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ABSTRACT

The velocity dispersion σ, radius R and luminosity L of elliptical galaxies are known to be related, leaving only two degrees of freedom and defining the so-called 'fundamental plane'. In this Letter, we present observational evidence that rich galaxy clusters exhibit a similar behaviour. Assuming a relation $L \propto R^{\alpha}\sigma^{2\beta}$, the best-fitting values of α and β are very close to those defined by galaxies. The dispersion of this relation is lower than 10 per cent, i.e. significantly smaller than the dispersion observed in the L–σ and L–R relations. We briefly discuss some possible implications for the spread of formation times of objects and for peculiar velocities of galaxy clusters.

Key words: galaxies: clustering – galaxies: formation – galaxies: fundamental parameters – large-scale structure of Universe.

1 INTRODUCTION

The distribution of structures in the Universe according to mass and size is believed to be intimately related to the conditions prevailing in the primordial Universe, and reflects the history of formation. The existence of a universal luminosity function (Schechter 1976), backed by a dynamical model (Press & Schechter 1974) of clustering, led to the belief that the luminosity (or the mass) is the major, if not the only, parameter describing a galaxy. Other properties such as the velocity dispersion (Faber & Jackson 1976; Tully & Fisher 1977) are then related to the luminosity (Cavaliere, Danese & De Zotti 1978) and provide important distance indicators. Analogous relations exist for smaller systems: for dwarf ellipticals, Held et al. (1992) found $L \propto \sigma^{2.5}$; for globular clusters, Meylan & Mayor (1986) and Paturel & Garnier (1992) found $L \propto \sigma^{2.5}$. However, the need for at least a second parameter for spiral galaxies (Bujarrabal, Guibert & Balkowski 1981), as well as for ellipticals (Terlevich et al. 1981; Tonry & Davis 1981), soon became apparent. Subsequently, elliptical galaxies were shown to form a genuine two-parameter family, the so-called 'fundamental plane' (Djorgovski & Davis 1987; Dressler et al. 1987), and the second parameter could then be identified as being the surface brightness (or equivalently the radius). Nieto et al. (1990) found that low-mass ellipticals, dwarf spheroidal galaxies and halo globular clusters are all near the plane defined by bright elliptical galaxies. Despite the identification of the relevant parameters, the reason why there are two is not known, except for an obvious relation with the virial theorem, which can be written in the form $M \propto R\sigma^2$. Internal dynamics in pressure-supported systems, dissipative merging, cooling and star formation processes have been invoked by various authors (Kashlinsky 1983; Dressler et al. 1987; Nieto et al. 1990), bearing in mind that this relation applies to extremely dense systems. A general discussion can be found in Kormendy & Djorgovski (1989), and references therein.

In this Letter, we show that

(i) the fundamental plane extends to large bound systems of much lower density, such as clusters of galaxies,

(ii) the fact that systems as small as globular clusters and as large as galaxy clusters lie on a fundamental plane suggests that stellar systems and galaxy systems were formed by the same process, thus supporting the hierarchical clustering scenario, and

(iii) the dispersion seen in the luminosity–radius or luminosity–velocity dispersion relation should reflect the dispersion in the epoch of formation.

2 DISCUSSION OF DATA

2.1 Abell clusters

In order to search for correlations between intrinsic parameters of galaxy clusters, it is essential to work on a homogeneous data set. We have therefore used the compilation of West, Oemler & Dekel (1989, hereafter WOD) of 29 Abell

