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New implementation of stability-based transition model by means of transport equations

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A new natural laminar-turbulent transition model compatible with Computation Fluid Dynamics is presented. This model accounts for longitudinal transition mechanisms (i.e. Tollmien-Schlichting induced transition) thanks to systematic stability computation on similar boundary profiles from Mach zero to four both on adiabatic and isothermal wall. The model embeds as well the so-called “C1-criterion” for transverse transition mechanisms (i.e. cross-flow waves induced transition). The transition model is written under transport equations formalism and has been implemented in the solver elsA (ONERA-Airbus-Safran property). Comparisons are performed on two-dimensional and three-dimensional configurations against transition database approach.

I. Nomenclature

γ	=	Intermittency
Λ_2	=	Polhausen parameter $\Lambda_2 = \frac{\theta^2}{\nu_e} \frac{dU_e}{ds}$
M	=	Mach number
Re_{δ_2}	=	Transverse displacement thickness based Reynolds number
Re_θ	=	Momentum thickness based Reynolds number
s	=	Curvilinear abscissa
T_u	=	Turbulence level

Subscript(s)

e	=	Edge of the boundary layer
i	=	Incompressible
cr	=	Critical point of the boundary layer
tr	=	Transition location

II. Introduction

Accurate computation of transport aircraft drag strongly relies on natural laminar-turbulent transition prediction capabilities. As computational fluid dynamics (CFD) is now a major component of industrial processes, it is necessary to develop accurate transition prediction techniques for RANS solvers both for aerodynamic performance prediction and design of future laminar transport aircraft concept.

The development of transition prediction methods compatible with Computation Fluid Dynamics (CFD) is a major research topic. A quite recent approach consists in using methods based on Partial Differential Equations (PDE). This approach consists in solving additional transport equations governing the dynamic of quantities that are related to transition. The most famous is probably the “ $\gamma - Re_\theta$ ” of Langtry and Menter [1] based on phenomenological reasoning.

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This method has demonstrated success on many configurations and has been extended to handle as well cross-flow transition [2]. The Amplification Factor Transport (AFT) method was derived more recently by Coder and Maughmer [3]. This promising method consists in writing under a transport equation the e^N method [4, 5] of Drela and Giles [6]. The $\gamma - Re_\theta$ and AFT methods are said to be “local” in the sense that the additional transport equations associated to transition only involve values available at RANS computational points. This property reduces much the implementation effort in a RANS solver.

As far as the elsA RANS solver (property of Airbus-Safran-ONERA) is concerned, developments have been conducted to give access to non local variables (for instance integral boundary layer variables) at grid point. This feature has been used to implement the AHD criterion evaluated along mesh lines [7] and the so-called “parabola method” [8] for transition prediction by means of transport equations [9].

This paper presents the implementation of the AHD and C1 criteria by means of transport equations and their comparison with the parabola method. These criteria are presented in section III while section IV deals with their implementation in a RANS solver. Numerical results are presented in section V.

III. AHD/C1 transition criterion and parabola method for transition prediction

A. AHD criterion

The Arnal-Habiballah-Delcourt (AHD) criterion [10] is derived by expressing N -factor associated to Tollmien-Schlichting instabilities obtained on similar profiles as curves of the form $N = N(R_\theta - Re_{\theta,cr}, \Lambda_2)$. The method consists then in replacing the Polhausen parameter Λ_2 by its average value $\bar{\Lambda}_2$ between the critical point (i.e. the location from which Tollmien-Schlichting instabilities start to grow) of curvilinear abscissa s_{cr} and the current location of curvilinear abscissa s (measured along the streamline at the edge of the boundary layer). Using the Mack’s relationship ($N_T = -2.4 \ln(T_u/100) - 8.43$) finally gives the transitional Reynolds number $Re_{\theta,tr}$ as:

$$Re_{\theta,tr} = Re_{\theta,cr} + A \exp(B\bar{\Lambda}_2) \left(\ln(CT_u) - D\bar{\Lambda}_2 \right) \quad (1)$$

where $Re_{\theta,cr}$ is the Reynolds number at the critical point and is given by $Re_{\theta,cr} = \exp(G/H_i^2 + E/H_i - F)$. This criterion account for receptivity through T_u and for flow history through $\bar{\Lambda}_2$.

In its compressible extension [11], the variables A, B, C, D, E, F, G are function of M_e , the Mach number at the edge of the boundary layer. Moreover, this criterion account for effects of wall temperature [11, section V].

