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Simplification of Multi-Criteria Decision-Making Using Inter-Criteria Analysis and Belief Functions

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Abstract—In this paper we propose a new Belief Function-based Inter-Criteria Analysis (BF-ICrA) for the assessment of the degree of redundancy of criteria involved in a multicriteria decision making (MCDM) problem. This BF-ICrA method allows to simplify the original MCDM problem by withdrawing all redundant criteria and thus diminish the complexity of MCDM problem. This is of prime importance for solving large MCDM problems whose solution requires the fusion of many belief functions. We provide simple examples to show how this new BF-ICrA works.

Keywords: Inter-Criteria Analysis, ICrA-BF, MultiCriteria Decision Making, MCDM, belief functions, information fusion.

I. INTRODUCTION

In a Multi-Criteria Decision-Making (MCDM) problem we consider a set of alternatives (or objects) $\mathbf{A} \triangleq \{A_1, A_2, \dots, A_M\}$ ($M > 2$), and a set of criteria $\mathbf{C} \triangleq \{C_1, C_2, \dots, C_N\}$ ($N \geq 1$). We search for the best alternative A^* given the available information expressed by a $M \times N$ score matrix (also called benefit or payoff matrix) $\mathbf{S} \triangleq [S_{ij} = C_j(A_i)]$, and (eventually) the importance factor $w_j \in [0, 1]$ of each criterion C_j with $\sum_{j=1}^N w_j = 1$. The set of normalized weighting factors is denoted by $\mathbf{w} \triangleq \{w_1, w_2, \dots, w_N\}$. Depending on the context of the MCDM problem, the score S_{ij} of each alternative A_i with respect to each criteria C_j can be interpreted either as a cost (i.e. an expense), or as a reward (i.e. a benefit). By convention and without loss of generality¹ we will always interpret the score as a reward having monotonically increasing preference. Thus, the best alternative A_j^* for a given criteria C_j will be the one providing the highest reward/benefit.

The MCDM problem is not easy to solve because the scores are usually expressed in different (physical) units and different scales. This necessitates a choice of score/data normalization yielding rank reversal problems [1], [2]. Usually there is no same best alternative choice A^* for all criteria, so a compromise must be established to provide a reasonable and acceptable solution of the MCDM problem for decision-making support.

¹because it suffices to multiply the scores values by -1 to reverse the preference ordering.

Many MCDM methods exist, see references in [3]. Most popular methods are AHP² [4], ELECTRE³ [5], TOPSIS⁴ [6], [7]. In 2016 and 2017, we did develop BF-TOPSIS methods [3], [8] based on Belief Functions (BF) to improve the original TOPSIS approach to avoid data normalization and to deal also with imprecise score values as well. It appears however that the complexity of these new BF-TOPSIS methods can become a bottleneck for their use in large MCDM problems because of the fusion step of basic belief assignments required for the implementation of the BF-TOPSIS. That is why a simplification of the MCDM problem (if possible) is very welcome in order to save computational time and resources. This is the motivation of the present work.

For this aim we propose a new Inter-Criteria Analysis (ICrA) based on belief functions for identifying and estimating the possible degree of agreement (i.e. redundancy) between some criteria driven from the data (score values). This permits to remove all redundant criteria of the original MCDM problem and thus solving a simplified (almost) equivalent MCDM problem faster and at lower computational cost. ICrA has been developed originally by Atanassov et al. [9]–[11] based on Intuitionistic Fuzzy Sets [12], and it has been applied in different fields like medicine [13]–[15], optimization [16]–[20], workforce planning [21], competitiveness analysis [22], radar detection [23], ranking [24]–[27], etc. In this paper we improve ICrA approach thanks to belief functions introduced by Shafer in [28] from original Dempster's works [29]. We will refer it as BF-ICrA method in the sequel.

After a short presentation of basics of belief functions in section II, we present Atanassov's ICrA method in section III and discuss its limitations. In Section IV we present the new BF-ICrA approach based on a new construction of Basic Belief Assignment (BBA) matrix from the score matrix and a new establishment of Inter-Criteria belief matrix. In section V a method of simplification of MCDM using BF-ICrA is proposed. Examples are given in VI with concluding remarks in Section VII.

²Analytic Hierarchy Process

³ELimination Et Choix Taduisant la REalité

⁴Technique for Order Preference by Similarity to Ideal Solution

II. BASICS OF THE THEORY OF BELIEF FUNCTIONS

To follow classical notations of the theory of belief functions, also called Dempster-Shafer Theory (DST) [28], we assume that the answer (i.e. the solution, or the decision to take) of the problem under concern belongs to a known finite discrete frame of discernment (FoD) $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$, with $n > 1$, and where all elements of Θ are exclusive. The set of all subsets of Θ (including empty set \emptyset and Θ) is the power-set of Θ denoted by 2^Θ . A BBA (or mass function) associated with a given source of evidence is defined [28] as the mapping $m(\cdot) : 2^\Theta \rightarrow [0, 1]$ satisfying $m(\emptyset) = 0$ and $\sum_{A \in 2^\Theta} m(A) = 1$. The quantity $m(A)$ is called the mass of A committed by the source of evidence. Belief and plausibility functions are usually interpreted respectively as lower and upper bounds of unknown (possibly subjective) probability measure [29]. They are defined by⁵

$$\text{Bel}(A) \triangleq \sum_{B \subseteq A, B \in 2^\Theta} m(B), \quad \text{and} \quad \text{Pl}(A) \triangleq 1 - \text{Bel}(\bar{A}). \quad (1)$$

If $m(A) > 0$, A is called a focal element of $m(\cdot)$. When all focal elements are singletons then $m(\cdot)$ is called a *Bayesian BBA* [28] and its corresponding $\text{Bel}(\cdot)$ function is homogeneous to a probability measure. The vacuous BBA, or VBBA for short, representing a totally ignorant source is defined as $m_v(\Theta) = 1$. The main challenge of the decision-maker consists to combine efficiently the possible multiple BBAs $m_s(\cdot)$ given by $s > 1$ distinct sources of evidence to obtain a global (combined) BBA, and to make a final decision from it. Historically the combination of BBAs is accomplished by Dempster's rule proposed by Shafer in DST. Because Dempster's rule presents several serious problems (insensitivity to the level of conflict between sources in some cases, inconsistency with bounds of conditional probabilities when used for belief conditioning, dictatorial behavior, counter-intuitive results), many fusion rules have been proposed in the literature as alternative to Dempster's rule, see [30], Vol. 2 for a detailed list of fusion rules. We will not detail here all the possible combination rules but just mention that the Proportional Conflict Redistribution rule no. 6 (PCR6) proposed by Martin and Osswald in [30] (Vol. 3) is one of the most serious alternative rule for BBA combination available so far.