clusters with reliable photometry. For a more exhaustive discussion of this sample, we refer the reader to their paper (and also to West 1990). High-quality profiles have been derived (West, Dekel & Oemler 1988), allowing good fits to a de Vaucouleurs law and thus determination of an accurate half-mass radius R_e. The photometry of these clusters, previously obtained by Oemler (1974), Butcher, Oemler & Wells (1983) and Dressler (1978), has been rereduced by extrapolating the previously calculated profiles to infinite radius, and using the luminosity function of Kirshner et al. (1983) for extrapolation to faint galaxies, where L is given in units of $L_* = 1.3 \times 10^{10}\ L_\odot$ in the V passband (with $H_0 = 100$ km s^{-1} Mpc^{-1}). Velocity dispersions have been derived from the compilation of Struble & Rood (1991), which allows us to build a subsample of 16 clusters with reliable effective radius R_e, total luminosity L and velocity dispersion σ: A154, 168, 194, 400, 401, 426, 539, 665, 1314, 1656, 1904, 2019, 2065, 2199, 2256 and 2670. σ has been measured with more than 50 galaxies per cluster for 75 per cent of the sample, and with between 13 and 50 galaxies per cluster for the remaining 25 per cent. These clusters are rich and essentially free from superposition effects, so that the uncertainty in σ is not critical. The values of χ^2 for this sample have been calculated using errors of 15 per cent of each variable (WOD).

2.2 Stellar systems

Data for ellipticals have been taken from Djorgovski & Davis (1987). We have converted their Lick r_G magnitudes (measured within R_e) to the V passband using Djorgovski's (1985) approximate relations and assuming $B-V=1$; we have adopted a luminosity a factor of 2 greater than that in R_e. The second sample of ellipticals used is that of Faber et al. (1989). We have used V magnitudes from the listed total B magnitudes and $B-V$ colours when available; otherwise, $B-V=1$ has been assumed. Being interested in a general comparison between elliptical galaxies and galaxy clusters, we will ignore more detailed corrections. We have also added a sample of low-luminosity and true dwarf elliptical galaxies, but uncertainties here are quite large. Data for low-luminosity ellipsoidal galaxies are taken from Bender & Nieto (1990), and for true dwarf elliptical galaxies from Bender, Paquet & Nieto (1991); we assume $B-V=0.7$ (as in Held et al. 1992).

Illingworth (1976) has studied 10 clusters, fitting them with both a King law and a de Vaucouleurs law, and listing their effective radius, total visual luminosity, central velocity dispersion and mass. The central velocity dispersions of other clusters have been taken from Gunn & Griffin (1979), Meylan & Mayor (1986), Lutpon, Gunn & Griffin (1987), Peterson & Latham (1986), Pryor et al. (1988), Rastorguev & Samus (1991) and Zaggia, Piotto & Capaccioli (1991). All of these authors except Illingworth and Zaggia et al. calculated velocity dispersions from radial velocities of giant stars in globular clusters (note that dispersions from integrated spectra appear to be systematically higher than those calculated from radial velocities of giant stars). Except for Illingworth's clusters, we have only core and tidal radii listed by Webbink (1985), who also gives integrated V absolute magnitudes (within r_t), but not de Vaucouleurs radii. Being interested in *global* quantities, we have estimated R_e as a function of core radius r_c and the concentration parameter $c = \log(r_t/r_c)$, where r_t is the tidal radius. Our final sample includes 33 globular clusters (13 with velocity dispersions derived from integrated spectra).

3 DISCUSSION OF RESULTS

3.1 Luminosity–radius and luminosity–velocity dispersion relations

WOD have shown the existence of a correlation between the total luminosity and the effective radius of Abell clusters. We obtain $R \propto L^{0.5\pm0.1}$, but with a high dispersion [$\chi^2 = 2.7$ per degree of freedom (d.o.f.)], in accordance with WOD. A fit of the inverse relation to the same data gives $L \propto R^{1.34\pm0.17}$ with $\chi^2 \sim 3.2$ per d.o.f. (Fig. 1). We note that the two fits are in principle not compatible. This is to be expected in a fit when the measurement errors are much smaller than the actual dispersion of the points, and just reflects the large value of χ^2.

