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Lattice Boltzmann Method & Mathematical Morphology

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Résumé – Cet article propose d’établir un lien direct entre la morphologie mathématique et la méthode de Boltzmann sur réseau (LBM), donnant lieu à la LB3M (Lattice Boltzmann Method & Mathematical Morphology). Pour ce faire, une démonstration de l’équivalence entre les équations composant la méthode proposée et les définitions de dilatation et d’érosion morphologiques en niveaux de gris est apportée. La LB3M permet d’embarquer dans un même formalisme le traitement des images et la simulation numérique. Afin d’illustrer l’efficacité de la LB3M, ces opérations élémentaires sont appliquées à l’image de Lena, puis un exemple de croissance osseuse est étudié.

Abstract – This paper suggests to build a direct link between the mathematical morphology and the Lattice Boltzmann Method (LBM) which leads to the LB3M (Lattice Boltzmann Method & Mathematical Morphology). To do so, a proof of the equivalence between the equations in the method and the definitions of morphological dilation and erosion in grey level is established. Thanks to the LB3M, the image processing and the physical simulation are embedded in the same formalism. In order to illustrate the efficiency of the suggested method, the elementary operations are applied to the Lena’s image, then an example of bone growth is studied.

1 Introduction

The lattice Boltzmann method is a numerical method from fluid mechanics [Wol00]. This method can be adapted to image processing problems, and some authors have proven its effectiveness in this field. One of the founding works of the LBM for image processing is the adaptation of the anisotropic diffusion of Perona-Malik for denoising applications [JLS99]. This work was, then, extended by Chen [CYQ08] with the introduction of a probability of bouncing during the process, allowing the denoising as well as the incorporation of an object in an image (inpainting). Subsequently, the work of Chang [CY09] made possible the detection by active contours and filtering [Che+14; NCR15; Ge+17].

Besides that, the mathematical morphology introduced by Matheron in 1967 is a mathematical theory used in image processing and still indispensable to this day. This theory is based on two elementary operations called erosion and dilation. From these two fundamental operators, many other more complex operators can be constructed, allowing in turn applications such as segmentation, denoising, skeletonization, gradient calculation, etc [Naj+10].

An interesting aspect of image processing related to applications in biology and medicine, is its intimate link with modelling. Indeed, biological modelling applications very often use images as a support for numerical simulations. In such a context, it is necessary to have a set of numerical methods that

are as compatible as possible. As such, the Lattice Boltzmann Method (LBM) is particularly interesting, because it offers the possibility of integrating image processing and numerical simulation on the same network [Noë+17].

In order to extend the compatibility cases of the LBM, this article proposes an adaptation of the method to mathematical morphology. Thus, the article is decomposed as follows: a brief review of the foundations of mathematical morphology and LBM is discussed, then the equivalence between the two is shown. Then, an illustration of the erosion and dilation operations on Lena’s image is proposed, then a biological growth phenomenon is simulated via an operator based on the mathematical morphology. Finally, these examples are followed by concluding discussions.

2 Backgrounds

A grey-level image ρ in n dimensions, is mathematically defined as a function of \mathbb{R}^n in $\overline{\mathbb{R}}$. Furthermore, a greyscale structuring element g is a function that goes from \mathbb{R}^n into $\overline{\mathbb{R}}$ and such as the set $\{x \in \mathbb{R}^n \mid g(x) \neq \{\infty\} \cup \{-\infty\}\}$ is bounded.

