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# A method to estimate horse speed at canter from IMU data with Machine Learning

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## 1 Introduction

According to Article 234 of the FEI Jumping Rules, horses speed for international competitions has to be 350 m per minute minimum and 400 m per minute maximum, with exceptions for different kinds of show conditions (FEI, FEI Jumping Rules, 26th edition, 2019). Speed is therefore a key parameter for success in show-jumping competitions and an important training input.

3D optical motion capture is currently the gold standard for horse gait analysis and can be therefore used for measuring stride parameters such as speed (Pfau, Witte, and Wilson, 2005). Nevertheless, the setting up of the measurement field is time consuming as well as data processing when your subject differs from the plug-in gait reference provided by the software (van der Kruk and Rejjine, 2018). Those aspects make its use impossible on a daily basis or during championships for a rider who wants descriptive results of his horse performance and locomotion parameters within a minute and potentially in real time without preliminary preparation.

New gait analysis techniques emerged and enabled the development of tools to provide objective parameters of horse's motion (Martin, Chèze, Pourcelot, Desquilbet, Duray, and Chateau, 2017) or to detect lameness (Pfau, Boulton, Davis, Walker, and Rhodin, 2016), using low-cost inertial measurement units (IMU), composed of two sensors: tri-axial accelerometer and tri-axial gyroscope. Those sensors can be coupled with tri-axial magnetometer, and are therefore called mIMU. Thanks to data fusion techniques, the use of a magnetometer helps reducing the IMU bias and leads to better estimation of distance (Filippeschi, Schmitz, Miezal, Bleser, Ruffaldi, and Stricker, 2017). IMUs can also be paired with Global Positioning System (GPS) unit, to improve the estimation of locomotion parameters such as speed (Tan, Wilson, and Lowe, 2008, Zihajehzadeh, Loh, Lee, Hoskinson, and Park, 2015). Nevertheless, GPS measurements can be badly influenced by the presence of obstacles (Wing, Eklund, and Kellogg, 2005) and it cannot be used indoors due to signal loss under roofing.

There exist three main families of methods developed to calculate motion characteristics from IMU signals. Firstly, model-based methods, like inverted pendulum models for speed estimation in human gait, which simplifies complex biomechanical behaviors with simple mechanical model and incorporates subject-specific information like the limb length (Duong and Suh, 2017, Murphy, Carr, and O'Neill, 2010). Secondly, signal-based methods which mainly rely on signal integration (Brzostowski, 2018) and use signal processing methods like Butterworth filter to

prevent drifting (Bosch, Serra Bragança, Marin-Perianu, Marin-Perianu, Van der Zwaag, Voskamp, Back, Van Weeren, and Havinga, 2018). Those methods need to formulate some realistic assumptions to correct sensors drift and need a zero-velocity phase within each stride to be able to apply the integration process. For example the method proposed by Pfau et al. (2005) estimates horse displacement from one IMU placed on the trunk, assuming that the horse is in a steady state because of a treadmill that constrained the horse motion. In this case, the IMU sensor displacement should follow a closed loop and then the average velocity over a stride should be zero, as well as the average forward-backward and side-to-side acceleration. Thus, in this context, stride-by-stride mean subtraction of acceleration and of the calculated velocity before integration enables determination of the integration constants. This assumption is often invalid in numerous experimental conditions, leading to the non-applicability of direct signal integration method. The third family of methods is more recent and based on statistical approaches, originally developed to estimate human speed from IMUs data (Sabatini and Mannini, 2016, Zihajehzadeh and Park, 2017). Those approaches provide accurate estimation of walking speed but regression models accuracy seems to be dependent of the range of motion. To prevent the drift, a first step is proposed where data are divided according to their speed regime, thanks to a Support Vector Machine (SVM, Bishop (2006)), before the computation of speed by regression model (Zihajehzadeh, Aziz, Tae, and Park, 2018). Thus, the regression model is fit independently to each range of speed. This method refines the regression model accuracy for slow speed regime. Statistical methods concept is simple: one has to provide a dataset, called training dataset, with known variable of interest value (for example, IMU signals matched to their associated speed) that will be used to build a model. The model will then be able to predict the value of the variable of interest for new data. The model has to be trained with cases that can be encountered in its future application, without which it will perform poorly. We want to extend those methods to the horse motion.