A correlation between richness and velocity dispersion for Abell clusters is known to exist (Danese, De Zotti & di Tullio 1980; Cole 1989). In the subsample that we have extracted from WOD, a relation between total luminosity and velocity dispersion has been sought. We obtain $L \propto \sigma^{1.87\pm0.44}$, but (see Fig. 2) the dispersion ($\chi^2 = 1.9$ per d.o.f.) is uncomfortably large for this relation to be statistically acceptable. Such deviations have often been interpreted as being the result of error underestimates due to the existence of subclustering or to a possible erroneous identification of cluster members.

3.2 The fundamental plane for clusters

Searching, on the other hand, for a relation $L \propto R^\alpha \sigma^{2\beta}$, we obtain $\alpha = 0.89 \pm 0.15$ and $\beta = 0.64 \pm 0.11$, with (Fig. 3) a considerable improvement ($\chi^2 = 0.38$ per d.o.f.). *We must emphasize that the evidence of the galaxy cluster fundamental plane relies on the careful photometric work done by WOD.* Were we to use richness as an approximate luminosity, the correlation would be nearly lost, and this is why such a relation has not been noticed before. The key of the following argument is the low relative value of χ^2 for the fit. It shows that the actual errors on the determination of L, R_e and σ are by no means larger than the estimations that we used. Simply, Abell clusters form a two-parameter family and follow a very tight relation. If, for instance, we remove from our sample the four clusters belonging to well-known superclusters such as Coma, A2199, Corona Borealis and Perseus, and also A194 which is elongated and A401 which may interact with A399, the values of α (0.90) and β (0.66) are found to be within the quoted errors, with a stable χ^2. Subclustering or intruders can give errors that are at most at the 10 per cent level. The values of α and β that we find for clusters are close to those obtained in the literature for the galaxies: $\alpha = 0.75$, $\beta = 0.83$ from Dressler et al. (1987), $\alpha = 0.92$, $\beta = 0.93$ or $\alpha = 0.89$, $\beta = 0.77$ from Djorgovski & Davis (1987), with uncertainties comparable to ours, as well as those that we have determined using the sample of 15 dwarf and low-luminosity ellipsoidal galaxies ($\alpha = 0.85$, $\beta = 1$), and those determined for 33 globular clusters, $\alpha = 0.68 \pm 0.12$, $\beta = 0.71 \pm 0.05$ (a fit to the globular clusters with dispersions calculated from individual stars gives

