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A simple model for the dynamics of contact lines freezing on a cold substrate

R. Herbaut^{1,2}, J. Dervaux¹, P. Brunet¹, L. Royon², L. Limat¹

¹Laboratoire Matière et Systèmes Complexes (MSC), UMR 7057 of CNRS and Université Paris Diderot, Université Sorbonne Paris Cité, 10 rue Alice Domon et Léonie Duquet, 75013 Paris, ...

²Laboratoire Interdisciplinaire des Energies de Demain (LIED), UMR 8236 of CNRS and Université Paris Diderot, Université Sorbonne Paris Cité, 10 rue Alice Domon et Léonie Duquet, 75013 Paris, France

Email: remy.herbaut@univ-paris-diderot.fr

Abstract

Starting from de Gennes model of contact line dynamics, we develop a simple, analytical, description of a four phase contact line, at which a liquid advances on a cold substrate (of infinite thermal conductivity) and at the same time freezes on the solid. The key idea consists in a subtle balance between thermal and hydrodynamical divergences at contact line, heat produced by dissipation in the liquid phase limiting the solid growth. We calculate the apparent liquid contact angle and its equivalent for the solid phase as a function of the contact line velocity U and the other parameters involved, in the limit of small angles. We show that the arrest condition ($U=0$) implies a divergence of these angles, that are presumably reaching 90° , while the liquid film thickness vanishes, which is reminiscent of previous works of Sonin et al in the 90's. Remarkably, at low contact line velocity, we show that a stick-slip behaviour is to be expected, which has been recently observed experimentally. The temperature dependence of the critical velocity differs from what is found in recent experiments, but this model is to our opinion a necessary first step to understand these complex behaviours.

Keywords: Contact lines, Phase Change, Wetting

1. Introduction

Contact line dynamics is still a disputed subject as classical hydrodynamical approaches are diverging near the contact line, where some cut-off needs to be introduced based on some microscopic physics that may depend on the system considered (free slip condition, molecular jumps, pinning/depinning events at small scale, etc)^{1,2,3}. The situation becomes even more complex when solidification of the liquid are involved at contact lines. Recent experimental studies performed in the 90's reveals an arrest condition at which contact line motion is blocked^{4,5,6,7}, and more recent ones the possibility of a stick-slip advance at low velocity⁸. Sonin⁹ proposed a model of this kind in the 90's, but were unable to develop something analytical. In the present paper, we show that, at least theoretically, it is possible to build a four phase model of a freezing contact line moving on a cold « perfect » substrate of infinite thermal conductivity, in contact with a liquid having no delay for solidification.

2. Quadruple dynamical contact lines and thermal fields in wedge

In our approach, we assume three distinct parts of a dynamic contact line (a nanometric-scaled region, intermediate region, a macroscopic quasi-static region) in a total wetting in the static case ($\theta_e=0$). An Approaches of dynamical contact lines are proposed in intermediate region which is inspired by de Gennes model. Let us establish the energy balance within the intermediate mesoscopic region, inspired from de Gennes's model for a dynamical triple line, here adapted for our situation of a quadruple line. We assume that the liquid and solid wedges (of angles respectively equal to θ_L - θ_s and θ_s) are in contact with each other and form altogether an apparent contact angle of θ_L , which holds until the nanometric scale, see Fig. 3. Both wedges advance at the same velocity U with respect to the substrate. A flow is established in the liquid wedge of height $h_L(x)$, and we assume that the liquid and solid wedge are undeformed by the flow yields. In this situation, the continuity equation can then be written as:

$$U_{\perp} \cos(\theta_L - \theta_s) = \langle U \rangle \tan(\theta_L - \theta_s)$$

Here, U_{\perp} is the advancing velocity normal to the free-surface of the wedge, which can be expressed as: $U_{\perp} = U \sin \theta_L$. Under the assumption of small angles ($\tan(\theta_L - \theta_s) \sim \theta_L - \theta_s$, $\sin \theta_L \sim \theta_L$ and $\cos(\theta_L - \theta_s) \sim 1$). The heat flux generated across the liquid-solid interface by the substrate at T_p , rules the solid angle θ_s within the liquid wedge. This flux at the interface liquid/solid is determined by the heat equation, which in a steady situation where convection is neglected, writes as the classical Laplacian equation. We calculate the average heat

flux of the solid/liquid interface inside intermediate region from the temperature field. Under the approximation of small angle and applying the Stefan condition, we estimate the angle θ_s . In this case, the de Gennes's model in total wetting applied to our geometry yields give:

$$\theta_L = \left(\frac{k\Delta T}{\rho L U (b-a)} \log \frac{b}{a} \right)^{\frac{1}{3}} + \left(6 Ca \log \left(\frac{b}{a} \right) \right)^{\frac{1}{3}}$$

where a and b are cut-off lengths corresponding respectively to the nanometric and micro-metric scales delimiting the intermediate region (hence, $a \sim 1\text{nm}$ and $b \sim 1\mu\text{m}$), Ca capillary number, k thermal conductivity, ρ density, L latent heat, T temperature.

