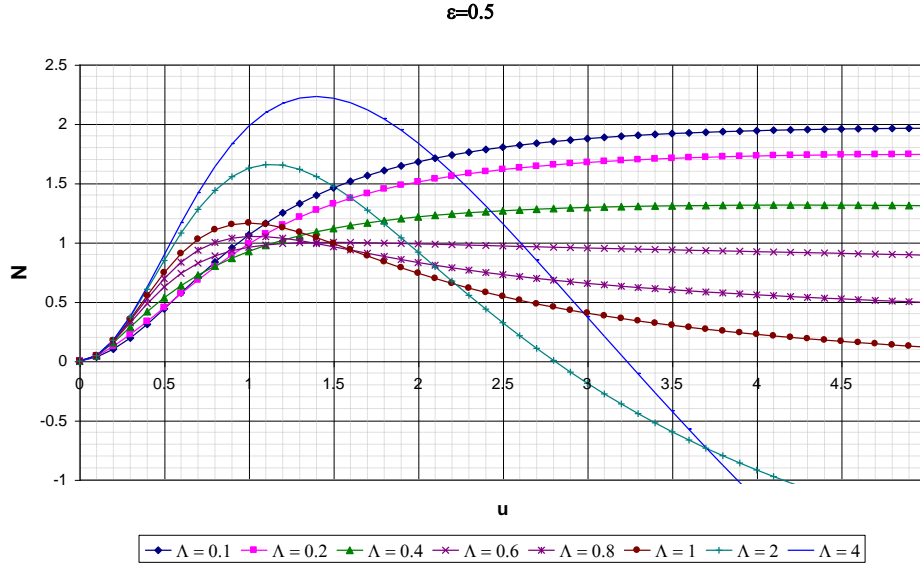
**Figure 9.** Evolution of coefficients A and B versus temperature

- For a fixed value of  $\Lambda$ , chosen between 0 and  $\infty$ , we have studied the effect of  $\varepsilon$  growing from 0 (one layer model) to 1.

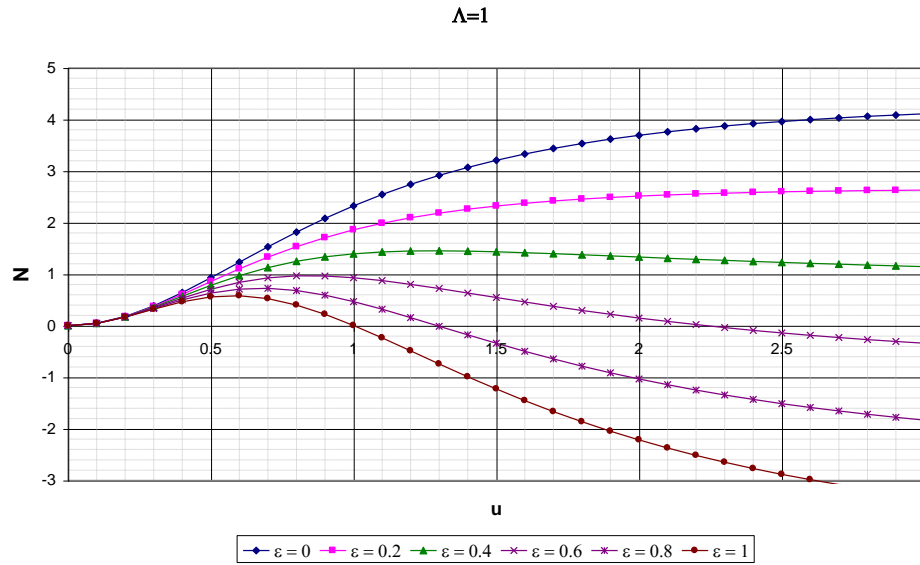
For  $\varepsilon = 1$ , equation (48) gives the square of the cut-off frequency  $u_c^{*2} = A^2(AB+A+B)/(A-A)(A+B)$ .

This value of  $u_c^*$  has a physical meaning only when  $\Lambda \geq A$ . For  $\Lambda = A$ , the obtained  $u_c^*$  is infinite.