III. ATANASSOV'S INTER-CRITERIA ANALYSIS (ICRA)

Atanassov's Inter-Criteria Analysis (ICrA) approach is based on a $M \times N$ score matrix⁶ $\mathbf{S} \triangleq [S_{ij} = C_j(A_i), i = 1, \dots, M, j = 1, \dots, N]$, and intuitionistic fuzzy pairs [12] including two membership functions $\mu(\cdot)$ and $\nu(\cdot)$. Mathematically, an intuitionistic fuzzy set (IFS) A is denoted by $A \triangleq \{(x, \mu_A(x), \nu_A(x)) | x \in E\}$, where E is the set of possible values of x , $\mu_A(x) \in [0, 1]$ defines the membership of x to the set A , and $\nu_A(x) \in [0, 1]$ defines the non-membership of x to the set A , with the restriction $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The ICrA method consists to build an $N \times N$ Inter-Criteria (IC)

matrix from the score matrix \mathbf{S} . The elements of the IC matrix consist of all intuitionistic fuzzy pairs $(\mu_{jj'}, \nu_{jj'})$ whose components express respectively the degree of agreement and the degree of disagreement between criteria C_j and $C_{j'}$ for $j, j' \in \{1, 2, \dots, N\}$. For a given column j (i.e. criterion C_j), it is always possible to compare with $>$, $<$ and $=$ operators all the scores S_{ij} for $i = 1, 2, \dots, M$ because the scores of each column are expressed in same unit. The construction of IC matrix can be used to search relations between the criteria because the method compares homogeneous data relatively to a same column. In [32] Atanassov prescribes to normalize the score matrix before applying ICrA as follows

$$S_{ij}^{\text{norm}} = (S_{ij} - S_j^{\text{min}}) / (S_j^{\text{max}} - S_j^{\text{min}}) \quad (2)$$

if one wants to apply it in the dual manner for the search of InterObjects analysis (IOBa).

Because we focus on ICrA only, we don't need to apply a score matrix normalization because each column of the score matrix represents the values of a same criterion for different alternatives, and the criterion values are expressed with the same unit (e.g. m , m^2 , sec , Kg , or € , etc).

A. Construction of Inter-Criteria matrix

The construction of the $N \times N$ IC matrix, denoted⁷ \mathbf{K} , is based on the pairwise comparisons between every two criteria along all evaluated alternatives. Let $K_{jj'}^\mu$ be the number of cases in which the inequalities $S_{ij} > S_{i'j}$ and $S_{ij'} > S_{i'j'}$ hold simultaneously, and let $K_{jj'}^\nu$ be the number of cases in which the inequalities $S_{ij} > S_{i'j}$ and $S_{ij'} < S_{i'j'}$ hold simultaneously. Because the total number of comparisons between the alternatives is $M(M-1)/2$ then one always has necessarily

$$0 \leq K_{jj'}^\mu + K_{jj'}^\nu \leq \frac{M(M-1)}{2} \quad (3)$$

or equivalently after the division by $\frac{M(M-1)}{2} > 0$

$$0 \leq \frac{2K_{jj'}^\mu}{M(M-1)} + \frac{2K_{jj'}^\nu}{M(M-1)} \leq 1 \quad (4)$$

This inequality permits to define the elements of $N \times N$ IC matrix $\mathbf{K} = [K_{jj'}]$ as intuitionistic fuzzy (IF) pairs $K_{jj'} = (\mu_{jj'}, \nu_{jj'})$ where

$$\mu_{jj'} \triangleq \frac{2K_{jj'}^\mu}{M(M-1)} \quad \text{and} \quad \nu_{jj'} \triangleq \frac{2K_{jj'}^\nu}{M(M-1)} \quad (5)$$

$\mu_{jj'}$ measures the degree of agreement between criteria C_j and $C_{j'}$, and $\nu_{jj'}$ measures their degree of disagreement. By construction the IC matrix \mathbf{K} is always a symmetric matrix. The computation of $K_{jj'}^\mu$ and $K_{jj'}^\nu$ can be done explicitly thanks to Atanassov's formulas [32]

$$K_{jj'}^\mu = \sum_{i=1}^{M-1} \sum_{i'=i+1}^M [\text{sgn}(S_{ij} - S_{i'j}) \text{sgn}(S_{ij'} - S_{i'j'}) + \text{sgn}(S_{i'j} - S_{ij}) \text{sgn}(S_{i'j'} - S_{ij'})] \quad (6)$$

⁵where the symbol \triangleq means *equal by definition*.

⁶called index matrix by Atanassov in [31].

⁷We use \mathbf{K} because it corresponds to the first letter of word Kriterium, meaning criteria in German. The letter C is being already in use.

and

$$K_{jj'}^\nu = \sum_{i=1}^{M-1} \sum_{i'=i+1}^M [\text{sgn}(S_{ij} - S_{i'j}) \text{sgn}(S_{i'j'} - S_{ij'}) + \text{sgn}(S_{i'j} - S_{ij}) \text{sgn}(S_{ij'} - S_{i'j'})] \quad (7)$$

where the signum function $\text{sgn}(\cdot)$ used by Atanassov is defined as follows

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases} \quad (8)$$

Actually the values of $K_{jj'}^\mu$ and $K_{jj'}^\nu$ depend on the choice of $\text{sgn}(x)$ function⁸. That is why in [21], [33], the authors propose different algorithms implemented under Java in an ICRA software yielding different $K_{jj'}^\mu$ and $K_{jj'}^\nu$ values for making the analysis and to reduce the dimension (complexity) of the original MCDM problem.

B. Inter-criteria analysis

Once the Inter-Criteria matrix $\mathbf{K} = [K_{jj'}]$ of intuitionistic fuzzy pairs is calculated one needs to analyze it to decide which criteria C_j and $C_{j'}$ are in strong agreement (or positive consonance) reflecting the correlation between C_j and $C_{j'}$, in strong disagreement (or negative consonance) reflecting non correlation between C_j and $C_{j'}$, or in dissonance reflecting the uncertainty situation where nothing can be said about the non correlation or the correlation between C_j and $C_{j'}$. If one wants to identify the set of criteria $C_{j'}$ for $j' \neq j$ that are strongly correlated with C_j then we can sort $\mu_{jj'}$ values in descending order to identify those in strong positive consonance with C_j . In [25], [26], the authors propose a qualitative scale to refine the levels of consonance and dissonance and for helping the decision making procedure. A dual approach based on $\nu_{jj'}$ values can be made to determine the set of criteria that are not correlated with C_j . An other approach [10], [27] proposes to define two thresholds $\alpha, \beta \in [0; 1]$ for the positive and negative consonance respectively against which the components $\mu_{jj'}$ and $\nu_{jj'}$ of $K_{jj'} = (\mu_{jj'}, \nu_{jj'})$ will be compared. The correlations between the criteria C_j and $C_{j'}$ are called "positive consonance", "negative consonance" or "dissonance" depending on their $\mu_{jj'}$ and $\nu_{jj'}$ values with respect to chosen thresholds α and β , see [22] for details. More precisely, C_j and $C_{j'}$ are in

- (α, β) positive consonance (i.e. correlated):
If $\mu_{jj'} > \alpha$ and $\nu_{jj'} < \beta$.
- (α, β) negative consonance (i.e. no correlated):
If $\mu_{jj'} < \beta$ and $\nu_{jj'} > \alpha$.
- (α, β) dissonance (i.e. full uncertainty): Otherwise.

