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Ability of a Robot to Travel Through its free Work Space in an Environment with Obstacles

Philippe Wenger, Patrick Chedmail

Laboratoire d'Automatique de Nantes

Ecole Nationale Supérieure de Mécanique de Nantes, France

Abstract

This article presents a new geometric analysis of the free work space of a robot among obstacles. The free work space (FW) is defined as the set of positions and orientations that the robot's end effector can reach, according to the joint limits and the various obstacles lying in the environment. The aim is to give global descriptions of the robot's ability to move in the operational space (which coincides with Cartesian space when only position coordinates are specified).

The main contribution of this work is the characterization of the effects of obstacles on the work space geometry, as well as on its topology. The ability of a robot to move freely in its work space (called the "moveability") is difficult to describe and needs stringent formalizations. The concept of moveability is introduced through various properties and their corresponding necessary and sufficient conditions. Using a Constructive Solid Geometry (CSG) Computer-Aided Design (CAD) description of robots and obstacles and an octree model of the FW, these properties permit characterization of selected moveability areas in the FW, where, for instance, any n points can be linked together or where any continuous trajectory can be achieved without changing configuration. This new global description is of great interest for the user of CAD systems when designing robotic cells.

1. Introduction

Automatic Design or Computer Aided Design of robotic cells is actually an important challenge in industry and involves several difficult geometric problems:

- the choice of a robot; the difficulty is to find the morphology (type and number of joints, length of

links, . . .) that is best suited to the family of tasks to be achieved.

- the geometric layout of the cell; one has to position the robot(s) and the other components of the cell in such a way that the robot is able to work conveniently in the environment (accessibility of the specified work areas, as well as mobility in them, should be ensured).
- Collision-free paths planning; the problem is to find a continuous path among obstacles between one specified location to another.

This third problem has drawn the interest of many authors, and its theoretical solution is now well known (Schwartz and Sharir 1982; Brady et al. 1982; Canny and Reif 1987). The techniques are various: for instance, the local approaches with potential methods (Khatib 1985; Koditschek 1987) or the global ones based on the configuration space analysis and using cell decomposition (Faverjon 1984; Brooks 1983a,b; Lozano-Pérez 1986) or retraction techniques such as stratified sets (Canny 1987) and Voronoi diagrams (O'Dunlaing et al. 1984; Canny and Donald 1988). These studies have already given rise to operational systems (Lozano-Pérez et al. 1987).

On the other hand, the first two problems have not been so extensively studied. Softwares of current robotics computer-aided design (CAD) systems (such as ROBCAD from Teknomatix, MACAUTO from McDonnell-Douglas, CATIA from Dassault System, or ROBOT PLUS from Computervision) all require the action of a human operator who selects the right solution, validates the choices, and modifies the parameters if necessary (Bernard 1984; Dombre et al. 1986; Deligneres 1987). This article deals with these first two problems. For both of them, characterization and analysis of the free work space are well-suited aids. The free work space (FW) of a robot among obstacles

is the space of positions and orientations of the end effector according to the joint limits and the various obstacles. Concerning the first problem, criteria such as volume, connectedness, compactness of the work space, or dexterity in it are very convenient for the optimization of the robot's morphology, as they give a good evaluation of the geometric performances of the manipulator (Vijaykumar et al. 1986; Lenarcic et al. 1988). A possible solution to the second problem is the characterization of selected regions of "moveability" in the work space. The aim is then to ensure the feasibility of tasks defined as areas to be freely reached (Deligneres 1987) and/or to be "travelled through" in a certain sense by the robot (e.g., areas of continuous welding or painting). In Figure 1, the piece cannot be placed anywhere in the free work space; it must be enclosed in a region reachable from configurations $q_2 > 0$ or $q_2 < 0$ separately.

The goal of our study is to give a global description of the obstacles in the operational space (space of positions and orientations of the end effector; this space coincides with the Cartesian space when only position coordinates are specified). This is achieved through geometric and topologic analysis of the FW. This new description is of great interest for the designer of robotic cells.

A lot of work has already dealt with work space analysis. Typical geometric characterizations of a work space are:

1. Its well-known projections in the Cartesian space: "reachable work space" (projection of the whole

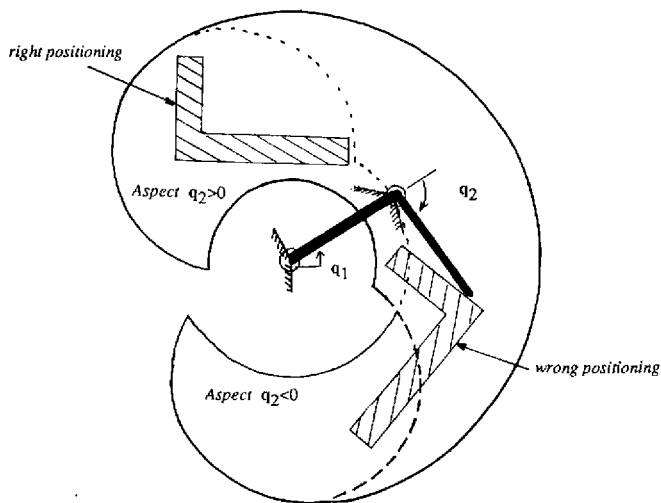


Fig. 1. The task to be achieved is continuous welding of a piece. The piece must be carefully placed in the work space to avoid configuration changing during the operation. The end effector must remain in the same aspect.

work space) and "dextrous work space" (projection of the work space part where every orientation of the end effector is possible) (Kumar and Waldron 1981).

2. The analysis of holes and voids in the work space (Gupta and Roth 1982).
3. The approach angles and lengths (Hansen et al. 1983).
4. Work area analysis (Yang and Chiueh 1986).
5. Characterization of the so-called "aspects" (set of positions and/or orientations reachable under given configurations) (Borrel 1986).
6. "Well-connectedness" (ability to move between any two points in the work space without changing configuration) (Paden and Sastry 1988).

However, none of these take into account the effects of obstacles. The proximity of obstacles not only modifies the shape of the work space, but also reduces the moveability of the robot within it, as has already been shown in Chedmail and Wenger (1987; 1988). Figure 2A shows a very simple example where the FW is connected but does not allow the robot to travel through it. In Figure 2B, the mobility between any two points within the FW is ensured, but not necessarily along a given continuous trajectory.

In this article, we propose five stringent characterizations of the moveability in the FW for a robot among obstacles. We attach five necessary and sufficient conditions to these characterizations. These conditions are tested using a constructive solid geometry (CSG) CAD technique (Martin et al. 1985) and an octree model of the FW. Finally, an algorithm is described to check the various properties, and results are presented.

2. Definitions

We give in this section some definitions before proceeding further.

Let a robot — called $ROBOT(q)$ — be defined by its n degrees of freedom:

$ROBOT(q)$, enclosed in \mathbb{R}^3 , is defined as a set of n solids, as in Wenger (1985) and Yu and Khalil (1986).

Let an environment with obstacles be defined by $Ob =$ set of the physical obstacles of the environment, enclosed in \mathbb{R}^3

Let $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be the geometric operator of the robot (Khalil and Kleinfinger 1986), where m is the number of operational coordinates of the end effector of the robot, $m \leq 6$.

