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# Numerical and Experimental Analysis of the Correlation Between EIT Data Eigenvalues and Two-phase Flow Phase Fraction

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## ABSTRACT

Electrical Impedance Tomography (EIT) is an imaging technique with advantages of non-intrusiveness, low-cost and high temporal resolution, which is promising for multiphase flow instrumentation. However, it produces smooth images with low spatial resolution where the interface between phases cannot be distinguished and from which the phase fraction cannot be estimated correctly. In this article, an eigenvalue analysis of EIT raw data is used to estimate the void fraction, i.e. the phase area ratio in 2D, without reconstructing images. For a given EIT sensor, each acquisition frame is represented by an impedance matrix whose eigenvalues are computed after normalization. The main characteristics of the eigenvalue distribution for different two-phase flow patterns within a cylindrical pipe are analyzed numerically. The behaviors of the leading eigenvalue and of the sum of the absolute values of the following ones are assessed as functions of the void fraction. This leads to an estimation of the two-phase flow void fraction based on the characteristics of the EIT sensor configuration. The presented numerical results highlight the existing correlation between the eigenvalues and the void fraction for the phase distribution patterns considered. These simulation results are compared with experimental static tests for validation.

## KEYWORDS

Electrical Impedance Tomography, impedance matrix, flow patterns, static test.

## 1 Introduction

In the research field of experimental multiphase flow, instrumentation techniques for local void fraction (phase area ratio) and phase distribution estimation are in great need. These parameters are essential to understand the heat transfer coefficient and assess the risk of critical boiling, for example in nuclear industry [1, 2]. Various techniques have been developed for multiphase flow phase fraction estimation, for example, wire-mesh sensors can provide information about local, cross-section or in-situ volume profiles and distributions of phase fraction, but they have disruptive effects on the flow [3]; optical probes are sensitive to

37 interfacial passages enabling to measure local void fraction with a high precision, however,  
38 they are intrusive [4], as are electrical probes as well [5]; X-ray and Gamma-ray tomography  
39 allow fast measurements of multiphase flows at high spatial resolution but they require high  
40 acceleration voltage (hundreds of kV) and radiation protection [6, 7]; Electrical Capacitance  
41 Tomography (ECT) has similar characteristics to the EIT but it requires high voltage exci-  
42 tation signals. Note that Huang et al. [8] used ECT to measure void fraction based on image  
43 reconstruction with a satisfactory accuracy. With the advantages of non-intrusiveness, high  
44 temporal resolution and low cost, the Electrical Impedance Tomography (EIT) technique  
45 seems to be promising for dynamic multiphase flow instrumentation.

46 EIT is a technique to determine the material distribution inside a 2D or 3D domain  
47 based on their constitutive electric properties, e.g. the electric admittivity (conductivity  
48 and permittivity), with an image acquisition rate that can be up to hundreds of frames  
49 per second [9]. For an EIT sensor, the measurements are acquired on the boundary and  
50 the typical way to handle the measurement data is through image reconstruction, which  
51 leads to an approximate reconstruction of the admittivity distribution within the domain.  
52 Theoretically, continuous boundary measurement determines a unique solution to the re-  
53 construction problem [10], while in practice the boundary measurement is limited by the  
54 number of electrodes.

55 A number of reconstruction methods have been proposed based on non-iterative or iter-  
56 ative algorithms. Some non-iterative algorithms are based on linear approximation, such as  
57 the linear back projection method [11], the one-step Newton method [12, 13] or the Calderon's  
58 approach [14]. Iterative methods tackle the reconstruction problem by minimizing some cost  
59 functions, e.g. the variational method [15] or the modified Newton-Raphson method [16].  
60 In general, non-iterative algorithms produce smooth images with low spatial resolution com-  
61 pared to the hard-field tomography techniques, like X-ray or Gamma-ray tomography [17].  
62 This makes it difficult to distinguish the phase interface in the reconstructed images. It-  
63 erative methods yield reconstructions at higher resolutions but they usually require high  
64 computation power. In the present article, an alternative methodology is proposed to esti-  
65 mate the void fraction of two-phase flows. It circumvents the need to reconstruct images in  
66 the extreme non-linear case of two-phase flows, which are characterized by a high resistivity  
67 contrast between phases, while it allows to consider a wide range of void fraction.

68 A typical EIT sensor has a set of electrodes installed on the boundary of a domain, Fig-  
69 ure 1 shows an EIT sensor with 16 electrodes. A stimulation current (or voltage) are exerted  
70 on one pair of selected electrodes (noted as source and drain electrode), the corresponding  
71 voltage (or current) at all the other electrodes are measured, this is called a stimulation  
72 pattern. This procedure continues until each independent pair of electrodes has been se-  
73 lected as source and drain [18, 19], a strategy referred to as the full scan stimulation strategy.  
74 Since source and drain are essentially the same, the measurements would be reciprocal for  
75 a specific pair of electrodes. For an EIT sensor with  $\ell$  electrodes, there are  $\ell(\ell - 1)/2$  inde-  
76 pendent stimulation patterns in total for one frame of image. Each stimulation pattern has  
77 a representative bulk impedance, which depends on the phase area ratio and distribution,  
78 as well as on the source/drain electrodes. The impedance array of each frame can be con-  
79 veniently arranged into a  $\ell \times \ell$  symmetric matrix, whose diagonal entries are related to the  
80 mutual impedances. In the work of Fang et al. [20], the relation between the void fraction  
81 and the eigenvalues of the capacitance matrix obtained from ECT data is investigated and

82 it is shown that the multiplicity of the eigenvalues is correlated to the degree of symmetry of  
83 the flow. Dupré et al. applied the eigenvalue analysis to ECT data for the purpose of flow  
84 regime identification as well as the estimation of two-phase flow void fraction, the result is  
85 promising [19, 21]. In this context, and up to the Author’s knowledge, the literature deal-  
86 ing with void fraction estimation from EIT data and through eigenvalue analysis is scarce.  
87 Therefore, in the present article, the impedance matrix is investigated and its eigenvalues  
88 are used to devise some metrics of the void fraction.

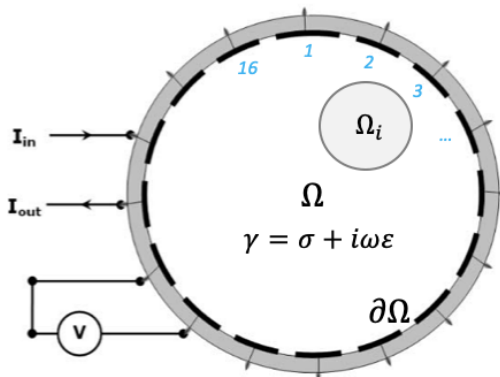


Figure 1. schematic of an EIT sensor with 16 electrodes.

89 The EIT problem is associated with an elliptic boundary-value problem and it aims at  
90 recovering the information on the admittivity distribution inside a domain of interest from  
91 boundary measurements. In Section 2, the mathematical basis of EIT is introduced with  
92 the Neumann-to-Dirichlet (NtD) operator along with the bulk impedance that is the dis-  
93 crete realization of the former. The impedance data can be computed numerically using the  
94 Finite Element Method (FEM) and arranged into a square impedance matrix, the eigen-  
95 values of which are the focus of this study. In Section 3, the typical regimes of two-phase  
96 flows are classified into three different canonical cases that are studied separately: the dis-  
97 tribution of the eigenvalues of the impedance matrix is studied for each pattern based on a  
98 suitable normalization, with the relation between eigenvalues and void fraction being inves-  
99 tigated numerically. In Section 4, the influence of noisy data and an extension to conductive  
100 inclusions are investigated. In Section 5, the EIT system developed in the laboratory is  
101 introduced and the simulation results are validated by experimental results associated with  
102 static configurations. These results are discussed and concluded in Section 6.

## 103 2 Mathematical basis

104 Considering the electric field in a two- or three-dimensional domain  $\Omega$ , from Maxwell’s equa-  
105 tion, the electric potential  $u$  inside is governed by,

$$\nabla \cdot (\gamma(x) \nabla u(x)) = 0, \quad x \text{ in } \Omega, \quad (1)$$

106 where  $\gamma(x) = \sigma(x) + i\omega\epsilon(x)$  is the isotropic admittivity distribution in  $\Omega$ , in which  $\sigma$  is  
107 the electric conductivity,  $\epsilon$  is the electric permittivity,  $\omega$  is the stimulation frequency. In the

108 application to multiphase flow instrumentation, usually only the conductivity  $\sigma$  is considered,  
 109 because the electric permittivity  $\epsilon$  of liquid phase can be neglected in the chosen working  
 110 frequency range of EIT sensors [22].

## 111 2.1 Neumann-to-Dirichlet map

112 The domain is assumed to be homogeneous except for a number of non-conductive inclusions,  
 113 which are denoted as  $\Omega_i$ . These inclusions have a conductivity significantly different with  
 114 the background, and they are simply connected domains contained in  $\Omega$ . Specifically for  
 115 two-phase flow, the conductivity  $\sigma$  of the water phase is within  $10^{-4} - 10^{-2}S/m$ , and that  
 116 of gas is around  $10^{-15} - 10^{-9}S/m$ , i.e.,

$$\sigma = \begin{cases} \sim 0 & \text{in } \Omega_i, \\ 1 & \text{in } \Omega \setminus \Omega_i, \end{cases} \quad (2)$$

117 with  $\sigma$  of the water phase normalized to 1.

118 Denoting as  $\mathbf{n}$  the unit outward normal vector on the boundary  $\partial\Omega$ , which is assumed  
 119 to be smooth, we have the Neumann boundary conditions

$$\sigma \nabla u \cdot \mathbf{n} = f \quad \text{on } \partial\Omega, \quad (3)$$

120 in which  $f \in L^2(\partial\Omega)$  represents the boundary current that satisfies  $\int_{\partial\Omega} f \, dS = 0$ . Note that  
 121 the model (2) entails that  $\sigma \nabla u \cdot \mathbf{n} \approx 0$  on the boundary  $\partial\Omega_i$  of the inclusions.

122 Introducing the functional space  $H_\diamond^1(\Omega) = \{\varphi \in H^1(\Omega) : \int_{\partial\Omega} \varphi \, dS = 0\}$ , the Neumann  
 123 boundary value problem is as follows: find  $u \in H_\diamond^1(\Omega)$  that satisfies

$$\int_{\Omega} \sigma \nabla u \cdot \nabla \varphi \, dV = \int_{\partial\Omega} f \varphi \, dS, \quad \forall \varphi \in H_\diamond^1(\Omega). \quad (4)$$

124 On denoting  $L_\diamond^2(\partial\Omega) = \{\varphi \in L^2(\Omega) : \int_{\partial\Omega} \varphi \, dS = 0\}$ , the Neumann-to-Dirichlet (NtD)  
 125 map is introduced as  $\Lambda_\sigma : L_\diamond^2(\partial\Omega) \rightarrow L_\diamond^2(\partial\Omega)$  so that the boundary potential can be written  
 126 as  $\Lambda_\sigma f = u|_{\partial\Omega}$ , where  $u \in H_\diamond^1(\Omega)$  is the solution to (4). The boundary potential can be  
 127 measured and compared with the boundary potential  $\Lambda_0 f = u_0|_{\partial\Omega}$  for the same  $f$  and  $\Omega$  but  
 128 without inclusions, i.e.  $\Omega_i = \emptyset$ , with  $u_0 \in H_\diamond^1(\Omega)$  being the solution of:

$$\int_{\Omega} \nabla u_0 \cdot \nabla \varphi \, dV = \int_{\partial\Omega} f \varphi \, dS, \quad \forall \varphi \in H_\diamond^1(\Omega), \quad (5)$$

129 which corresponds to the reference problem with a homogeneous conductivity distribution  
 130 inside the domain  $\Omega$ . The relative NtD map is denoted as  $\Pi = \Lambda_\sigma - \Lambda_0$ . In the study  
 131 by M. Hanke et al. [23], the eigenvalues of  $\Lambda_\sigma - \Lambda_0$  are used to locate the inhomogeneities  
 132 non-iteratively.

