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# Differential Equivalences on SBoxes

**Valentin SUDER**

(Université Rouen Normandie, France)

Joint work with **Christina BOURA** (UVSQ), **Anne CANTEAUT** (inria Paris, France) and  
**Jérémy JEAN** (ANSSI, France)

**CAEN2018**, Cryptography and Algorithmic Number Theory  
June 21, 2018

# Outline

## Introduction

Symmetric cryptography

Differentiality

Equivalences

## Results

Equivalences

Main contributions

First Step for a classification

Algorithm

Experimental Results

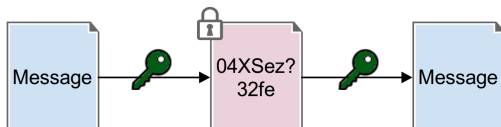
Special case

## Conclusion

# Symmetric Cryptography

## Symmetric Cryptography

Cryptographic **algorithms** and **protocols** where the secret is **identical** between communicating parties.



- ▶ **Block ciphers**
- ▶ Stream ciphers
- ▶ Hash functions

# Design of Block ciphers

## Block cipher

A **block cipher** is a function

$$\begin{aligned} E : \mathbb{F}_2^k \times \mathbb{F}_2^m &\rightarrow \mathbb{F}_2^m \\ (k, P) &\mapsto C = E(k, P) := E_k(P) \end{aligned}$$

such that for any **fixed key**  $k \in \mathbb{F}_2^k$ ,  $E_k$  is *bijective* over  $\mathbb{F}_2^m$ .

Problem?

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$$\text{SB} : \mathbb{F}_2^m = \mathbb{F}_2^n \times \mathbb{F}_2^n \times \dots \rightarrow \mathbb{F}_2^m = \mathbb{F}_2^n \times \mathbb{F}_2^n \times \dots$$

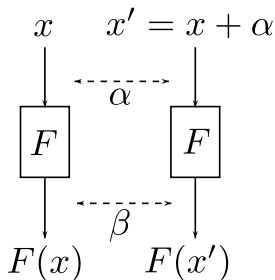
$$X = (x_0, x_1, \dots) \mapsto (F_0(x_0), F_1(x_1), \dots)$$

Functions  $F_i$ 's are usually called *SBoxes*.



## Differential Criteria

$$F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$$



Derivatives in direction  $\alpha \in \mathbb{F}_2^n$

$$\begin{aligned} \Delta_\alpha F : \mathbb{F}_2^n &\rightarrow \mathbb{F}_2^n \\ x &\mapsto F(x) + F(x + \alpha) \end{aligned}$$

Differential Uniformity [Nyberg 94]

$$\delta_F = \max_{\alpha \in \mathbb{F}_2^n \setminus \{0\}, \beta \in \mathbb{F}_2^n} \# \{x \mid \Delta_\alpha F(x) = \beta\}$$

Whenever  $\delta_F = 2$  (**minimal** value), the function  $F$  is said to be **APN** ('Almost Perfect Nonlinear').

# Example

## Differences Distribution Table (DDT)

Table of size  $2^n \times 2^n$ , with entries:

$$\mathcal{D}_F(\alpha, \beta) = \#\{x \in \mathbb{F}_2^n \mid \Delta_\alpha F(x) = \beta\}, \forall \alpha, \beta \in \mathbb{F}_2^n.$$

$\alpha$	$\mathcal{D}_F$	$\beta$								$\alpha$	$\mathcal{D}_{F'}$	$\beta$							
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
	0	8	.	.	.	.	.	.	.		0	8	.	.	.	.	.	.	.
	1	.	2	.	2	.	2	.	2		1	.	.	8	.	.	.	.	.
	2	.	.	2	2	.	.	2	2		2	.	.	.	4	.	.	4	.
	3	.	2	2	.	.	2	2	.		3	.	2	.	.	6	.	.	.
	4	.	.	.	.	2	2	2	2		4	.	.	.	.	2	.	2	4
	5	.	2	.	2	2	.	2	.		5	.	2	.	2	.	.	2	2
	6	.	.	2	2	2	2	.	.		6	.	2	.	2	.	4	.	.
	7	.	2	2	.	2	.	.	2		7	.	2	.	.	.	4	.	2

# Big APN Problem

**APN functions**  $\Rightarrow$  Best resistance to **differential cryptanalysis!**

1. **Very few** known APN functions . . .
2. . . . even fewer known **bijjective** APN functions (none when  $n = 4$ )

## 'Big APN Problem' :

Is there any **bijjective APN functions** over  $\mathbb{F}_{2^n}$ , when  $n$  is **even** ?