Results are shown in figure 11 for  $\Lambda = 1$ . This figure shows that, for a given  $\Lambda$ , the maximum response factor decreases, and the instability region is restricted, as  $\varepsilon$  increases.

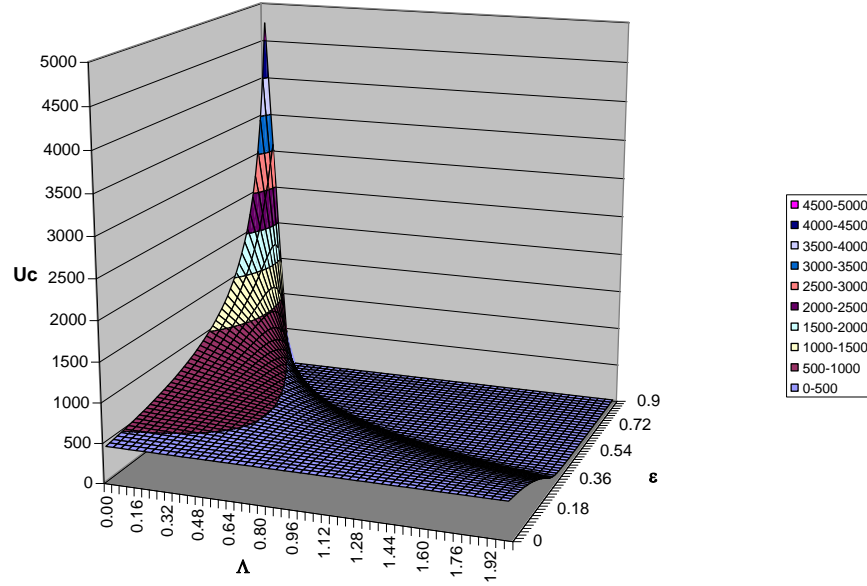


**Figure 10.** Influence of the reduced heat exchange coefficient  $\Lambda$  between the two liquid layers, on the reduced response factor  $N/\alpha$ , for  $\varepsilon = (r_L/r_S)^3 = 0.5$ ,  $T=3000K$ ,  $p=10b$ ,  $Y_{AC}=0.9$ .



**Figure 11.** Influence of the volume ratio  $\varepsilon=(r_L/r_S)^3$ , on the reduced response factor  $N/\alpha$ , for  $\Lambda=1$ ,  $T=3000K$ ,  $p=10b$ ,  $Y_{AC}=0.9$ .

- Figure 12 shows the combined effects of  $\varepsilon$  and  $\Lambda$  on the reduced cut-off frequency  $u$ . In this figure, the cut-off frequency exhibits a maximum when  $\varepsilon$  is varied.



**Figure 12.** Effects of  $\varepsilon$  and  $\Lambda$  on the reduced cut-off frequency  $u_c$ .

### 3.3.4. Influence of $\varepsilon$ for liquid Nusselt number equal to 2.

The mean life time of the droplet as defined in (31) can be written

$$\bar{\tau}_v = \frac{\bar{M}}{M} = \frac{(4/3)\pi \rho_L \bar{r}_s^3}{4\pi \bar{\rho} \bar{D} \bar{r}_s \ln(1 + \bar{B}_M)} = \frac{\rho_L \bar{r}_s^2}{3\bar{\rho} \bar{D} \ln(1 + \bar{B}_M)} \quad (50)$$

From (22), (44) and (50), we deduce

$$\Lambda = \frac{\bar{\tau}_v}{\bar{\tau}_T} = \frac{3k_L Nu_L}{2c_L \bar{\rho} \bar{D} \ln(1 + \bar{B}_M)} \varepsilon^{-2/3} \quad (51)$$

The liquid Nusselt number  $Nu$  characterizes the degree of agitation inside the liquid droplet, which may be induced by a small velocity difference between droplet and gas, or by a non-homogeneous temperature at the droplet surface which can generate Marangoni effect.