The objective of this work is to develop a smart device that can provide the rider with the movement parameters of his horse, in daily routine such as during training sessions as well as during competition events, using only one IMU fixed in the saddle. The idea is also to propose a tool that overcomes the limits imposed by the use of GPS or 3D optical motion capture systems. This user-case differs from existing published work for sports (Camomilla, Bergamini, Fantozzi, and Vannozzi, 2018) by not using sensors fusion, not being in a steady state that allows an easy use of direct integration of acceleration signals, nor using a sensor on the limb which allows to reset errors at each cycle on short time periods.

In order to do so, several machine learning models and different data conformations will be tested. The obtained results will be compared to those of one signal based method, already used for speed estimation in animal locomotion, the Overall

Dynamic Body Acceleration. The aimed accuracy is 0.6 m/s (36 m/min) in order to meet the expectations of the show jumping professionals. As far as the authors know, this accuracy has not been reached for horses by the previously mentioned methods using data from one IMU only (Pfau et al., 2005, Bosch et al., 2018).

## **2 Materials and methods**

### **2.1 Data**

The database used for model development is made of 3221 canter strides from 58 ridden jumping horses of different breeds, height (129-176 cm), age (5-18 years old) and different levels of competition (amateur or professional). One IMU (LSM6DSL, STMicroelectronics) placed in the saddle close to the horse's withers was used to measure tri-axial acceleration (range  $\pm 8$  g) and tri-axial rotation rate (range  $\pm 2000$  dps) at a sampling frequency of 100 Hz. Data collected by the IMU were sent via a Bluetooth® antenna to a smartphone (iPhone X, Apple Inc.) and then stored on an online server. Two different protocols, which will be detailed here-after, were used to collect data: the first one was speed measurement for a straight path and the second one was speed measurement for a curved path. For both protocols, reference speed was measured by video cameras or chronometer and matched to each stride signal to train the machine learning model. For each protocol, "strides" were defined from the maximum peak on the Z-axis (cf. Figure 1) of the raw acceleration data to the next 100 samples, in order to have the same number of points for each individual regardless the speed, a necessary condition to use machine learning methods. So, depending on the horse's speed, this data segmentation may include more than one cycle. We chose not to re-sample a cycle in order to keep the information on the duration of the stride to estimate speed. Values from the three axes of the gyroscope and the accelerometer were extracted according to this cutting process with an automated detection algorithm written in MATLAB (R2014b).

#### **2.1.1 Straight path**

To get reference stride speed, IMU data were synchronized to a 4-cameras 2D tracking system (UI-5240CP-M-GL, IDS) which had a measuring field of 26 meters. Horses were equipped with ten 2D-reflecting markers on anatomical landmarks (Figure 2) and their speed in the camera's field was derived from the markers' 2D trajectories using a custom software written on MATLAB® (R2014b). The accuracy of this system is of 1.4% of the measured distance (Martin, 2015), which