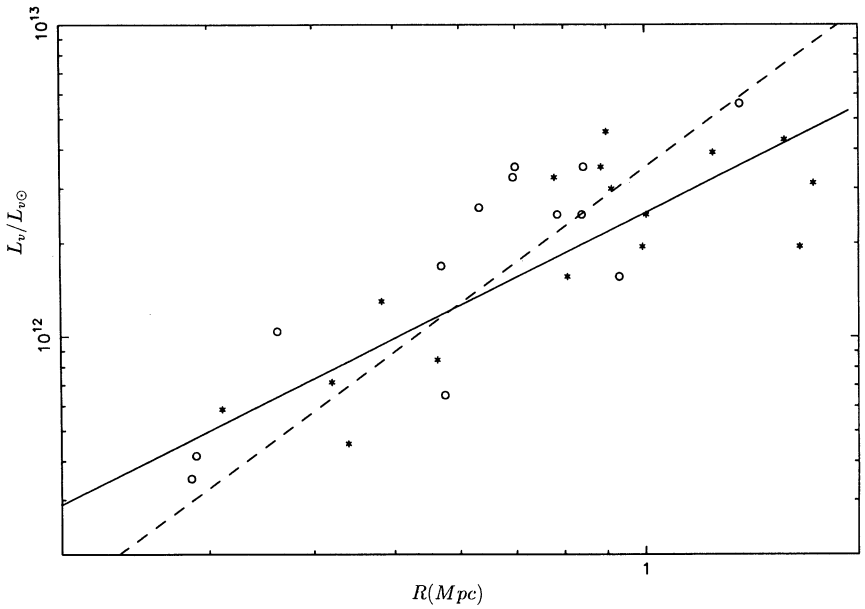

Figure 1. Luminosity–radius relation for the compilation (West et al. 1989) of 29 Abell clusters (R is the effective radius in Mpc, assuming $H_0 = 100$ km s^{-1} Mpc^{-1}). The star symbols represent the 16 clusters for which the velocity dispersion is known. The fits for an $R \propto L^{\delta}$ relation with $\delta = 0.5 \pm 0.1$ as in West et al. (1989) (dashed line), and for an $L \propto R^{\alpha}$ relation with $\alpha = 1.34 \pm 0.17$ (full line), are shown. Because the dispersion of the points is much larger than the measurement errors, the two procedures are not equivalent.

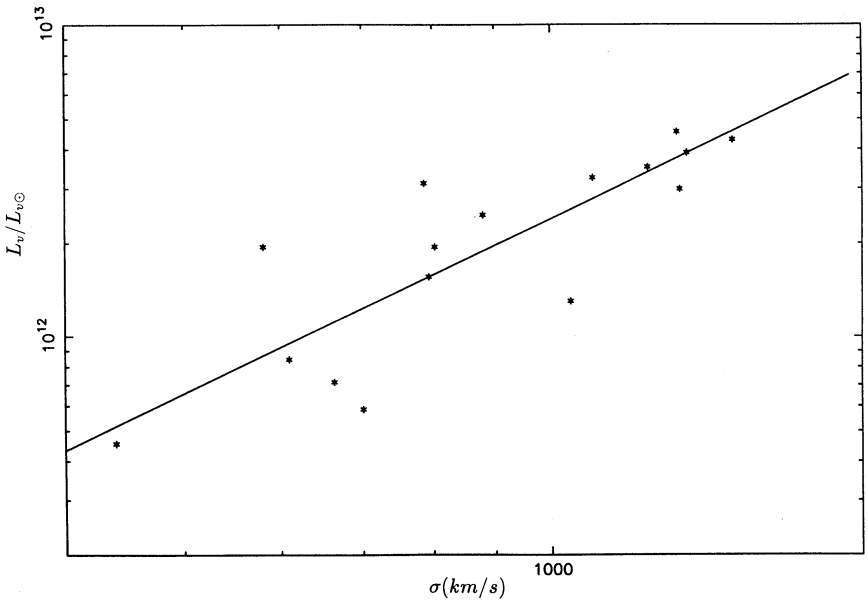

Figure 2. Luminosity–velocity dispersion relation, with a fit (full line) for $L \propto \sigma^{2\beta}$, $\beta = 0.94$.

$\alpha = 0.70$ and $\beta = 0.73$). In this sense, our conclusion is that a fundamental plane exists for *matter condensations with extremely different masses, i.e. globular clusters, galaxies and rich clusters* (Fig. 4). As α and β are not very different, we have fitted the relation $L = K(R\sigma^2)^{\gamma}$. This allows us to determine directly the constant K, which characterizes the gap between the planes, and to find the average mass-to-light (M/L) ratios for the different classes of object. The resulting values are summarized in Table 1, which shows the stability of the slope and the consistency of the average M/L ratios with the previous estimations of Oemler (1974) and Illingworth (1976).

3.3 Implications for structure formation

A natural outcome of structure formation models (Schaeffer & Silk, 1985, 1988) based on hierarchical clustering is that all mass condensations form a two-parameter family (see also Peacock & Heavens 1985, Kaiser 1988 and Peacock 1990, who addressed the same question). Primordial mass