## 133 2.2 Electrode models

134 In practical implementation, the current density cannot be measured, only the current or  
 135 voltage at discrete electrodes could be obtained. There are various electrode models available

136 depending on their assumptions on current density, i.e. the gap model, the shunt model and  
 137 the complete model [24, 25]. The gap model assumes that the current density is constant  
 138 over electrodes, while the shunt model considers that the integral of the current density over  
 139 the electrode equals to the total current flowing through that electrode. Furthermore, the  
 140 complete model is based on the shunt model, but takes into account the electrochemical  
 141 effect at the interface between the electrode and the probed medium, which is called the  
 142 “contact impedance”. Compared to the gap model, the shunt and complete models are  
 143 closer to reality [18]. In the numerical simulation part, the shunt model is used, while in  
 144 the static tests, the complete model is considered, the electrode specific contact impedance  
 145 is computed and excluded from the measurement data.

146 In the shunt model, considering a number  $\ell$  of identical electrodes placed on  $\partial\Omega$  equally  
 147 spaced, the integral of the current density over the electrode is equal to the current through  
 148 this electrode, while the current density at the isolated gaps between electrodes is zero, i.e.

$$\int_{e_k} \sigma \nabla u \cdot \mathbf{n} \, ds = I_k \text{ for } k = 1, \dots, \ell \text{ while } \sigma \nabla u \cdot \mathbf{n} = 0 \text{ on } \partial\Omega \setminus \bigcup_k e_k, \quad (6)$$

149 where  $I_k$  is the current passing through the  $k^{\text{th}}$  electrode and  $e_k$  is the surface of the  $k^{\text{th}}$   
 150 electrode. Besides, the electrodes are assumed to be perfectly conducting so that the elec-  
 151 trostatic potential  $u|_{e_k}$  is constant at each electrode. Recall that the electrostatic potential  
 152 inside the domain satisfies the Laplace equation (since  $\sigma = 1$  in  $\Omega \setminus \Omega_i$ ),

$$\Delta u = 0, \quad \text{in } \Omega \setminus \Omega_i. \quad (7)$$

153 The shunt model has a unique solution. This allows to define a matrix  $\mathbf{Z}$ , which is the  
 154 discrete mapping from the boundary current stimulations to the boundary voltage measure-  
 155 ments. The matrix  $\mathbf{Z}$  is referred to as the *impedance matrix* and it takes over the role of  
 156 the NtD operator  $\Lambda_\sigma$  as its discrete representation. Defining the discrete version of  $L_\diamond^2(\partial\Omega)$   
 157 as the  $\ell$ -dimensional vector space  $\mathbb{R}_\diamond^\ell = \{y = [y_1 \ \cdots \ y_\ell] \in \mathbb{R}^\ell, \sum_{i=1}^\ell y_i = 0\}$ , one gets

$$\begin{aligned} \mathbf{Z} : \quad \mathbb{R}_\diamond^\ell &\rightarrow \mathbb{R}_\diamond^\ell \\ [I_0 \ I_1 \ \cdots \ I_\ell] &\longmapsto [V_0 \ V_1 \ \cdots \ V_\ell] \end{aligned} \quad (8)$$

158 where  $V_k = u|_{e_k}$  for  $k = 1, \dots, \ell$  are the boundary measurements of the electric potential  
 159 solution  $u$  corresponding to the set of imposed currents  $I_k$  considered.

160 Each element in  $\mathbf{Z}$  is a *bulk impedance* corresponding to a certain imposed current and  
 161 boundary measurement. when the continuous liquid phase is the only conducting phase,  
 162 given the mixture conductivity  $\sigma_m$ , the liquid conductivity  $\sigma_l$  and the liquid volume fraction  
 163  $\alpha_l$ , the Maxwell-Hewitt relation [26] reads:

$$\alpha_l = 1 - \frac{1 - \sigma_m/\sigma_l}{1 + \sigma_m/\sigma_l}, \text{ in 2D,} \quad \alpha_l = 1 - \frac{1 - \sigma_m/\sigma_l}{1 + 0.5 \cdot \sigma_m/\sigma_l}, \text{ in 3D.} \quad (9)$$

164 This relation yields an approximation of the volume fraction of each phase, as shown in [27].  
 165 The water-gas two-phase flow system conforms this relation, so that the measured impedance  
 166 can be correlated to the void fraction. In the present study, the impedance matrix contains  
 167 the impedance of all the possible electrode separations. In this framework, the impedance

168 matrix is assumed to be directly correlated to the void fraction and our objective is to  
 169 investigate this correlation based on an eigenvalue analysis of numerical and experimental  
 170 standardized tests.

## 171 2.3 Impedance matrix and normalization

172 For each stimulation pattern, denoting the source and drain electrodes pair as  $(i, j)$ , there  
 173 is a corresponding bulk impedance  $Z_{i,j}$ , which is governed by the conductivity distribution  
 174  $\sigma(x)$  in  $\Omega$ , as well as the selected source and drain electrodes. The impedance between  
 175 source electrode  $j$  and drain electrode  $i$  is the mutual impedance of  $Z_{i,j}$ , so that they satisfy  
 176 the reciprocity relation  $Z_{i,j} = Z_{j,i}$ . According to the definition of  $\mathbb{R}_{\diamond}^{\ell}$ , the diagonal term  
 177  $Z_{i,i}$  is not measured but computed by summing up the corresponding mutual impedances as  
 178  $Z_{i,i} = -\sum_{j \neq i} Z_{i,j}$ , which finally yields the  $\ell \times \ell$  square impedance matrix  $\mathbf{Z} = (Z_{i,j})_{1 \leq i,j \leq \ell}$ .

179 The relative NtD map  $\Pi$  is commonly used in EIT to retrieve information on the probed  
 180 medium. This amounts to consider the matrix  $\Delta \mathbf{Z} = \mathbf{Z} - \mathbf{Z}^0$ , where  $\mathbf{Z}^0$  denotes the  
 181 impedance matrix of the homogeneous conductivity distribution for which  $\Omega_i = \emptyset$ . The  
 182 matrix  $\Delta \mathbf{Z}$  is referred to as the Differential Impedance matrix (DIM). To perform a non-  
 183 dimensional analysis, a normalization method of the matrix entries  $Z_{i,j}$  is needed to reduce  
 184 the dependencies on the geometric parameters of the sensor and on the background conduc-  
 185 tivity, as well as to enhance the presence of inhomogeneities. In this study, the normalization  
 186 method is proposed as:

$$\hat{Z}_{i,j} = \frac{Z_{i,j} - Z_{i,j}^0}{Z_{i,j}^0} \quad \text{for } i, j \in \{1, \dots, \ell\}. \quad (10)$$

187 The associated matrix  $\hat{\mathbf{Z}} = (\hat{Z}_{i,j})_{1 \leq i,j \leq \ell}$  is referred to as the Normalized Impedance Matrix  
 188 (NIM). Note that  $\hat{Z}_{i,j}$  is equal to zero for all  $i, j$  when  $\Omega_i = \emptyset$  and is infinite when  $\Omega_i = \Omega$ .

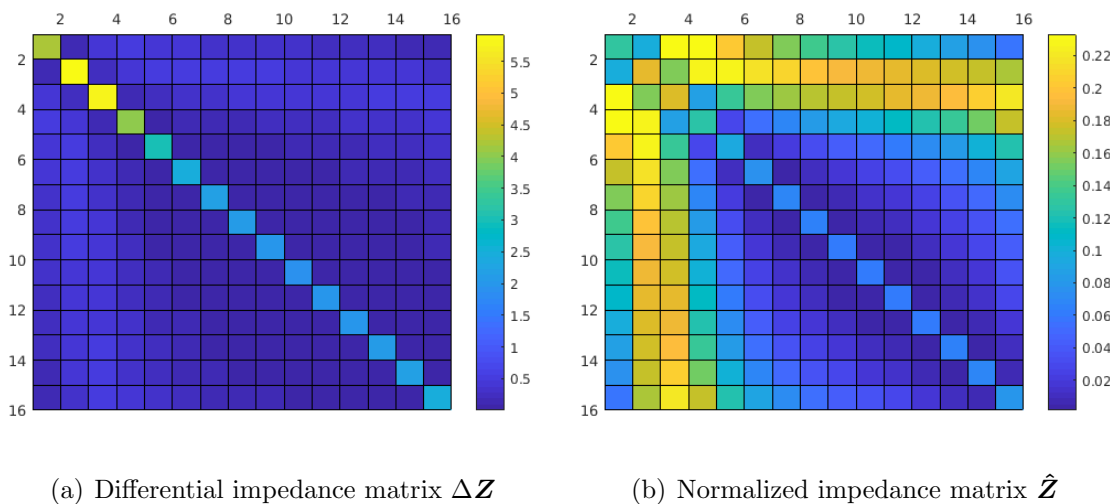


Figure 2. Effect of the normalization on the entries of the impedance matrices in the case of a single bubble (Case 1).

189 Considering the case shown in Figure 1 with a single bubble of radius 0.3 and an EIT  
 190 system with 16 electrodes, the corresponding DIM and NIM are visualized with the horizon-  
 191 tal axes as the matrix entry indices and the vertical axis for the associated entry value, as in  
 192 Figure 2. The effect of normalization is highlighted: The DIM has significant values on the  
 193 diagonal, which is consistent with the computation of its diagonal terms  $Z_{i,i}$ . The normal-  
 194 ization diminishes this effect and enhances the impedance discrepancies between electrodes,  
 195 see for example the entries associated with the electrodes 2 and 3 that are the closest to the  
 196 bubble.