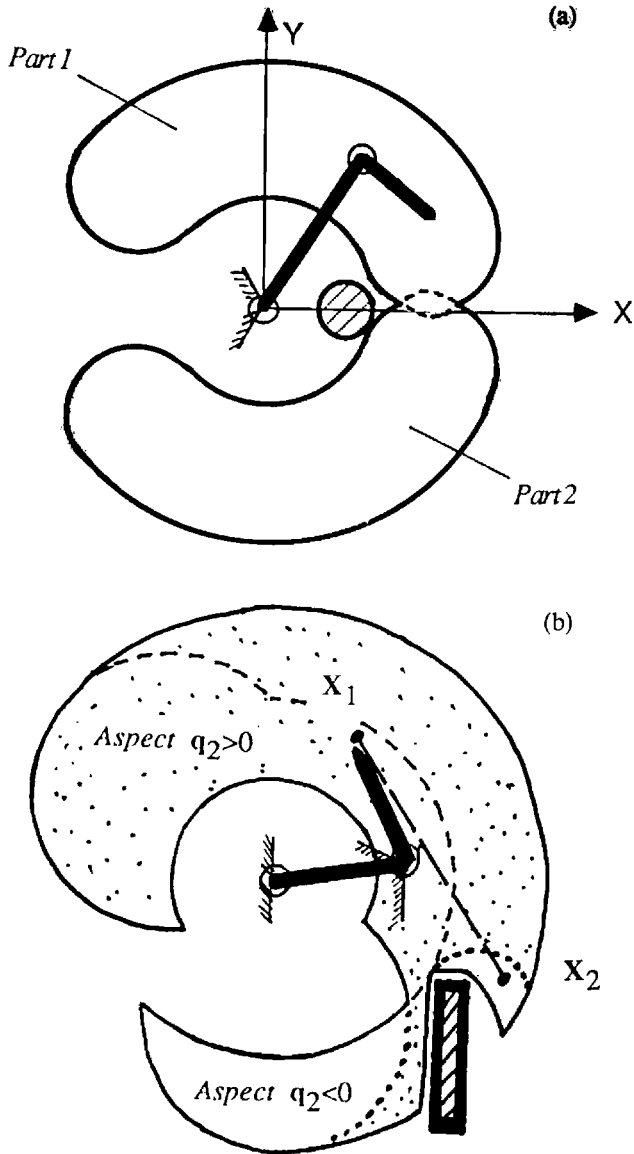


Fig. 2A, A two-DOF planar linkage with one circular obstacle and $-120^\circ \leq q_1 \leq 120^\circ, -180^\circ \leq q_2 \leq 180^\circ$. The free work space is connected; however, the robot cannot travel through it but can only move separately within part 1 or part 2B, A two-DOF planar linkage with one rectangular obstacle and $-90^\circ \leq q_1 \leq 90^\circ, -100^\circ \leq q_2 \leq 120^\circ$. The robot can join any two points in its free work space but not along any trajectory (the given continuous trajectory is unfeasible; point X_1 can be reached only in aspect 1 because of the joint limits, and point X_2 can be reached only in aspect 2 because of the obstacle).

Let $\mathcal{D} = \{\mathbf{q} \in \mathbb{R}^n / \forall i, q_{i\max} \leq q_i \leq q_{i\min}\}$ be the configuration space, according to the joint limits.

Let Qf be the configuration collision-free space of $ROBOT(\mathbf{q})$:

$$Qf = \{\mathbf{q} \in \mathcal{D} / ROBOT(\mathbf{q}) \cap Ob = \emptyset\}$$

Let Wf be the FW of $ROBOT(\mathbf{q})$. It is the image of Qf under the geometric operator:

$$Wf = f(Qf)$$

The *free work space* is the space of positions and orientations of the end effector according to the joint limits and the various obstacles in the environment. A point \mathbf{X} in Wf is a vector of m operational coordinates.

We suppose that there are N connected components of Qf . We shall denote them as:

$$\{Qf_i; i \in I\} \text{ where } I \text{ is the set } \{1, \dots, N\}.$$

The following relations hold:

$$Qf = \cup_{i \in I} (Qf_i)$$

and $\forall i \neq j, i \in I, j \in I; Qf_i \cap Qf_j = \emptyset$; and $Qf_i \cup Qf_j$ is not connected.

Then:

$\forall i \in I; Wf_i = f(Qf_i)$ is the image of Qf_i under the geometric operator.

The following relation holds:

$$Wf = \cup_{i \in I} Wf_i$$

Define: The aspects are defined as in Borrel (1986); an aspect is a subspace A enclosed in Qf such that:

1. A is connected,
2. $\forall \mathbf{q} \in A$; the determinant of any $m \times m$ matrix extracted from the Jacobian matrix \mathbf{J} is not equal to zero, except if this minor is equal to zero everywhere in \mathcal{D} . The components of \mathbf{J} are:

$$J_{ij} = \frac{\partial x_i}{\partial q_j}$$

where q_j is the j th component of \mathbf{q} , and x_i is the i th operational coordinate of $\mathbf{X} = f(\mathbf{q})$ ($1 \leq i \leq m$ and $1 \leq j \leq n$).

The number of aspects is finite. $\{A_j\}_{j \in J}$ is a partition of \mathcal{D} .

3. Characterization of the Robot's Ability to Move Through Wf

We expose in this section the five characterizations of the moveability in the FW of a robot, and their corresponding necessary and sufficient conditions.

3.1. First Characterization and Corresponding Necessary and Sufficient Condition

The FW can be travelled through by the robot in the sense of P_1 if, by definition, any two points in Wf can

be joined by the end effector. This means that for any two points in the FW, a connected component of the configuration collision-free space exists such that its image under f contains both of these points:

$$(P_1) \quad \forall \mathbf{X}_1, \mathbf{X}_2 \in Wf, \exists i \in I, \mathbf{X}_1 \in Wf_i \text{ and } \mathbf{X}_2 \in Wf_i$$

Note: This first property can be expressed in a more succinct way:

$$(P_1) \Leftrightarrow Wf^2 = \cup_{i \in I} Wf_i^2$$

Wf satisfies P_1 if and only if Wf is connected, and there exists a family $\{I_1, I_2, \dots, I_p\} = \{I_k / k \in K\}$ of subsets of I such that:

- i. $\forall k \in K: Wf = \cup_{j \in I_k} Wf_j$
- ii. $Wf = \cup_{k \in K} (\cap_{j \in I_k} Wf_j)$

(See the demonstration in Appendix A.)

Example: If Qf is composed of three connected components, the following relations hold:

$$\begin{aligned} I &= \{1, 2, 3\} \\ Qf &= Qf_1 \cup Qf_2 \cup Qf_3 \\ Wf &= Wf_1 \cup Wf_2 \cup Wf_3 \end{aligned}$$

Let $\{I_k / k \in K\} = \{I_1, I_2, I_3\}$ with:

$$I_1 = \{1, 2\}, I_2 = \{2, 3\}, I_3 = \{1, 3\}.$$

Assume:

$$Wf = Wf_1 \cup Wf_2, Wf = Wf_2 \cup Wf_3, Wf = Wf_1 \cup Wf_3$$

and

$$Wf = (Wf_1 \cap Wf_2) \cup (Wf_2 \cap Wf_3) \cup (Wf_1 \cap Wf_3)$$

Then Wf satisfies P_1 (Fig. 3).