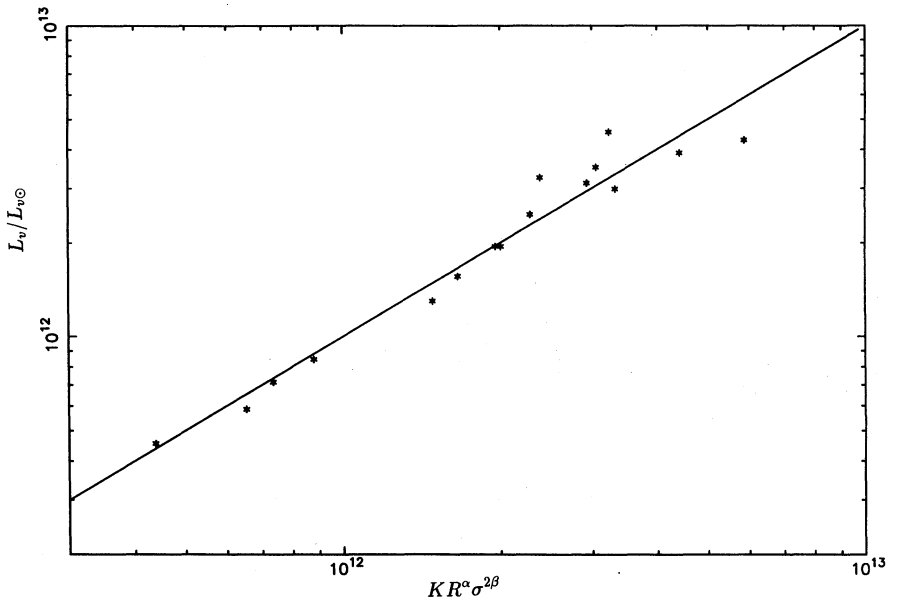

Figure 3. Relation between luminosity L and the product $R^{\alpha}\sigma^{2\beta}$. Note the excellent fit ($\alpha = 0.89$, $\beta = 0.64$ with a constant factor $K = 4 \times 10^8$), for which the χ^2 per degree of freedom is improved by a factor of 8 compared to the previous cases.