## 197 2.4 Eigenvalue analysis

198 As, by definition, the normalized impedance matrix  $\hat{\mathbf{Z}}$  is a real-valued symmetric matrix,  
 199 it is diagonalizable and its eigenvalues  $\lambda^{(i)}$  are real. Therefore, it has a set of  $\ell$  pairs of  
 200 eigenvectors  $\mathbf{v}^{(i)} \in \mathbb{R}_{\diamond}^{\ell}$  and eigenvalues  $\lambda^{(i)} \in \mathbb{R}$  satisfying

$$\hat{\mathbf{Z}} \cdot \mathbf{v}^{(i)} = \lambda^{(i)} \mathbf{v}^{(i)}, \quad i \in \{1, \dots, \ell\}. \quad (11)$$

201 Sorting the eigenvalues in an increasing order, the resulting eigenvalue array is defined as

$$\lambda = [\lambda^{(1)} \ \lambda^{(2)} \ \dots \ \lambda^{(\ell)}] \text{ with } \lambda^{(1)} < \lambda^{(2)} < \dots < \lambda^{(\ell)}, \quad (12)$$

202 where  $\lambda^{(\ell)}$  is the leading eigenvalue. Moreover, considering the rotational symmetry of the  
 203 electrodes placement, the eigenvalues of  $\hat{\mathbf{Z}}$  are invariant under similarity transformations of  
 204 the electrodes, such as rotation or renumbering.

205 The largest eigenvalues contain the most information about the matrix  $\hat{\mathbf{Z}}$  and represent  
 206 its main features, while  $\hat{\mathbf{Z}}$  is decided by the conductivity distribution within the probed  
 207 medium, the other smallest eigenvalues are also affected by it but contain qualitatively less  
 208 information. Hereafter, the characteristics of these eigenvalues are investigated numerically.  
 209 Note that in the configurations where different numbers of electrodes are used to impose the  
 210 current and to measure the potential on the domain boundary, then the associated impedance  
 211 matrices are not square. In such cases, their singular value decompositions can be considered  
 212 alternatively.

## 213 3 Numerical simulations

214 In the previous section, the mathematical model of EIT and the normalized impedance  
 215 matrix are introduced. In this section, numerical simulations are considered to investigate  
 216 the relation between the eigenvalues and the conductivity distribution within the probed  
 217 medium.

### 218 3.1 Canonical cases considered

219 The typical regimes of two-phase flows include bubbly flow, stratified flow, slug/plug/churn  
 220 flow and annular flow, see [28–30]. The phase distributions can be classified into three  
 221 classes, regardless of pipe placement and gas/liquid flow directions, that is, (i) Single bubble

222 case, representing the water slug in slug/plug/churn flow and the annular flow as well; (ii)  
 223 Stratified case; (iii) Multiple bubbles case, relevant to the bubbly flow and the bubble cloud  
 224 that follows the slug in slug/plug/churn flow.

225 Consider a 2D circular pipeline  $\Omega$  with a radius  $r_0 = 1$ , i.e.  $\Omega = \{(x, y) : x^2 + y^2 < 1\}$ .  
 226 Given the rotational symmetry of the impedance matrix, three classes of bubble distribution  
 227 patterns are defined as follows:

- 228 • Case 1 (single bubble):  $\Omega_i = \{(x, y) : (x - d \cos \theta_0)^2 + (y - d \sin \theta_0)^2 \leq r^2\}$  with  
 229  $0 \leq \theta_0 \leq 2\pi$  and such that  $0 \leq r + d \leq 1.0$ .
- 230 • Case 2 (stratified):  $\Omega_i = \{(x, y) : y \geq r_0 \cdot \cos \theta\}$ , with  $r_0 = 1$  and  $0 \leq \theta \leq \pi$ .
- 231 • Case 3 (multiple bubbles): a number  $n_{bub} \in \{1, 3, 11, 36, 62, 84\}$  of uniformly  
 232 distributed bubbles of radius  $r_{dis}$  is considered. To fix the void fraction relatively to  
 233 an equivalent concentric bubble of radius  $r_e$  then  $r_{dis}$  is defined as  $r_{dis} = r_e / \sqrt{n_{bub}}$ .

234 The conductivity distributions of the three cases are depicted in Figure 3, the area in  
 235 light blue represents the water phase and the one in white is the gas phase. The electrical  
 236 conductivity of gas is set to be eight magnitudes lower than that of water, which is in  
 237 accordance with the physical conductivity values of water and gas. Each case corresponds  
 238 to specific regimes of two phase flow: in Case 1, one single bubble with various diameter is  
 239 varied spatially along one specific diameter inside the domain; Case 2 represents stratified  
 240 flow with various water level, in which  $h = 1 - d = 1 - \cos \theta$ ; Case 3 is characterized by  
 241 multiple bubbles uniformly distributed inside the domain, Figure 3 shows the case with 11  
 242 bubbles.

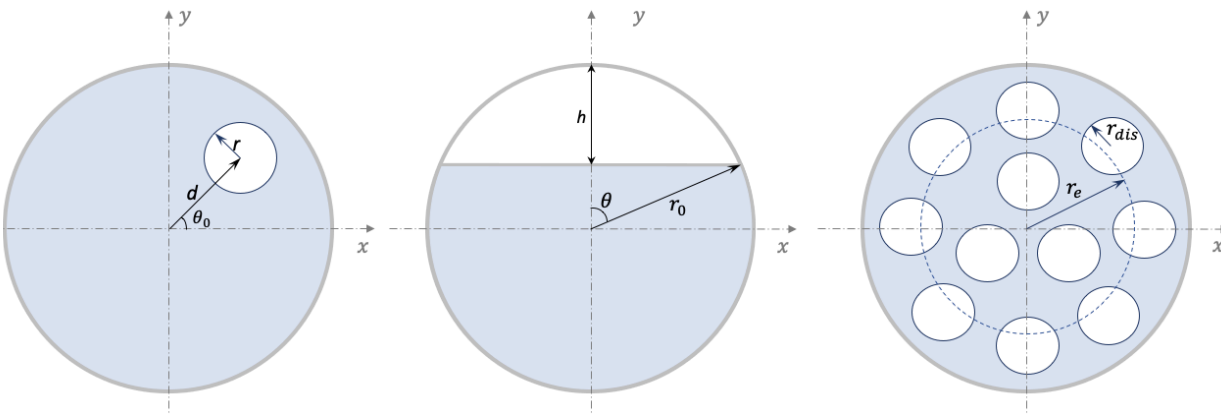


Figure 3. Conductivity distribution patterns for the three canonical cases considered.

243 To investigate the eigenvalues of the impedance matrix for these three canonical cases,  
 244 boundary measurements have to be computed. The open source code Electrical Impedance  
 245 Tomography and Diffuse Optical Tomography Reconstruction Software (EIDORS) is used.  
 246 It is dedicated to solve EIT forward and inverse problems by the Finite Element Method  
 247 (FEM) [31].

248 The numerical model is a 2D circular model with unit radius and 16 electrodes placed  
 249 evenly at the boundary, the width of the electrode is chosen to have the ratio between the

250 area occupied by the electrodes and the total area the same than the practical EIT system  
 251 used in our experimental setting. A triangle mesh with 54177 nodes is used in the FEM  
 252 model.

253 A full scan stimulation strategy with a constant current of  $I_{stimu} = 1A$  at source and  
 254 drain electrodes is employed. For each stimulation pattern, the difference voltage between  
 255 adjacent electrodes are computed, denoting the selected source and drain electrodes as  $(i, j)$ ,  
 256 the voltage difference between source and drain  $V_{i,j}$  could be obtained by summing up the  
 257 voltage differences at all adjacent electrodes between the source and drain. Since the injected  
 258 current is a constant, the corresponding bulk impedance  $Z_{i,j}$  can be obtained by Ohm's law:

$$Z_{i,j} = V_{i,j}/I_{stimu}. \quad (13)$$

260 With 16 electrodes  $(i, j = 1, \dots, 16)$ , there are 120 independent stimulation patterns,  
 261 giving 120 different bulk impedances for one frame of acquisition. After normalization, the  
 262 square matrix  $\hat{\mathbf{Z}}$  of size  $16 \times 16$  is obtained, along with the  $\ell = 16$  pairs of eigenvalues and  
 263 eigenvectors, see Eqn. (12).

264 The NIM of the three simulated cases considered are visualized to highlight their correla-  
 265 tion with the phase pattern. Case 1 is shown in Figure 2(b) with  $r = 0.3$  and the cases 2 and  
 266 3 are shown in Figure 4, in which Case 2 is computed with  $h = 0.8$  and Case 3 is computed  
 267 with 3 bubbles and  $r_e = 0.3$ . These parameters are chosen to obtain the same phase fraction  
 268 for all three cases. As it can be seen, the main features of the NIM vary significantly with  
 269 the phase pattern, even at the same phase fraction. This characteristic is at the foundation  
 270 of the present study.

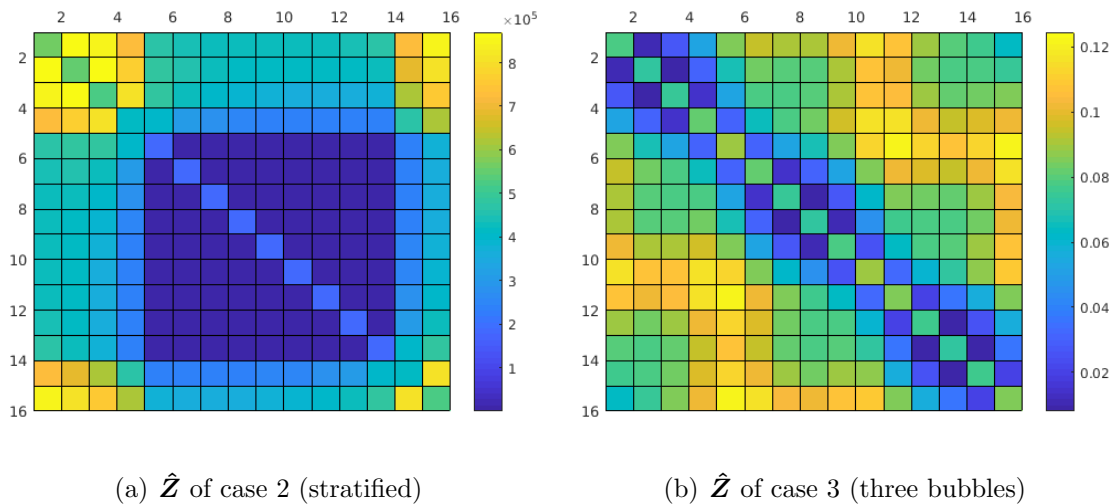


Figure 4. Entries of the normalized impedance matrix of the cases 2 and 3.

271 In the following sections, the eigenvalues of various simulation cases are investigated  
 272 and compared to study their dependence on the bubble distribution. In particular, the void  
 273 fraction is plotted with the leading eigenvalue and the sum of the absolute value of the first 15  
 274 eigenvalues, to assess their correlations. For convenience, we denote the leading eigenvalue as  
 275  $\lambda^{(16)}$  and the sum of the absolute value of the first 15 eigenvalues as  $\sum_{|\lambda|}^{15} = |\lambda^{(1)}| + \dots + |\lambda^{(15)}|$ .

276 Moreover, the void fraction  $\alpha$  is defined in 2D by the area ratio occupied by the bubble inclusion  
 277 over the total area, i.e.  $\alpha = |\Omega_i|/|\Omega| = |\Omega_i|/\pi$ .