B. C1 criterion

The C1 criterion [12] is used to predict natural transition induced by cross-flow instabilities. This criterion defines the transition point as the location where the transverse incompressible displacement thickness Reynolds number $R_{\delta_{2i}}$ equals a threshold given by:

$$Re_{\delta_{2,i,tr}} = \begin{cases} 150 & H_i \leq 2.31 \\ \frac{300}{\pi} \arctan \left(\frac{0.106}{(H_i - 2.3)^{2.052}} \right) \left(1 + \frac{\gamma - 1}{2} M_e^2 \right) & 2.31 < H_i < 2.65 \end{cases} \quad (2)$$

(the criterion should not be applied for $H_i > 2.65$).

IV. Implementation in a CFD solver

A. Non local variables

Evaluating eqs. (1) or (2) requires the knowledge of boundary layer variables Re_θ , Re_{δ_2} , M_e , etc Contrary to the approaches presented in Ref. [1] or [3], the present method does not rely on correlations between on local variables. Non local variables are evaluated and made available at each cell point in elsA thanks to the fact that it is possible to: i) to get for a cell in the volume the associated wall interface (if there is any) and ii) to know which cells in the volume form the line normal to a given wall interface.

B. “Transition lines method”

An implementation of the AHD criterion has been proposed by Cliquet et al. [13]. It consists in assuming that streamlines at the boundary layer edge might be approximated by mesh lines. The implementation is denoted thereafter as “transition lines method”. This method has been implemented in the elsA CFD solver and has shown good results on aircraft configuration [14], helicopter blades flow [15], etc . . . However, this method requires some effort from the user as the latter is asked to prescribe the starting points of each transition line.

C. Transport equations approach

To alleviate user effort and to account with higher fidelity for three dimensional geometries where streamlines directions might strongly differ from mesh lines, a new implementation of the AHD/C1 criteria has been derived. This implementation is based on transport equations and shares similarities with the method derived by Bégou et al. [9].

The first prerequisite to estimate the transition threshold $Re_{\theta, tr}$ following eq. (1) is to know the value of the critical Reynolds number downstream of the critical location. To answer this need, the transported variable $\widetilde{Re}_{\theta, cr}$ governed by:

$$\partial_t (\rho \widetilde{Re}_{\theta, cr}) + \nu_{cr} \nabla \cdot (\rho \widetilde{Re}_{\theta, cr} \underline{U}) = (1 - \nu_{cr}) \Gamma_{\widetilde{Re}_{\theta, cr}} (\widetilde{Re}_{\theta, cr} - Re_{\theta, cr}) \quad (3)$$

is introduced. ν_{cr} equals one where $Re_{\theta} \geq \widetilde{Re}_{\theta, cr, e}$ and zero elsewhere. As a consequence, as long as the boundary layer is not critical the source term forces $\widetilde{Re}_{\theta, cr}$ to equal $Re_{\theta, cr}$ and if the boundary layer becomes critical $\widetilde{Re}_{\theta, cr}$ is simply convected.

The second prerequisite is to compute the value of $\widetilde{\Lambda}_2$. To do so, a second transport equation is introduced:

$$\partial_t (\rho \widetilde{\Lambda}_2) + \nabla \cdot (\rho \widetilde{\Lambda}_2 \underline{U}) = \nu_{cr} \frac{\rho \|\underline{U}\|}{\tilde{s}} (\Lambda_2 - \widetilde{\Lambda}_2) + \Gamma_{\widetilde{\Lambda}_2} (1 - \nu_{cr}) \rho (\widetilde{\Lambda}_2 - \Lambda_2) \quad (4)$$

where \tilde{s} is an additional transported variable corresponding to the curvilinear abscissa measured from the critical point (upstream of the critical point \tilde{s} equals zero). \tilde{s} is governed by:

$$\partial_t (\rho \tilde{s}) + \nabla \cdot (\rho \tilde{s} \underline{U}) = \rho \|\underline{U}\| - \Gamma_s \rho (1 - \nu_{cr}) \tilde{s}. \quad (5)$$

The transition threshold on Re_{θ} given by (1) can then be evaluated from $\widetilde{\Lambda}_{2, e}$ and $\widetilde{Re}_{\theta, cr, e}$ obtained by extracting $\widetilde{\Lambda}_2$ and $\widetilde{Re}_{\theta, cr}$ at the edge of the boundary layer. A last equation is then added to set the intermittency of the transition point given by $Re_{\theta} > Re_{\theta, cr}$:

$$\partial_t (\rho \widetilde{I}) + \nabla \cdot (\rho \widetilde{I} \underline{U}) = \nu_{tr} \rho \|\underline{U}\| - \Gamma_I \rho (1 - \nu_{tr}) \widetilde{I}. \quad (6)$$