Then the morphological dilation of a grey-level image ρ by a functional structuring element g , is denoted $(\rho \oplus g)$ and is defined by the relation

$$(\rho \oplus g)(x) = \sup_y (\rho(y) + g(x - y)). \quad (1)$$

Otherwise, a morphological erosion of a grey-level image ρ by a functional structuring element g , is denoted $(\rho \ominus g)$ and is defined such as

$$(\rho \ominus g)(x) = \inf_y (\rho(y) - g(y - x)) \quad (2)$$

The linearization of the collision operator [BGK54] in the Boltzmann equation in its discretized form with a space step Δx and a time step Δt , allows to express it with the form

$$f_i(x + \xi_i \Delta t, t + \Delta t) - f_i(x, t) + F_i = -\omega (f_i - f_i^{(0)}) \quad (3)$$

with f_i the discretized distribution over the phase space, ξ_i the discretized velocity, F_i the external force experienced by the distribution, ω a relaxation parameter towards the equilibrium distribution $f_i^{(0)}$. The f_i can be interpreted as the information (greyscale) conveyed by the neighbours of the current point x .

The resolution of this differential equation usually goes through an explicit time scheme. In addition, from this scheme, a separation into a collision step and a streaming step is performed, giving rise to the equations

$$f_i^c(x, t) = f_i(x, t) - \omega (f_i - f_i^{(0)}) \quad (4)$$

$$f_i^s(x + \xi_i \Delta t, t + \Delta t) = f_i^c(x, t). \quad (5)$$

In order to find the macroscopic equations and especially the variables they describe, it is necessary to work with the discrete integral. Here a macroscopic variable is the greyscale ρ obtained from the distributions f_i using the following equation

$$\rho = \sum_i f_i. \quad (6)$$

Thus for the case of diffusion (possibly anisotropic), the use of the LBM allows the resolution of the following diffusion equation

$$\frac{\partial \rho}{\partial t} = \frac{\Delta x^2}{3\Delta t} \left(\frac{1}{\omega} - 0.5 \right) \nabla_x^2 (\rho). \quad (7)$$

It is also possible to add a boundary conditions, through the operation called "bounce back", which is written

$$\text{Bounce-Back}(f_i) = f_{\bar{i}} = f(-\xi_i). \quad (8)$$

3 Equivalence between MM and LBM operators – LB3M

Proposition 1. *One increment of time in the LBM at the equilibrium using the following definitions of the first moment and the equilibrium function:*

$$\rho(x, t) = \max_i \{f_i(x, t)\} \quad (9)$$

$$f_i^{(0)}(x, t) = \rho(x, t) + g(\xi_i); \quad (10)$$

is equivalent to a morphological grey-level dilation of the image ρ by the structuring element g .

Proof. In order to construct the equivalency of the greyscale morphological dilation operator with the LBM, it is necessary to adapt some operations in the different step of the LBM. Thus, by considering the density ρ no more as the sum over the probabilities f_i , but as the maximum over these probabilities, this leads to the writing of the equation eq. (9). It is also necessary to integrate the evaluation of the image through the structuring element, which is introduced via the equation eq. (10).

These simple modifications, without any changes in the collision and streaming steps, allow to obtain a new density after one time step; this latter density can be expressed in terms of the different steps constituting the LBM at equilibrium (i. e. for $\omega = 1$). Thus, by rolling back successively the equations composing the LBM, one can obtain

$$\begin{aligned} \rho(x, t + \Delta t) &= \max_i \{f_i^s(x, t + \Delta t)\} \\ &= \max_i \{f_i^c(x - \xi_i \Delta t, t)\} \\ &= \max_i \left\{ \left[f_i - \omega (f_i - f_i^{(0)}) \right] (x - \xi_i \Delta t, t) \right\} \\ &= \max_i \{f_i^{(0)}(x - \xi_i \Delta t, t)\} \\ &= \max_i \{\rho(x - \xi_i \Delta t, t) + g(\xi_i)\} \end{aligned} \quad (11)$$

Moreover, by reusing the definition of the morphological dilation and by applying the change of variable $z = x - y$, it becomes possible to write this definition in its discrete form

$$(\rho \oplus g)(x) = \max_z (\rho(x - z) + g(z)) \quad (12)$$

This last expression shows the equivalence between the suggested method and the greyscale morphological dilation. \square

In the same manner, it is possible to define the greyscale morphological erosion through the LBM.