**Answer (?)** : In 2009, Dillon et al. found **only one** example for  $n = 6$ . Since then, **nothing more** (or very little) . . .

3. **Few** (bijjective) functions are known with  $\delta = 4$ .



K. A. Browning, J. F. Dillon, M. T. McQuistan and A. J. Wolfe,  
*An APN Permutation in Dimension Six*,  
Fq9 (Selected Papers), Contemporary Mathematics, 2010.

## Some equivalence classes

$$F : \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n}$$

The **differential uniformity**  $\delta(F)$  of  $F$  is an **invariant** under:

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- ▶ Graph equivalence:  $\mathcal{G}_{F'} = \{\mathcal{L}(x, F(x))\} = \mathcal{L}(\mathcal{G}_F)$ ,  $\delta(F) = \delta(F')$   
 $\Rightarrow$  **CCZ-equivalence** ( $\sim_{CCZ}$ )

$$\mathcal{M}\mathcal{G}_F = \begin{pmatrix} \begin{array}{c} | \\ 1 \\ | \end{array} & \dots & \begin{array}{c} | \\ x \\ | \end{array} & \dots & \begin{array}{c} | \\ 2^n - 1 \\ | \end{array} \\ \hline \begin{array}{c} | \\ F(1) \\ | \end{array} & \dots & \begin{array}{c} | \\ F(x) \\ | \end{array} & \dots & \begin{array}{c} | \\ F(2^n - 1) \\ | \end{array} \end{pmatrix}$$

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## Link with other areas

$$F : \mathbb{F}_{2^n} \rightarrow \mathbb{F}_{2^n}$$

Carlet-Charpin-Zinoviev (1998)

$F$  is **APN if and only if** the **minimal distance** of the linear code with **parity check matrix**  $\mathcal{MG}_F$  is 5.



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Let  $F$  be **quadratic**, i.e.  $F(x) = \sum_{i,j} c_{i,j} x^{2^i+2^j} \in \mathbb{F}_{2^n}[x]$ .

$F$  **APN**  $\Rightarrow$  **semifields, dual hyperoval, rank-metric code,...**

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not only **cryptography** benefits from research on **APN functions**.

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# Differential equivalences of SBoxes

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Definitions :

## Differences Distribution Table (DDT)

Table of size  $2^n \times 2^n$ , with entries:

$$\mathcal{D}_F(\alpha, \beta) = \# \{x \in \mathbb{F}_2^n \mid \Delta_\alpha F(x) = \beta\}, \forall \alpha, \beta \in \mathbb{F}_2^n.$$

## Indicator of a DDT

Boolean function  $\gamma_F : \mathbb{F}_2^{n \times n} \rightarrow \mathbb{F}_2$ ,

$$\gamma_F(\alpha, \beta) = 0 \Leftrightarrow \delta_F(\alpha, \beta) = 0 \text{ or } \alpha = 0.$$

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$\Rightarrow$  Two equivalences:

$$\blacktriangleright \quad F \sim_{\mathcal{D}} G \quad \Leftrightarrow \quad \mathcal{D}_F = \mathcal{D}_G,$$

$$\blacktriangleright \quad F \sim_{\gamma} G \quad \Leftrightarrow \quad \gamma_F = \gamma_G.$$

# Motivation, Origin

## Differential Equivalence

In **odd** characteristic, **Perfect Nonlinear** (PN) functions are such that all of their derivatives are **bijective**  $\Rightarrow$  all PN functions have the **same DDT**.

Gorodilova introduced the  **$\gamma$ -equivalence**, while working on **APN monomial quadratic** functions (aka **Gold** functions, aka APN functions of the type  $x \mapsto x^{2^i+1}$ ).