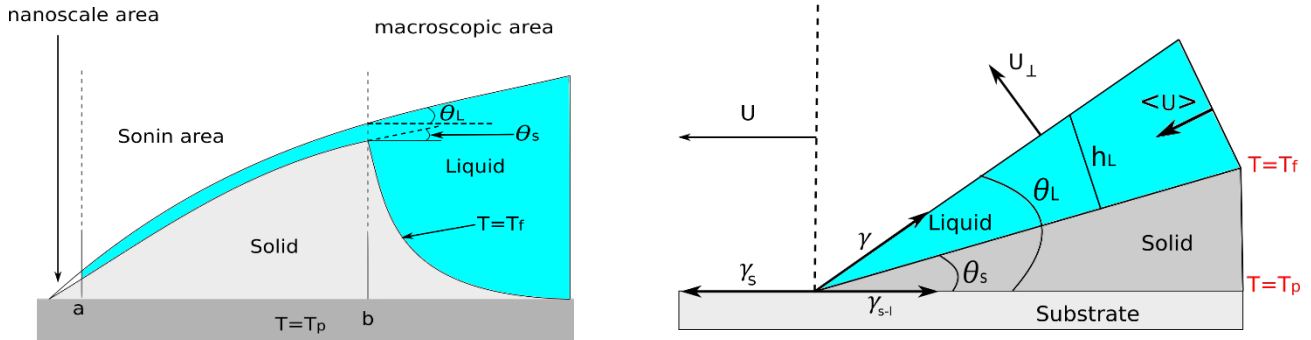


Figure 1. Sketch of the quadruple line geometry and the quadruple contact-line in the intermediate mesoscopic

3. Results, comparison and discussion

Figure 2a plots θ_L versus U for various ΔT , the evolution of θ_L follows a trend similar to the isothermal situation for relatively large U , with an increase of θ_L with U . However, at relatively low velocity ($U < U^*$), θ_L decrease sharply with U . This decrease corresponds to an unstable situation for the contact-line dynamics. We assume that U^* (the location of the minimum of $\theta_L(U)$) corresponds to a critical velocity which could delimitate the transition between continuous and stick-slip dynamics. The critical velocity U^* defined in the previous section, is determined by the minimum of $\theta_L(U)$ and combined with θ_L leads to an estimation of the arrest angle θ_a :

$$\theta_a = \left(\left(\frac{3}{2} \right)^{-\frac{2}{5}} + \left(\frac{3}{2} \right)^{\frac{2}{5}} \right) \left(\frac{6\eta k \Delta T}{\gamma \rho L (b-a)} \right)^{\frac{1}{5}} \left(\log \frac{b}{a} \right)$$

which is plotted and compared to the experimental data of De Ruiter and Tavakoli^{6,7} in figure 2b.

6. References

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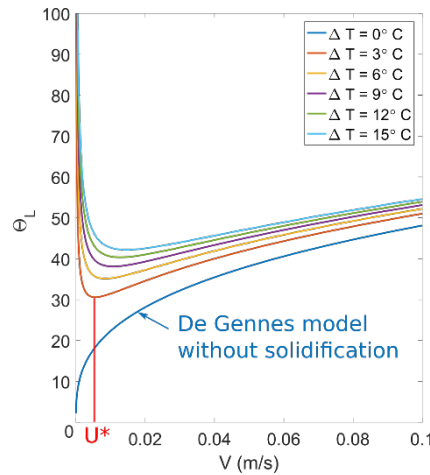


Figure 2a: Dynamical contact angle versus U

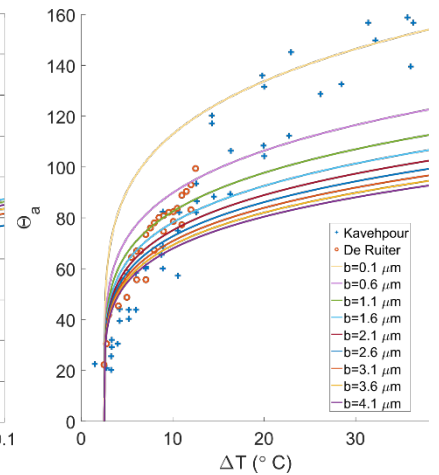


Figure 2b: Arrest angle versus ΔT