COROLLARY 1 $Wfp \subset Wf$ satisfies the generalized property $P_1(Wfp)$ where:

$$(P_1(Wfp)) \quad \forall (\mathbf{X}_1, \mathbf{X}_2) \in Wfp^2, \exists i \in I, \mathbf{X}_1 \in Wf_i \cap Wfp \text{ and } \mathbf{X}_2 \in Wf_i \cap Wfp$$

if and only if there exists a family $\{I_1, I_2, \dots, I_p\} = \{I_k / k \in K\}$ of subsets of I such that:

- i. $Wfp \subset \cap_{k \in K} (\cup_{j \in I_k} Wf_j)$
- ii. $Wfp \subset \cup_{k \in K} (\cap_{j \in I_k} Wf_j)$

which can be written in the following more succinct way:

$$Wfp \subset (\cap_{k \in K} (\cup_{j \in I_k} Wf_j)) \cap (\cup_{k \in K} (\cap_{j \in I_k} Wf_j))$$

Note: Because any subset of set satisfying $P_1(Wfp)$ also satisfies $P_1(Wfp)$, such subsets are not necessarily connected.

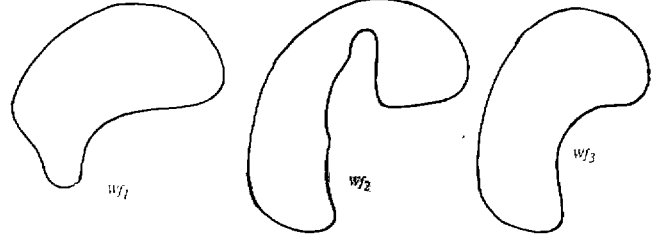
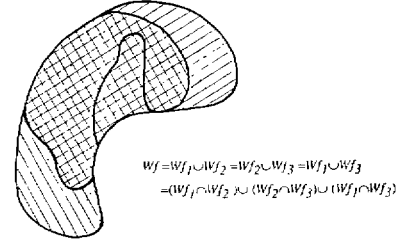


Fig. 3. A case where the free work space can be traveled through in the sense of P_1 .

Definition 1

The maximal parts Wfp of Wf satisfying $P_1(Wfp)$ are defined as follows:

There exists a family $\{I_1, I_2, \dots, I_p\} = \{I_k / k \in K\}$ of subsets of I such that:

$$Wfp = (\cap_{k \in K} (\cup_{j \in I_k} Wf_j)) \cap (\cup_{k \in K} (\cap_{j \in I_k} Wf_j)) \text{ is nonempty.}$$

Note 1: Any part satisfying $P_1(Wfp)$ is enclosed in such a maximal part. The converse being true, it is interesting to find the maximal parts satisfying P_1 (see Fig. 4A; in the case where $\text{Card}(I) = 3$, there are four maximal parts satisfying $P_1(Wfp)$).

Note 2: The properties P_1 and $P_1(Wfp)$ do not take into account initial and final configurations. Moreover, every trajectory in the FW between \mathbf{X}_1 and \mathbf{X}_2 is not necessarily achievable. That is why we will complete this definition in the following property; then, in the next two properties, we will successively take into account the configuration at point \mathbf{X}_1 (or \mathbf{X}_2) and then at points \mathbf{X}_1 and \mathbf{X}_2 ; finally, we will propose a property of mobility along any continuous trajectory through the FW.

3.2. Second Definition (P_2) and Corresponding Necessary and Sufficient Condition

The FW can be traveled through by the robot in the sense of P_2 if, by definition, every discrete trajectory T_d in Wf is achievable, which means:

$$(P_2) \quad \forall T_d \text{ discrete trajectory in } Wf, \exists i \in I \text{ such that } T_d \subset Wf_i$$

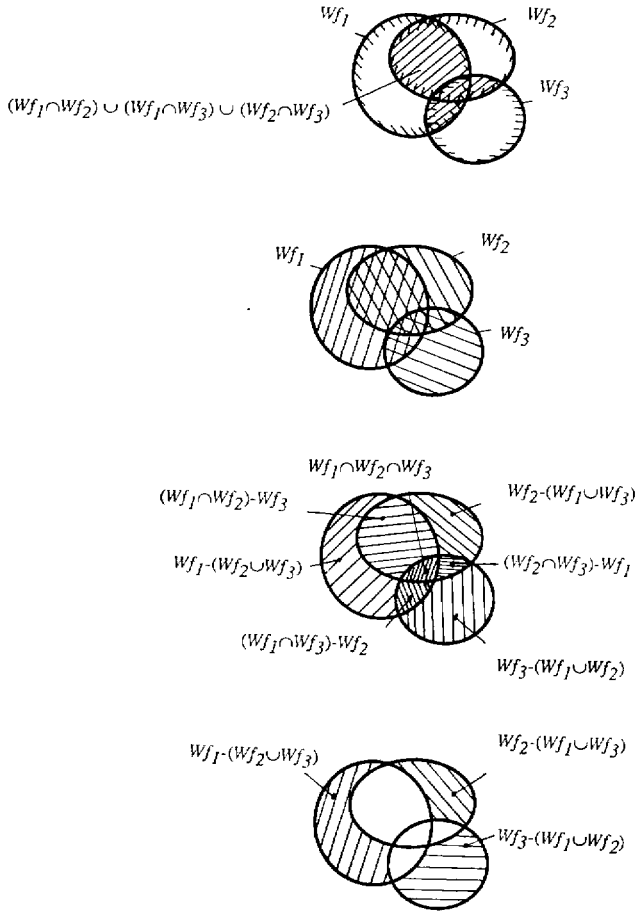


Fig. 4. The different maximal parts of the free work space satisfying (A) $P_1(Wfp)$, $P_2(Wfp)$ (B), $P_3(Wfp)$ (C) and $P_4(Wfp)$ (D) when the configuration collision-free space has three connected components.

Note: A discrete trajectory is an arranged sequence of points.

Wf satisfies P_2 if and only if:

$$\exists i \in I, Wf_i = Wf$$

(See the demonstration in Appendix B.)

COROLLARY 2 $Wfp \subset Wf$ satisfies the generalized property $P_2(Wfp)$, where:

$(P_2(Wfp))$

$\forall T_d$ discrete trajectory in Wfp , $\exists i \in I$ such that $T_d \subset Wf_i \cap Wfp$

if and only if:

$$\exists i \in I, Wfp \subset Wf_i$$

Definition 2

The maximal parts Wfp satisfying $P_2(Wfp)$ are defined as follows:

$$\exists i \in I, Wfp = Wf_i$$

(see Fig. 4B; in the case where $\text{Card}(I) = 3$, there are three maximal parts satisfying $P_2(Wfp)$).

3.3. Third Characterization and Corresponding Necessary and Sufficient Condition

The FW can be traveled through by the robot in the sense of P_3 if, by definition, any two points in Wf can be joined by the end effector, whatever the initial or (exclusive or) final configuration:

$(P_3) \forall (q, X) \in Qf \times Wf, \exists i \in I, q \in Qf_i$ and $X \in Wf_i$

Note: This third property can be expressed in a more succinct way:

$$(P_3) \Leftrightarrow Wf \times Qf = \bigcup_{i \in I} (Wf_i \times Qf_i)$$

Wf satisfies P_3 if and only if:

$$\forall i \in I, Wf_i = Wf$$

(See the demonstration in Appendix C.)