Figure 1: Orientation of IMUs axes on an equipped horse

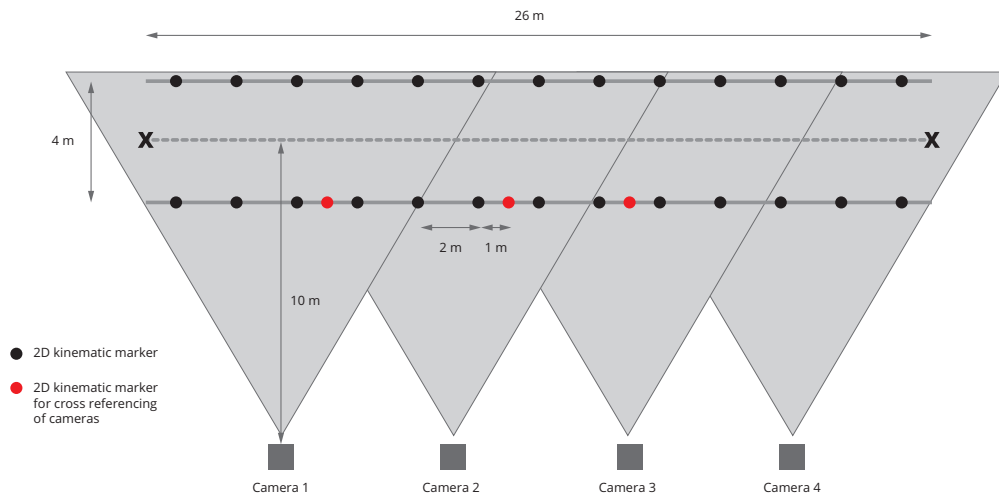


Figure 2: Field of measurement with 2D video cameras

would correspond to  $\pm 2.8$  cm for a 2 m measured distance for example. Data were gathered for different speeds chosen by the rider (normal, slow and fast), with and without jumps before and after the camera's field and with or without ground bars, spaced from 2.5 m to 4.5 m, in the field of measurement. Those various conditions were chosen in order to expand the range of canter within the training set, in order to get closer to daily training conditions of jumping horses.

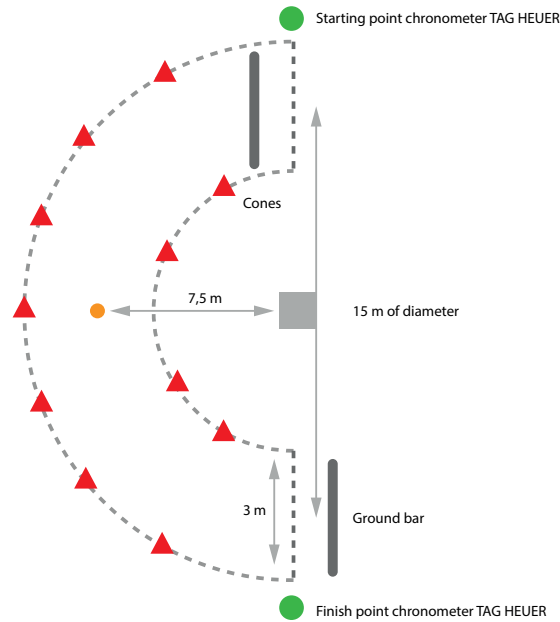


Figure 3: Plan of speed measurement on curves for a horse at left-hand canter

### 2.1.2 Curved path

Because the 2D tracking system has a good accuracy only when the horse displacement is perpendicular to the cameras field, another measurement protocol was designed for curve displacement. A curved path of known perimeter was defined with cones and with a width small enough to make the horses pass the measuring equipment in a rather narrow path (Figure 3). The traveled distance was calculated as  $distance = 2\pi r$ , with  $r$  the radius of the circle. Time spent in the curve by the horse was measured with an automatic chronometer (CP 520, Tag Heuer) triggered at the entrance and at the exit of the curved path (Figure 3). The average speed of the horse was then derived as  $speed = distance/time$ . Each stride of the horse within the curve was then matched with the average speed. For example, if the average speed in the curve was 6 m/s and the horse did 5 strides, then a speed of 6 m/s was assumed for all these strides. In order to mimic real-life conditions of a jumping course the whole database is composed of 2906 strides collected in straight path and 315 strides in curved path.

## **2.2 Speed measurement methods**

### **2.2.1 Overall Dynamic Body Acceleration method**

Overall Dynamic Body Acceleration (ODBA) method is a signal-based method proposed in Wilson, White, Quintana, Halsey, Liebsch, Martin, and Butler (2006) that does not rely on signal integration. They developed a parameter named ODBA, calculated from acceleration in the 3 space directions, which is closely linked to the speed of a walking animal.

In our case, acceleration signals are low-pass filtered using a fourth-order Butterworth filter with a cut-off frequency of 10 Hz. After that, an angle correction is applied to align the Z-axis with the gravity vector. Then for each axis, as specified by Wilson et al. (2006), the signal mean value is subtracted from smoothed data. Those values are then converted to absolute positive units. Finally, resulting signals are summed up and a mean ODBA value is calculated for each stride. A linear regression is then used to link the mean ODBA for one stride to the speed of the stride. The linear regression is performed with R software (v.3.4.0) (R Core Team, 2017a) and the `lm` function of stats package (R Core Team, 2017b).