At the beginning of ICRA development it was not very clear how these intuitionistic fuzzy (IF) pairs $(\mu_{jj'}, \nu_{jj'})$ had to be used and that is why Atanassova [34], [35] proposed to handle both components of the IF pair. For this, she interpreted pairs $(\mu_{jj'}, \nu_{jj'})$ as points located in the elementary TFU

triangle, where the point T of coordinate $(1, 0)$ represents the maximal positive consonance (i.e. the true consonance), the point F with coordinate $(0, 1)$ represents the maximal negative consonance (i.e. the falsity), and the point U with coordinates $(0, 0)$ represents the maximal dissonance (i.e. the uncertainty). From this interpretation it becomes easy to identify the top of consonant IF pairs $(\mu_{jj'}, \nu_{jj'})$ that fall in bottom right corner of (TFU) triangle limited by vertical line from x -axis $x = \alpha$, and horizontal line from y -axis $y = \beta$. The set of consonant IF pairs are then ranked according to their Euclidean distance $d_{C_j C_{j'}}^T$ with respect to T point of coordinate $(1, 0)$ defined by

$$d_{C_j C_{j'}}^T = d((1, 0), (\mu_{jj'}, \nu_{jj'})) = \sqrt{(1 - \mu_{jj'})^2 + \nu_{jj'}^2} \quad (9)$$

In the MCDM context only the criteria that are negatively consonant (or uncorrelated) must be kept for solving MCDM and saving computational resources because they have no (or only very low) dependency with each other, so that each uncorrelated criterion provides useful information. The set of criteria that are positively consonant (if any), called the consonant set, indicates somehow a redundancy of information between the criteria belonging to it in term of decisional behavior. Therefore all these positively consonant criteria must be represented by only one representative criterion that will be kept in the MCDM analysis to simplify MCDM problem. Also all the criteria that are deemed strongly dissonant (if any) could be taken out of the original MCDM problem because they only introduce uncertainty in the decision-making.

C. General comments on ICRA

Although appealing at the first glance, the classical ICRA approach induces the following comments:

- 1) The IF values $\mu_{jj'}$ and $\nu_{jj'}$ can be easily interpreted in the belief function framework. Indeed, the belief and plausibility of (positive) consonance between criteria C_j and $C_{j'}$ can be directly linked to the values $\mu_{jj'}$ and $\nu_{jj'}$ by taking $Bel_{jj'}(\theta) = \mu_{jj'}$ and $Pl_{jj'}(\theta) = 1 - \nu_{jj'}$. Moreover $U_{jj'}(\theta) = Pl_{jj'}(\theta) - Bel_{jj'}(\theta) = 1 - \nu_{jj'} - \mu_{jj'}$ represents the dissonance (the uncertainty about the correlation) of the criteria C_j and $C_{j'}$. Here the proposition θ means: the criteria C_j and $C_{j'}$ are totally positively consonant (i.e. totally correlated) and the frame of discernment is defined as $\Theta \triangleq \{\theta, \bar{\theta}\}$, where $\bar{\theta}$ means: the criteria C_j and $C_{j'}$ are totally negatively consonant (uncorrelated). From this, one can define any BBA $m_{jj'}(\theta)$, $m_{jj'}(\bar{\theta})$ and $m_{jj'}(\theta \cup \bar{\theta})$ of 2^Θ by

$$m_{jj'}(\theta) = \mu_{jj'} \quad (10)$$

$$m_{jj'}(\bar{\theta}) = \nu_{jj'} \quad (11)$$

$$m_{jj'}(\theta \cup \bar{\theta}) = 1 - \mu_{jj'} - \nu_{jj'} \quad (12)$$

- 2) The construction of $\mu_{jj'}$ and $\nu_{jj'}$ proposed in the classical ICRA is disputable because it is only based on counting the valid ">" or "<" inequalities but it doesn't exploit how bigger and how smaller the scores values are in each comparison done in the construction of the Inter-Criteria Matrix \mathbf{K} . Therefore the construction of

⁸for instance if we use $\text{sgn}(x) = 1$ if $x \geq 0$ and $\text{sgn}(x) = 0$ if $x < 0$, we will obtain, in general, other $K_{jj'}^\mu$ and $K_{jj'}^\nu$ values.

$\mu_{jj'}$ and $\nu_{jj'}$ is actually only a very crude method to estimate IF pairs.

- 3) The construction of the Inter-Criteria Matrix \mathbf{K} is in fact not unique as reported in [33]. This will yield different results in general.
- 4) The exploitation of the ICRA method depends on the choice of α and β thresholds that will impact the final result.
- 5) The classical ICRA method cannot deal directly with imprecise or missing score values.

IV. A NEW ICRA METHOD BASED ON BELIEF FUNCTIONS

In this paper we propose a new ICRA method, called BF-ICRA for short, based on belief functions that circumvents most of the aforementioned drawbacks of classical ICRA. Here we show how to get more precisely the Inter-Criteria Belief Matrix and how to exploit it for MCDM simplification.