COROLLARY 3 Let $Wfp \subset Wf$ and define:

$$Qfp = f^{-1}(Wfp)$$

then Wfp satisfies the generalized property $P_3(Wfp)$, where:

$(P_3(Wfp)) \forall (q, X) \in Qfp \times Wfp, \exists i \in I, q \in Qfp \cap Qf_i$
and $X \in Wfp \cap Wf_i$

if and only if:

$$Wfp \subset (\bigcap_{i \in I'} Wf_i) - (\bigcup_{k \notin I'} Wf_k)$$

for some sets I' of indices in I . (See the demonstration in Appendix D.)

Definition 3

The maximal parts Wfp satisfying $P_3(Wfp)$ are defined as follows:

$$Wfp = (\bigcap_{i \in I'} Wf_i) - (\bigcup_{k \notin I'} Wf_k)$$

for any subsets I' of I such that Wfp is nonempty. (See Fig. 4C; when $\text{Card}(I) = 3$, there are seven maximal parts satisfying $P_3(Wfp)$.)

3.4. Fourth Characterization and Corresponding Necessary and Sufficient Condition

The FW can be traveled through by the robot in the sense of P_4 if, by definition, any two points in Wf can be joined by the end effector, whatever the initial and final configurations:

$$(P_4) \quad \forall (q_1, q_2) \in Qf^2, \exists i \in I, q_1 \in Qf_i \text{ and } q_2 \in Qf_i$$

Wf satisfies P_4 if and only if Qf is connected (the proof is obvious).

COROLLARY 4 Let Qfp be defined as in previous corollary 3. $Wfp \subset Wf$ satisfies the generalized property $P_4(Wfp)$ where:

$$(P_4(Wfp)) \quad \forall (q_1, q_2) \in Qfp^2, \exists i \in I, q_1 \in Qfp \cap Qf_i \text{ and } q_2 \in Qfp \cap Qf_i$$

if and only if:

$$\exists i \in I / Wfp \subset Wf_i - (\cup_{k \neq i} Wf_k)$$

(See the demonstration in Appendix E.)

Definition 4

The maximal parts Wfp satisfying $P_4(Wfp)$ are defined as follows:

$$\exists i \in I / Wfp = Wf_i - (\cup_{k \neq i} Wf_k)$$

(See Fig. 4D; in the case where $\text{Card}(I) = 3$, there are three maximal parts satisfying $P_4(Wfp)$.)

Note: Properties P_4 up to P_1 characterize four increasing levels describing point-to-point motions in the FW of a robot:

$$P_4 \Rightarrow P_3 \Rightarrow P_2 \Rightarrow P_1.$$

3.5. Fifth Characterization and Corresponding Necessary and Sufficient Condition

This fifth property will concern only nonredundant robots, for which the geometric operator is bijective on the aspects.

The FW can be traveled through by a nonredundant robot in the sense of P_5 if, by definition, any two points in Wf can be joined with any continuous trajectory T_c without changing configuration and regardless of the initial or final configuration.

For any j in J (set describing the aspects A_j), define the single partition $\{A_{jk}\}_{k \in I_j}$ of the aspects A_j in connected components A_{jk} as:

$$A_j \cap Qf = \cup_{k \in I_j} A_{jk}$$

We note, for $j \in J$ and $k \in I_j$:

$$WA_{jk} = f(A_{jk})$$

is the image under f of the connected component k of the aspect j in the operational space.

Similarly, we note, for $j \in J$:

$$WA_j = f(A_j)$$

is the image of the aspect j in the operational space.

According to the results obtained by Borrel (1986) and generalizing them in the case of environments with obstacles, this fifth property can be expressed as:

$$(P_5) \quad \forall (q, X) \in Qf \times Wf, \exists j \in J, \exists k \in I_j / q \in A_{jk} \text{ and } X \in WA_{jk}$$

Wf satisfies property P_5 if and only if:

$$\forall j \in J, \forall k \in I_j WA_{jk} = Wf$$

The proof is quite analogous to that of theorem 3.

COROLLARY 5 Let Qfp be defined as in previous corollary 3. Then $Wfp \subset Wf$ satisfies the property $P_5(Wfp)$, where:

$$(P_5(Wfp)) \quad \forall (q, X) \in Qfp \times Wfp, \exists j \in J, \exists k \in I_j / q \in Qfp \cap A_{jk} \text{ and } X \in Wfp \cap WA_{jk}$$

if and only if:

$$Wfp \subset WA_{ji} - (\cup_{j \in J', k \in I'(j)} WA_{jk}) - (\cup_{j \in J', i \notin I'(j)} WA_{ji'}) - (\cup_{m \notin J'} WA_m)$$

where J' is a set of indices in J , and, for any j in J' , $I'(j)$ is a set of indices in $I(j)$.

The proof is quite analogous to that of corollary 3.

Definition 5

The maximal parts Wfp satisfying $P_5(Wfp)$ are defined as the connected components of:

$$WA_{ji} - (\cup_{j \in J', k \in I'(j)} WA_{jk}) - (\cup_{j \in J', i \notin I'(j)} WA_{ji'}) - (\cap_{m \notin J'} WA_m)$$

4. Algorithmic Analysis of the Robot's Ability to Travel Through Wf

Let Wfp be enclosed in (or equal to) Wf . There exists an infinity of parts Wfp that verify $P_1(Wfp)$ or $P_2(Wfp)$ (any subset of any Wf_i , for instance). The following algorithm tests the properties P_1 , P_2 , P_3 , and P_4 for Wf and leads to all the maximal parts Wfp satisfying $P_1(Wfp)$ or $P_2(Wfp)$.

The robot and its environment are modeled using a CSG CAD technique. It makes it possible to perform collision detection between the robot and its environment (Yu and Khalil 1986; Deligneres 1987).

1. Determination of I , $\{Qf_i\}_{i \in I}$, $\{Wf_i\}_{i \in I}$, Wf , Qf : in the case where $n \leq 3$ and $m \leq 3$, we use an octree (octal tree) description of these spaces in a similar way as in Faverjon (1986). If $n \geq 3$, the wrist may be modeled by a circumscribing sphere. The FW and the configuration collision-free space are obtained by sweeping the configuration space \mathcal{D} according to the joint limits. Collision detection is performed using an efficient algorithm described in Yu (1987). Finally, analysis of the connected components of Qf , as described in Samet (1979) and Chedmail and Wenger (1988) leads to I and to the octree definition of Qf_i and Wf_i .
 2. Verification of the property P_4 : If $\text{Card}(I) = 1$, then Qf is connected; therefore P_4 (and so P_1 , P_2 , P_3) is true for Wf ; end of the algorithm.
else
 3. Verification of the property P_3 : If $\forall i \in I$, $Wf_i = Wf$, then P_3 is true for Wf ; end of the algorithm.
else
 4. Verification of the property P_2 and determination of the maximal Wfp satisfying $P_2(Wfp)$: If $\exists i \in I$, $Wf_i = Wf$, the P_2 is true for Wf ; the maximal parts satisfying $P_2(Wfp)$ are the Wf_i ; end of the algorithm.
else
 5. Verification of the property P_1 and determination of the maximal Wfp satisfying only $P_1(Wfp)$:
 - a. Enumerate all possible families of nonempty subsets of I . For each such family, say $\{I_k / k \in K\}$, determine
$$B = (\bigcap_{k \in K} (\bigcup_{j \in I_k} Wf_j)) \cap (\bigcup_{k \in K} (\bigcap_{j \in I_k} Wf_j))$$

Note: the determination of B is performed using the octree model with very simple and fast boolean operations.
 - b. If B is nonempty, then it satisfies $P_1(Wfp)$; go to 5a.
- End of the algorithm.