## 278 3.2 Result analysis

### 279 3.2.1 Case 1

280 Due to the fact that the properties of the impedance matrix are invariant with the rotation  
 281 of the electrodes, we only need to simulate the bubble inclusion moving along one radius of  
 282 the domain. We set  $\theta_0 = 0$ , which is representative of all cases of a single bubble moving  
 283 within the domain. Applying the restriction condition  $r + d \leq 0.95$  and an increment step  
 284 of 0.05, we get 19 different bubble radius  $r$  and placements  $d$ , separately, which will give 190  
 285 different simulation cases (190 pairs of  $(r, d)$ ) in total. Here,  $r + d \leq 0.95$  is chosen because of  
 286 the extremely high sensitivity near the boundary, which could cause an abnormal deviation,  
 287 besides, in practical applications it is also rare to have a bubble occupying 95% of the pipe  
 288 diameter.

289 The eigenvalues are computed for each case and plotted. Figure 5(a) shows the 16  
 290 eigenvalues of concentric bubble inclusions with  $d = 0$  and  $0.05 \leq r \leq 0.95$ , while Figure 5(b)  
 291 shows the 16 eigenvalues of eccentric bubble inclusions with  $d = 0.35$  and  $0.05 \leq r \leq 0.6$ .  
 292 In both figures, each curve represents an eigenvalue  $\lambda^{(i)}$ , and they are evolving with the  
 293 bubble radius. As we can see, the leading eigenvalue is much larger than the others, and its  
 294 amplitude is correlated to the bubble size, especially when the bubble edge is close to the  
 295 domain boundary. Besides, for concentric bubble configurations, the first 15 eigenvalues are  
 296 symmetrical around zero due to the symmetry of the conductivity distribution.

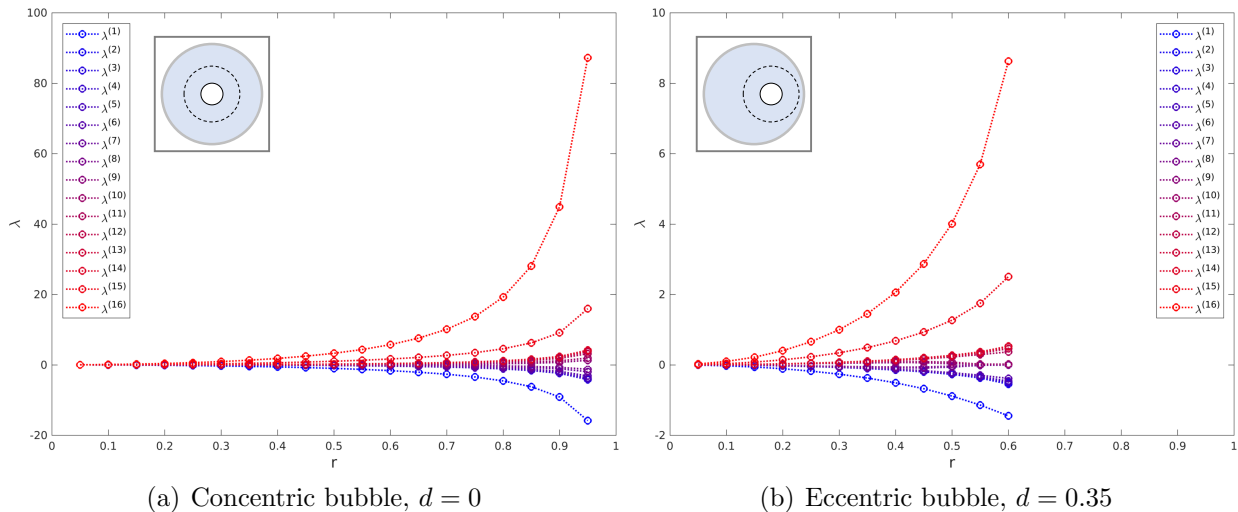


Figure 5. Eigenvalue array for some configurations corresponding to Case 1.

297 Figure 6 shows the 16 eigenvalues of a bubble inclusion with  $r = 0.35$  moving from  
 298  $d = 0$  to  $d = 0.6$ . As it can be seen, as the bubble moves toward the boundary, the leading  
 299 eigenvalue increases rapidly, while the other eigenvalues vary little with  $d$ .

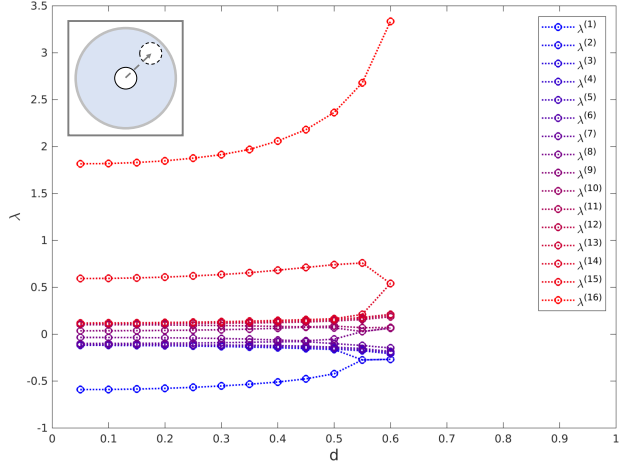


Figure 6. Eigenvalue array of one bubble inclusion ( $r = 0.35$ ) moving from  $d = 0$  to  $d = 0.6$ .

300 As we know, adding bubble inclusions increases the overall bulk impedance, hence the  
 301 normalized impedance  $\tilde{Z}_{i,j}$ , which in turn affects the eigenvalues. That is why the eigenvalues  
 302 are changing with bubble size. To study the influence of bubble location, the sensitivity map  
 303 of EIT has to be considered. The sensitivity map  $s(x)$  of an EIT sensor quantifies the  
 304 relation between the change of boundary measurement  $\delta V_{meas}$  caused by a local conductivity  
 305 perturbation  $\delta\sigma(x)$ , that is,  $s(x) = \delta V_{meas} / \delta\sigma(x)$ . It describes how effectively each region is  
 306 contributing to the measurement [32].

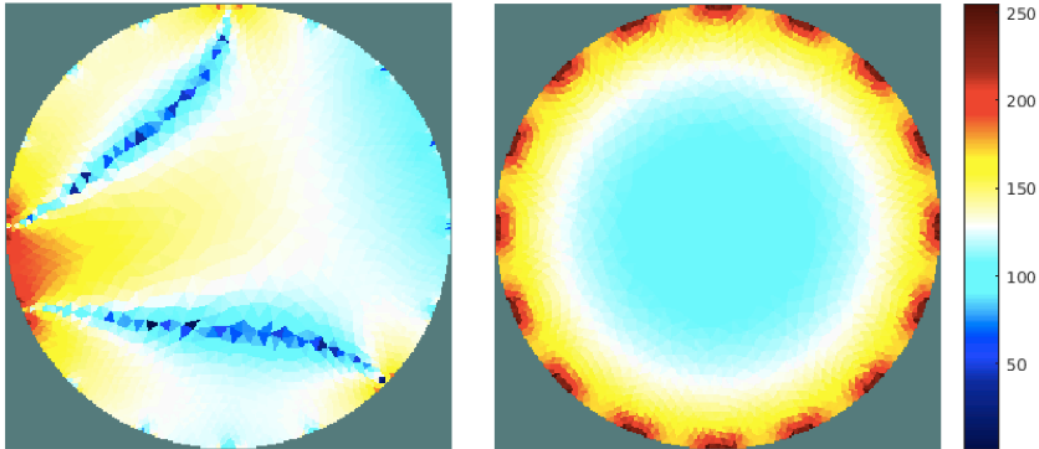


Figure 7. Map of the logarithm of the sensitivity distribution: (*left*) source/drain at electrodes (1,7) and measurement at electrodes (12,13); (*right*) full scan strategy.

307 EIT is a soft-field imaging technique for which high sensitivity areas concentrate near  
 308 the boundary and the vicinity of the active electrodes while the domain interior contains  
 309 low sensitivity regions. The sensitivity maps for one single measurement and for the full  
 310 scan strategy are shown in Figure 7. The color map in the left panel shows the sensitivity  
 311 distribution of one stimulation pattern with source and drain at electrodes 1 and 7, and one

312 single measurement at electrodes 12 and 13. The sensitivity map of the full scan strategy is  
 313 shown in the right panel. Considering the same bubble inclusion placed either in the center  
 314 or near the boundary, the boundary voltage turns out to be more affected on average in  
 315 the latter configuration, and so are the impedance matrix and its eigenvalues. Thus, the  
 316 eigenvalues are affected by both the size and position of the bubble inclusion.

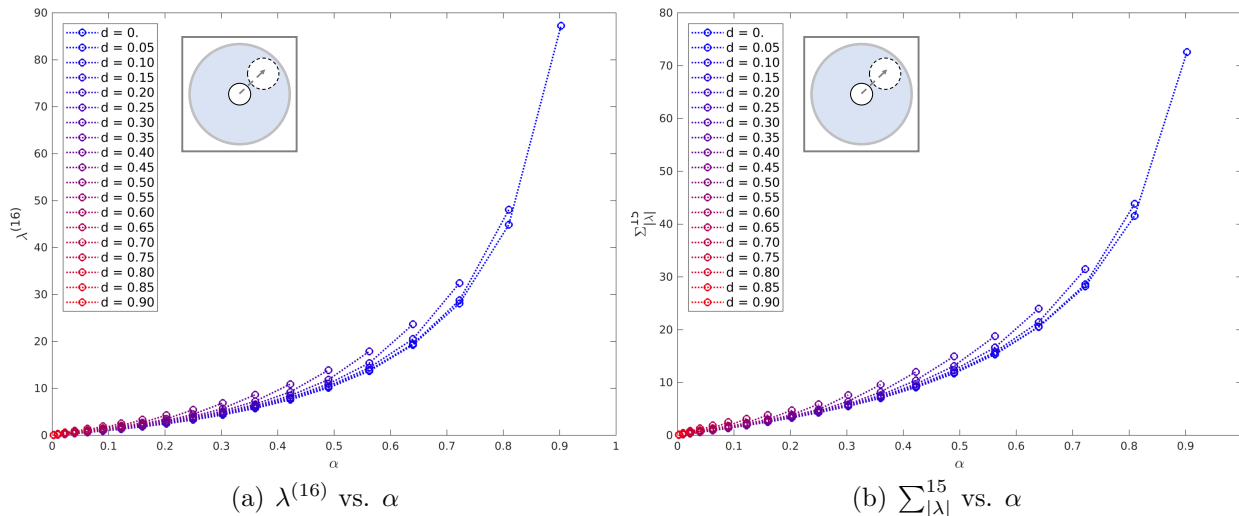


Figure 8. Eigenvalues vs. void fraction for Case 1.

