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► **To cite this version:**

A. Morison, S. Labrosse, R. Deguen, T. Alboussière. Timescale of overturn in a magma ocean cumulate. *Earth and Planetary Science Letters*, 2019, 516, pp.25-36. 10.1016/j.epsl.2019.03.037 . hal-02343575

HAL Id: hal-02343575

<https://hal.science/hal-02343575v1>

Submitted on 2 Nov 2019

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Timescale of overturn in a magma ocean cumulate

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Abstract

The formation and differentiation of planetary bodies are thought to involve magma oceans stages. We study the case of a planetary mantle crystallizing upwards from a global magma ocean. In this scenario, it is often considered that the magma ocean crystallizes more rapidly than the time required for convection to develop in the solid cumulate. This assumption is appealing since the temperature and composition profiles resulting from the crystallization of the magma ocean can be used as an initial condition for convection in the solid part. We test here this assumption with a linear stability analysis of the density profile in the solid cumulate as crystallization proceeds. The interface between the magma ocean and the solid is a phase change interface. Convecting matter arriving near the interface can therefore cross this boundary via melting or freezing. We use a semi-permeable condition at the boundary between the magma ocean and the solid to account for that phenomenon. The timescale with which convection develops in the solid is found to be several orders of magnitude smaller than the time needed to crystallize the magma ocean as soon as a few hundreds kilometers of cumulate are formed on a Mars- to Earth-size planet. The phase change boundary condition is found to decrease this timescale by several orders of magnitude. For a Moon-size object, the possibility of melting and freezing at the top of the cumulate allows the overturn to happen before complete crystallization. The convective patterns are also affected by melting and freezing at the boundary: the linearly most-unstable mode is a degree-1 translation mode instead of the approximately aspect-ratio-one convection rolls found with classical non-penetrative boundary conditions. The first overturn of the crystallizing cumulate on Mars and the Moon could therefore be at the origin of their observed degree-1 features.

Keywords: magma ocean, overturn, mantle dynamics, linear stability

1. Introduction

2 A common scenario considered for the formation of terrestrial planets is the
3 crystallization of a global magma ocean from the bottom-up, because the liq-

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4 uidus of silicate magmas increases with pressure more steeply than the isentropic
5 temperature, at least at low to moderate mantle pressure (Andrault et al., 2011;
6 Fiquet et al., 2010; Thomas and Asimow, 2013; Boukaré et al., 2015). The
7 crystallization of the surface magma ocean is expected to be rapid, around 1
8 Myr (e.g. Abe, 1997; Lebrun et al., 2013). This has led several authors to as-
9 sume convection in the solid part of the crystallizing mantle does not start until
10 the mantle is entirely crystallized (e.g. Hess and Parmentier, 1995; Abe, 1997;
11 Parmentier et al., 2002; Elkins-Tanton et al., 2003, 2005; Zhang et al., 2013).
12 However, this assumption deserves scrutiny since the compositional and thermal
13 structure of the mantle after complete crystallization could be widely different
14 if solid-state convection does set in during its crystallization.

15 Two processes might lead to the destabilization of the solid mantle during its
16 crystallization. First, assuming fractional crystallization, the surface magma
17 ocean gets enriched in incompatible elements. As a secondary result, the new
18 solid formed at the solid/liquid boundary gets richer and richer in these elements
19 as crystallization progresses. Iron is such an element and its abundance is such
20 that it affects significantly the density of both the solid and the liquid. The solid
21 formed at the end of the crystallization is richer in iron than the solid formed at
22 the beginning of the crystallization, leading to an unstable setup with material
23 denser at the top than at the bottom of the solid mantle.

24 The second process that can further destabilize the solid mantle is the tem-
25 perature gradient in the solid. The solidus temperature increases with pressure,
26 and is steeper than the isentropic temperature profile. Assuming the tempera-
27 ture in the solid stays close to the solidus, the resulting profile is hence unstable.
28 This effect is enhanced by fractional crystallization and the associated enrich-
29 ment of the solid in incompatible elements: their presence further decreases the
30 solidus temperature and the compositional gradient discussed above induces an
31 even steeper solidus.

