How Do Universities Compete for Students?
Two Competing Strategies and Their Impact on Capacity and Tuition Fees

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EconomiX, Université Paris X, 7 April 2016
Motivation for the paper

- Main question: how can universities’ strategic behavior affect the provision of higher education and at what price?
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  - increased student mobility,
  - a greater autonomy granted to universities.

- These changes are likely to get universities to engage in competition on the enrollment market.
Introduction

Research issues

- Issue 1: how does competition between universities affect:
  - the size of universities (capacity),
  - the choice of the number of students enrolled and/or the level of fees?

- Issue 2: how does the number of universities impact social welfare: is it better to have many small universities or a few large universities?
**Observation**: two different university systems

- **Anglo-Saxon system**: universities can set their own fees (as in the UK, the USA, and Canada, among others).
- **European system**: universities cannot use a variable pricing strategy as a response to their new environment (as in France, Spain, Italy, and Germany). Tuition fees are set by public authorities, without control over the number of students enrolled.
Introduction

Stylized facts

<table>
<thead>
<tr>
<th>Country</th>
<th>Annual average fees (€) a</th>
<th>Entry rates into higher education b</th>
<th>Average nbr of students per university</th>
<th>% of small univ. (&lt; 10000)</th>
<th>% of great univ. (&gt; 25000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>No fees</td>
<td>53,15%</td>
<td>22 646</td>
<td>13%</td>
<td>42,5%</td>
</tr>
<tr>
<td>France</td>
<td>[189-261]</td>
<td>40,85%</td>
<td>21 648</td>
<td>19%</td>
<td>33,5%</td>
</tr>
<tr>
<td>Italy</td>
<td>[200-1000]</td>
<td>47,2%</td>
<td>22 564</td>
<td>31%</td>
<td>38%</td>
</tr>
<tr>
<td>Spain</td>
<td>[1000-2000]</td>
<td>52,1 %</td>
<td>27 547</td>
<td>17,7%</td>
<td>46%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2000</td>
<td>65,27</td>
<td>17 066</td>
<td>21%</td>
<td>21%</td>
</tr>
<tr>
<td>Canada</td>
<td>[3000-4000]</td>
<td>-</td>
<td>13 670</td>
<td>54%</td>
<td>22%</td>
</tr>
<tr>
<td>UK</td>
<td>[5000-11000]</td>
<td>67,44%</td>
<td>14 678</td>
<td>38%</td>
<td>15,7%</td>
</tr>
<tr>
<td>USA</td>
<td>[3500-20000]</td>
<td>71,02%</td>
<td>12 422</td>
<td>58,5%</td>
<td>14,6%</td>
</tr>
</tbody>
</table>

Table 1: Fees and size of public universities in 2014-15

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a National Student Fee and Support Systems 2014/15, European Commission
Introduction

1. a greater (smaller) number of small (large) universities in Anglo-Saxon countries than in old European countries,
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2. higher tuition fees in Anglo-Saxon countries compared with European countries,
3. Anglo-Saxons countries have a much higher proportion of students enrolling in higher education compared with old European countries,
4. the average number of students per university is lower in the Anglo-Saxon system.
Another stylized fact:
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In the UK: following the reform (2012), fees have more than tripled. Over this period, no improvement in teaching has been noted.
Introduction

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Is it preferable to have many small universities or a few large ones?
Introduction

Preview of the model

- Two competition strategies:

  1. Bertrand competition: universities compete in terms of price; they set their own fees (as in the UK, the USA, Canada, and Australia).
  2. Cournot competition: universities compete in terms of quantity; they decide on the number of students enrolled (as in France, Italy, and Germany, among others).