A Gorodilova,

*On a remarkable property of APN Gold functions,*  
[Cryptography ePrint Archive, 2016.](#)







# Examples

n=4

$$F = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 14],$$

$$G = [0, 1, 3, 2, 5, 4, 7, 6, 8, 9, 10, 11, 12, 13, 14, 15].$$

$$\gamma_G = \begin{pmatrix} 0 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & 1 & 1 & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & 1 & 1 & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & 1 & 1 & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & 1 & 1 & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & 1 & 1 & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & 1 & 1 & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & 1 & 1 & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & 1 & 1 & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & 1 & 1 & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & 1 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & 1 \end{pmatrix}$$

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# Properties

$$F, G : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$$

Basic:

$$\blacktriangleright F \sim_{\mathcal{D}} G \Rightarrow F \sim_{\gamma} G \quad (\not\Leftarrow)$$

$\blacktriangleright$  for any  $a, b \in \mathbb{F}_2^n$ ,

$$F(x) \sim_{\mathcal{D}} G(x) = F(x + a) + b$$

in that case,  $F$  and  $G$  are said to be **trivially equivalent**.

A little more advanced:

$\blacktriangleright$  if  $F$  and  $G$  are either **APN** or **quadratic** (or both), then

$$F \sim_{\mathcal{D}} G \Leftrightarrow F \sim_{\gamma} G$$

What are the functions that share the same **DDT**?  
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the same **indicator**? How many of them exist?

### Contributions :

- Using **EA-Eq** and **CCZ-Eq** to *help* the **classification**
- a **recursive algorithm** to compute **equivalence classes**.  
*Input* : a **DDT** (resp. **indicator**),  
*Output* : **every functions** having this **DDT** (resp. **indicator**)
- **research** for some **known functions**.
- **new fact** about **APN permutations**.



C. Boura, A. Canteaut, J. Jean and V. Suder,  
*Two Notions of Differential Equivalence on SBoxes*,  
DCC, Design, Codes and Cryptography, To appear.

## Relation to CCZ-Eq and EA-Eq

Gorodilova (2016)

For  $F \sim_{EA} G$ ,

$$\forall F' \sim_{\gamma} F \Rightarrow \exists G' \sim_{\gamma} G \text{ s.t. } G' \sim_{EA} F'$$

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For  $F \sim_{CCZ} G$ ,

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*Input* : a DDT (resp. indicator),

*Output* : **every functions** having this DDT (resp. indicator)

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  - find **all possible values** for  $F(i)$
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**Pruning trick:** We can **fix**  $F(0)$  and  $F(1)$ :

$$G(x+1) + G(1) := F(x) \sim_{\mathcal{D}} G(x)$$

so that

$$F(0) = 0, \text{ and } F(1) = \Delta_1 F(0) = \Delta_1 G(0)$$

# Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
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7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$
1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$   
Set  $F(1) = 1$

# Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$
1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$   
Set  $F(1) = 1$
2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$



# Example

 $\mathbb{F}_2^3$ 

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$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$

# Example

 $\mathbb{F}_2^3$ 

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$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$
1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$   
Set  $F(1) = 1$
2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus  
 $F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

$$3. F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$$

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1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

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 $\mathbb{F}_2^3$ 

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0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$

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1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$   
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0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

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2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus  
 $F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

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 $\mathbb{F}_2^3$ 

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0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus  
 $F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

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 Set  $F(1) = 1$
2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus  
 $F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$

1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$

## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$

1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$  and  
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$



## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$

1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$  and  
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$  and  
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$

## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

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1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$  and  
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$  and  
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$  and  
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$

## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$

1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and

$F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$  and  
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$  and  
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$  and  
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$  thus

$F(4) \in \{\emptyset\}$  if  $F(2) = 3$  and  $F(3) = 2$

## Example

 $\mathbb{F}_2^3$ 

$$\mathcal{R}_i := \{j \mid \mathcal{D}(i,j) \neq 0\}$$

$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$

1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$  and  
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$  and  
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$  and  
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$  thus