Proposition 2. *One increment of time with the LBM at the equilibrium using the following definitions of the moments and the equilibrium function:*

$$\rho(x, t) = \min_i \{f_i(x, t)\} \quad (13)$$

$$f_i^{(0)}(x, t) = \rho(x, t) - g(-\xi_i) \quad (14)$$

is equivalent to a morphological grey-level erosion of the image ρ by the structuring element g .

Proof. By proceeding like in the previous demonstration, one can roll back the constituting LBM equations. Thus, one obtains under the same hypothesis than previously ($\omega = 1$), the expression of the density after a time step

$$\rho(x, t + \Delta t) = \min_i \{\rho(x - \xi_i \Delta t, t) - g(-\xi_i)\}. \quad (15)$$

This last expression also coincides with the discrete expression of the greyscale morphological erosion after the change of variable $z = x - y$. \square

Therefore, it is possible to define rigorously the two operations funding the mathematical morphology through the LBM formalism. One can also notice the element $g(-\xi_i)$ is nothing more than bounce-back operator applied to $g(\xi_i)$.

Whole these set provides the elementary operation constituting the link between the LBM and the Mathematical Morphology (MM), which leads to the Lattice Boltzmann Method & Mathematical Morphology (LB3M). Those equations are summed up in the following algorithm 1.

```

1.1 function LBM_DilationErosion() :
    Data : Lattice and initial conditions
    Result : Density field
1.2 InitiateLattice()
1.3 if erosion then
1.4     |  $g \leftarrow$  ApplyBounceBack( $g$ ) (eq. (8))
1.5 end
1.6 for  $tStep = 1$  to  $tMax$  do
1.7     |  $\rho \leftarrow$  ComputeMacros( $f_i$ ) (eq. (9) or eq. (13))
1.8     |  $f_i^{(0)} \leftarrow$  ComputeFeq( $\rho, g$ ) (eq. (10) or eq. (14))
1.9     |  $f_i \leftarrow$  Collide( $f_i, f_i^{(0)}$ ) (eq. (4))
1.10    |  $f_i \leftarrow$  Stream( $f_i$ ) (eq. (5))
1.11 end
1.12 end_function

```

Algorithm 1 : Pseudo-code LB3M for elementary mathematical morphology operations.

4 Application interests

To study the efficiency of the suggested method, a comparison between the LBM approach and the *SciPy* library was conducted. This comparison confirms the perfect correspondence between the methods, for both dilation and erosion.

To illustrate these results, 30 dilations with a structuring element of size 3×3 were performed on Lena's image and the output is represented on the fig. 1a. To achieve these dilations, a Gaussian structuring element was used, so discretely one can write

$$g(\xi_i) = g \begin{pmatrix} \xi_7 & \xi_3 & \xi_6 \\ \xi_4 & \xi_0 & \xi_2 \\ \xi_8 & \xi_1 & \xi_5 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 1 \\ 4 & 16 & 4 \\ 1 & 4 & 1 \end{pmatrix}. \quad (16)$$

In accordance with the result stated by the proven equivalence, there is no difference between the classical dilation obtained from the library *SciPy* and that obtained with the LB3M. This is confirmed by a zero quadratic error.

In order to illustrate the results of the comparison around erosion, 30 morphological erosions with a structuring element of size 3×3 were also performed on the image of Lena (see fig. 1b).

Once again, the erosion obtained with the LB3M and the one resulting from the library *SciPy* are identical, which is characterized by a zero quadratic error.



(a) Dilatation.

(b) Érosion.

Figure 1 – Morphological dilation and erosion in grey-level using LBM applied to Lena's image.

After these introductory examples on Lena's image and in order to show the use of mathematical morphology related to modelling with the LBM, an example of bone growth modelling is suggested.

Indeed, a major advantage of the LB3M is that it can easily couple mathematical morphology and numerical simulation such as fluid dynamics.

For this purpose, a bone growth study is carried out. This occurs when some cells, transported by a biological fluid, succeed to anchor themselves to a scaffold. This fixing operates according to certain physical parameters such as the shear stress at the solid-fluid boundary, not taken into account in this illustration. Once the maximum cell density is reached, a new extracellular matrix layer is created on the previous one, creating a region growth [BWD13].