A. Construction of BBA matrix from the score matrix

From any non-zero score matrix $\mathbf{S} = [S_{ij}]$, we can construct the $M \times N$ BBA matrix $\mathbf{M} = [m_{ij}(\cdot)]$ as follows

$$m_{ij}(A_i) = Bel_{ij}(A_i) \quad (13)$$

$$m_{ij}(\bar{A}_i) = Bel_{ij}(\bar{A}_i) = 1 - Pl_{ij}(A_i) \quad (14)$$

$$m_{ij}(A_i \cup \bar{A}_i) = Pl_{ij}(A_i) - Bel_{ij}(A_i) \quad (15)$$

Assuming $A_{\max}^j \neq 0$ and $A_{\min}^j \neq 0$, we take⁹

$$Bel_{ij}(A_i) \triangleq Sup_j(A_i)/A_{\max}^j \quad (16)$$

$$Bel_{ij}(\bar{A}_i) \triangleq Inf_j(A_i)/A_{\min}^j \quad (17)$$

where $A_{\max}^j \triangleq \max_i Sup_j(A_i)$ and $A_{\min}^j \triangleq \min_i Inf_j(A_i)$ and with

$$Sup_j(A_i) \triangleq \sum_{k \in \{1, \dots, M\} | S_{kj} \leq S_{ij}} |S_{ij} - S_{kj}| \quad (18)$$

$$Inf_j(A_i) \triangleq - \sum_{k \in \{1, \dots, M\} | S_{kj} \geq S_{ij}} |S_{ij} - S_{kj}| \quad (19)$$

The entire justification of these formulas can be found in our previous works [3]. For example, consider the j -th column corresponding to a criterion C_j of a score matrix $\mathbf{S} = [S_{ij}]$ with seven rows given by $\mathbf{s}_j = [10, 20, -5, 0, 100, -11, 0]^T$, where T denotes the transpose. Then based on above formula we get the BBA values listed in Table I.

Table I
BBAS CONSTRUCTED FROM SCORE VALUES.

	$m_{ij}(A_i)$	$m_{ij}(\bar{A}_i)$	$m_{ij}(A_i \cup \bar{A}_i)$
A_1	0.0955	0.5236	0.3809
A_2	0.1809	0.4188	0.4003
A_3	0.0102	0.8115	0.1783
A_4	0.0273	0.6806	0.2921
A_5	1.0000	0	0
A_6	0	1.0000	0
A_7	0.0273	0.6806	0.2921

⁹If $A_{\max}^j = 0$ then $Bel_{ij}(A_i) = 0$, and if $A_{\min}^j = 0$ then $Pl_{ij}(A_i) = 1$.

For another criterion $C_{j'}$ and the j' -th column of the score matrix we will obtain another set of BBA values $m_{ij'}(\cdot)$. Applying this method for each column of the score matrix we are able to compute the BBA matrix $\mathbf{M} = [m_{ij}(\cdot)]$ whose each component is in fact a triplet $(m_{ij}(A_i), m_{ij}(\bar{A}_i), m_{ij}(A_i \cup \bar{A}_i))$ of BBA values in $[0, 1]$ such that $m_{ij}(A_i) + m_{ij}(\bar{A}_i) + m_{ij}(A_i \cup \bar{A}_i) = 1$ for all $i = 1, \dots, M$ and $j = 1, \dots, N$.

B. Construction of Inter-Criteria Matrix from BBA matrix

The next step of BF-ICRA approach is the construction of the $N \times N$ Inter-Criteria Matrix $\mathbf{K} = [K_{jj'}]$ from $M \times N$ BBA matrix $\mathbf{M} = [m_{ij}(\cdot)]$ where elements $K_{jj'}$ corresponds to the BBA $(m_{jj'}(\theta), m_{jj'}(\bar{\theta}), m_{jj'}(\theta \cup \bar{\theta}))$ about positive consonance θ , negative consonance $\bar{\theta}$ and uncertainty between criteria C_j and $C_{j'}$ respectively. The principle of construction of the triplet $K_{jj'} = (m_{jj'}(\theta), m_{jj'}(\bar{\theta}), m_{jj'}(\theta \cup \bar{\theta}))$ is based on two steps that will be detailed in the sequel:

- Step 1: For each alternative A_i , we first compute the BBA $(m_{jj'}^i(\theta), m_{jj'}^i(\bar{\theta}), m_{jj'}^i(\theta \cup \bar{\theta}))$ for any two criteria $j, j' \in \{1, 2, \dots, N\}$.
- Step 2: The BBA $(m_{jj'}(\theta), m_{jj'}(\bar{\theta}), m_{jj'}(\theta \cup \bar{\theta}))$ is then obtained by the combinations of the M BBA $m_{jj'}^i(\cdot)$.

Construction of BBA $m_{jj'}^i(\cdot)$

The mass of belief $m_{jj'}^i(\theta)$ represents the degree of agreement between the BBA $m_{ij}(\cdot)$ and $m_{ij'}(\cdot)$ for the alternative A_i , and $m_{jj'}^i(\bar{\theta})$ represents the degree of disagreement between $m_{ij}(\cdot)$ and $m_{ij'}(\cdot)$. The mass $m_{jj'}^i(\theta \cup \bar{\theta})$ is the degree of uncertainty about the agreement (or disagreement) between $m_{ij}(\cdot)$ and $m_{ij'}(\cdot)$ for the alternative A_i . The calculation of $m_{jj'}^i(\theta)$ could be envisaged in several manners.

The first manner would consist to consider the degree of conflict [28] $k_{jj'}^i \triangleq \sum_{X, Y \subseteq \Theta | X \cap Y = \emptyset} m_{ij}(X) m_{ij'}(Y)$ and consider the Bayesian BBA $m_{jj'}^i(\theta) = 1 - k_{jj'}^i$, $m_{jj'}^i(\bar{\theta}) = k_{jj'}^i$, and $m_{jj'}^i(\theta \cup \bar{\theta}) = 0$. Instead of using Shafer's conflict, the second manner would consist to use a normalized distance $d_{jj'}^i = d(m_{ij}, m_{ij'})$ to measure the closeness between $m_{ij}(\cdot)$ and $m_{ij'}(\cdot)$, and then consider the Bayesian BBA modeling defined by $m_{jj'}^i(\theta) = 1 - d_{jj'}^i$, $m_{jj'}^i(\bar{\theta}) = d_{jj'}^i$, and $m_{jj'}^i(\theta \cup \bar{\theta}) = 0$. These two manners however are not very satisfying because they always set to zero the degree of uncertainty between the agreement and disagreement of the BBA, and the second manner depends also on the choice of the distance metric. So, we propose a more appealing third manner of the BBA modeling of $m_{jj'}^i(\theta)$, $m_{jj'}^i(\bar{\theta})$, and $m_{jj'}^i(\theta \cup \bar{\theta})$. For this, we consider two sources of evidences (SoE) indexed by j and j' providing the BBA m_{ij} and $m_{ij'}$ defined on the simple FoD $\{A_i, \bar{A}_i\}$ and denoted $m_{ij} = [m_{ij}(A_i), m_{ij}(\bar{A}_i), m_{ij}(A_i \cup \bar{A}_i)]$ and $m_{ij'} = [m_{ij'}(A_i), m_{ij'}(\bar{A}_i), m_{ij'}(A_i \cup \bar{A}_i)]$. We also denote $\Theta = \{\theta, \bar{\theta}\}$ the FoD about the relative state of the two SoE, where θ means that the two SoE agree, $\bar{\theta}$ means that they disagree and $\theta \cup \bar{\theta}$ means that we don't know. Then the BBA modeling is based on the important remarks