$F(4) \in \{\emptyset\}$  if  $F(2) = 3$  and  $F(3) = 2$

$F(4) \in \{7\}$  if  $F(2) = 3$  and  $F(3) = 6$

$F(4) \in \{5\}$  if  $F(2) = 7$  and  $F(3) = 2$

$F(4) \in \{\emptyset\}$  if  $F(2) = 7$  and  $F(3) = 6$

## Example

 $\mathbb{F}_2^3$ 

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$\mathcal{D}$	0	1	2	3	4	5	6	7
0	8	.	.	.	.	.	.	.
1	.	2	.	2	.	2	.	2
2	.	.	2	2	.	.	2	2
3	.	2	2	.	.	2	2	.
4	.	.	.	.	2	2	2	2
5	.	2	.	2	2	.	2	.
6	.	.	2	2	2	2	.	.
7	.	2	2	.	2	.	.	2

0. Set  $F(0) = 0$

1.  $F(0) + F(1) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$

Set  $F(1) = 1$

2.  $F(0) + F(2) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(1) + F(2) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  thus

$F(2) \in F(0) + \mathcal{R}_1 \cap F(1) + \mathcal{R}_3 = \{3, 7\}$

3.  $F(0) + F(3) \in \mathcal{R}_3 = \{1, 2, 5, 6\}$  and  
 $F(1) + F(3) \in \mathcal{R}_2 = \{2, 3, 6, 7\}$  and  
 $F(2) + F(3) \in \mathcal{R}_1 = \{1, 3, 5, 7\}$  thus

$F(3) \in \{2, 6\}$  (OR  $\{2, 6\}$ )

4.  $F(0) + F(4) \in \mathcal{R}_4 = \{4, 5, 6, 7\}$  and  
 $F(1) + F(4) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$  and  
 $F(2) + F(4) \in \mathcal{R}_6 = \{2, 3, 4, 5\}$  and  
 $F(3) + F(4) \in \mathcal{R}_7 = \{1, 2, 4, 7\}$  thus

$F(4) \in \{\emptyset\}$  if  $F(2) = 3$  and  $F(3) = 2$

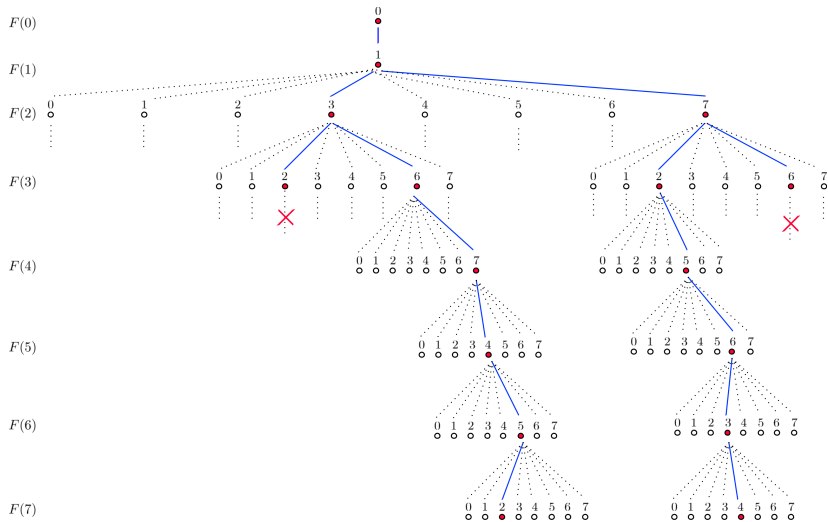
$F(4) \in \{7\}$  if  $F(2) = 3$  and  $F(3) = 6$

$F(4) \in \{5\}$  if  $F(2) = 7$  and  $F(3) = 2$

$F(4) \in \{\emptyset\}$  if  $F(2) = 7$  and  $F(3) = 6$

5.  $F(0) + F(5) \in \mathcal{R}_5 = \{1, 3, 4, 6\}$

$\vdots$



## Known functions

$F$  and  $G$  are *trivial* if  $F(x) \sim_{\mathcal{D}} G(x) = F(x + a) + b$

[Gorodilova'16]  $\rightarrow$  (non-bijective) APNs with **non-trivial**  $\gamma$ -classes.

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What are the DDT-classes of '*optimal*' bijective SBoxes?



## Known functions

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[Gorodilova'16]  $\rightarrow$  (non-bijective) APNs with **non-trivial**  $\gamma$ -classes.

What are the DDT-classes of '*optimal*' bijective SBoxes?