If we suppose the liquid at rest and the Nusselt number  $Nu=2$ , then  $\Lambda$  and  $\varepsilon$  become dependent parameters. We have

$$\Lambda \varepsilon^{2/3} = 3k_L / c_L \bar{\rho} \bar{D} \ln(1 + \bar{B}_M) \quad (52)$$

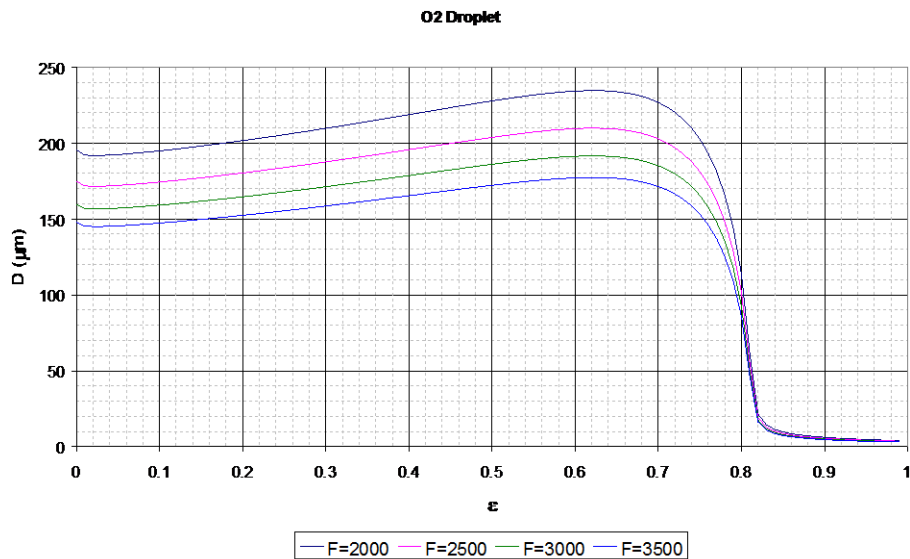
The volume ratio  $\varepsilon = (r_L / r_s)^3$  is then the single parameter from which  $\Lambda$  and  $u_c = 3\omega_c \bar{\tau}_v = 6\pi F_c \bar{\tau}_v$  can be deduced, using (52).

For a given pressure perturbation frequency in the combustion chamber, one can determine the diameter under which evaporation is unstable

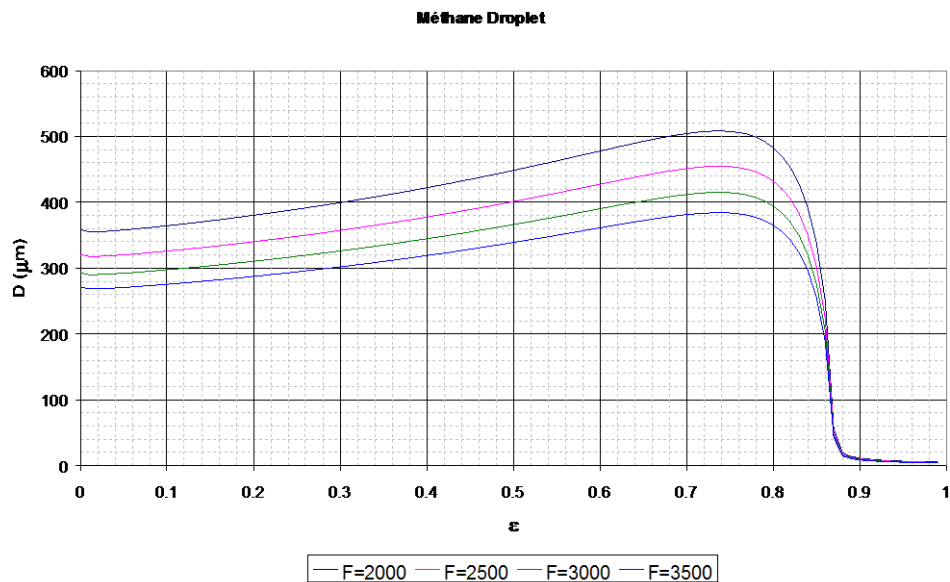
$$D_c = \sqrt{\frac{2u_c \bar{\rho} \bar{D} \ln(1 + \bar{B}_M)}{\pi \rho_L F_c}} \quad (53)$$

This stability limit diameter has been calculated for LOX droplets and for liquid methane droplets. We find LOX droplets are more stable than liquid methane droplets for frequencies orders between 1000 and 10000 Hz (Figure 13) which is in accordance with experimental observations.