We can make the following remarks concerning the part 5 of the algorithm:

Note 1: It is possible to considerably reduce the number of cases as a lot of them do not have to be studied:

1. $\text{Card}(I_k) = 1$. Then the element i of I_k satisfies $Wf_i = Wfp$, already seen

2. There exists j_0 such that $\forall k \in K$, $j_0 \in I_k$. Then $Wfp \subset Wf_{j_0}$ and Wfp is not maximal. In particular, if $\text{Card}(K) = 1$: there exists i in I such that $Wfp \subset Wf_i$, and Wfp is not maximal.
3. A great number of sequences $\{I_k\}_{k \in K}$ do not have to be tested as they generate nonmaximal parts. For instance, in the case where $N = 4$, the sequence $\{(1, 2), (2, 3), (1, 3, 4)\}$ does not have to be studied, because the subspace generated by it is enclosed in the maximal Wfp generated by the sequence $\{(1, 2), (2, 3), (1, 3)\}$.

Note 2: it is useless to study the case where, in a sequence $\{I_k\}_{k \in K}$, there exists k and k' such that $I_k \subset I_{k'}$. Indeed, let $A_k = B$ for any $k \in K$; then

$$B = \bigcup_{j \in K} (\bigcap_{i \in I_j} Wf_i) = \bigcup_{j \in K - \{k'\}} (\bigcap_{i \in I_j} Wf_i)$$

and so Wfp generated by

$$\{I_k\}_{k \in K}$$

is equal to Wfp , generated by

$$\{I_k\}_{k \in K - \{k'\}}$$

and do not have to be tested. For example, in the case where $N = 3$, it is useless to test the sequence:

$$\{(1, 2), (2, 3), (1, 3), (1, 2, 3)\}.$$

Therefore we verify that for $N = 2$, out of the cases where $Wf_i = Wf$, no subset of Wf exists that satisfies $P_1(Wfp)$. For $N = 3$, then $\mathcal{N} = 127$. Out of the cases where $Wf_i = Wf$, one single set of subsets of I may lead to parts of Wf that satisfy the condition $P_1(Wfp)$: $\{(1, 2), (1, 3), (2, 3)\}$ where $\text{card}(K) = 3$ (see Fig. 4A).

Similarly, for $N = 4$, then $\mathcal{N} = 32767$. Out of the cases where $Wf_i = Wf$, we verify that only 27 sets of subsets of I may lead to parts of Wf that satisfy the condition $P_1(Wfp)$.

5. Results

We have developed a package that enables us to study any type of robot in any environment, by only dealing with three-dimensional spaces. In this section, we present some examples, using planar robots for graphic convenience, that illustrate the algorithmic analysis of the moveability in the FW of a robot among obstacles.

5.1. Example 1

The FW is not connected (Fig. 5). In this example, the algorithm detects two connected components in the Cartesian space and concludes that the FW cannot be traveled through in the sense of P_1 (it cannot enter the box). Two maximal parts Wf_1 and Wf_2 are detected as satisfying $P_1(Wfp)$ and $P_2(Wfp)$.

5.2. Example 2

The FW is connected, and the configuration free space is not connected (Fig. 6). In this example, the algorithm detects two connected components Qf_1 and Qf_2 in the configuration space. It verifies that FW cannot be traveled through in the sense of P_1 (the robot cannot move completely around the spherical obstacle), but the algorithm exhibits two parts Wf_1 and Wf_2 as satisfying $P_1(Wfp)$ and $P_2(Wfp)$.

5.3. Example 3

The FW is connected, and the configuration free space is not connected (Fig. 7). In this case, the algorithm

detects two connected components Qf_1 and Qf_2 in the configuration free space and exhibits two parts Wf_1 and Wf_2 such that $Wf_2 \subset Wf$ and $Wf_1 = Wf$. So the algorithm concludes that the FW can be traveled through in the sense of $P_2(Wfp)$ (and $P_1(Wfp)$).

We verify that Wf_2 can be traveled through in the sense of $P_3(Wfp)$ and the subspace $Wf - Wf_2$ in the sense of $P_4(Wfp)$.

5.4. Example 4

The FW and the configuration collision-free space are both connected (Fig. 8). In this example, the algorithm finds only one single connected component in the sense of P_4 (not in the sense of P_3 , as we could verify that the configuration space is composed of two aspects, whose images in the operational space are different).

5.6. Comments

As it has been shown in these examples, a simple connectivity analysis of the configuration collision-free space is not sufficient to characterize the geometric