### **2.2.2 Statistical models**

In a mathematical point of view monitored signals can be considered into two different ways. The first approach is as a multi-dimensional vector of size 606 (101 values per measured signal). The second approach is a vector of six functional variables, one per measured signal. The main advantage of considering collected data as a vector of functional variables is that we keep the temporal dynamic. We refer to Ramsay and Silverman (2005) for univariate and bivariate examples.

The statistical methods we have tested can be divided into two categories. The first one, high dimensional models which deal with data sets where the number of columns can be large. Among those methods we tested Ridge regression, Lasso regression, Principal Component Regression (PCR), Partial Least Square regression (PLS), Elastic net regression, neural network, random forest and Support Vector Machine (SVM). For a complete review about these models refer to (Bishop, 2006, Hastie, Tibshirani, and Friedman, 2001). Secondly, the functional approaches with a parametric regression model for functional data (Ramsay and Silverman, 2005) and a non parametric regression model (Ferraty and Vieu, 2006).

As in Zihajehzadeh et al. (2018), we wonder if the division of the database into smaller homogeneous subgroups, before the computation of speed by the regression model may improve the model accuracy. Thus, each previously described model will be tested on raw data and on the database divided into two subgroups.

These subgroups has been built thanks to a clustering method for multivariate functional data (Schmutz, Jacques, Bouveyron, Cheze, and Martin, 2018).

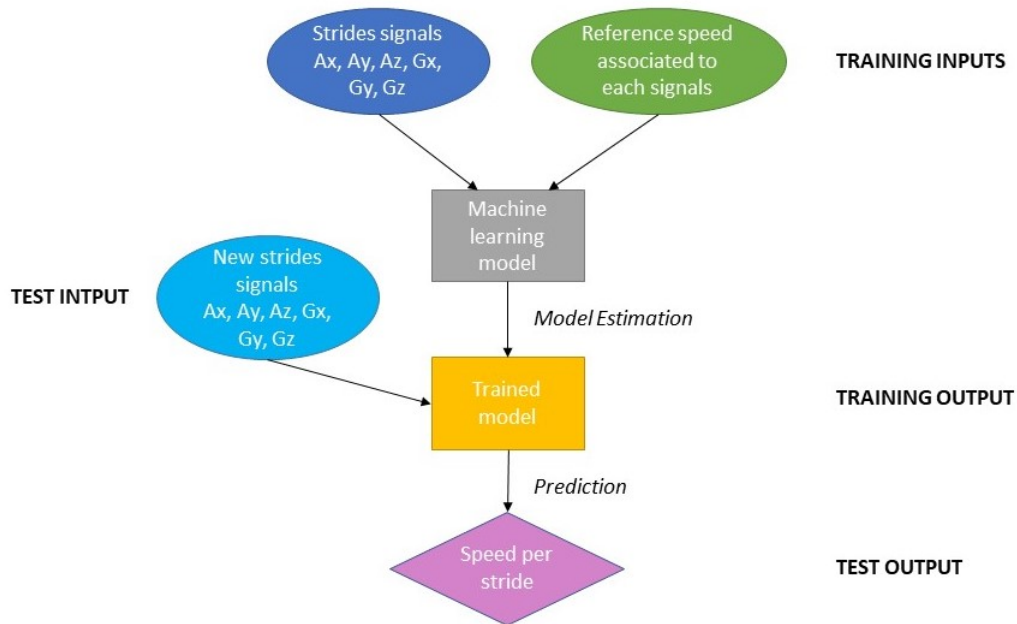


Figure 4: Diagram of statistical methods process from training to the speed prediction

To sum up, the signals collected with the accelerometer and the gyroscope for each stride are matched with the measured reference speed. This is used as input data to train the model in order to obtain the best speed estimation for new data in the future. All this process is illustrated on Figure 4. Models are developed with R software (v.3.4.0) and packages glmnet (Friedman, Hastie, and Tibshirani, 2010), pls (Mevik, Wehrens, and Liland, 2016), neuralnet (Fritsch and Guenther, 2016), randomForest (Liaw and Wiener, 2002), e1071 (Meyer, Dimitriadou, Hornik, Weingessel, and Leisch, 2017), fda.usc (Febrero-Bande and Oviedo de la Fuente, 2012), funHDDC (Schmutz and Bouveyron, 2019) and function funopare.knn.lcv (Ferraty and Vieu, 2006).