317 Figure 8 shows the relation between the void fraction and the leading eigenvalue  $\lambda^{(16)}$   
 318 and the sum of the absolute value of the first 15 eigenvalues  $\sum_{|\lambda|}^{15}$ . For Case 1, the void  
 319 fraction satisfies  $\alpha = \pi r^2 / \pi r_0^2 = r^2$ , which is invariant with  $d$ . In both figures, each curve  
 320 corresponds to a specific value  $d$  with varying  $r$ , for example, the curve at the right most  
 321 represents a bubble in the center ( $d = 0$ ), its radius  $r$  increases from 0.05 to 0.95, hence the  
 322 void fraction ranges from 0 to 0.9025. The figures show that both  $\lambda^{(16)}$  and  $\sum_{|\lambda|}^{15}$  increase  
 323 with the void fraction and vary with the distance to the center. The void fraction can be  
 324 estimated by  $\lambda^{(16)}$  or  $\sum_{|\lambda|}^{15}$ , from Figure 8 we can estimate the void fraction with an error  
 325 less than 10%. Furthermore, the error can be reduced drastically when the position of the  
 326 bubble is known a priori (distance to the center  $d$ ).

### 327 3.2.2 Case 2

328 In the stratified configuration, a horizontal liquid-gas interface is considered since the impedance  
 329 matrix is invariant with electrodes rotation. Note that the void fraction  $\alpha$  in Case 2 satisfies  
 330 the relation

$$\alpha = (\theta - \sin \theta \cdot \cos \theta) / \pi \quad \text{with} \quad \theta = \arccos(1 - h), \quad (14)$$

331 where  $h$  is the *thickness* of the gas layer as in Figure 3.

332 The eigenvalue array of Case 2 is plotted in Figure 9(a). A linear approximation can be  
 333 found for the relation between  $h$  and the leading eigenvalue, while the data points at high  
 334 void fraction ( $h > 1.99$ ) are considered to be abnormal and discarded. The other eigenvalues  
 335 are not strongly correlated to  $h$ . In Figure 9(b),  $\lambda^{(16)}$  is plotted with  $h$ , and the curve is

336 fitted linearly with a fitting measure  $R^2$  of 0.9905. The deviation between the fitting line  
 337 and the data points may originate from the finite number and size of the electrodes and from  
 338 their limited geometrical extent too.

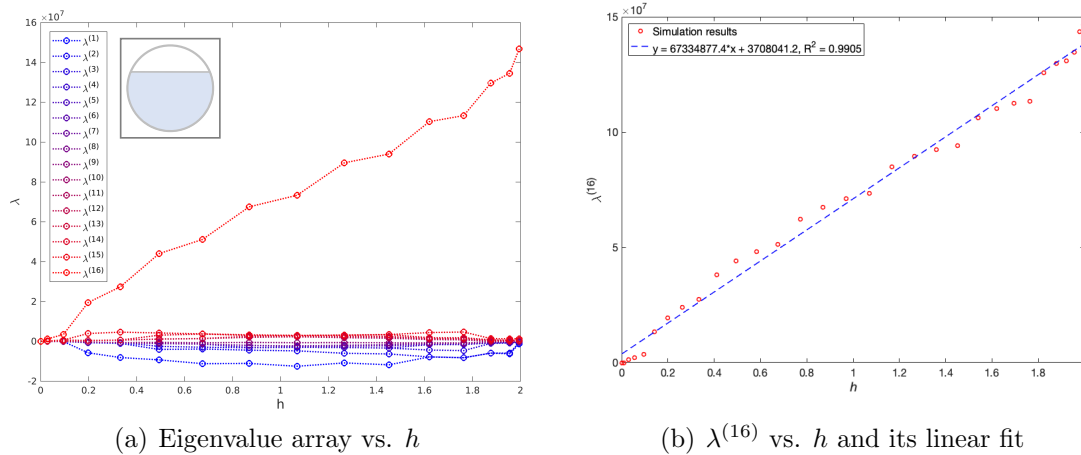


Figure 9. Eigenvalue arrays and  $\lambda^{(16)}$  as a function of  $h$  for Case 2.

### 339 3.2.3 Case 3

340 In Case 3, a number  $n_{bub} \in \{1, 3, 11, 36, 62, 84\}$  of bubble inclusions are considered with  
 341 a radius adjusted to get the same water fraction as an equivalent single concentric bubble of  
 342 radius  $r_e$ . The void fraction of Case 3 is  $\alpha = r_e^2$ , which is restricted by  $n_{bub}$  to avoid contact  
 343 among bubbles. With  $n_{bub} = 3$  the top limit of void fraction is 0.5, for  $n_{bub} = 11$  it is 0.6,  
 344 while for other cases it ranges from 0.65 to 0.7. For practical two-phase flows, the intensive  
 345 bubbly flow rarely reaches a void fraction over 0.5, otherwise the bubbles would collapse and  
 346 form bigger bubbles and the flow regime would change.

347 The eigenvalue  $\lambda^{(16)}$  is plotted with the void fraction and the bubbles radius in Figure  
 348 10 respectively. In Figure 10(a) the right most curve represents the single concentric bubble  
 349 case, from right to left the curves correspond to an increasing value of  $n_{bub}$ . As we can  
 350 see, at the same void fraction the leading eigenvalue  $\lambda^{(16)}$  is increasing with  $n_{bub}$ . The same  
 351 behaviour is also observed in Figure 10(b): at the same void fraction,  $\lambda^{(16)}$  is decreasing with  
 352 the bubbles radius. Besides, the smaller the bubbles are, the stronger the effect is on  $\lambda^{(16)}$ ,  
 353 which is shown by the drop of  $\lambda^{(16)}$  when  $r_{dis}$  is close to zero. After that point  $\lambda^{(16)}$  decreases  
 354 smoothly as  $r_{dis}$  increases.

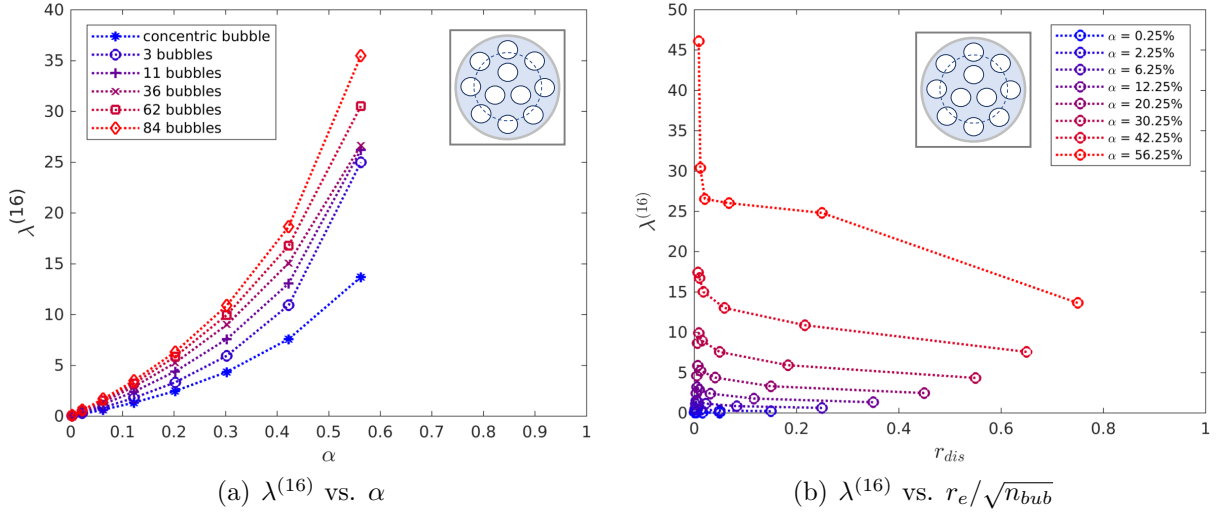


Figure 10. Eigenvalues vs. void fraction for Case 3.

355 For the multiple bubble case, two parameters have to be known to predict the void  
 356 fraction: the eigenvalues and the number of bubbles within the sensor area. However, it is  
 357 not practical to observe the bubble distribution inside a pipe. In the work of Bruhl et al. [33],  
 358 the eigenvalues of  $\Lambda_\sigma - \Lambda_0$  are used to locate the inhomogeneities and estimate the number  
 359 of inhomogeneities non-iteratively, which is a very promising method to be combined with  
 360 the approach proposed here, to obtain a reliable and accurate void fraction estimation for  
 361 two-phase flows.

## 362 4 Robustness of the proposed approach

363 The properties of the eigenvalue distribution of the normalized impedance matrix  $\hat{\mathbf{Z}}$  are ana-  
 364 lyzed in the previous section based on 2D simulations. In the present section, the robustness  
 365 of this methodology is investigated by studying the case where the data is noisy and when  
 366 the inclusions are conductive.

### 367 4.1 Noisy data

368 In practical EIT systems, the boundary measurements are always polluted by some noise,  
 369 including background white noise or the cross-talk of electrical devices. Thus, the robustness  
 370 of the eigenvalue analysis to noisy data has to be assessed. The Signal-to-Noise Ratio (SNR)  
 371 of the EIT system developed for the present study at the Laboratory of Thermal-Hydraulics  
 372 in Core and Circuits (LTHC) is higher than 60dB for all channels at a stimulation frequency  
 373 of 20kHz and amplitude of 1V. Given this SNR value then adding a noise of 10dB to the  
 374 measurement data from numerical simulations is conservative.

375 The noise is added to the measurement data from both the homogenous and inhomoge-  
 376 neous configurations. The eigenvalues of Case 1 are computed and compared to the results  
 377 without noise. Again,  $\lambda^{(16)}$  and  $\sum_{|\lambda|}^{15}$  are the metrics this study is focusing on, so they are plot-

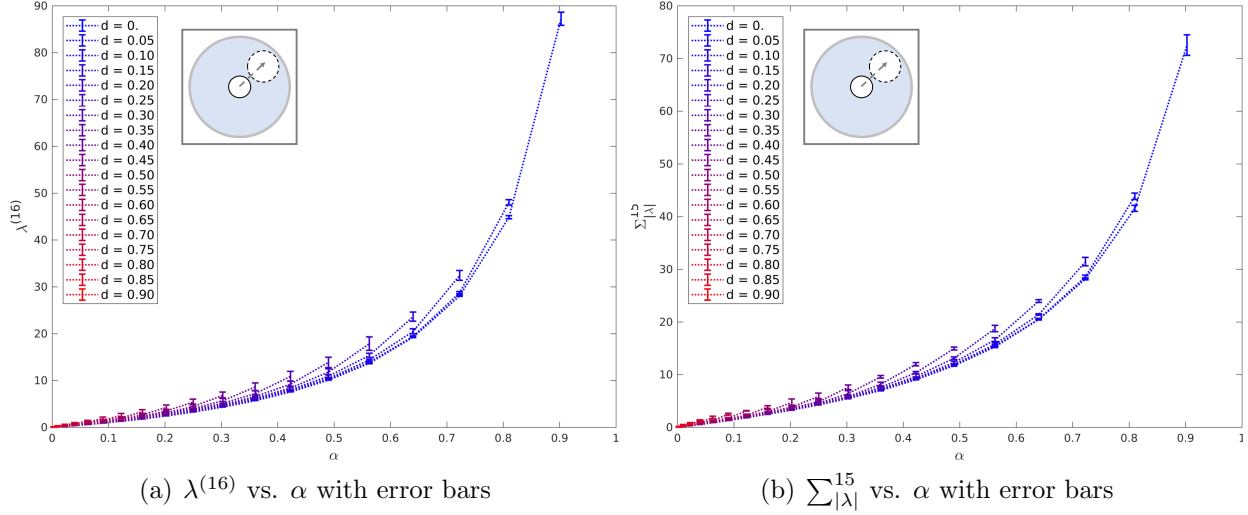


Figure 11. Eigenvalue trends obtained with noisy data in Case 1.