32 Numerical simulations including these processes suggest it is possible for
33 solid-state convection to set in prior to the entire crystallization of the surface
34 magma ocean (e.g. Maurice et al., 2017; Boukaré et al., 2018). Whether con-
35 vection in the mantle starts during or after the crystallization of the surface
36 magma ocean is found to have profound implications on the preservation of
37 compositional heterogeneities as well as the dynamics of the mantle (Ballmer
38 et al., 2017; Tosi et al., 2013). These results further confirm the need to assess
39 the parameters controlling the onset of convection in the primitive mantle.

40 A dynamical feature of the solid cumulate in contact with a magma ocean
41 that has not been accounted for in the past studies is the possibility of exchange
42 of matter at the boundary between the solid and the ocean via melting and
43 freezing. We use a boundary condition developed for the inner core boundary
44 (Deguen et al., 2013) to take this effect into account. This boundary condition
45 is expected to have important effects on the convection pattern and heat flux
46 as well as the timescale with which convection sets in (Deguen, 2013; Labrosse
47 et al., 2018).

48 Our aim is to assess how the timescale at which convection starts in the
49 solid cumulate compares with the time needed to crystallize a surface magma

50 ocean. Different scenarios are explored to determine the parameters controlling
51 the onset of convection in the magma ocean cumulate. We consider the case
52 where fractional crystallization happens during the entire cooling history of the
53 magma oceans as well as the case where no compositional fractionation occurs.
54 We explore the classical case for which no matter crosses the boundary between
55 the magma ocean and the solid cumulate, and also the case with a boundary
56 that allows matter transfer across it. The study is applied to the Earth, Mars,
57 and the Moon.

58 2. Methods

59 We consider a mantle that is initially fully molten and crystallizes from the
60 bottom or some intermediate depth upward. The goal of the present study is to
61 determine the timescale for convection to start in the solid part of the mantle
62 as the magma ocean crystallizes.

63 For the sake of simplicity, we assume the compaction length to be small and
64 neglect the thickness of a mush layer at the phase change interface. Matter on
65 one side of the boundary is entirely liquid while matter on the other side is
66 entirely solid. We nonetheless allow for compositional fractionation to occur
67 as the mantle crystallizes. The temperature at the solid/liquid boundary is
68 denoted \mathcal{T}_m and referred to as the melting temperature.

69 Depending on how the temperature profile in the magma ocean compares
70 with the profile of the melting temperature, two situations can occur. Either the
71 solidification of the ocean progresses from the bottom up, or the solidification
72 starts from an intermediate depth leading to a setup in which the solid part of
73 the mantle is surrounded by two magma oceans. In this second scenario, the
74 crystallization of the surface magma ocean (SMO) is thought to be a lot faster
75 than the crystallization of the basal magma ocean (BMO) (Labrosse et al., 2007).

76 We assume the solid mantle is a spherical shell of internal radius R^- and
77 external radius R^+ . Since the crystallization of the BMO is much slower than the
78 crystallization of the SMO, we assume R^- to be constant even for the case where
79 the solid shell is surrounded by two magma oceans. The presence or absence of a
80 BMO however affects the boundary condition applied at the bottom boundary
81 of the solid mantle (see section 2.4).

82 As the magma ocean cools down, R^+ increases to reach the total radius of the
83 planetary body, denoted by R_T . The temperature at the top boundary of the
84 solid follows the melting temperature. The composition of the solid changes as
85 well with the radius if we assume fractional crystallization occurs. For the sake
86 of simplicity, we only consider fractionation of iron. The mass fraction of FeO,
87 denoted by C , varies between 0 (e.g. Forsterite) and 1 (e.g. Fayalite). Although
88 simplistic, such a model allows us to study the effect of the density gradient
89 due to fractional crystallization on the dynamics of the solid. Figure 1 shows
90 the composition and temperature profiles at two different times. We assume
91 the velocity of the freezing front \dot{R}^+ does not vary laterally and that the SMO
92 is well mixed, the temperature and compositional fields in the resulting solid