At a given frequency  $F$ , we can consider the limiting diameter as a function of the parameter  $\varepsilon$ . Figure 13a shows the obtained curves for oxygen and Figure 13b those for methane, Figure 13b.



a)



b)

**Figure 13.** Influence of parameter  $\varepsilon$  on stability limit of methane droplet diameter in the case of dependent parameters  $\varepsilon$  and  $\Lambda$  (Equ. (52))

#### 4. CONCLUSIONS AND PROSPECTS

Basic considerations on transient heating and dynamic response of an evaporating droplet are discussed in this article. For transient regime study the quasi-steady hypotheses are assumed for

the gaseous phase. For the liquid phase the heat transfer in the droplet is commonly described by a uniform temperature model (assuming an infinite conductivity coefficient). A two-layer model is proposed here to take into account heat transfer inside the droplet (assuming a finite heat exchange coefficient between both layers).

A linearized model is developed on the basis of Heidmann and Wieber theory [18].

A dynamic model is investigated on the basis of the two previous models, and is then used in the stability analysis to determine the complex transfer function and the response factor of a vaporizing droplet submitted to small periodic perturbations. Only the case of a pressure perturbation is considered.

The driving parameters are the four reduced coefficients ( $A, B, \varepsilon, \Lambda$ ). The parameters  $A$  and  $B$ , characterize the processes in the gas phase and at the droplet surface, and the two other characterize the liquid layer volumes and the heat exchange in the liquid phase. These parameters ( $\varepsilon, \Lambda$ ) are only present in the two-layer model.

In the two-layer model the transfer function  $Z$  of the evaporating droplet depends on both the relative volume  $\varepsilon$  of the inner layer and the ratio  $\Lambda = \bar{\tau}_v / \bar{\tau}_T$  of the mean droplet life time on a thermal exchange time, which characterizes the heat exchange inside the liquid droplet.

The influence of  $\varepsilon$  and  $\Lambda$ , considered as independent parameters, is studied for liquid oxygen droplet and for liquid methane droplet in water vapor. The cut-off reduced frequency  $u_c$ , which characterizes the extension of the instability domain, and the maximum of the response factor  $N$ , appear to be affected by the values of  $\varepsilon$  and  $\Lambda$ . It should be noticed that the case  $\varepsilon=0$  and the case  $\Lambda=\infty$  (for  $\varepsilon$  chosen between 0 and 1) are limiting cases corresponding to the simple droplet with uniform temperature. The case  $\varepsilon=1$  corresponds to a drop with an internal uniform temperature not equal to its surface temperature.

If  $\varepsilon$  and  $\Lambda$  are no more assumed independent, i.e. for a Nusselt number of the liquid equal to 2, the stability limit droplet diameter is a function of the relative volume  $\varepsilon$  which depends of the considered perturbation frequency.

In every case the results shows the important effect of thermal exchange inside the droplet on the vaporization stability.

Perhaps will we need to take into account variations of  $\varepsilon$  during vaporization, but this is not obvious, and probably one value of  $\varepsilon$  will be sufficient to improve the results in comparison to the uniform temperature model.

In practice, we have to chose representative values of  $\varepsilon$  and  $\Lambda$ :  $\Lambda$  depends on conduction and convection coefficients inside the droplet, i.e. on  $k_L, \chi, Nu_L$  and  $\varepsilon$  should be deduced from a comparison with results of more accurate calculations (i.e. Law and Sirignano).