ν_{tr} equals one if $Re_{\theta} \geq Re_{\theta, tr}$ or $Re_{\delta_{2, i}}$ reaches $Re_{\delta_{2, i, tr}}$ (given by Eq. (2)). As a consequence \widetilde{I} corresponds to a curvilinear abscissa measured from the transition point (either induced by longitudinal or transverse instabilities). Its value at the edge of the boundary layer \widetilde{I}_e is then used to evaluate the intermittency following (in the current implementation, the intermittency is set constant in the whole boundary layer profile):

$$\gamma(\widetilde{I}_e) = 1 - \exp\left(-5 \left(\frac{\widetilde{I}_e}{l_{tr}}\right)^2\right). \quad (7)$$

Eq. (7) is derived from Refs. [16, Eqs. (1,6)]. Currently, the user is asked to prescribe l_{tr} .

Further details will be given in the final paper

V. Validations

Flows over two-dimensional and three-dimensional geometries are considered to validate the prediction method over a wide-range of Mach numbers. Validations are performed against results obtained with the 3C3D solver of ONERA which has been shown to give excellent results for instance in Ref. [14]. The latter solves the boundary layer equations on three dimensional geometries. It embeds the AHD/C1 transition criteria and the parabola method.

Computations are performed as well with the method derived by Bégou et al. [9]. *In the final paper, further details will be given about this latter method.*

eISA computations are performed with a second order Roe spatial scheme and a backward Euler time scheme. $k - \omega$ SST turbulence of Menter [17] is chosen.

1. Nacelle transition prediction

While numerical validations of transition models in CFD are usually made on wings, numerical results on a generic nacelle configuration are shown in this section. A cut view of the geometry and of the surface mesh is shown Fig. 1. There are about 3×10^6 points in the mesh. The turbulence level is set to $T_u = 0.1\%$.

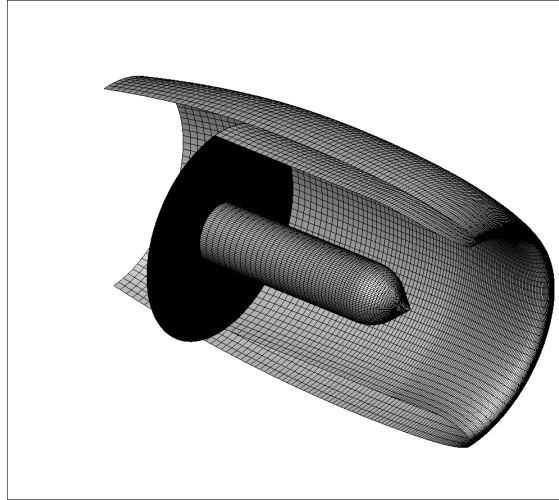


Fig. 1 Cut view of the nacelle. Pressure boundary condition is imposed on the black surface.

The computed intermittency is plotted Figs. 2(a,b) at the outer and inner sides of the nacelle. For the sake of visibility, both sides are “unrolled”.

Good agreement is obtained against the transition line (defined as the location where γ starts to grow) given by the boundary layer equations solver 3C3D.

Further comparisons and validations cases will be presented in the final paper

VI. Conclusion

A conclusion will be given here in the final paper

References

- [1] Langtry, R. B., and Menter, F. R., “Correlation-Based Transition Modeling for Unstructured Parallelized Computational Fluid Dynamics Codes,” *AIAA Journal*, Vol. 47, No. 12, 2009, p. 2894–2906.
- [2] Grabe, C., Shengyang, N., and Krumbein, A., “Transport Modeling for the Prediction of Crossflow Transition,” *AIAA Journal*, Vol. 56, No. 8, 2018, pp. 3167–3178.
- [3] Coder, J. G., and Maughmer, M. D., “Computational Fluid Dynamics Compatible Transition Modeling Using an Amplification Factor Transport Equation,” *AIAA Journal*, Vol. 52, No. 11, 2014, p. 2506–2512.
- [4] Van Ingen, J. L., “A suggested semi-empirical method for the calculation of the boundary layer transition region,” Tech. Rep. VTH-74, Dept. of Aerospace Engineering, Delft Univ. of Technology, 1956.
- [5] Smith, A. M. O., and Gamberoni, N., “Transition, pressure gradient, and stability theory,” Tech. Rep. ES-26388, Douglas Aircraft, 1956.