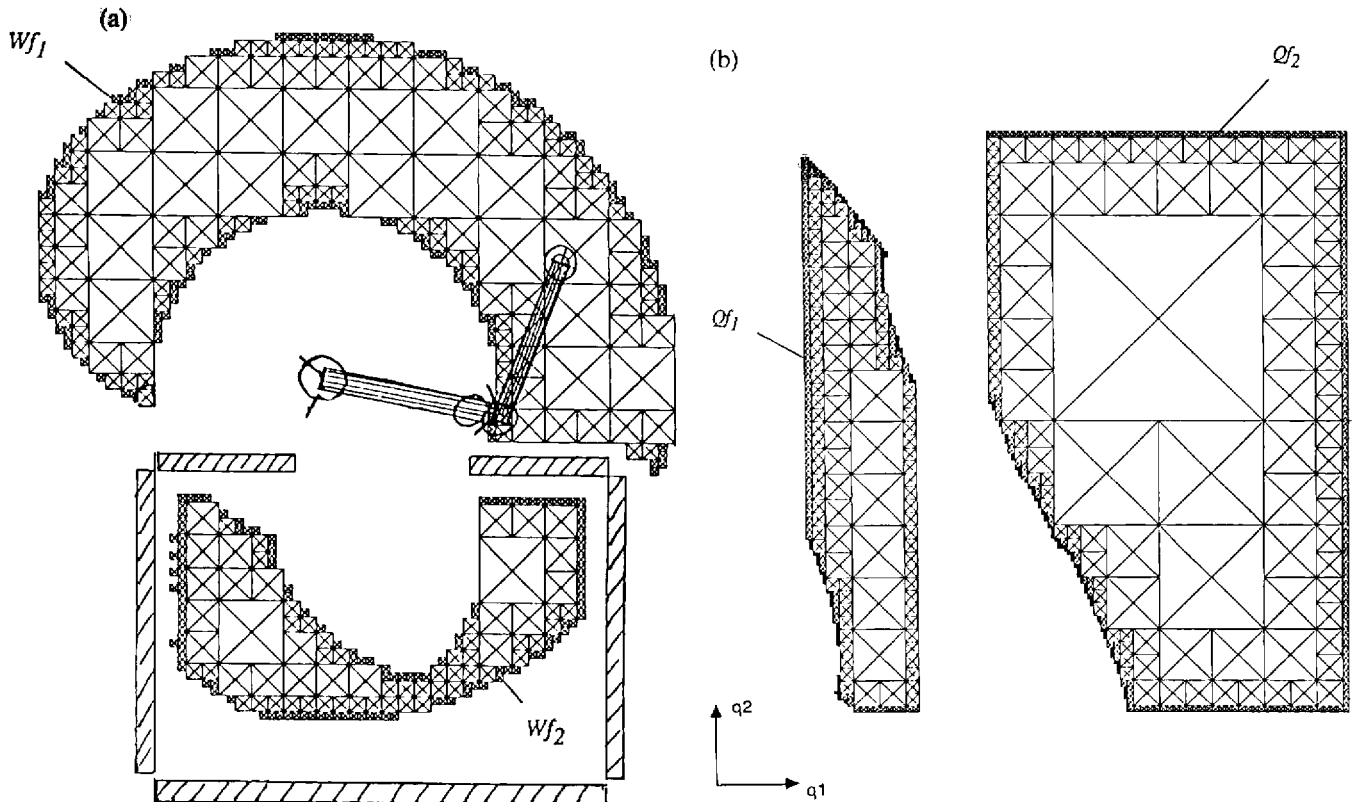


Fig. 5. The free work space is not connected (A, Cartesian space); the configuration collision-free space is composed of two connected components Qf_1 and Qf_2 (B, configuration

space), and their image in the operational space (Wf_1 and Wf_2) satisfies property $P_1(Wfp)$.

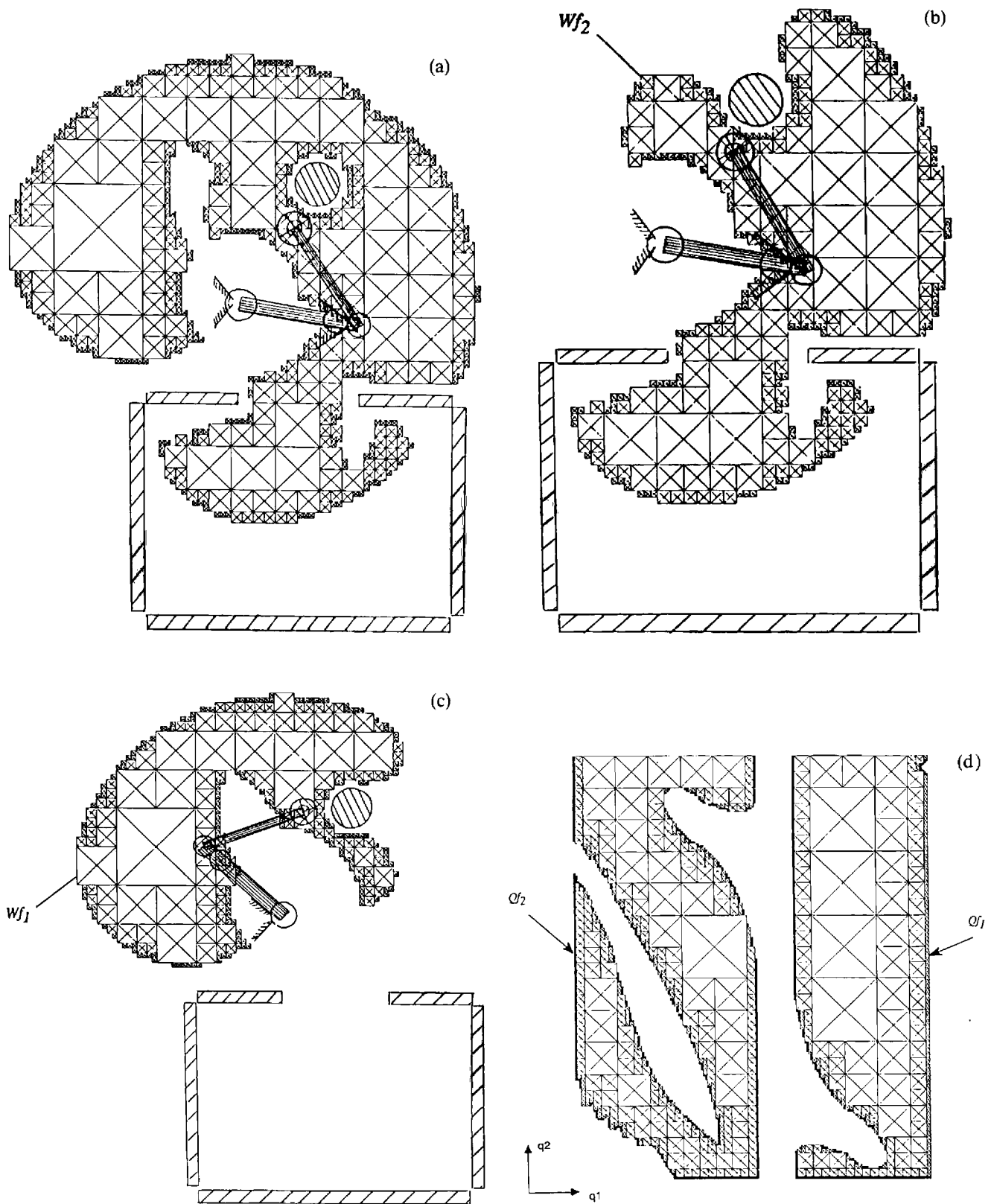


Fig. 6. The free work space is connected (A, Cartesian space); the configuration collision-free space is composed of two connected components Q_1 and Q_2 (D, configuration space)

and their image in the operation space (Wf_1 and Wf_2) satisfies property $P_f(Wfp)$ (B and C).