- Two SoE are in total agreement if both commit their maximum belief mass to the element A_i or to element \bar{A}_i . So they perfectly agree if $m_{ij}(A_i) = m_{ij'}(A_i) = 1$,

or if $m_{ij}(\bar{A}_i) = m_{ij'}(\bar{A}_i) = 1$. Therefore the pure degree of agreement¹⁰ between two sources is modeled by

$$m_{jj'}^i(\theta) = m_{ij}(A_i)m_{ij'}(A_i) + m_{ij}(\bar{A}_i)m_{ij'}(\bar{A}_i) \quad (20)$$

- Two SoE are in total disagreement if each one commits its maximum mass of belief to one element and the other to its opposite, that is if one has $m_{ij}(A_i) = 1$ and $m_{ij'}(\bar{A}_i) = 1$, or if $m_{ij}(\bar{A}_i) = 1$ and $m_{ij'}(A_i) = 1$. Hence the pure degree of disagreement¹¹ between two sources is modeled by

$$m_{jj'}^i(\bar{\theta}) = m_{ij}(A_i)m_{ij'}(\bar{A}_i) + m_{ij}(\bar{A}_i)m_{ij'}(A_i) \quad (21)$$

- All possible remaining products between components of m_{ij} and $m_{ij'}$ reflect the part of uncertainty we have about the SoE (i.e. we don't know if they agree or disagree). Hence the degree of uncertainty between the two sources is modeled by

$$\begin{aligned} m_{jj'}^i(\theta \cup \bar{\theta}) &= m_{ij}(A_i)m_{ij'}(A_i \cup \bar{A}_i) + m_{ij}(\bar{A}_i)m_{ij'}(A_i \cup \bar{A}_i) \\ &+ m_{ij}(A_i \cup \bar{A}_i)m_{ij'}(A_i) + m_{ij}(A_i \cup \bar{A}_i)m_{ij'}(\bar{A}_i) \\ &+ m_{ij}(A_i \cup \bar{A}_i)m_{ij'}(A_i \cup \bar{A}_i) \quad (22) \end{aligned}$$

By construction $m_{jj'}^i(\cdot) = m_{jj'}^i(\cdot)$, hence this BBA modeling permits to build a set of M symmetrical Inter-Criteria Belief Matrices (ICBM) $\mathbf{K}^i = [K_{jj'}^i]$ of dimension $N \times N$ relative to each alternative A_i whose components $K_{jj'}^i$ correspond to the triplet of BBA values $m_{jj'}^i = (m_{jj'}^i(\theta), m_{jj'}^i(\bar{\theta}), m_{jj'}^i(\theta \cup \bar{\theta}))$ modeling the belief of agreement and of disagreement between C_j and $C_{j'}$ based on A_i . One has also¹² $m_{jj'}^i(\theta), m_{jj'}^i(\bar{\theta}), m_{jj'}^i(\theta \cup \bar{\theta}) \in [0, 1]$ and $m_{jj'}^i(\theta) + m_{jj'}^i(\bar{\theta}) + m_{jj'}^i(\theta \cup \bar{\theta}) = 1$. This BBA construction can be easily extended for modeling the agreement, disagreement and uncertainty of $n > 2$ criteria C_{j_1}, \dots, C_{j_n} altogether if needed by taking

$$\begin{aligned} m_{j_1 \dots j_n}^i(\theta) &= \prod_{k=1}^n m_{ij_k}(A_i) + \prod_{k=1}^n m_{ij_k}(\bar{A}_i) \\ m_{j_1 \dots j_n}^i(\bar{\theta}) &= \sum_{\substack{X_{j_1}, \dots, X_{j_n} \in \{A_i, \bar{A}_i\} \\ X_{j_1} \cap \dots \cap X_{j_n} = \emptyset}} \prod_{k=1}^n m_{ij_k}(X_{j_k}) \\ m_{j_1 \dots j_n}^i(\theta \cup \bar{\theta}) &= 1 - m_{j_1 \dots j_n}^i(\theta) - m_{j_1 \dots j_n}^i(\bar{\theta}) \end{aligned}$$

Construction of BBA $m_{jj'}(\cdot)$

Once all the BBAs $m_{jj'}^i(\cdot)$ ($i = 1, \dots, M$) are calculated one combines them to get the component $K_{jj'} = (m_{jj'}(\theta), m_{jj'}(\bar{\theta}), m_{jj'}(\theta \cup \bar{\theta}))$ of the Inter-Criteria Belief matrix (ICBM) $\mathbf{K} = [K_{jj'}]$. This fusion step can be done in many ways depending on the combination rule chosen by the user. If the number of alternatives M is not too large we recommend to combine the BBAs $m_{jj'}^i(\cdot)$ with PCR6 fusion rule [30] (Vol. 3) because of known deficiencies of Dempster's rule. If M is too large to prevent PCR6 working on computer, we can just use the simple averaging rule of combination in these high dimensional MCDM problems.

¹⁰or positive consonance according Atanassov's terminology.

¹¹or negative consonance according Atanassov's terminology.

¹²because $(m_{ij}(A_i) + m_{ij}(\bar{A}_i) + m_{ij}(A_i \cup \bar{A}_i))(m_{ij'}(A_i) + m_{ij'}(\bar{A}_i) + m_{ij'}(A_i \cup \bar{A}_i)) = 1 \cdot 1 = 1$.

V. SIMPLIFICATION OF ORIGINAL MCDM

Once the global Inter-Criteria Belief Matrix $\mathbf{K} = [K_{jj'} = (m_{jj'}(\theta), m_{jj'}(\bar{\theta}), m_{jj'}(\theta \cup \bar{\theta}))]$ is calculated, we need to identify and cluster the criteria that are in strong agreement, in strong disagreement, and those on which we are uncertain. For identifying the criteria that are in very strong agreement, we evaluate the distance of each component of $K_{jj'}$ with the BBA representing the best agreement state and characterized by the specific BBA¹³ $m_T(\theta) = 1$. From a similar approach we can also identify, if we want, the criteria that are in very strong disagreement using the distance of $m_{jj'}(\cdot)$ with respect to the BBA representing the best disagreement state characterized by the specific BBA $m_F(\bar{\theta}) = 1$. As alternative of Jousselme's distance [37], we use the $d_{BI}(\cdot, \cdot)$ distance based on belief interval [36] because it is a good method for measuring the distance $d(m_1, m_2)$ between the two BBAs¹⁴ $m_1(\cdot)$ and $m_2(\cdot)$ over the same FoD. It is defined by