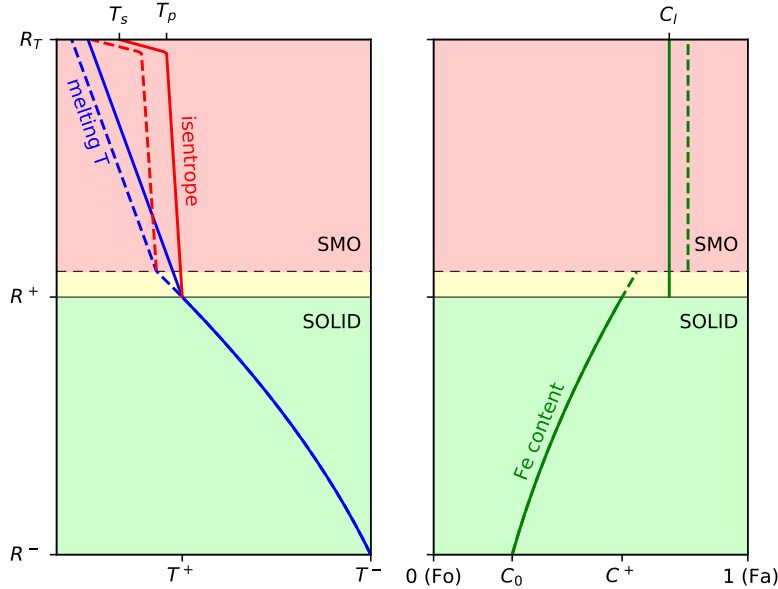


Figure 1: Temperature and composition reference profiles. Solid lines are the profiles at time t , dashed lines the profiles at time $t + \delta t$. The green area is the solid mantle at time t . The yellow area represents the part of the surface magma ocean (SMO, in red) that has crystallized during δt . All the annotations on the axes are written at time t (see table 1 for the meaning of symbols). Notice how the melting temperature decreases between the two instants owing to the enrichment in iron of the surface magma ocean. The slopes of the curves are exaggerated for readability purpose.

93 hence only vary with the radial position (as long as no solid-state convection
 94 operates).

95 In this section, we introduce the simple phase diagram we use to compute the
 96 resulting temperature and composition profiles in the solid under the assumption
 97 that no convection occurs in the solid (section 2.1). This serves as base state
 98 which stability against overturning motion is studied. We don't treat the full
 99 dynamics of the overturn but compute, using a linear stability analysis, the
 100 growth rate of an overturning instability to compare it to the crystallization
 101 rate of the magma ocean. The latter is computed using a magma ocean cooling
 102 model which gives R^+ as a function of time, as described in section 2.2.

103 2.1. Composition and temperature reference profiles

104 Under the assumption that no convection occurs during crystallization, one
 105 can determine the resulting temperature and compositional profiles in the cu-
 106 mulate. These profiles are used as reference profiles in order to perform the
 107 linear stability analysis (section 2.5).