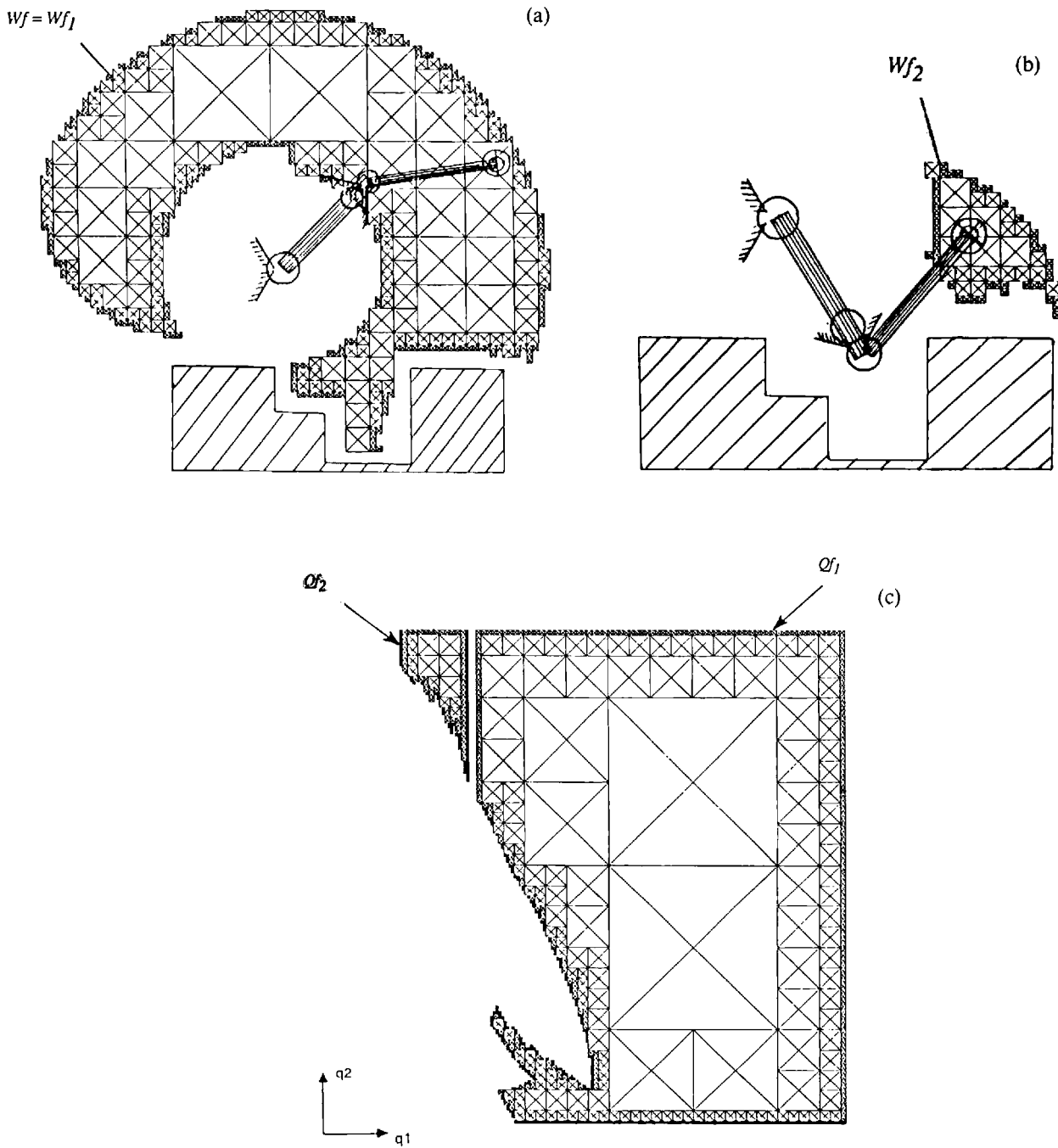


Fig. 7. The robot is able to travel through its free work space in the sense of P_2 (A, Cartesian space) and through the subspace Wf_2 in the sense of P_3 (Wfp) (B). The configuration

collision-free space is composed of two connected components (C, configuration space).

properties of the free work space. The configuration collision-free space may be nonconnected, whereas the FW can be traveled through (in the sense of P_1 , P_2 , or P_3 ; see example 3). On the other hand, the configuration collision-free space may be connected, whereas

the FW cannot be traveled through (in the sense of P_5 ; see example 4).

The analysis of this paradox and the characterizations of the moveability of the robot in its FW are the main contributions of the present work.

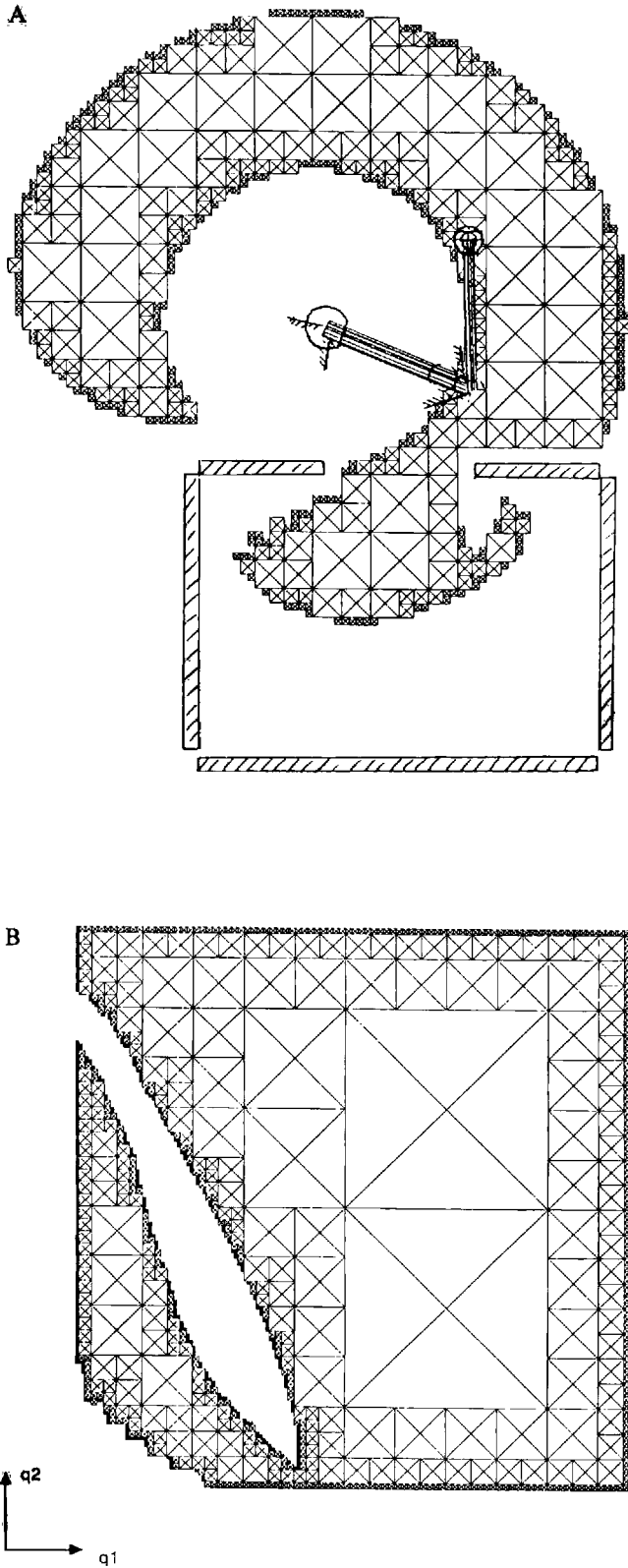


Fig. 8. The robot is able to travel through its free work space in the sense of P_4 (A, Cartesian space) as the configuration free space is connected (B, configuration space).

6. Conclusion

This article presents a classification, in the operational space, of regions where the robot is able to achieve motions according to its joint limits and the obstacles lying in the environment. Five characterizations of the moveability in the work space have been explained, as well as the five corresponding necessary and sufficient conditions. Based on these conditions, an algorithm has been developed that tests whether Wf satisfies P_1 or P_2 or P_3 or P_4 and finds all the parts of Wf satisfying $P_1(Wfp)$ or $P_2(Wfp)$. Any robot in any environment can be studied, provided that the dimensions of the spaces are not greater than three (our algorithm is quite general and could theoretically deal with k -dimensional trees, with $k > 3$, but this would lead to prohibitive computational time; when $k \leq 3$, the number of tests is reasonable and compatible with the capacity of current minicomputers).

This work is a new contribution to the design of robotic cells. The geometric performances of the robot among the various obstacles of the cell can be evaluated through the five characterizations of its FW. This is an interesting aid for the morphology choice of a robot. In addition, the geometric layout of the cell may be realized by determining the suitable moveability areas in the environment. Independently, we have developed a methodology that performs the automatic positioning of a robot in its environment in order to reach a given area (Chedmail and Wenger 1989). This methodology and the present study are currently integrated in a software package that defines the automatic positioning of a robot among the various obstacles.