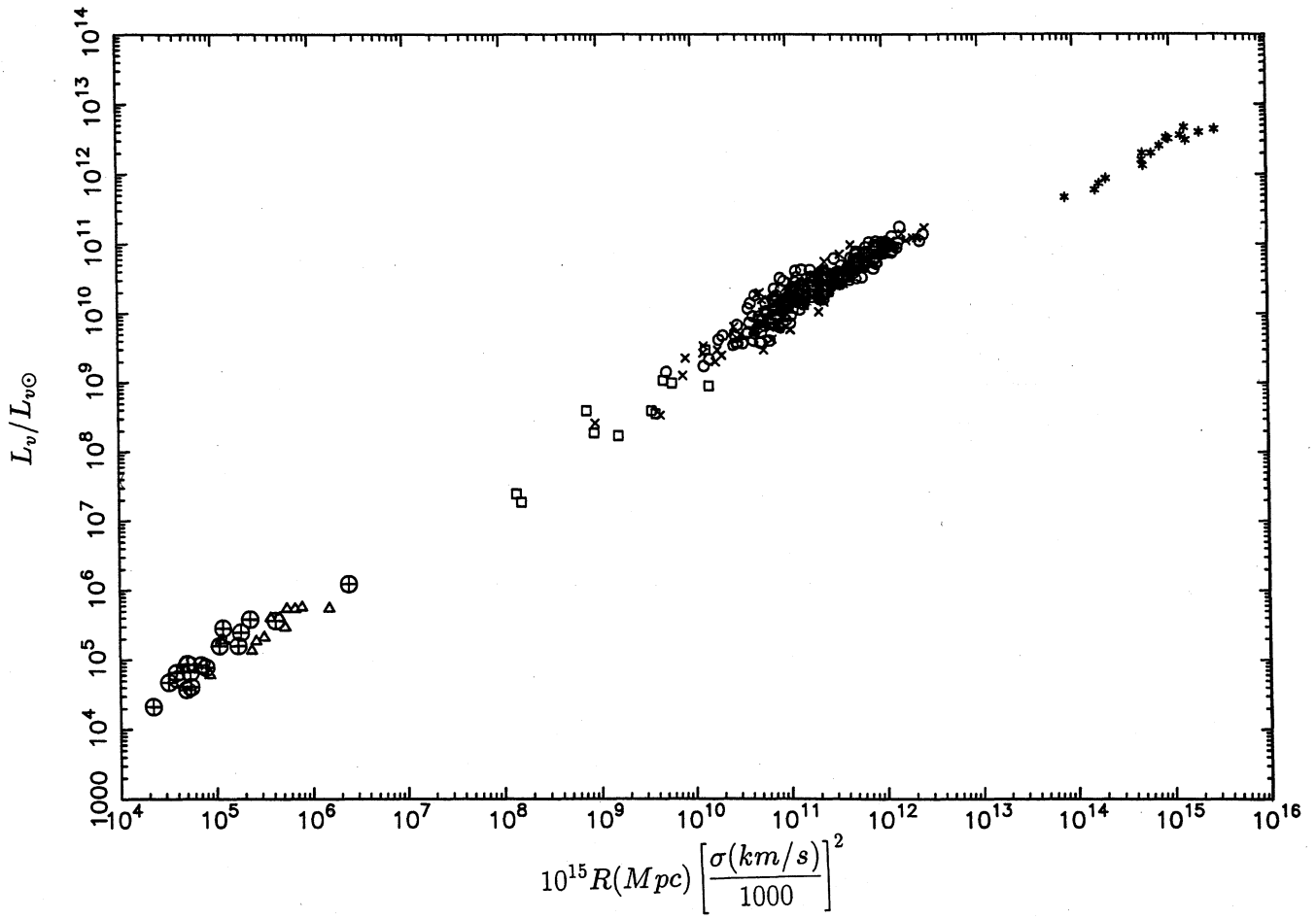

Figure 4. The 'fundamental plane' seen edge-on for different systems. Crossed circles: globular clusters with individual stellar spectra; triangles: globular clusters with integrated spectra; squares: dwarf and low-luminosity ellipsoidal galaxies (Bender & Nieto 1990; Bender et al. 1991); crosses: elliptical galaxies (Djorgovski & Davis 1987); circles: elliptical galaxies (Faber et al. 1989); stars: galaxy clusters (West et al. 1989; Struble & Rood 1991).

Table 1. Best-fitting values for the relation $L=K(R\sigma^2)^{\gamma}$ and relative average M/L ratios.

	γ	K	$\langle M/L\rangle/\langle M/L\rangle_{gal}$
Abell clusters	0.73	$1.1\ 10^8$	40
ellipticals [1]	0.78	$3.8\ 10^8$	1
ellipticals [2]	0.82	$3.0\ 10^8$	1
globular clusters	0.70	$7.4\ 10^7$	$0.1(H_0/100)^{-1}$

Notes: [1]Faber et al. (1989); [2]Djorgovski & Davis (1987).