The fact that universities pursue both teaching and research objectives affects how they compete for students: students are at once inputs and clients of the educational process (Rotchschild and White, 1995). Research requires money; more students enrolled may imply an increased research budget. However, increasing the size of the student population may be costly.
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  - Students are at once inputs and clients of the educational process (Rotcheschild and White, 1995)
  - Research requires money; more students enrolled may imply an increased research budget. *However*, increasing the size of the student population may be costly.
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Related literature
Theoretical papers focus on:

1. the performance of universities in terms of costs and returns to higher education (Garcia-Penalosa and Walde, 2000, Gary-Bobo and Trannoy, 2004, Kemnitz, 2007),
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2. the link between the quality of educational provision, mobility costs and student choice (De Fraja and Iossa, 2002, Del Rey and Wauthy, 2006),


4. the impact of different funding systems (Beath et al, 2012).
Our approach:

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- a university can enroll beyond capacity but at a strictly convex cost (Dastidar, 1995, Chowdhury, 2009, Jacqmin and Wauthy, 2014)
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- we assume the existence of capacities in the production of education,
- a university can enroll beyond capacity but at a strictly convex cost (Dastidar, 1995, Chowdhury, 2009, Jacqmin and Wauthy, 2014)
- each university’s capacity level is chosen endogenously (Cabon and Drouhin, 2014).
Introduction

Results:

- *Bertrand competition* always implies a lower capacity than *Cournot competition*. 

Universities adopt a high level of fees in Bertrand competition which can rise with the number of universities. Cournot competition leads to a higher number of students enrolled in each university than Bertrand competition. The equilibrium adopted in Bertrand competition may be more efficient in terms of cost minimization than Cournot competition. An increase in the number of universities has a positive impact on social welfare, except when the capacity is too high in Cournot competition.

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- An increase in the number of universities has a positive impact on social welfare, except when the capacity is too high in Cournot competition.
The model

The utility function

In line with Jacqmin & Wauthy (2014), the objective function of universities is specified as follows:

\[ \text{Max } U_i(T, R) \] (1)

with \( \frac{\partial U_i}{\partial T} > 0 \) and \( \frac{\partial U_i}{\partial R} > 0 \)

- Students are required for the provision of education: the level of teaching is equal to the enrollment of students \( n_i \) weighted according to the parameter \( 0 < \gamma < 1 \):

\[ T = \gamma n_i \]
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$$T = \gamma n_i$$

- Students provide the funds a university needs to operate either directly through fees \( f \) or indirectly via the government (per-student subsidy \( s \)). \( R \) represents each university’s expenditure on research. Here, the research output \( S_i \) depends only on the money invested in it:

$$R = S_i$$
The cost function

The cost function of each university $i$ is given by

$$C_i(n_i, k_i) = \begin{cases} 
\delta k_i & \text{if } 0 \leq n_i \leq k_i \\
\delta k_i + \mu(n_i - k_i)^2 & \text{if } n_i > k_i
\end{cases}$$

(2)

where $\mu > 0$. Cost parameters ($\delta$ and $\mu$) are similar for all universities and constant.

$n_i$: the number of students enrolled at a university $i$,

$k_i$: the capacity of a university $i$. 
The optimization problem of a university $i$

The optimization problem of a university $i$ is thus defined as:

$$\text{Max } U_i = \gamma n_i + S_i \quad \text{s.t.} \quad S_i + C_i(n_i, k_i) = n_i(f_i + s)$$

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- There are $m$ identical universities: there is no differentiation in curriculum, and/or in other non-price dimensions (location, admission standard or financial endowment...)

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- We assume that potential students will not differ in their willingness to enroll and each student gets one unit of education.
**The Cournot competition Model**

- **Sequential game in Cournot competition:**

\[
U_i(n_i; N - i; k_i) = n_i ((1 - n_i - N - i) + s + \gamma) - C_i(n_i, k_i)
\]

with \(N - i = \sum_{j=2}^{m} n_j \).
Sequential game in Cournot competition:

1. First stage: each university $i$ chooses its level of capacity $k_i$ taking into account the threshold capacity of other universities $k_{-i}$,
The Cournot competition Model

- Sequential game in Cournot competition:

  1. First stage: each university $i$ chooses its level of capacity $k_i$ taking into account the threshold capacity of other universities $k_{-i}$,
  2. Second stage: each university $i$ selects a number of students enrolled $n_i$ for a given level of capacity $k_i$ and for a given number of students at other universities.