378 ted with error bars as in Figure 11. The error  $\xi$  on  $\lambda^{(16)}$  is computed as  $\xi = |\lambda_{noisy}^{(16)} - \lambda^{(16)}| / \lambda^{(16)}$   
 379 where  $\lambda^{(16)}$  is computed without noise and  $\lambda_{noisy}^{(16)}$  is computed with a 10dB noise. The error  
 380 on  $\sum_{|\lambda|}^{15}$  is computed in the same fashion. As we can see, the relative error for both  $\lambda^{(16)}$  and  
 381  $\sum_{|\lambda|}^{15}$  are very small, the maximum values being 4.3% and 2.0% respectively.

## 382 4.2 Extension to conductive inclusions

383 Apart from the application in water-gas two phase flow measurements, EIT sensors can also  
 384 be used in various industrial processes, which may concern conductive inclusions rather than  
 385 non-conductive bubbles. Here, the definition of the conductivity contrast  $\mathcal{R} = \sigma_{high} / \sigma_{low}$   
 386 from Seagar et al. [34] is used to represent the conductivity difference between two phases.  
 387 For water-gas two-phase flows,  $\mathcal{R}$  tends to infinity. In the section, some configurations with  
 388  $\mathcal{R} \in \{100, 10, 5, 2.5, 1.25\}$  are simulated and compared to the results associated with  
 389  $\mathcal{R} \approx \infty$  (i.e. water-gas two-phase flows), to assess the general applicability of the proposed  
 390 methodology.

391 All three cases are simulated in 2D when varying  $\mathcal{R}$  and the results for the cases 1 and 2  
 392 are plotted in Figures 12 to 13. In Figure 12(a), the eigenvalue  $\lambda^{(16)}$  for the single concentric  
 393 bubble case is plotted with  $\alpha$  and each curve represents different values of  $\mathcal{R}$ . As we can see,  
 394 the trends of  $\lambda^{(16)}$  changing with  $\alpha$  are similar for different values of  $\mathcal{R}$ , while the amplitude  
 395 are increasing globally with  $\mathcal{R}$ . Also, as  $\mathcal{R}$  increases to large values,  $\lambda^{(16)}$  increases less and  
 396 less, especially after  $\mathcal{R} = 100$ , i.e. there exists a threshold effect. The same conclusions are  
 397 obtained for  $\sum_{|\lambda|}^{15}$ , see Figure 12(b).

398 For the stratified case, the same results can be obtained, while the contrast  $\mathcal{R}$  has a much  
 399 stronger effect on the eigenvalues, especially for large values. As shown in Figure 13,  $\lambda^{(16)}$  is  
 400 5 to 6 magnitudes higher for  $\mathcal{R} = \infty$  than for the other cases, so that  $\lambda^{(16)}$  vs.  $h$  is plotted  
 401 in Figure 13(b) excluding the case  $\mathcal{R} = \infty$ . Nevertheless, a linear correlation can be found  
 402 between  $\lambda^{(16)}$  and  $h$  for varying  $\mathcal{R}$ .

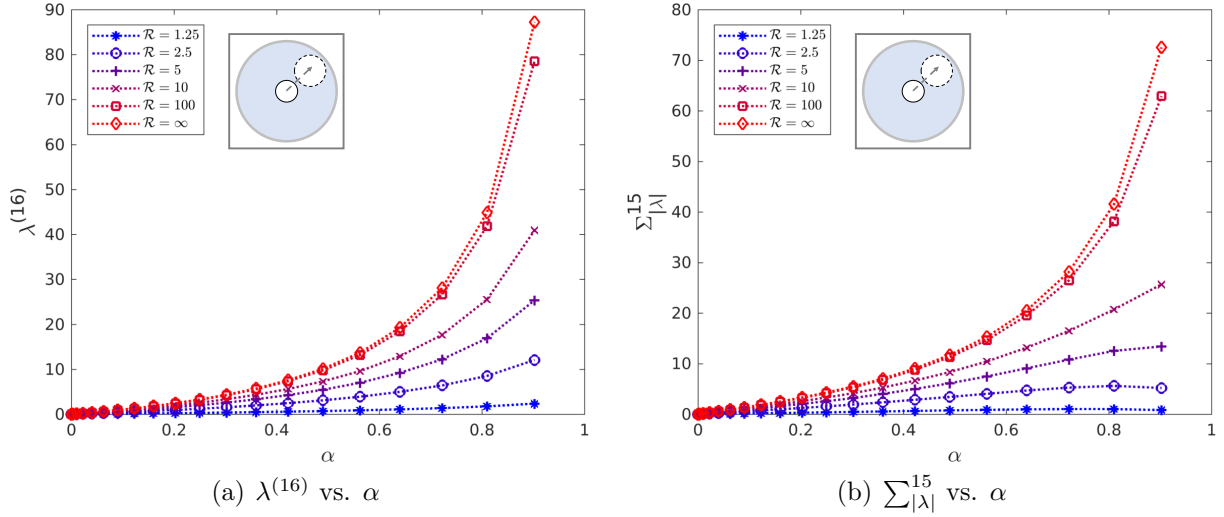


Figure 12.  $\lambda^{(16)}$  and  $\sum_{|\lambda|}^{15}$  vs.  $\alpha$  for various values of  $\mathcal{R}$  in Case 1.

403 Lastly, the results for the multiple bubble case have been found to be similar to the single  
 404 bubble case. The applicability of the eigenvalue analysis to conductive inclusions is therefore  
 405 emphasized but the conductivity contrast  $\mathcal{R}$  between phases is an important information to  
 406 be known beforehand.

## 407 5 Validation by experimental static tests

408 In this section, the methodology for void fraction estimation, which has been devised based  
 409 on numerical simulations, is validated experimentally. We perform static tests, with a pipe  
 410 containing still water, for a number of configurations corresponding to the three inclusion  
 411 patterns considered previously. The motivations for performing such static tests are as  
 412 follows: (i) they are easily implementable using standard laboratory equipment and allow to  
 413 control the phase distribution accurately, which is crucial to validate the proposed approach,  
 414 (ii) EIT measurements can be performed at a high frame rate (of the order of 800fps for the  
 415 system considered), which allows to treat a dynamic flow as if it were quasi-static on each  
 416 frame. These experimental tests are carried out with the EIT system developed in LTHC.

### 417 5.1 EIT system in LTHC

418 The prototype EIT sensor has 16 electrodes on the boundary of the test section with an  
 419 angular separation of  $22.5^\circ$ . The test section is a pipe with diameter of  $80\text{mm}$  and height of  
 420  $300\text{mm}$  while the electrode size is  $170\text{mm} \times 5\text{mm}$  [9], as shown in Figure 14.

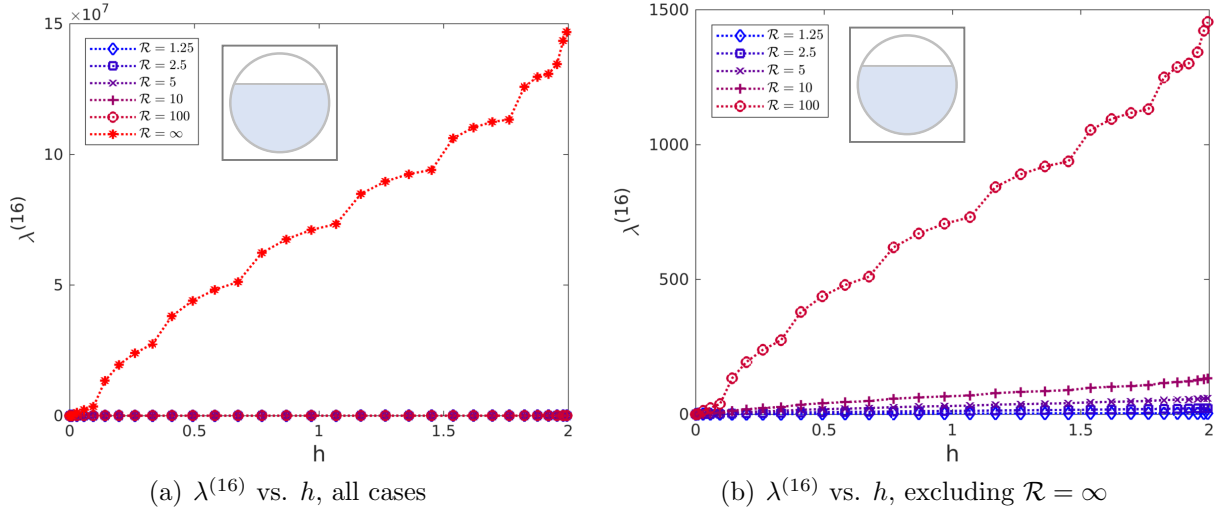


Figure 13.  $\lambda^{(16)}$  vs.  $h$  in Case 2.

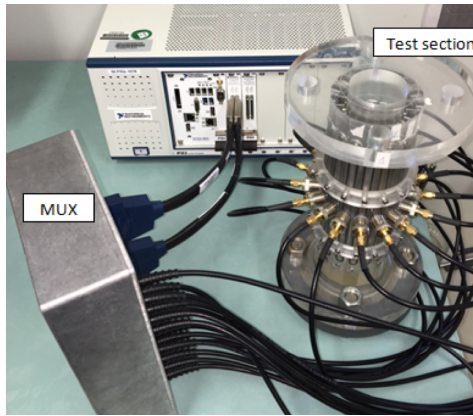


Figure 14. EIT system in LTCH.

421 Stimulation and signal acquisition are both performed using the PXIe system from Na-  
 422 tional Instrument. The stimulation signal is a sinuous voltage with amplitude of  $1V$  to avoid  
 423 the electrolysis reaction of water. The signal is generated by the analogue output channel of  
 424 the PXIe system, the sample rate ranges from  $1\text{ Hz}$  to  $3.33\text{ MHz}$ . Data acquisition is done  
 425 by the differential analogue input channels of the system, the simultaneous sample rate of  
 426 all channels reaches up to  $2\text{ MHz}$  [35]. A multiplexer with 16 ports is used to route the  
 427 stimulation signal to a selected pair of electrodes. A discrete Fourier transform is performed  
 428 on the measurement data to obtain amplitudes and phases at the stimulation frequency.

429 Static tests are carried out with still water filling the test section and a number of plastic  
 430 rods emulating bubbles. Different diameters and numbers of rods are placed into the water  
 431 to get various void fractions. In a first step, the impedance of each stimulation pattern has  
 432 to be obtained from practical measurements. The prototype EIT uses voltage of  $\pm 1V$  as a  
 433 stimulation signal, which is different from the current stimulation considered in the numerical  
 434 simulations but we consider the two settings as being equivalent. Each stimulation pattern

435 can be assumed as a closed circuit as in Figure 15, in which the bulk impedance  $Z^{bulk}$  and  
 436 the contact impedances at the source/drain electrodes  $Z^{source}$  and  $Z^{drain}$  are connected as

$$Z^{tot} = Z^{source} + Z^{drain} + Z^{bulk} = V/I. \quad (15)$$

437 The terms  $Z^{source}$  and  $Z^{drain}$  are essentially equal, they are electrode-specific and invariant  
 438 with phase distributions. In (15),  $V$  is the stimulation voltage and  $I$  is the sensor current.