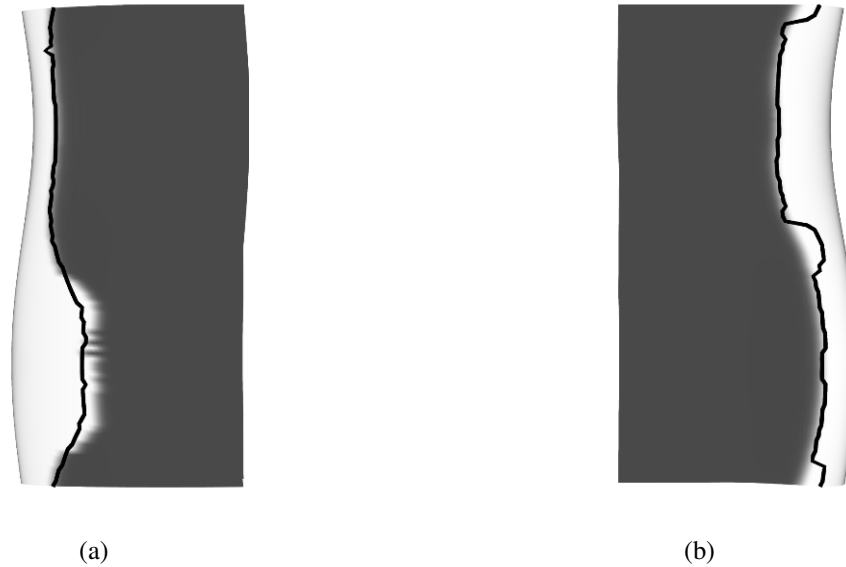


Fig. 2 Intermittency contours (light and dark corresponds respectively to $\gamma = 0$ and $\gamma = 1$) at the outer (a) and inner (b) sides of the nacelle. The black line depicts the transition location predicted by 3C3D.

- [6] Drela, M., and Giles, M. B., “Viscous-inviscid analysis of transonic and low Reynolds number airfoils,” *AIAA journal*, Vol. 25, No. 10, 1987, pp. 1347–1355.
- [7] Perraud, J., Deniau, H., and Casalis, G., “Overview of transition prediction tools in the elsA software,” *ECCOMAS*, 2014.
- [8] Perraud, J., Arnal, D., Casalis, G., Archambaud, J.-P., and Donelli, R., “Automatic transition predictions using simplified methods,” *AIAA journal*, Vol. 47, No. 11, 2009, pp. 2676–2684.
- [9] Bégou, G., Deniau, H., Vermeersch, O., and Casalis, G., “Database Approach for Laminar-Turbulent Transition Prediction: Navier–Stokes Compatible Reformulation,” *AIAA Journal*, 2017, pp. 3648–3660.
- [10] Arnal, D., “Transition prediction in transonic flow,” *Symposium Transsonicum III*, Springer, 1989, pp. 253–262.
- [11] Perraud, J., and Durant, A., “Stability-Based Mach Zero to Four Longitudinal Transition Prediction Criterion,” *Journal of Spacecraft and Rockets*, 2016, pp. 730–742.
- [12] Arnal, D., Habiballah, M., and Coustols, E., “Théorie de l’instabilité laminaire et critères de transition en écoulement bi et tridimensionnel,” *La Recherche Aéronautique*, Vol. 2, 1984, pp. 125–143.
- [13] Cliquet, J., Houdeville, R., and Arnal, D., “Application of Laminar-Turbulent Transition Criteria in Navier-Stokes Computations,” *AIAA Journal*, Vol. 46, No. 5, 2008, pp. 1182–1190.
- [14] Hue, D., Vermeersch, O., Duchemin, J., Colin, O., and Tran, D., “Wind-Tunnel and CFD Investigations Focused on Transition and Performance Predictions of Laminar Wings,” *AIAA Journal*, Vol. 56, No. 1, 2018, pp. 132–145.
- [15] Richez, F., Nazarians, A., and Lienard, C., “Assessment of laminar-turbulent transition modeling methods for the prediction of helicopter rotor performance,” *43rd European Rotorcraft Forum*, 2017.
- [16] Stock, H. W., and Haase, W., “Navier-Stokes Airfoil Computations with e Transition Prediction Including Transitional Flow Regions,” *AIAA journal*, Vol. 38, No. 11, 2000, pp. 2059–2066.
- [17] Menter, F. R., “Two-equation eddy-viscosity turbulence models for engineering applications,” *AIAA journal*, Vol. 32, No. 8, 1994, pp. 1598–1605.