fluctuations in an Einstein–de Sitter universe, $\delta(M)=\delta_0(M)(1+z)^{-1}$ at scale M and epoch t [related to the redshift by $t\propto(1+z)^{-3/2}$] are believed to follow a Gaussian random distribution with $\langle\delta_0^2(M)\rangle^{1/2}=\Sigma(M)$. A given fluctuation collapses when $\delta(M)\sim 1$, at an epoch $(1+z_{\rm form})\sim\delta_0(M)$, to an object with a final density $\rho\propto(1+z_{\rm form})^3\propto\langle\delta_0^3(M)\rangle$ which is proportional to the density of the universe at the formation time (Gott & Rees 1975; Peebles 1980). This implies $R\propto M^{1/3}/\delta_0(M)$ and $\sigma^2\propto M^{2/3}\delta_0(M)$. As a result of the fluctuations of δ_0, both R and σ fluctuate and are not simply a function of mass. The product $R\sigma^2\propto M$, however, is independent of δ_0. The relation between mass and luminosity, which originates from the intrinsic physical processes that govern star formation, has a very low dispersion. This implies that $M/L\propto L^{\varepsilon}$, where $\varepsilon=2/(\alpha+\beta)-1$ or $L\propto(R\sigma^2)^{1/(1+\varepsilon)}$, and is consistent with the previous findings provided that the difference $\alpha-\beta$ is compatible with zero. Indeed, we have $\alpha-\beta=0.2\pm0.2$ (Abell clusters), -0.1, 0.0 or 0.1 (galaxies: Dressler et al. 1987; Djorgovski & Davis 1987), -0.1 (dwarf galaxies: Bender & Nieto 1990) and 0.1 ± 0.2 (globular clusters). The mass is found to vary in nearly the same way as the luminosity: $\varepsilon=0.3\pm0.1$ (Abell clusters), 0.3, 0.1, 0.2 (galaxies), -0.1 (dwarf galaxies) and 0.4 ± 0.1 (globular clusters) respectively. Except for dwarf galaxies, the mass has a slight tendency to increase faster than the luminosity does. To show that M/L deviates significantly from a constant value would require further work. The point here is simply that M is much more tightly correlated to L than R or σ are: *the dispersion seen in the luminosity–radius or luminosity–velocity dispersion relation should reflect the dispersion in the epoch of formation.* Indeed, the values of χ^2 can be used to obtain the dispersion of δ_0, whence of $z_{\rm form}$: $\Delta_z\equiv[\langle(1+z_{\rm form})^2\rangle-\langle1+z_{\rm form}\rangle^2]^{1/2}/\langle1+z_{\rm form}\rangle$. The calculation can be schematically summarized as follows. We assume that the dispersion in radius is due to random observational errors with dispersion $\Delta_{\rm obs}$ as well as to a random redshift of formation with dispersion Δ_z, and link the latter to the observed dispersion in excess of the expected one. This procedure is obviously sensitive to the adopted observational errors which are usually not determined with an extremely high accuracy. We find $\Delta_z\sim0.2$–0.3 for Abell clusters and $\Delta_z\sim0.4$–0.6 for elliptical galaxies.

A theoretical formulation (Schaeffer & Silk 1985) based on hierarchical clustering, along the lines developed by Press & Schechter but with the difference in formation times explicitly taken into account, leads to $\Delta_z=[\Sigma(M)/\delta_c]^2\sim0.1$, with $\delta_c\sim1.7$, for rich clusters [$\Sigma(M)\sim0.6$], a value that increases for the smaller mass objects to $\Delta_z=\sqrt{\pi/2-1}=0.76$ at scales where the fluctuations $\Sigma(M)$ are large, as is expected for galaxies. These theoretical results are comparable to the observed values. In any case, more work is required to take into account not only observational errors, but also the effect of the distinct dynamical processes which led to the formation of galaxy clusters, galaxies and globular clusters.

Moreover, if we assume that the distance of each cluster from the fit of the fundamental plane is entirely due to its pecular motion, it follows that most of the cluster peculiar velocities relative to the cosmic microwave background are less than 1000 km s^{-1}.

4 CONCLUSIONS

We have shown that galaxy clusters lie in the fundamental plane. This common property of stellar systems and galaxy clusters suggests a similar process of formation, favouring the hierarchical clustering scenario; the dispersions in the observed relations L–R and L–σ should reflect the dispersion in the epoch of formation.

The small dispersion (8 per cent) could allow a direct estimation of distances for galaxy clusters, giving access to their large-scale motions. Moreover, the existence of the fundamental plane gives us further information on the distribution of luminous and dark components of matter in clusters. We shall analyse these subjects in future work.

Accurate measurements on a larger sample are urgently required to have better estimates of the relevant parameters; in particular, a careful determination of the measurement errors is important. An observational programme is in progress in order to test the relation on 10 more clusters.

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