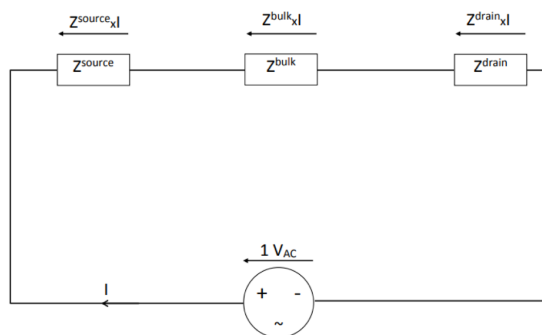


Figure 15. Closed circuit between the source and drain electrodes.

439 In the EIT system, a constant resistor  $R_0 = 200\Omega$  is included in the closed circuit and the  
 440 voltage difference  $\Delta V_{R_0}$  across  $R_0$  is measured. Thus, the current  $I$  through the circuit can  
 441 be obtained as  $I = \Delta V_{R_0}/R_0$ . Considering the homogeneous case, there are 120 independent  
 442 equations as Eqn. (15). As  $Z^{bulk}$  depends only on the angular separation of source/drain  
 443 electrodes, there are 8 different values of  $Z^{bulk}$  overall. Together with the 16 different values  
 444 of  $Z^{source}$  (or  $Z^{drain}$ ), the set of 120 linear equations contains 24 unknown parameters. Thus  
 445 the values of  $Z^{source}$  and  $Z^{drain}$  can be obtained and used to apply the proposed methodology.  
 446 For inhomogeneous configurations, the bulk impedances are computed as in the homogeneous  
 447 case.

## 448 5.2 Result comparison

449 The static test results are compared to numerical simulations for validation. Simulations  
 450 in 2D can be considered as being equivalent to configurations with infinite long electrodes  
 451 and inclusions, while in practical 3D implementations, the electrodes and the inclusions have  
 452 finite extents. Therefore, it is more relevant to compare the results from a practical EIT  
 453 system with 3D simulations. As a consequence, we consider here a 3D cylindrical model  
 454 implemented in EIDORS. The radius of the numerical model is 1 while all other parameters  
 455 are normalized from the prototype EIT system so that the length of the model is 7.5, the  
 456 electrodes have a width of 0.125 and a length of 4.25. The bubble inclusion is emulated  
 457 by a non-conductive rod of the height of the model. The 3D numerical model has 198730  
 458 tetrahedral mesh elements.

459 As of the experimental tests, a series of configurations are investigated to obtain data for  
 460 all of the three patterns considered. For Case 1, a single non-conductive rod with different  
 461 diameters is placed in the test section at different distances to the center: the diameter of the

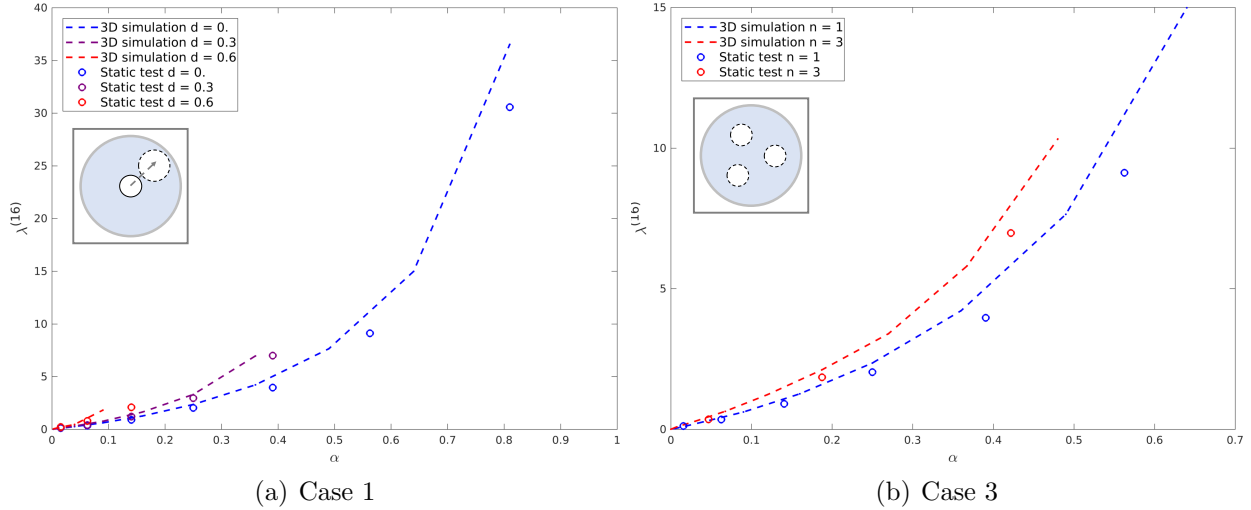


Figure 16. Comparison between 3D simulations and experimental static tests for the cases 1 and 3: eigenvalue  $\lambda^{(16)}$  as a function of the void fraction  $\alpha$ .

462 rod ranges from  $10\text{mm}$  to  $70\text{mm}$ , the distance to the center being  $d \in \{0, 12\text{mm}, 24\text{mm}\}$ ,  
 463 corresponding to  $d \in \{0, 0.3r_0, 0.6r_0\}$  in the 3D simulations. For Case 2, the test section  
 464 is placed horizontally with different water levels. In Case 3, different tests with three rods  
 465 placed symmetrically are carried out, the diameter of the rods ranges from  $10\text{mm}$  to  $40\text{mm}$ .  
 466 The normalized impedance matrix and its eigenvalues are computed for each test. The  
 467 evolutions of the eigenvalue  $\lambda^{(16)}$  with the void fraction are shown along with 3D simulation  
 468 results in Figure 16 for the cases 1 and 3, and in Figure 17 for Case 2.

469 For the cases 1 and 3, the trends of the eigenvalue  $\lambda^{(16)}$  for the static tests agree well with  
 470 the 3D simulations, which validates the proposed methodology. Deviations of low relative  
 471 amplitude can be observed, which may be due to the following facts: (i) the simulation  
 472 model is non-dimensional with normalized size and conductivity; (ii) the stimulation signal  
 473 is a constant current in the simulations while it is a constant voltage in the experiments.

474 For Case 2, the trends of the eigenvalue  $\lambda^{(16)}$  with the thickness  $h$  of the gas layer, see  
 475 (14), for the experiments and the 3D numerical simulations are very different, as seen in  
 476 Figure 17(a), especially at small  $h$ . The magnitudes in the experiments are also significantly  
 477 lower than in the simulations. These discrepancies may come from: (i) the surface tension of  
 478 water causes a rise at the interface between the plastic pipe and the water, especially at low  
 479  $h$ ; (ii) a conductive water film may be present on the uncovered part of the pipe surface, while  
 480 in the simulation there is no such film. Moreover, the magnitude of the eigenvalue  $\lambda^{(16)}$  is  
 481 remarkably high for all values of  $h$  compared to the other phase distribution patterns (at the  
 482 same void fraction), as shown in Figure 17(b). Lastly, the electrodes that are immersed in  
 483 water can be directly revealed by the distribution of the entries of the normalized impedance  
 484 matrix, as seen in Figure 4(a), which could therefore be used as an indicator of the water  
 485 level.

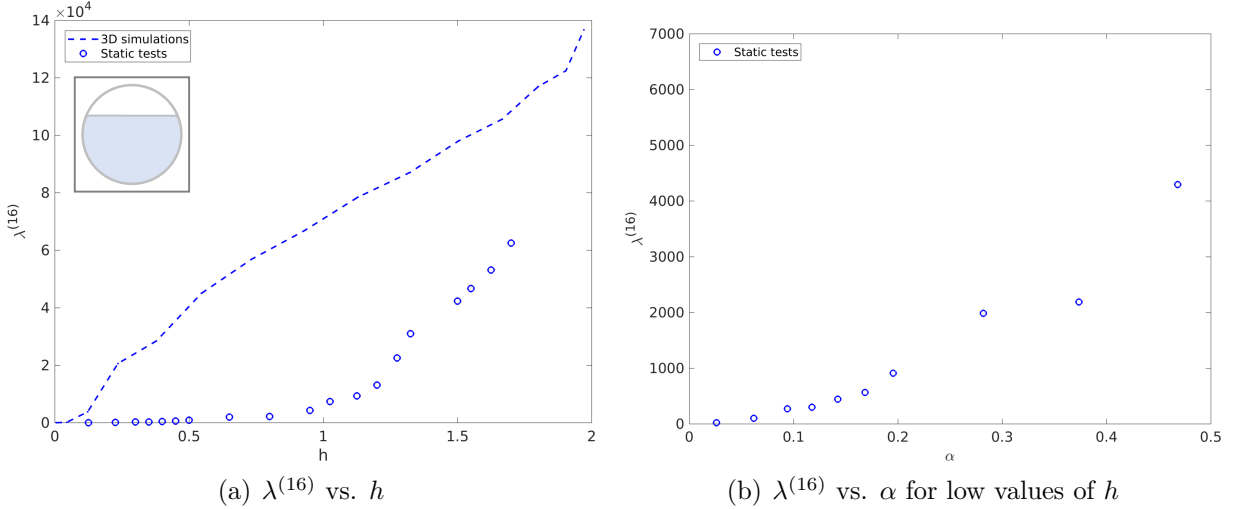


Figure 17. Comparison between 3D simulations and experimental static tests for Case 2.

## 6 Conclusion

In this article, the impedance data from electrical impedance tomography sensor and its relation to the void fraction of two-phase flows is investigated numerically and experimentally for a cylindrical pipe configuration. The forward problem associated with EIT is a boundary value problem: prescribing Neumann boundary conditions, the Dirichlet boundary measurements depend on the internal conductivity distribution. In practical implementations, discrete electrodes are used for stimulation and boundary measurements, leading to one bulk impedance matrix for each stimulation pattern. A normalization of the impedance matrix is considered to extract information from the data and reduce the influences of factors other than the bubble inclusion distribution, such as the diameter of the sensor, the background medium conductivity or the stimulation signal.

Numerical simulations are carried out for three different canonical cases that cover a diversity of bubble distribution patterns for two-phase flows. Synthetic boundary measurements are computed to obtain normalized impedance matrices, the eigenvalues of which are investigated in the different configurations considered. The robustness of the proposed approach to noisy data is assessed by adding 10dB of noise to the measurement data and the presented results highlight some satisfying performances. Moreover, this methodology can be extended to case of conductive inclusions.