Consider the utility of university $i$ in the second stage, conditional on $k_i$:

$$U_i(n_i; N_{-i}; k_i) = n_i((1 - n_i - N_{-i}) + s + \gamma) - C_i(n_i, k_i)$$

with $N_{-i} = m \sum_{j=2}^{k_{-i}} n_j$. 

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The Bertrand competition model

Sequential game in Bertrand competition:
1. First stage: universities determine their capacity levels anticipating the effect on the price equilibria in the second stage of the game.
2. Second stage: universities determine their tuition fees for a given capacity.

The utility function of each university can be rewritten:

\[ U_i(f_i, f_{-i}, k_i) = \begin{cases} 
-\delta k_i & \text{if } f_i > f_{-i}^{min} \\
(f_i + \gamma + s) \frac{N(f_i)}{m} - C_i \left( \frac{N(f_i)}{m}, k_i \right) & \text{if } f_i = f_{-i} \\
(f_i + \gamma + s) N(f_i) - C_i (N(f_i), k_i) & \text{if } f_i < f_{-i}^{min}
\end{cases} \]

with \( f_i \), the fee of the university \( i \), and \( f_{-i}^{min} = \text{Min}\{ f_1, \ldots, f_{i-1}, f_{i+1}, \ldots, f_m \} \)

\( N(f_i) \): total number of students enrolled when the university \( i \) sets \( f_i \)
Equilibria in the Cournot competition model

The symmetric subgame perfect equilibrium values are:

\[ n_c^*(m) > k_c^*(m) \]

if and only if \( \delta > \bar{\delta} \)
Equilibria in the Cournot competition model: comparative statics

Impact of the number of universities on the equilibrium results:

- the capacity $k^*_c(m)$ is decreasing with $m$, 

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Impact of the number of universities on the equilibrium results:

- the capacity $k_c^*(m)$ is decreasing with $m$,  
- the number of students enrolled at a university $n_c^*(m)$ is decreasing with $m$,  
- the total number of students enrolled $N_c^*(m)$ is increasing with $m$. 
Proposition 1

In the second stage, \((f_1, f_2, \ldots, f_m)\) is a pure-strategy Nash equilibrium if and only if \(f_1(k_1) = f_2(k_2) = \ldots = f_m(k_m) = f^N(k_i)\), such that

<table>
<thead>
<tr>
<th>If (\mu &lt; \frac{m}{m-1}) Case 1</th>
<th>If (\mu = \frac{m}{m-1}) Case 2</th>
<th>If (\mu &gt; \frac{m}{m-1}) Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f^N = \tilde{f}(k_i, m) = \frac{\mu(m+1) - m(\gamma + s) - 2m\mu k_i}{m + \mu m + \mu})</td>
<td>(f^N = \tilde{f}(k_i, m) = \frac{(m+1)(m-1)(\gamma + s) - 2mk_i}{2m})</td>
<td>(f^N = f^*(k_i, m) = \frac{m(1-\gamma - s) + 2\mu(1-mk_i)}{2(m + \mu)})</td>
</tr>
</tbody>
</table>
Equilibria in the Bertrand competition model

Corollary 2

We verify that: \( \forall \mu, \gamma, s, \delta > 0 \) and \( m \geq 3 \),

\[
\begin{align*}
\bar{k}_b^*(m) &< \bar{n}_b^*(m) \\
\tilde{k}_b^*(m) &< \tilde{n}_b^*(m) \\
\hat{k}_b^*(m) &< \hat{n}_b^*(m) \\
k_b^*(m) &< n_b^*(m)
\end{align*}
\]
Equilibria in the Bertrand competition model: comparative statics

Impact of the number of universities on the equilibrium results:

- **Case 1**: $\mu < \frac{m}{m-1}$
Equilibria in the Bertrand competition model: comparative statics

Impact of the number of universities on the equilibrium results:

- **Case 1**: $\mu < \frac{m}{m-1}$
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Cases 2 and 3:

- $\mu \geq \frac{m}{m-1}$
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Equilibria in the Bertrand competition model: comparative statics

Impact of the number of universities on the equilibrium results:

**Case 1:** $\mu < \frac{m}{m-1}$
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Cases 2 and 3:

- the capacity $\bar{k}_b^*(m)$ is decreasing with $m$,
- the level of fees $\bar{f}_b^*(m)$ is constant with $m$,
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The model

Equilibria in the Bertrand competition model: comparative statics

Impact of the number of universities on the equilibrium results:

- **Case 1**: \( \mu < \frac{m}{m-1} \)
  - the capacity \( \bar{k}_b^*(m) \) is decreasing with \( m \),
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  - the total number of students enrolled \( \bar{N}_b^*(m) \) is decreasing with \( m \).