Symbol	Description	Earth	Moon	Mars
Input parameters				
R^-	Internal radius of the solid shell	3871 km*	737 km	2090 km
R_T	Total radius of the planet	6371 km	1737 km	3390 km
\mathcal{T}^-	Temperature at the bottom boundary [†]	4500 K	1500 K	2400 K
\mathcal{T}_∞	Black body equilibrium temperature	255 K	255 K	212 K
ε	Emissivity [¶]	10^{-4}	1	10^{-3}
g	Gravity acceleration	9.81 m/s ²	1.62 m/s ²	3.71 m/s ²
Ra_S	Rayleigh number of SMO	10^{30}	10^{28}	5×10^{28}
α	Thermal expansion coefficient		10^{-5} K^{-1}	
C_p	Heat capacity		10^3 J K^{-1}	
κ	Thermal diffusivity		$10^{-6} \text{ m}^2/\text{s}$	
L_h	Latent heat		$4 \times 10^5 \text{ J kg}^{-1}$	
σ	Stefan-Boltzmann constant		$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$	
ρ	Reference density		$4 \times 10^3 \text{ kg/m}^3$	
$\Delta\rho_m$	Solid/liquid density contrast		$2 \times 10^2 \text{ kg/m}^3$	
η	Viscosity in the solid		10^{18} Pa s	
C_{l_0}	Iron content of the primitive SMO [†]		0.1	
D	Solid/liquid partition coefficient of iron [‡]		0.6	
β	Compositional expansion coefficient		-0.33	
$\partial\mathcal{T}_m/\partial P$	Clapeyron slope		$2 \times 10^{-8} \text{ K Pa}^{-1}$	
$\partial\mathcal{T}_m/\partial C$	Dependence of \mathcal{T}_m on iron content		-700 K	
Computed dimensional variables				
L_M	Final thickness of solid mantle $R_T - R^-$	2500 km	1000 km	1300 km
\mathcal{T}_m	Melting temperature	$\mathcal{T}_m(P, C)$ described by eq. (2.4)		
\mathcal{T}^+	Temperature at the top boundary	$\mathcal{T}^+(t)$ with eq. (2.5)		
\mathcal{T}_p	Potential temperature at the surface	$\mathcal{T}_p(t)$ with eq. (2.8)		
\mathcal{T}_s	Temperature at the surface of the planet	$\mathcal{T}_s(t)$ with eq. (2.9)		
R^+	External radius of the solid shell	$R^+(t)$ with eq. (2.10)		
L	Thickness of the solid shell	$L = R^+ - R^-$		
C_0	Iron content of the first solid	$K C_{l_0} = 0.06$		
C_l	Iron content of the SMO	$K C_l(t) = C^+(t)$ with eq. (2.2)		
τ_{Stokes}	Stokes time	$\eta L^2 / (\Delta\rho g L_M^3)$		
Dimensionless numbers				
$\text{Ra}(t)$	Thermal Rayleigh number	$\rho g \alpha \Delta T L^3 / (\eta \kappa)$		
$\text{Rc}(t)$	Compositional Rayleigh number	$\rho g \beta L^3 / (\eta \kappa)$		
$W(t)$	Freezing front velocity (Peclet number)	$L \dot{R}^+ / \kappa$		
$\Gamma(t)$	Thickness of the solid part	L / L_M		
$\Gamma_S(t)$	Thickness of the SMO	$(R_T - R^+) / L_M$		
Φ^\pm	Phase change number [§]	$10^{-2}; \infty$		

Table 1: Symbols used in this paper. All quantities with a + superscript are evaluated at the top boundary (R^+), while quantities with a - superscript are evaluated at the bottom boundary (R^-). [¶] The emissivity values for the Earth and Mars are chosen so that the crystallization time scale of the SMO is of the order of 1 Myr (Lebrun et al., 2013). For the Moon, we neglect the effects of the atmosphere and assume a black body cooling. * This choice assumes a 400 km thick basal magma ocean. Using $R^- = 3471$ km does not change significantly the results. [†] From Andrault et al. (2011), [‡] from Andrault et al. (2012). [§] 10^{-2} : flow-through, ∞ : non-penetrative. For the Moon and Mars, the possibility of a BMO is not considered and $\Phi^- = \infty$ (see section 2.4 for details).

108 We consider a magma ocean crystallizing from some depth R^- up to its top
 109 radius R_T . The mass fraction of the heavy component (FeO) is $C(r)$ in the solid
 110 and $C_l(t)$ in the liquid, assuming that no diffusion (nor convection) operates in
 111 the solid (therefore C does not depend on time) and convection mixes the liquid
 112 efficiently (therefore C_l depends only on time). At the freezing front, the phase
 113 relation is

$$C(R^+(t)) = DC_l(t) \quad (2.1)$$

114 with D the partition coefficient (considered constant) and $R^+(t)$ the time-
 115 evolving radius of the freezing interface.

116 Assuming the magma ocean undergoes fractional crystallization, the com-
 117 position profile in the cumulate is exponential. At the radial position r it is

$$C(r) = \begin{cases} C_0 \left(\frac{R_T^3 - (R^-)^3}{R_T^3 - r^3} \right)^{1-D} & \text{if } r < R_s \\ 1 & \text{if } r > R_s, \end{cases} \quad (2.2)$$

118 with

$$R_s = \left((R^-)^3 C_0^{\frac{1}{1-D}} + R_T^3 \left(1 - C_0^{\frac{1}{1-D}} \right) \right)^{1/3} \quad (2.3)$$

119 the value of R^+ at which C_l reaches 1 (see appendix A for more details).