From the numerical simulations, it is found that the leading eigenvalue  $\lambda^{(16)}$  and the sum of eigenvalues  $\sum_{|\lambda|}^{15} = |\lambda^{(1)}| + \dots + |\lambda^{(15)}|$  are strongly correlated to the void fraction for all of the cases considered: (i) in the single bubble case (Case 1), the simulation results are encapsulated by the curves of  $\lambda^{(16)}$  and  $\sum_{|\lambda|}^{15}$  vs.  $\alpha$  excluding the data point at  $\alpha > 0.9025$ . The void fraction can be estimated by  $\sum_{|\lambda|}^{15}$  with an error of 10% regardless of the bubble position; (ii) in the stratified case (Case 2), a linear correlation is found between  $\lambda^{(16)}$  and  $h$  with a fitting measure  $R^2 = 0.9905$ ; (iii) in the multiple bubble case (Case 3), the relation between the void fraction and the eigenvalues are strongly dependent on the number  $n_{bubble}$

512 of inclusions and an accurate estimation of the void fraction requires the knowledge of both  
513  $\lambda^{(16)}$ , or  $\sum_{|\lambda|}^{15}$ , and  $n_{bub}$ .

514 Experimental static tests corresponding to the three cases considered are carried out and  
515 the eigenvalues of the NIM from experimental data are compared with 3D simulations. The  
516 trends for the eigenvalue  $\lambda^{(16)}$  with  $\alpha$  agree well for the cases 1 and 3, although there are  
517 small deviations in terms of the magnitudes; for Case 2 there is a large discrepancy between  
518 the static tests and the simulations, while  $\lambda^{(16)}$  is remarkably high at all  $h$  compared to other  
519 phase distribution patterns at the same phase fraction.

520 For a given EIT sensor with specific electrode size and placement, the void fraction can  
521 be estimated from the eigenvalues of the NIM. The estimation error would be reduced by  
522 incorporating a priori knowledge on the flow regime, which can be identified too from EIT  
523 data, see [19]. This overall approach provides a good estimation of the void fraction in two-  
524 phase flows without performing image reconstruction, especially for annular flow (concentric  
525 bubble column) and stratified flow. While the eigenvalue-based indicators considered here  
526 constitute valuable metrics for the state of the system, there is a need for an in-depth math-  
527 ematical analysis of the relation between the system and the eigenvalues of the impedance  
528 matrix. Such an analysis would allow to determine which information can be retrieved from  
529 such a matrix and which metrics are suitable to do so. Lastly and as in [36], this methodology  
530 can be used to devise an initial guess for iterative image reconstruction algorithms thereby  
531 improving their convergence. For further improvements, the Multiple Signal Classification  
532 (MUSIC) algorithm, see [33], could be employed to obtain the number and positions of  
533 bubble inclusions within the probed medium to improve the estimation of the void fraction.

## 534 References

- 535 [1] Neil E Todreas and Mujid S Kazimi. *Nuclear Systems Volume I: Thermal Hydraulic Funda-*  
536 *mentals*. CRC press, 2011.
- 537 [2] Guillaume Ricciardi, M Pettigrew, and Njuki Mureithi. Fluidelastic instability in a normal tri-  
538 angular tube bundle subjected to air-water cross-flow. *Journal of Pressure Vessel Technology*,  
539 133:9, 10 2011.
- 540 [3] HF Velasco Peña and OMH Rodriguez. Applications of wire-mesh sensors in multiphase flows.  
541 *Flow measurement and instrumentation*, 45:255–273, 2015.
- 542 [4] J Enrique Julia, Wouter K Harteveld, Robert F Mudde, and Harrie EA Van den Akker. On  
543 the accuracy of the void fraction measurements using optical probes in bubbly flows. *Review*  
544 *of scientific instruments*, 76(3):035103, 2005.
- 545 [5] Owen C Jones Jr and Jean-Marc Delhaye. Transient and statistical measurement techniques  
546 for two-phase flows: a critical review. *International Journal of Multiphase Flow*, 3(2):89–116,  
547 1976.
- 548 [6] Theodore J Heindel. A review of x-ray flow visualization with applications to multiphase flows.  
549 *Journal of Fluids Engineering*, 133(7):074001, 2011.
- 550 [7] César Marques Salgado, Cláudio MNA Pereira, Roberto Schirru, and Luis EB Brandão. Flow  
551 regime identification and volume fraction prediction in multiphase flows by means of gamma-

- 552 ray attenuation and artificial neural networks. *Progress in Nuclear Energy*, 52(6):555–562,  
553 2010.
- 554 [8] Zhiyao Huang, Baoliang Wang, and Haiqing Li. Application of electrical capacitance tomog-  
555 raphy to the void fraction measurement of two-phase flow. *IEEE Transactions on Instrumen-*  
556 *tation and Measurement*, 52(1):7–12, 2003.
- 557 [9] Antoine Dupré, Guillaume Ricciardi, and Salah Bourennane. Novel approach for analysis and  
558 design of high-speed electrical impedance tomographic system for void fraction measurements  
559 in fast two-phase flows. *IEEE Sensors Journal*, 17(14):4472–4482, 2017.
- 560 [10] Erkki Somersalo, Margaret Cheney, and David Isaacson. Existence and uniqueness for elec-  
561 trode models for electric current computed tomography. *SIAM Journal on Applied Mathemat-*  
562 *ics*, 52(4):1023–1040, 1992.
- 563 [11] Fadil Santosa and Michael Vogelius. A backprojection algorithm for electrical impedance  
564 imaging. *SIAM Journal on Applied Mathematics*, 50(1):216–243, 1990.
- 565 [12] Margaret Cheney, David Isaacson, Jonathan C Newell, S Simske, and J Goble. Noser: An  
566 algorithm for solving the inverse conductivity problem. *International Journal of Imaging*  
567 *Systems and Technology*, 2(2):66–75, 1990.
- 568 [13] Luiz Felipe Fuks, Margaret Cheney, David Isaacson, David G Gisser, and JC Newell. Detection  
569 and imaging of electric conductivity and permittivity at low frequency. *IEEE Transactions on*  
570 *Biomedical Engineering*, 38(11):1106–1110, 1991.
- 571 [14] Alberto P Calderón. On an inverse boundary value problem. *Computational & Applied Math-*  
572 *ematics*, 25(2-3):133–138, 2006.
- 573 [15] Robert V Kohn and Alan McKenney. Numerical implementation of a variational method for  
574 electrical impedance tomography. *Inverse Problems*, 6(3):389, 1990.
- 575 [16] Thomas J Yorkey, John G Webster, and Willis J Tompkins. Comparing reconstruction algo-  
576 rithms for electrical impedance tomography. *IEEE Transactions on Biomedical Engineering*,  
577 BME-34(11):843–852, 1987.
- 578 [17] Liliana Borcea. Electrical impedance tomography. *Inverse problems*, 18(6):R99, 2002.
- 579 [18] Margaret Cheney, David Isaacson, and Jonathan C Newell. Electrical impedance tomography.  
580 *SIAM review*, 41(1):85–101, 1999.
- 581 [19] Antoine Dupre. *Electrical impedance tomography for void fraction measurements of harsh two-*  
582 *phase flows: prototype development and reconstruction techniques*. PhD thesis, Ecole Centrale  
583 Marseille, 2017.
- 584 [20] Weifu Fang and Ellis Cumberbatch. Matrix properties of data from electrical capacitance  
585 tomography. *Journal of engineering mathematics*, 51(2):127–146, 2005.
- 586 [21] Antoine Dupré, Guillaume Ricciardi, Salah Bourennane, and Saba Mylvaganam. Electrical  
587 capacitance-based flow regimes identification - multiphase experiments and sensor modeling.  
588 *IEEE Sensors Journal*, 17(24):8117–8128, 2017.

- 589 [22] Chunhui Dang, Mathieu Darnajou, Guillaume Ricciardi, Achim Beisiegel, Salah Bourennane,  
590 and Cédric Bellis. Performance analysis of an electrical impedance tomography sensor with  
591 two sets of electrodes of different sizes. In *Proceedings of WCIPT-9, Bath, UK*, 2018.
- 592 [23] Martin Hanke and Martin Brühl. Recent progress in electrical impedance tomography. *Inverse*  
593 *Problems*, 19(6):S65, 2003.
- 594 [24] Kuo-Sheng Cheng, David Isaacson, JC Newell, and David G Gisser. Electrode models for elec-  
595 tric current computed tomography. *IEEE Transactions on Biomedical Engineering*, 36(9):918–  
596 924, 1989.
- 597 [25] M Wang. Electrode models in electrical impedance tomography. *Journal of Zhejiang*  
598 *University-SCIENCE A*, 6(12):1386–1393, 2005.
- 599 [26] SL Ceccio and DL George. A review of electrical impedance techniques for the measurement  
600 of multiphase flows. *Journal of fluids engineering*, 118(2):391–399, 1996.
- 601 [27] DL George, JR Torczynski, KA Shollenberger, TJ O’Hern, and SL Ceccio. Validation of  
602 electrical-impedance tomography for measurements of material distribution in two-phase flows.  
603 *International Journal of Multiphase Flow*, 26(4):549–581, 2000.
- 604 [28] Yemada Taitel and Abe E Dukler. A model for predicting flow regime transitions in horizontal  
605 and near horizontal gas-liquid flow. *AIChE journal*, 22(1):47–55, 1976.
- 606 [29] L Rossi, R De Fayard, and S Kassab. Measurements using x-ray attenuation vertical distribu-  
607 tion of the void fraction for different flow regimes in a horizontal pipe. *Nuclear Engineering*  
608 *and Design*, 336:129–140, 2018.
- 609 [30] Geoffrey Frederick Hewitt and DN Roberts. Studies of two-phase flow patterns by simultane-  
610 ous x-ray and fast photography. Technical report, Atomic Energy Research Establishment,  
611 Harwell, England (United Kingdom), 1969.
- 612 [31] Nick Polydorides and William RB Lionheart. A matlab toolkit for three-dimensional elec-  
613 trical impedance tomography: a contribution to the electrical impedance and diffuse optical  
614 reconstruction software project. *Measurement science and technology*, 13(12):1871, 2002.
- 615 [32] P Kauppinen, J Hyttinen, and J Malmivuo. Sensitivity distribution simulations of impedance  
616 tomography electrode combinations. In *BEM & NFSI Conference Proceedings*, volume 7, pages  
617 344–347, 2005.
- 618 [33] Martin Brühl. Explicit characterization of inclusions in electrical impedance tomography.  
619 *SIAM Journal on Mathematical Analysis*, 32(6):1327–1341, 2001.
- 620 [34] AD Seagar, DC Barber, and BH Brown. Theoretical limits to sensitivity and resolution in  
621 impedance imaging. *Clinical Physics and Physiological Measurement*, 8(4A):13, 1987.
- 622 [35] NI PXIe. *DEVICE SPECIFICATIONS NI 6368 X Series Data Acquisition*. National Instru-  
623 ment, 2016.
- 624 [36] Cédric Bellis, Andrei Constantinescu, Thomas Coquet, Thomas Jaravel, and Armin Lechleiter.  
625 A non-iterative sampling approach using noise subspace projection for EIT. *Inverse Problems*,  
626 28(7):075015, 2012.