### 2.2.3 Methods comparison

To compare the accuracy of methods, the database is cut into two parts: a training dataset which is composed of a random sampling of 80% of the database, and the remaining 20% forms the test dataset. The models are built on the training dataset



and their accuracy is then evaluated on the test dataset. Evaluating methods on an independent test dataset prevents over-fitting.

Comparison between models is done with the calculation of the percentage error in the estimated speed above 0.6 m/s. This threshold is the minimum satisfactory for this parameter to make sense for the professionals. Percentage error is computed as:

$$\% \text{ error} = 100 \times \sum_i \frac{|\text{Measured speed at stride } i - \text{Predicted speed at stride } i| > 0.6}{\text{Total number of strides}},$$

with  $i$  corresponding to each stride of the test dataset.

Then, the best machine learning model and ODBA model are also compared with Bland and Altman plots and its 95% limits of agreement (Bland and Altman, 1999), which allows the evaluation of differences between two methods used on the same individuals (here strides). In our case, we examine the average difference between each method and reference values obtained with 2D tracking system for straight path and chronometer for curve path. Bland and Altman analysis and graphs are built with the `bland.altman.plot` function from the `BlandAltmanLeh` R package (Lehnert, 2015).

### 3 Results

To avoid results fluctuation due to random sampling of the test dataset, the random sampling process of the database is repeated 50 times and the average, minimum and maximum of percentage error are estimated for each repetition on raw data. When data are divided into two subgroups, the weighted mean, minimum and maximum is computed.

Table 1 shows the mean results for each model. The division of raw data into subgroups improves results of percentage of error for all high dimensional regression models, ODBA method and, to a lesser extend, neural network and parametric functional regression. Nevertheless, the best results are obtained with SVM method applied on raw data. SVM clearly outperforms its competitors.

The Bland and Altman plot of one SVM repetition is shown on Figure 5 (top), where one point corresponds to one stride. The speed predicted by the model and the measured speed of the stride are compared. The mean bias is 0, which means that in average the SVM model output is close to the measured speed. If the model predictions were perfect, all the points would be aligned on the zero line. The points that are the farthest from the zero line are the worst predictions. We can see that for some strides of low speed (below 5 m/s), the SVM model has a tendency to overestimate their speed. Whereas for some strides of high speed (above 5 m/s),

the SVM model has a tendency to underestimate them Whereas ODBA estimations (Figure 5, bottom) are more variable than SVM ones. The mean bias is also 0 but the 95% confidence interval is twice the size of SVM one (cf. Table 1), that is to say high above our objective value. The ODBA method is more variable than the SVM one, with 95% of strides bias lower than 2.5 m/s and a clear tendency to underestimate strides of high speed.