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Equilibria in the Bertrand competition model: comparative statics

Impact of the number of universities on the equilibrium results:

- **Case 1:** $\mu < \frac{m}{m-1}$
  - the capacity $\bar{k}_b^* (m)$ is decreasing with $m$,
  - the level of fees $\bar{f}_b^* (m)$ is increasing with $m$,
  - the total number of students enrolled $\bar{N}_b^* (m)$ is decreasing with $m$.

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- **Cases 2 and 3:** $\mu \geq \frac{m}{m-1}$
  - the capacity $k_b^*(m)$ is decreasing with $m$,
  - the level of fees $\tilde{f}_b^*(m)$ is constant with $m$,
  - the total number of students enrolled $N_b^*(m)$ is constant with $m$. 
Comparison of equilibria: Cournot vs Bertrand

Proposition 3

$\forall \gamma, s, \delta > 0$ and $m \geq 2$, we verify that:

- The equilibrium level of capacity:
  \[ k_b^*(m) < k_c^*(m) \]

- The total number of students enrolled:
  1. if $\mu \leq \mu = 0.4589$, $N_c^*(m) \leq \bar{N}_b^*(m)$,
  2. if $\mu > \mu = 0.4589$, $N_c^*(m) > N_b^*(m)$. 
Because of the U-shaped average cost in $k$, there exists an efficient capacity which minimizes the average cost:

$$\text{Min}_{n_i} AC(k_i, n_i) = \text{Min}_{n_i} \left[ \frac{C(k_i, n_i)}{n} \right]$$

The capacity which minimizes the average cost for a given number of students enrolled is:

$$k_{\text{min}} = \frac{2\mu n_i - \delta}{2\mu}$$
Efficient capacity and comparison with equilibria

Proposition 4

Comparison between the equilibrium results of the two competition strategies with the efficient solution:

<table>
<thead>
<tr>
<th></th>
<th>Cournot equilibrium vs efficient solution</th>
<th>Bertrand equilibrium vs efficient solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>capacities</td>
<td>$k_c^{\text{min}} &lt; k_c^*$</td>
<td>$k_b^{\text{min}} &gt; k_b^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$k_b^{\text{min}} = k_b^* = k_b$</td>
</tr>
<tr>
<td>number of students/fees</td>
<td>$n_c^{\text{min}} &lt; n_c^*$</td>
<td>$f_b^{\text{min}} &lt; f_b^*$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_b^{\text{min}} = f_b^* = \tilde{f}_b$</td>
</tr>
</tbody>
</table>
Impact of the number of universities on social welfare

Proposition 5

In the **Bertrand competition**, social welfare is always increasing with the number of universities $m$

Proposition 6

In the **Cournot competition**, social welfare may decrease with the number of universities when the capacity is too high:

$$\delta \leq \bar{\delta} \iff n_c^*(m) \leq k_c^*(m)$$

otherwise, social welfare is increasing with $m$. 
Conclusion

Regarding our results:

- Higher tuition in most countries which have adopted a deregulated system can now be explained by our model,
- Cournot competition is likely to induce too high a capacity level in terms of the minimization of the average cost for the university,
- Bertrand competition seems more efficient from this point of view, but only if the cost is sufficiently convex.
- An increase in the number of universities improves social welfare, except in Cournot competition when the capacity is too high.
Conclusion

Some limits and possible extensions:

- the price elasticity of demand,
- the access to financial support or other aid for students,
- differentiation in curriculum, and/or other non-price dimensions,
- student selection.