Appendix A. Demonstration of Property P_1

Sufficient Condition

Let $X_1 \in Wf$, $X_2 \in Wf$:

$$\exists k \in K / X_1 \in \bigcap_{j \in k} Wf_j \quad (\text{from ii}),$$

and so

$$\exists j \in I_k / X_1 \in Wf_j, \quad \text{and} \quad X_2 \in Wf_j \quad (\text{from i}),$$

therefore:

$$\forall X_1, X_2 \in Wf, \exists j \in I, X_1 \in Wf_j \quad \text{and} \quad X_2 \in Wf_j,$$

That is the property P_1 .

Necessary Condition

Suppose Wf is not connected, then no trajectory exists between the different connected components of it, and so it does not satisfy the property P_1 .

So we suppose that Wf is connected and satisfies the property P_1 :

$$(P_1): \quad \forall \mathbf{X}_1, \mathbf{X}_2 \in Wf, \exists j \in I, \mathbf{X}_1 \in Wf_j \text{ and } \mathbf{X}_2 \in Wf_j.$$

Let

$$I(\mathbf{X}_1) = \{j \in I / \mathbf{X}_1 \in Wf_j\}.$$

Hence

$$Wf = \bigcup_{j \in I(\mathbf{X}_1)} Wf_j$$

and

$$\mathbf{X}_1 \in \bigcap_{j \in I(\mathbf{X}_1)} Wf_j$$

as P_1 is satisfied.

Therefore

$$\forall \mathbf{X}_1 \in Wf, \exists I(\mathbf{X}_1) \subset I$$

such that

$$Wf = \bigcup_{j \in I(\mathbf{X}_1)} Wf_j \quad (A1)$$

and

$$\mathbf{X}_1 \in \bigcap_{j \in I(\mathbf{X}_1)} Wf_j. \quad (A2)$$

Elsewhere

$$\exists K / (I_k)_{k \in K} = \{I(\mathbf{X}_1)_{\mathbf{X}_1 \in Wf}\}. \quad (A3)$$

K is obtained by sweeping the finite set of the parts of I , the cardinal of which is finite: $\text{Card}(K) \leq 2^N - 1$.

Finally

$$\forall k \in K, Wf = \bigcup_{j \in I_k} Wf_j$$

from (1) and (3), and

$$Wf \subset \left(\bigcap_{j \in I_k} Wf_j \right)$$

from (2) and (3); therefore i and ii are satisfied.

Appendix B. Demonstration of Property P_2

Sufficient Condition

It is obvious.

Necessary Condition

Suppose that: $\forall i \in I, Wf_i \subset Wf$ (strictly). Thus, for any i in I , $Df_i = Wf_i - Wf$ is nonempty.

Let $\mathbf{T}_d = \{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p\}$ be a discrete trajectory in Wf , where p is the number of connected components of the configuration collision-free space: $p = \text{Card}(I)$. This trajectory is chosen such that for any $j \leq p$, \mathbf{X}_j is in Df_j . Thus there exists a discrete trajectory in Wf such that $\forall i \in I, \mathbf{T}_d \cap Df_i \neq \emptyset$, which means:

$$\exists \mathbf{T}_d \subset Wf / \forall i \in I, \mathbf{T}_d \not\subset Wf_i.$$

Appendix C. Demonstration of Property P_3

Sufficient Condition

Let $\mathbf{X}_1 \in Wf$ and $\mathbf{q}_1 \in Qf$ such that $\mathbf{X}_1 = f(\mathbf{q}_1)$.

$$\forall i \in I, \mathbf{X}_1 \in Wf_i \text{ and } \mathbf{q}_1 \in Qf_i.$$

Similarly, let $\mathbf{X}_2 \in Wf$, then $\forall i \in I, \mathbf{X}_2 \in Wf_i$, and so P_3 is satisfied.

Necessary Condition

Suppose $\exists i \in I$ such that $Wf_i \subset Wf$ (strictly); let $\mathbf{q}_1 \in Qf_i$ and let $\mathbf{X}_2 \in Wf - Wf_i$; then P_3 is false.

Appendix D. Demonstration of Corollary 3 (Property $P_3(Wfp)$)

Sufficient Condition

Let $Wfp \subset (\bigcap_{i \in I'} Wf_i) - (\bigcup_{k \notin I'} Wf_k)$ for some I' in I . Let \mathbf{q}_1 in Qfp and let $\mathbf{X}_1 = f(\mathbf{q}_1)$.

As $\mathbf{X}_1 \notin Wf_k$ for any k not belonging to I' , there exists i_0 in I' such that $\mathbf{q}_1 \in Qfp \cap Qf_{i_0}$.

Let \mathbf{X}_2 in Wfp : $\forall i \in I', \mathbf{X}_2 \in Wfp \cap Wf_i$ and thus $\mathbf{X}_2 \in Wfp \cap Wf_{i_0}$.

Necessary Condition

Let $D(I') = (\bigcap_{i \in I'} Wf_i) - (\bigcup_{k \notin I'} Wf_k)$. The sets $D(I')$ make a partition of Wf , when considering all possible subsets I' in I . Indeed, they are clearly all disjointed, and moreover:

$$\bigcup_{I' \subset I} D(I') \supset \bigcup_{i \in I} Wf_i - \bigcup_{i \in I} (\bigcap_{k \notin I} Wf_k) = Wf.$$

Now, let Wfp in Wf , and suppose $\forall I' \subset I, Wfp \not\subset D(I')$. Then, there exists I'_1 and I'_2 in I such that $Wfp \cap D(I'_1) \neq \emptyset$ and $Wfp \cap D(I'_2) \neq \emptyset$. If $I'_1 \not\subset I'_2$, then there exists \mathbf{q}_1 in $f^{-1}(Wfp \cap D(I'_1))$ such that \mathbf{q}_1 belongs to Qf_{i_0} for i_0 in $I'_1 - I'_2$. Let \mathbf{X}_2 in $Wfp \cap D(I'_2)$; then the property $P_3(Wfp)$ is false. Similarly, if $I'_1 \subset I'_2$, there exists \mathbf{q}_1 in $f^{-1}(Wfp \cap D(I'_1))$ such that \mathbf{q}_1 belongs to Qf_{i_0} for i_0 in $I'_2 - I'_1$. Let \mathbf{X}_2 in $Wfp \cap D(I'_2)$; then the property $P_3(Wfp)$ is false.

Appendix E. Demonstration of Corollary 4 (Property $P_4(Wfp)$)

Sufficient Condition

Let $(\mathbf{X}_1, \mathbf{X}_2) \in Wfp \times Wfp$. Then

$$\exists i \in I / (\mathbf{X}_1, \mathbf{X}_2) \in (Wf_i \cap Wfp) \times (Wf_i \cap Wfp),$$

and

$$(X_1, X_2) \notin (\cup_{k \neq i} Wf_k \cap Wfp) \times (\cup_{k \neq i} Wf_k \cap Wfp),$$

which proves that $\exists i \in I / q_1 \in Qfp \cap Qf_i$ and $q_2 \in Qfp \cap Qf_i$.

Necessary Condition

Suppose $\forall i \in I, Wfp \not\subset Wf_i - (\cup_{k \neq i} Wf_k)$. Let $X_1 \in Wf_i - (\cup_{k \neq i} Wf_k)$; then $q_1 \in Qf_i$. Let $X_2 \in Wfp - (Wf_i - (\cup_{k \neq i} Wf_k))$; then $q_2 \notin Qf_i$, which proves that Wfp does not satisfy $P_4(Wfp)$.

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