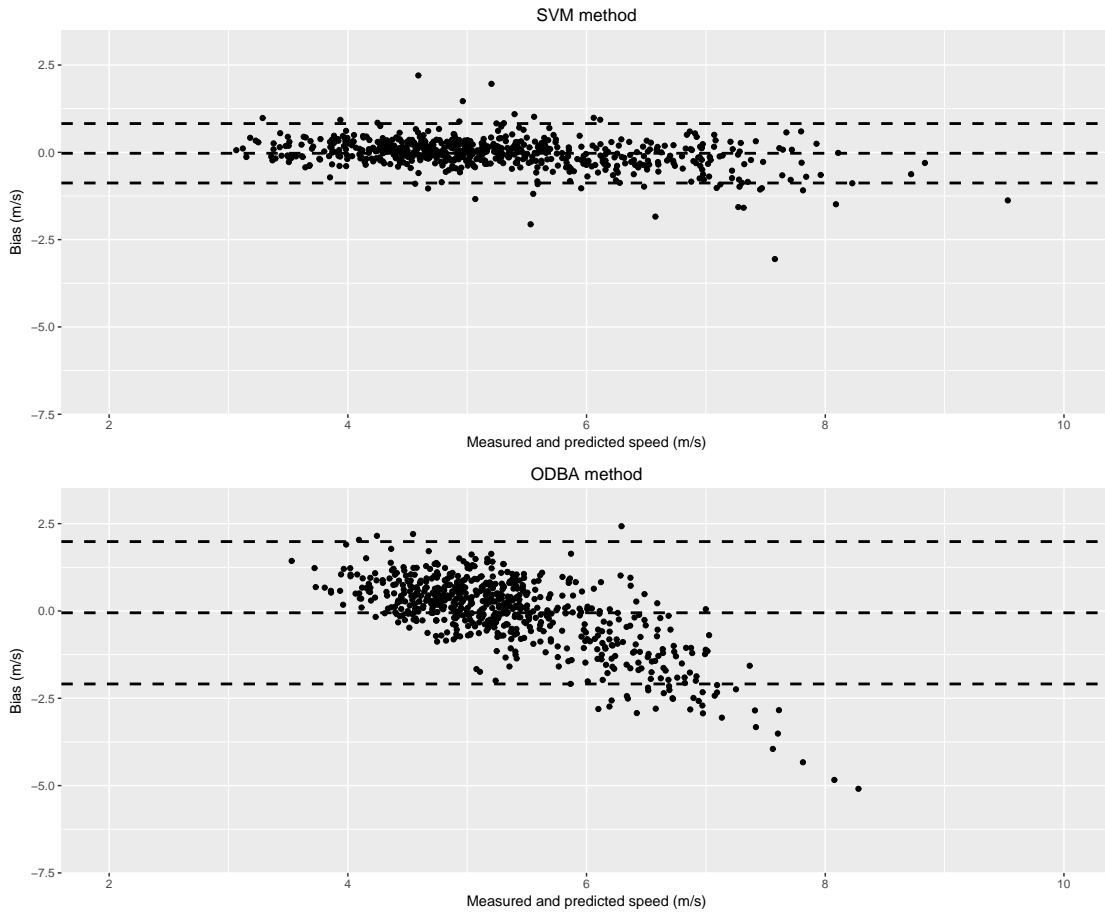


Figure 5: Bland and Altman plot for one repetition of SVM model with its 95% confidence interval (top) and ODBA method (bottom)

Method	Raw data			Subgroups		
	Mean	Min	Max	Mean	Min	Max
Ridge	27.7	24.2	30.7	24.0	21.2	27.6
Lasso	29.1	25.9	31.8	25.0	22.0	28.1
PCR	28.5	25.5	31.1	26.4	21.2	34.8
PLS	28.8	25.0	32.2	26.4	21.0	33.8
Elastic net $\alpha = 0.3$	27.9	24.6	30.9	24.1	21.0	27.7
Parametric functional regression	26.9	23.6	31.7	26.4	23.1	29.4
Non parametric functional regression	16.7	13.0	20.1	17.2	14.9	20.3
SVM	<b>9.9</b>	6.7	11.9	10.7	7.8	12.8
Neural network	29.9	25.7	33.7	28.7	24.8	34.8
Random Forest	17.5	14.3	20.7	18.3	14.7	20.7
ODBA	51.4	47.8	55.1	47.8	28.6	68.6

Table 1: Mean, minimum and maximum values of percentage of error above 0.6 m/s for speed estimation

## 4 Discussion and Conclusion

The objective of our study was to develop a model which can be integrated into a smart device in order to accurately provide horse speed per stride to the rider. This smart device is made of only one IMU situated on the horse wither. The number of sensors is kept to a minimum in order to facilitate the daily use of this tool and the non-use of GPS is due to our willingness to make this tool work both indoors and outdoors.