To continue these investigations, it should be interesting:

- To conceive a N-layer model to compare with the two-layer model and find the volume fraction
- To study the effect of convection due to relative velocity gas/droplet
- For the future, to validate numerically the results

### **Acknowledgements**

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## LIST OF SYMBOLS

### *Latin characters*

$\bar{a}$ :	coefficient defined by (33)
$\bar{b}$ :	derivative $d \ln p_{sat} / d \ln T$
$a, b, c$ :	coefficients of saturated vapor pressure as a function of temperature
$A$ :	diluent; coefficient defined in (18)
$B$ :	Spalding parameter, coefficient defined in (18)
$B_M$ :	Spalding parameter for mass exchange
$B_T, B'_T$ :	Spalding parameter for heat exchange, respectively from Sirignano and from Lefebvre
$c_L$ :	specific heat of liquid
$c_p$ :	specific heat of gas at constant pressure per unit mass
$D$ :	droplet diameter : $D=2 r_S$ ; diffusion coefficient
$F$ :	gaseous fuel; frequency
$k$ :	thermal conductivity
$k_{eff}$ :	effective thermal conductivity of liquid droplet
$K$ :	rate of evaporation $dD^2/dt$
$\ell$ :	latent heat of evaporation per unit mass
$L$ :	latent heat of evaporation per mole
$Le$ :	Lewis number $k c_p / \rho D c_{pf}$
$M$ :	mass of droplet
$M_L$	internal liquid sphere mass
$M_S$	difference $M-M_L$
$M_j$ :	molar mass of species j
$\dot{M}$ :	mass flow rate (evaporation rate) of droplet
$\dot{m}$ :	unit mass flow rate
$N, N_0$ :	response factor
$Nu$ :	Nusselt number
$p$ :	pressure
$p_{sat}$ :	saturated vapor pressure of liquid
$Pr$ :	Prandtl number
$q$ :	parameter
$\bar{q}$ :	mean value of the parameter $q$
$q'$ :	reduced perturbation $q'=(q-\bar{q})/\bar{q}$
$\hat{q}$ :	complex modulus of $q'$
$Q_L$ :	heat brought to the drop, other than latent heat of evaporation
$Q_{SL}$ :	heat exchanged between the internal liquid sphere and its complement
$r$ :	radius
$r', r''$ :	characteristic roots
$r_L$ :	internal liquid sphere radius
$r_S$ :	droplet radius
$R$ :	universal gas constant
$Sh$ :	Sherwood number
$t$ :	time
$T$ :	temperature
$T_R$ :	reduced temperature
$u$ :	reduced frequency
$v$ :	radial velocity
$V$ :	volume

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$X_j$  : molar fraction of species j  
 $Y_j$  : mass fraction of species j  
 $Z$  : complex transfer function of evaporating droplet

***Greek characters***

$\alpha$  : defined in (32)  
 $\beta, \beta', \beta_L, \beta_S$  : reduced thermal exchange coefficients  
 $\gamma$  : isentropic coefficient  
 $\varepsilon$  : volume ratio  $(r_L/r_S)^3$   
 $\Delta t_{hu}$  : droplet heating time  
 $\Delta t_{ex}$  : droplet life time  
 $\Lambda$  : time ratio  $\bar{\tau}_v/\bar{\tau}_T$   
 $\lambda, \mu, \varphi$  : coefficients defined in (33)  
 $\mu_j, \mu_L$  : chemical potential of gaseous species j, of liquid  
 $\tau$  : characteristic time  
 $\bar{\tau}_T$  : mean heat transfer time  
 $\bar{\tau}_v$  : mean droplet life time equal to  $\bar{M}/\dot{\bar{M}}$   
 $\rho$  : density  
 $\theta, \theta_0$  : phase difference  
 $\chi$  : ratio  $k_{eff}/k_L$   
 $\omega$  : pulsation

***Subscripts and exponents***

c: cut-off  
C: combustion chamber  
g: relative to gas  
hu : transient heat-up period  
i : initial conditions  
j : chemical species  
L : liquid, in the bulk of liquid phase  
M : relative to mass exchanges  
QS : quasi-steady  
S : on, or at the vicinity of the droplet surface  
SL: corresponds to the heat exchange between two liquid layers  
Sat: saturated vapor  
T : relative to thermal exchanges  
0 : relative to the case  $\varepsilon=0$   
 $\infty$  : at infinity  
- : mean value

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