- ▶  $n = 6$ : Dillon permutation  $\rightarrow$  **trivial** DDT-class
- ▶  $n = 5$ : five APN permutations  $\rightarrow$  **trivial** DDT-classes
- ▶  $n = 4$ : 16 4-differentially permutations  $\rightarrow$  **trivial** DDT-classes
- ▶  $n = 3$ : Gold permutation ( $x \mapsto x^3$ )  $\rightarrow$  **trivial** DDT-class

## Known functions

$F$  and  $G$  are *trivial* if  $F(x) \sim_{\mathcal{D}} G(x) = F(x + a) + b$

[Gorodilova'16]  $\rightarrow$  (non-bijective) APNs with **non-trivial**  $\gamma$ -classes.

What are the DDT-classes of '*optimal*' bijective SBoxes?

- ▶  $n = 6$ : Dillon permutation  $\rightarrow$  **trivial** DDT-class
- ▶  $n = 5$ : five APN permutations  $\rightarrow$  **trivial** DDT-classes
- ▶  $n = 4$ : 16 4-differentially permutations  $\rightarrow$  **trivial** DDT-classes
- ▶  $n = 3$ : Gold permutation ( $x \mapsto x^3$ )  $\rightarrow$  **trivial** DDT-class

**Question:** Are DDT-classes for permutations always trivial? **NO**

## Known functions and conjecture

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### Conjecture

For a bijective function,  
the **rows** of its DDT are all **distinct**  $\Rightarrow$  its **DDT-class** is **trivial**.

# Bijjective APN functions and their DDTs

## Theorem

Let  $F : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$  be a **bijjective APN function**

**Then**, the rows of its DDT are pairwise **distinct**.

(the image sets of the derivatives of  $F$  are pairwise distinct.)

**Proof Idea:** by contradiction. We suppose two rows match, and then by induction on the images of the derivatives, we show that  $F$  is either not APN, or not a permutation.

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(!!) if the conjecture is **true**, DDT-classes of **APN permutations** are always **trivial**.

# Outline

## Introduction

- Symmetric cryptography
- Differentiability
- Equivalences

## Results

- Equivalences
- Main contributions
- First Step for a classification
- Algorithm
- Experimental Results
- Special case

## Conclusion

► **Contributions:**

- Characterize and extend [Gorodilova'16] equivalence
- an algorithm that computes classes
- a new property about APN permutations
- a conjecture on what are the functions with non-trivial classes
- ...

► **Future works:**

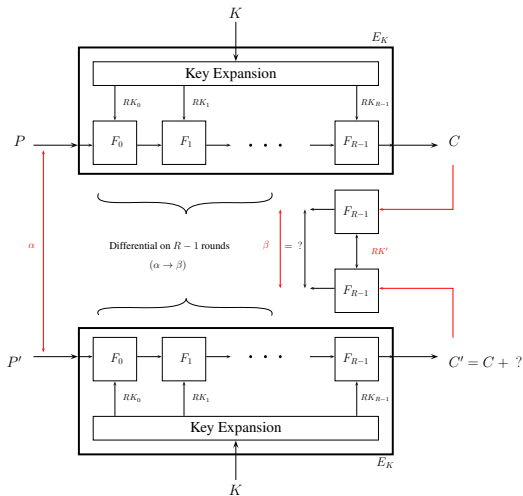
- ... prove the conjecture ('easier' cases, e.g. quadratic APN)
- what does it mean for a function to have derivatives with the same image set?
- what are 'possible' DDTs? (use and improve the algorithm)



C. Boura, A. Canteaut, J. Jean and V. Suder,  
*Two Notions of Differential Equivalence on SBoxes*,  
DCC, Design, Codes and Cryptography, To appear.



# Differential Cryptanalysis of the last round



1. Encrypt  $N$  pairs  $P$ ,  
 $P' = P + \alpha$
2. For each pairs, decrypt  
the last round with  $RK'$
3. Increment a counter if  
difference is  $\beta$
4. Check which counter is  
the higher (closer from  
 $N \times \mathbb{P}$ )



Eli Biham and Adi Shamir,  
*Differential Cryptanalysis of DES-like Cryptosystems*,  
J. Cryptology 4(1), 1991.