120 Since the diffusion timescale is much larger than the other time scales con-
 121 sidered here, we assume the temperature profile in the cumulate stays close to
 122 the melting temperature. We take into account variations of the melting tem-
 123 perature \mathcal{T}_m due to both the pressure and the composition. A higher concentra-
 124 tion in iron leading to a lower melting temperature, the resulting temperature
 125 profile in the solid is steeper than a constant-concentration solidus when frac-
 126 tional crystallization is accounted for (Figure 1). The melting temperature \mathcal{T}_m
 127 verifies:

$$\frac{d\mathcal{T}_m}{dr} = \frac{\partial \mathcal{T}_m}{\partial P} \frac{\partial P}{\partial r} + \frac{\partial \mathcal{T}_m}{\partial C} \frac{\partial C}{\partial r}. \quad (2.4)$$

128 With $\frac{\partial P}{\partial r} = -\rho g$ and eq. (2.2), one obtains

$$\frac{d\mathcal{T}_m}{dr} = -\rho g \frac{\partial \mathcal{T}_m}{\partial P} + 3C(1-D) \frac{r^2}{R_T^3 - r^3} \frac{\partial \mathcal{T}_m}{\partial C}. \quad (2.5)$$

129 For the sake of simplicity, we assume $\frac{\partial \mathcal{T}_m}{\partial P}$ and $\frac{\partial \mathcal{T}_m}{\partial C}$ to be constant (see table 1
 130 for values).

131 We denote $T = \mathcal{T} - \mathcal{T}_{\text{isen}}$ the superisentropic temperature in the solid, with

$$\mathcal{T}_{\text{isen}} = \mathcal{T}^- \exp \left(\frac{\alpha g (R^- - r)}{C_p} \right) \quad (2.6)$$

132 the isentropic temperature profile in the solid, with α the coefficient of ther-
 133 mal expansion, g the acceleration of gravity and C_p the heat capacity. We

134 assume the variations of α , C_p and g with depth to be negligible. The reference
 135 superisentropic temperature (denoted \bar{T}) gradient is then:

$$\frac{d\bar{T}}{dr} = -\rho g \frac{\partial \mathcal{T}_m}{\partial P} + 3C(1-D) \frac{r^2}{R_T^3 - r^3} \frac{\partial \mathcal{T}_m}{\partial C} + \frac{\alpha g}{C_p} \mathcal{T}^- \exp\left(\frac{\alpha g(R^- - r)}{C_p}\right). \quad (2.7)$$

136 2.2. Crystallization time scale

137 Assuming the temperature profile in the SMO to be isentropic and neglecting
 138 variations of α , g and C_p with depth, the potential temperature at the surface is:

$$\mathcal{T}_p = \mathcal{T}^+ \exp\left(\frac{-\alpha g(R_T - R^+)}{C_p}\right). \quad (2.8)$$

139 Note that we are neglecting the temperature drop across the boundary layer at
 140 the bottom of the magma ocean. This is justified by the very small viscosity of
 141 the magma and the main buoyancy force coming from cooling to the atmosphere
 142 at the top surface.

143 King et al. (2012) showed that the scaling law for the heat flux in a ro-
 144 tating fluid (such as the surface magma ocean) depends on how the quantity
 145 $\text{Ra}_S \text{E}_S^{3/2} = \frac{\alpha g \Delta T \nu^{1/2}}{\kappa (2\Omega)^{3/2}}$ compares to 1, with E_S the Ekman number and Ra_S
 146 the Rayleigh number in the SMO. A conservative lower bound with the ther-
 147 mal expansivity $\alpha \sim 10^{-5} \text{K}^{-1}$, the gravity $g \sim 10 \text{m/s}^2$, the super-isentropic
 148 temperature difference $\Delta T \sim 1 \text{K}$, the kinematic viscosity $\nu \sim 10^{-5} \text{m}^2/\text{s}$,
 149 the thermal diffusivity $\kappa \sim 10^{-6} \text{m}^2/\text{s}$ and the rotation rate $\Omega \sim 10^{-4} \text{s}^{-1}$ is
 150 $\text{Ra}_S \text{E}_S^{3/2} \sim 10^5 \gg 1$. We then consider the heat flux is not controlled by rota-
 151 tion and scales as $\text{Nu} = 0.16 \text{Ra}_S^{2/7} \Gamma_S^{6/7}$ with $\Gamma_S = (R_T - R^+)/L$ the dimension-
 152 less thickness of the SMO (King et al., 2012). Note that this scaling does not
 153 depend on the Prandtl number in the range of values explored by King et al.
 154 (2012), i.e. $1 \leq \text{Pr} \leq 100$. Since $\text{Pr} \sim 10$ is a reasonable value for a magma
 155 ocean, we assume this scaling holds for our study. We neglect variations of Ra_S
 156 with time and assume the magma ocean behaves like a gray body at its upper
 157 surface. Heat flow conservation at the surface gives the following equation for
 158 the surface temperature \mathcal{T}_s :