$$d_{BI}(m_1, m_2) \triangleq \sqrt{N_c \cdot \sum_{X \in \Theta} d_W^2(BI_1(X), BI_2(X))} \quad (23)$$

where the *Belief-Intervals* are defined by $BI_1(X) \triangleq [Bel_1(X), Pl_1(X)]$ and $BI_2(X) \triangleq [Bel_2(X), Pl_2(X)]$ and computed from $m_1(\cdot)$ and $m_2(\cdot)$ thanks to formula (1). $d_W(BI_1(X), BI_2(X))$ is Wassertein's distance between intervals calculated by

$$d_W([a_1, b_1], [a_2, b_2]) = \sqrt{\left[\frac{a_1 + b_1}{2} - \frac{a_2 + b_2}{2} \right]^2 + \frac{1}{3} \left[\frac{b_1 - a_1}{2} - \frac{b_2 - a_2}{2} \right]^2}$$

and $N_c = 1/2^{|\Theta|-1}$ is a factor to get $d_{BI}(m_1, m_2) \in [0, 1]$.

Because all criteria that are in strong agreement somehow contain redundant (correlated) information and behave similarly from decision-making standpoint, we propose to simplify the original MCDM problem by keeping in the MCDM only criteria that are non redundant. The remaining criteria can be eventually weighted by their degree of importance reflecting the number of different criteria that are in agreement through this BF-ICrA approach.

For instance, if one has a seven criteria MCDM problem and if criteria C_1, C_2 and C_3 are in strong agreement we will only select one remaining criterion among $\{C_1, C_2, C_3\}$ and we give it a weight of $w_1 + w_2 + w_3$. Moreover if C_4 and C_5 are in strong agreement also we will only select one remaining criterion among $\{C_4, C_5\}$ and we give it a weight of $w_4 + w_5$, and we will use the weight w_6 for C_6 , and w_7 for C_7 . Hence the original MCDM problem will reduce to a four simplified MCDM problem that can be solved using BF-TOPSIS method already presented in details in [3] and in [8], or with AHP [4] if one prefers, or with any other chosen method that the system-designer may prefer.

¹³We use the index T in the notation $m_T(\cdot)$ to refer that the agreement is true, and F in $m_F(\cdot)$ to specify that the agreement is false.

¹⁴Here $m_1(\cdot) = m_{jj'}(\cdot)$, and $m_2(\cdot) = m_T(\cdot)$ or $m_2(\cdot) = m_F(\cdot)$

The strategy for selecting the most representative criterion among a set of redundant criteria is not unique and depends mainly on the cost necessary (i.e. human efforts, data mining, computational resources, etc) for getting the values of the score matrix of the problem under concern. The least costly criteria may be a good option of selection. In the next section we provide simple examples for BF-ICrA and, for simplicity, we will select the representative criterion as being the one with smallest index. So in the aforementioned example the simplified MCDM problem will reduce to a $M \times 4$ MCDM problem involving only four criteria C_1, C_4, C_6 and C_7 .

The BF-ICrA method proposed in this work allows also, in principle, to make a refined analysis (if necessary) based on IC matrices $\mathbf{K}_{jj'}^i$ about the origin of disagreement between criteria with respect to each alternative A_i in order to identify the potential inconsistencies in original MCDM problem. This aspect is not developed in this paper and has been left for future investigations. It is worth mentioning that the analysis of the number of redundant criteria versus time improvements that could be proposed as an effective measure of performance of this approach depends highly of the application under consideration and the difficulty (and cost) to get the value of each criteria. For convenience the Figure 1 shows the flow chart of BF-ICrA to help the reader to have a better understanding of this new proposed method.

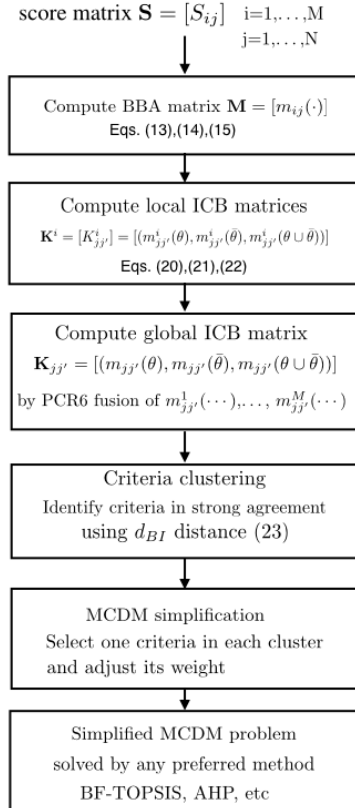


Figure 1. Flow chart of BF-ICrA method.

VI. EXAMPLES

A. Example 1 (Comparison of \mathbf{K} matrices)

Here we compare the construction of the global IC matrix \mathbf{K} based on Atanassov ICrA and our new BF-ICrA approach. For this, we use the 5×4 MCDM example given in [33] based on the following score matrix (called sample data matrix in [33]). Each row of \mathbf{S} corresponds to an alternative, and each column to a criterion. In [33], the authors use rows for criteria and columns for alternatives so they work with \mathbf{S}^T .

$$\mathbf{S} = [S_{ij}] = \begin{bmatrix} 6 & 7 & 4 & 4 \\ 5 & 7 & 3 & 5 \\ 3 & 8 & 5 & 6 \\ 7 & 1 & 9 & 7 \\ 6 & 3 & 1 & 8 \end{bmatrix}$$

Based on Atanassov's ICrA method (using unbiased algorithm presented in details in [33]) we will get the following 4×4 global Inter-Criteria \mathbf{K}^μ and \mathbf{K}^ν matrices

$$\mathbf{K}^\mu = [K_{jj'}^\mu] = \begin{bmatrix} 0.9 & 0 & 0.5 & 0.5 \\ 0 & 0.9 & 0.5 & 0.3 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.3 & 0.5 & 1 \end{bmatrix}$$

$$\mathbf{K}^\nu = [K_{jj'}^\nu] = \begin{bmatrix} 0 & 0.8 & 0.4 & 0.4 \\ 0.8 & 0 & 0.4 & 0.6 \\ 0.4 & 0.4 & 0 & 0.5 \\ 0.4 & 0.6 & 0.5 & 0 \end{bmatrix}$$