In this work, we propose to use machine learning models to predict horse speed from IMU data. On the collected dataset, Support Vector Machine gave the best results which the lowest percentage of error (above 0.6 m/s). SVM method outperformed other machine learning models and ODBA method. ODBA is a signal based method that does not need external inputs to estimate speed per stride from new IMU data. Indeed, direct signal integration methods, which are commonly used in human case or in horses running on a treadmill case, need strong assumptions to calculate an integration constant for speed estimation that cannot be made when the horse moves in real conditions and when the IMU is not located on the limb (Filippeschi et al., 2017, Camomilla et al., 2018). Moreover, biomechanical models have not been developed for an asymmetrical gait such as horse canter.

The superiority of machine learning models in this work can be partly explain by the fact that, usually, models for speed estimation first detect a stride, then

cut the collected signal according to this stride and apply the calculation model (Bichler, Ogris, Kremser, Schwab, Knott, and Baca, 2012, Mannini and Sabatini, 2014, Zhao, Brahm, Gerhard, and Barden, 2016). These methods standardized the stride duration for all individuals and few of them take into account the stride duration to calculate speed parameter leading to a loss of information. In the present work, we choose a different way of pre-processing the collected signals. Indeed, the stride is detected on the Z-axis but 101 points are kept from the maximal peak on Z-axis. This change allows to keep the stride duration information: for high speed the "stride" will have a signal that contain more canter cycles than low speed "strides". This data pre-processing helps increasing the accuracy of the statistical model.

The novelty of the present work is to propose a model for speed estimation that relies on one IMU only. The integration of machine learning model in a device for equestrian sport is innovative in comparison with other existing systems for equestrian sports based on GPS or in comparison with human tracking motion systems that are mainly based on the use of a magnetometer or several IMUs (Filippeschi et al., 2017). The machine learning approach allows the development of a smart device that does not rely on a GPS for the estimation of a physical phenomenon, here the horse speed at each stride, with an accuracy of 0.6 m/s. This accuracy meets the expectations of professionals of show jumping discipline, which was their main concern about using or not connected devices. Indeed, as show jumping can be practiced both indoors and outdoors, our tool overcomes the GPS systems limitations. Moreover, the accuracy of our model can easily be refined by performing more campaigns of measurement with the reference systems. Indeed, a panel of 58 horses is not sufficient to model the behavior of all horses due to individual's diversity. It is also necessary to do more measurements on curves of various diameters, since it greatly influences the horse's behavior, the corresponding collected signals and therefore the horse speed (Greve and Dyson, 2016).

We cannot benchmark our model to other works on horses because no one else provides a speed per stride estimation. Indeed, Pfau et al. (2005) and Bosch et al. (2018) calculate traveled distance with its preciseness but Bosch et al. (2018) aim to provide a speed estimation in future work. Whereas in human researches, a wide literature exists on computing human walking speed from data collected by one IMU placed on the foot, as for example Mannini and Sabatini (2014) who compare two methods of walking speed estimation, whose accuracy is between 0.5 km/h (0.14 m/s) and 0.7 km/h (0.19 m/s) depending on the walking speed and the method used. Zihajehzadeh and Park (2016) develop a model for walking speed estimation based on a regression model which use data from one wrist-worn inertial sensor. In their paper, the Bland and Altman limits of agreements are lower than 0.2 m/s. Sabatini and Mannini (2016) estimate an instantaneous velocity decomposed

in the three space directions from two IMUs placed one on the pelvis and the other on the shank of the subject, its accuracy is in the same range than the one of previous studies. For instance, considering a walking man of 3 km/h (0.8 m/s), the error of the previous models is around 27% whereas for a running show jumping horse of 350 m/min (5.8 m/s) our model error is about 10%. Thus, our model is more accurate than the existing ones for human walking.

Another advantage of our model is that it can be transferred to another discipline than show jumping as long as consistent data are provided to train the model. Indeed, show jumping canter is specific to the discipline where the bounce is important whereas in endurance horses, flat canter is preferred. Therefore, in order to adapt our tool to other disciplines, the model has to be expanded with more data gathered in new situations. The SVM model is transposable to the other equestrian sports and to bipedal locomotion, as long as consistent data are provided to train the model.

To conclude, the predictive method shown herein above is accurate because the reference data, used as a training dataset, were collected with great precision and are consistent with the model application (show jumping).

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