So, this is characterized by an increase in the number of osteoblastic cells and the mineral matrix on the bone surface, until a new portion of bone is created. Indeed, these cells synthesize different elements and participate in the mineralization of the matrix, until they become encrusted (are walled themselves). Then once fully enclosed, they become osteocytes and have a much lower activity. This has the consequence of stabilizing bone density around a maximum. Once this maximum is reached, new osteoblastic cells can be grafted onto the newly created matrix, resulting in further growth of the bone area.

For bone density characterized by the grayscale of the image $glsdens$, this leads to a region growth. This growth is achieved by a temporal evolution at the periphery of the bone (the adherence of ρ obtained by mathematical morphology). This morphological adherence is multiplied by a factor α depending on the growth rate of bones, and added to the initial density. Thus, this growth is simulated by using the following morphological equation

$$\rho(x, t + \Delta t) = \rho(x, t) + \alpha ((\rho \oplus g)(x) - \rho). \quad (17)$$

where α is a selected setting parameter equal to 0,5. To ensure that growth does not occur until the maximum density is reached, it is possible to use the parameter ω , previously assumed to be equal to 1. Indeed, this parameter allows a weighting of the morphological operation. So here it is suggested to use the following function as a choice (illustrative, as there are

other solutions) for the parameter

$$\omega(\rho) = \begin{cases} 1 & \text{if } \rho = 1.0 \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Thus, fig. 2 represents the growth of cells on the bone substrate at three given times.

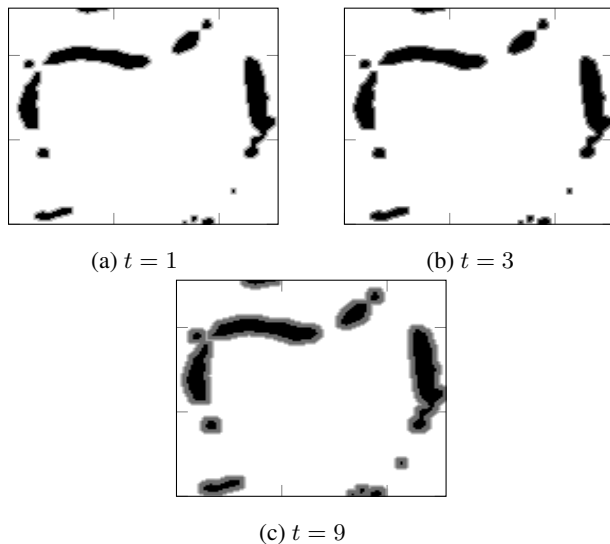


Figure 2 – Use of mathematical morphology in LBM to simulate bone growth from images.

The fig. 2a shows a slight densification in light grey on the border of the black substrate. Conversely, the fig. 2b shows a more pronounced densification with a dark grey at the border. While fig. 2c illustrates well the growth in an area with a high density of extracellular matrix (dark grey) wider than for fig. 2b, and a slight cellular growth on the edge of the latter (light grey). This application remains illustrative because it is not dimensioned; however, it is quite possible to link biological parameters to LBM parameters to simulate progressive densification.

5 Conclusions

This article proposes an adaptation of the fundamental functions of the LBM to rigorously recover the elementary operations of mathematical morphology. But other methods are imaginable such as integrating the structuring element in an external force.

The obtained method is illustrated through Lena’s image and shows very accurate results. Then an example of use for the mathematical morphology using LBM is performed by simulating biological growth; the latter also shows one of the advantages of the method, namely, to have an additional parameter controlling the speed of the morphological operation. Another interest of the method is to be able to link morphological operations with other parameters resulting from the simulation

of physical phenomena thanks to the LBM, associated with the fact that the LBM is intrinsically parallelizable allowing a significant reduction in computation time. As such, a comparison of computation times between the traditional approach and the LB3M approach is research to be carried out.

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