Regrouping these two matrices into one matrix $\mathbf{K} = [K_{jj'}]$ with components $K_{jj'} = (K_{jj'}^\mu, K_{jj'}^\nu, 1 - K_{jj'}^\mu - K_{jj'}^\nu)$, one gets the following global Inter-Criteria matrix \mathbf{K}

$$\mathbf{K} = \begin{bmatrix} (0.9, 0, 0.1) & (0, 0.8, 0.2) & (0.5, 0.4, 0.1) & (0.5, 0.4, 0.1) \\ (0, 0.8, 0.2) & (0.9, 0, 0.1) & (0.5, 0.4, 0.1) & (0.3, 0.6, 0.1) \\ (0.5, 0.4, 0.1) & (0.5, 0.4, 0.1) & (1, 0, 0) & (0.5, 0.5, 0) \\ (0.5, 0.4, 0.1) & (0.3, 0.6, 0.1) & (0.5, 0.5, 0) & (1, 0, 0) \end{bmatrix}$$

According to this \mathbf{K} matrix it appears intuitively that none of the criterion is in strong agreement with others. We observe that criteria C_1 and C_2 are in relatively strong disagreement because $K_{12}^\nu = K_{21}^\nu = 0.8$ which is quite close to one. Criteria C_2 and C_4 are in relatively medium disagreement because $K_{24}^\nu = K_{42}^\nu = 0.6$. In this example no MCDM simplification is prescribed based on Atanassov's ICrA. To get a more precise evaluation of degree of agreement between criteria based on Atanassov's ICrA we apply formula (23) to get the \mathbf{D}_{BI}^θ distance matrix from each component of \mathbf{K} to the total agreement state $m_T = [m(\theta), m(\bar{\theta}), m(\theta \cup \bar{\theta})] = [1, 0, 0]$. Hence we get

$$\mathbf{D}_{BI}^\theta = \begin{bmatrix} 0.0577 & \mathbf{0.9018} & 0.4509 & 0.4509 \\ \mathbf{0.9018} & 0.0577 & 0.4509 & 0.6506 \\ 0.4509 & 0.4509 & 0 & 0.5000 \\ 0.4509 & 0.6506 & 0.5000 & 0 \end{bmatrix}$$

As we see from this \mathbf{D}_{BI}^θ matrix, the distance of the inter-criteria BBA for C_1 and C_2 with respect to the total agreement state $m_T(\theta) = 1$ is very large (i.e. 0.9018) which means that C_1 and C_2 strongly disagree in this example as we expect from a more intuitive reasoning based on $K_{12}^\nu = 0.8$ value. Similar analyses can be done for all (non diagonal) elements

of \mathbf{D}_{BI}^θ to identify which criteria are in strong agreement, or not (if any).

Based on our new BF-ICrA method we first compute the 5×4 BBA matrix $\mathbf{M} = [m_{ij}(\cdot)]$ from the score matrix \mathbf{S} based on formulas (13)-(15). We get (all the values of results have been rounded at their second digit)

$$\mathbf{M} \approx \begin{bmatrix} (0.5, 0.08, 0.42) & (0.71, 0.05, 0.24) & (0.18, 0.35, 0.47) & (0, 1, 0) \\ (0.25, 0.33, 0.42) & (0.71, 0.05, 0.24) & (0.09, 0.53, 0.38) & (0.1, 0.6, 0.3) \\ (0, 1, 0) & (1, 0, 0) & (0.30, 0.24, 0.46) & (0.3, 0.3, 0.4) \\ (1, 0, 0) & (0, 1, 0) & (1, 0, 0) & (0.6, 0.1, 0.3) \\ (0.5, 0.09, 0.41) & (0.14, 0.62, 0.24) & (0, 1, 0) & (1, 0, 0) \end{bmatrix}$$

The construction of Inter-Criteria Matrices $\mathbf{K}^i = [K_{jj'}^i]$ (for $i = 1, \dots, 5$) from the BBA matrix \mathbf{M} based on formulas (20)-(22) yields the following five matrices

$$\mathbf{K}^1 \approx \begin{bmatrix} (0.26, 0.08, 0.66) & (0.36, 0.08, 0.56) & (0.12, 0.19, 0.69) & (0.08, 0.5, 0.42) \\ (0.36, 0.08, 0.56) & (0.51, 0.07, 0.42) & (0.14, 0.26, 0.6) & (0.05, 0.71, 0.24) \\ (0.12, 0.19, 0.69) & (0.14, 0.26, 0.6) & (0.15, 0.13, 0.72) & (0.35, 0.18, 0.47) \\ (0.08, 0.5, 0.42) & (0.05, 0.71, 0.24) & (0.35, 0.18, 0.47) & (1, 0, 0) \end{bmatrix}$$

$$\mathbf{K}^2 \approx \begin{bmatrix} (0.17, 0.17, 0.66) & (0.19, 0.25, 0.56) & (0.20, 0.16, 0.64) & (0.23, 0.18, 0.59) \\ (0.19, 0.25, 0.56) & (0.51, 0.07, 0.42) & (0.09, 0.38, 0.53) & (0.10, 0.43, 0.47) \\ (0.20, 0.16, 0.64) & (0.09, 0.38, 0.53) & (0.29, 0.09, 0.62) & (0.33, 0.10, 0.57) \\ (0.23, 0.18, 0.59) & (0.10, 0.43, 0.47) & (0.33, 0.10, 0.57) & (0.37, 0.12, 0.51) \end{bmatrix}$$

$$\mathbf{K}^3 \approx \begin{bmatrix} (1, 0, 0) & (0, 1, 0) & (0.24, 0.3, 0.46) & (0.3, 0.3, 0.4) \\ (0, 1, 0) & (1, 0, 0) & (0.30, 0.24, 0.46) & (0.3, 0.3, 0.4) \\ (0.24, 0.3, 0.46) & (0.30, 0.24, 0.46) & (0.15, 0.14, 0.71) & (0.16, 0.16, 0.68) \\ (0.3, 0.3, 0.4) & (0.3, 0.3, 0.4) & (0.16, 0.16, 0.68) & (0.18, 0.18, 0.64) \end{bmatrix}$$

$$\mathbf{K}^4 \approx \begin{bmatrix} (1, 0, 0) & (0, 1, 0) & (1, 0, 0) & (0.6, 0.1, 0.3) \\ (0, 1, 0) & (1, 0, 0) & (0, 1, 0) & (0.1, 0.6, 0.3) \\ (1, 0, 0) & (0, 1, 0) & (1, 0, 0) & (0.6, 0.1, 0.3) \\ (0.6, 0.1, 0.3) & (0.1, 0.6, 0.3) & (0.6, 0.1, 0.3) & (0.370, 0.12, 0.51) \end{bmatrix}$$

$$\mathbf{K}^5 \approx \begin{bmatrix} (0.26, 0.08, 0.66) & (0.12, 0.32, 0.56) & (0.08, 0.5, 0.42) & (0.5, 0.08, 0.42) \\ (0.12, 0.32, 0.56) & (0.40, 0.18, 0.42) & (0.62, 0.14, 0.24) & (0.14, 0.62, 0.24) \\ (0.08, 0.5, 0.42) & (0.62, 0.14, 0.24) & (1, 0, 0) & (0, 1, 0) \\ (0.5, 0.08, 0.42) & (0.14, 0.62, 0.24) & (0, 1, 0) & (1, 0, 0) \end{bmatrix}$$

The componentwise PCR6 fusion of all five \mathbf{K}^i matrices provides the following global Inter-Criteria matrix \mathbf{K}_{PCR6}

$$\mathbf{K}_{\text{PCR6}} \approx \begin{bmatrix} (0.90, 0.02, 0.08) & (0.06, 0.83, 0.11) & (0.55, 0.15, 0.30) & (0.49, 0.25, 0.26) \\ (0.06, 0.83, 0.11) & (0.95, 0.01, 0.04) & (0.18, 0.58, 0.24) & (0.08, 0.80, 0.12) \\ (0.55, 0.15, 0.30) & (0.18, 0.58, 0.24) & (0.89, 0.02, 0.09) & (0.22, 0.48, 0.30) \\ (0.49, 0.25, 0.26) & (0.08, 0.80, 0.12) & (0.22, 0.48, 0.30) & (0.90, 0.02, 0.08) \end{bmatrix}$$

Applying formula (23) we get the following \mathbf{D}_{BI}^θ distance matrix from each component of \mathbf{K}_{PCR6} to the total agreement state $m_T = [m(\theta), m(\bar{\theta}), m(\theta \cup \bar{\theta})] = [1, 0, 0]$

$$\mathbf{D}_{BI}^\theta = \begin{bmatrix} 0.0601 & 0.8845 & 0.3124 & 0.3909 \\ 0.8845 & 0.0327 & 0.7037 & 0.8618 \\ 0.3124 & 0.7037 & 0.0668 & 0.6355 \\ 0.3909 & 0.8618 & 0.6355 & 0.0622 \end{bmatrix} \quad (24)$$

We see that \mathbf{K} and \mathbf{K}_{PCR6} are different specially $K_{23} = K_{32} = (0.5, 0.4, 0.1)$ with respect to $K_{23}^{\text{PCR6}} = K_{32}^{\text{PCR6}} = (0.18, 0.58, 0.24)$. Based on \mathbf{D}_{BI}^θ matrix (24) it is obvious that no criteria strongly agree in this example so that no judicious MCDM simplification is recommended according to BF-ICrA.

B. Example 2 (MCDM simplification)

Here we consider a more interesting example showing how an MCDM simplification is possible. We consider a 6×5 MCDM problem with the following score matrix.

$$\mathbf{S} = [S_{ij}] = \begin{bmatrix} 7.5914 & 18.1828 & 18.3221 & 95.6739 & 4.5674 \\ 8.7753 & 20.5506 & 20.8240 & 48.0229 & -0.1977 \\ -1.3492 & 0.3017 & 0.7804 & 79.8283 & 2.9828 \\ 8.8739 & 20.7478 & 21.2302 & 13.3305 & -3.6669 \\ 5.2207 & 13.4413 & 13.5201 & 41.5979 & -0.8402 \\ -1.7320 & -0.4639 & 0.0213 & 91.4893 & 4.1489 \end{bmatrix}$$

It is not very obvious to identify the closeness of these criteria (if any) to know if there is some underlying relationship between them. For the analysis, we apply the BF-ICrA approach proposed in this work. After applying all derivations (similarly to those presented in Example 1), we finally get the following \mathbf{D}_{BI}^θ distance matrix from each component of \mathbf{K}_{PCR6} to the total agreement state $m_T = [m(\theta), m(\bar{\theta}), m(\theta \cup \bar{\theta})] = [1, 0, 0]$

$$\mathbf{D}_{BI}^\theta = \begin{bmatrix} 0.0239 & \mathbf{0.0239} & \mathbf{0.0250} & 0.7512 & 0.7512 \\ \mathbf{0.0239} & 0.0239 & \mathbf{0.0250} & 0.7512 & 0.7512 \\ \mathbf{0.0250} & \mathbf{0.0250} & 0.0262 & 0.7595 & 0.7595 \\ 0.7512 & 0.7512 & 0.7595 & 0.0568 & \mathbf{0.0568} \\ 0.7512 & 0.7512 & 0.7595 & \mathbf{0.0568} & 0.0568 \end{bmatrix}$$

From the analysis of upper off-diagonal components of \mathbf{D}_{BI}^θ (put in boldface for convenience) it is clear that criteria C_1 , C_2 and C_3 are in almost total agreement because their distance is close to zero. Also we can observe from \mathbf{D}_{BI}^θ that criteria C_4 and C_5 are also very close. So the original 6×5 MCDM problem in this example can be simplified into a 6×2 MCDM problem considering only the simplified score matrix involving only C_1 and C_4 because C_2 and C_3 behave similarly to C_1 for decision-making, and C_5 behaves similarly to C_4 . Then the simplified MCDM will have to be solved by any preferred technique.

Does the BF-ICrA make sense in this example? The answer is positive because it suffices to remark that the columns of the score matrix are not totally independent because $C_2(A_i) = 2 \cdot C_1(A_i) + 3$, $C_3(A_i) = C_2(A_i) + \epsilon$ (ϵ being a small contamination noise), and $C_5(A_i) = 0.1 \cdot C_4(A_i) - 5$. Hence the decision based either on C_1 , C_2 or C_3 will be very close, as well as the decision based on C_4 or C_5 . Therefore the result of BF-ICrA makes sense and the expected simplification of MCDM is well obtained from BF-ICrA. If we apply AHP, which is nothing but the weighted arithmetic average and we use the normalized score matrix based on (2), or BF-TOPSIS methods to solve original MCDM (assuming equal importance of criteria), or if we apply simplified MCDM based on BF-ICrA, we will get same preference order $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_6 \succ A_3$. So, the best decision to make is A_1 in this example.

VII. CONCLUSION

In this paper we have proposed a new method called BF-ICrA to simplify (when it is possible) Multi-Criteria Decision-Making problems based on inter-criteria analysis and belief functions. This method is in the spirit of Atanassov's method but proposes a better construction of Inter-Criteria Matrix that fully exploits all information of the score matrix, and the closeness measure of agreement between criteria based on belief interval distance. This BF-ICrA approach for simplifying MCDM could deal also with imprecise or missing score values using the technique presented in [8]. An application of BF-ICrA for GPS surveying problem is presented in [38], and applications of BF-ICrA for simplifying and solving real MCDM problems for the prevention of natural risks in mountains will be the object of forthcoming investigations.

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