$$\frac{k(\mathcal{T}_p - \mathcal{T}_s)}{L_M} 0.16 \text{Ra}_S^{2/7} \Gamma_S^{-1/7} = \varepsilon \sigma (\mathcal{T}_s^4 - \mathcal{T}_\infty^4) \quad (2.9)$$

159 where \mathcal{T}_∞ is the black body equilibrium temperature, σ is the Stefan-Boltzmann
 160 constant and ε the emissivity. The emissivity should depend on the atmosphere
 161 dynamics and composition (particularly its water content) and vary with time.
 162 Taking this effect into account would require an atmosphere model (e.g. Abe,
 163 1997; Lebrun et al., 2013). For the sake of simplicity, we assume the emissivity
 164 to be constant, tuning its value to obtain a crystallization timescale that matches
 165 the ones of Lebrun et al. (2013) (see table 1 for values).

166 As the SMO crystallizes (i.e. R^+ increases with time), we assume the tem-
 167 perature at the top of the solid mantle \mathcal{T}^+ follows the solidus (eq. (2.5)), and

168 the temperature profile in the SMO follows an isentropic profile. As R^+ grows,
 169 two phenomena produce heat that should be evacuated: the crystallization it-
 170 self with an associated latent heat L_h , and the cooling of the magma ocean.
 171 Assuming this heat is entirely evacuated through radiation in the atmosphere
 172 modeled as a gray body, one obtains the following equation:

$$\varepsilon\sigma R_T^2(\mathcal{T}_s^4 - \mathcal{T}_\infty^4) = \rho L_h R^{+2} \frac{dR^+}{dt} - \rho C_p \frac{d}{dt} \left(\int_{R^+}^{R_T} \mathcal{T}^+ \exp\left(\frac{\alpha g(R^+ - r)}{C_p}\right) r^2 dr \right). \quad (2.10)$$

173 The last term of this equation can be developed (keeping in mind that the lower
 174 bound of the integral R^+ depends on time). This yields the time derivative
 175 of \mathcal{T}^+ , which is written as a derivative with respect to R^+ using the chain
 176 rule. One obtains an ordinary differential equation on $R^+(t)$ whose numerical
 177 integration gives the position of the interface between the solid and the surface
 178 magma ocean as a function of time.

179 2.3. Set of dimensionless equations

180 $L = R^+ - R^-$, L_M^2/κ , κ/L , $\eta L^3/\kappa$, $\Delta T = T^- - T^+$ are used as scales for
 181 length, time, velocity, mass and temperature respectively. Note that R^+ and T^+
 182 vary with time as the surface magma ocean crystallizes. $L_M = R_T - R^-$ is the
 183 thickness of the solid mantle once the SMO is entirely crystallized. Note that
 184 all scales depend on time except the one for time itself, which is why $\Gamma = L/L_M$
 185 appears in the following equations. The dimensionless radial position is built
 186 as $1 + (r - R^-)/L$ so that it is between 1 and 2 at all times. Similarly, the
 187 dimensionless temperature is chosen as $(T - T^+)/\Delta T$ so that it is between 0
 188 and 1 at all times.

Using the same symbols for dimensionless quantities, dimensionless conser-
 vation equations of mass, momentum, heat and iron fraction are written as:

$$\nabla \cdot \mathbf{u} = 0 \quad (2.11)$$

$$\mathbf{0} = -\nabla p + \nabla^2 \mathbf{u} + \text{Ra} (\Theta - \langle \Theta \rangle) \hat{\mathbf{r}} + \text{Rc} (c - \langle c \rangle) \hat{\mathbf{r}} \quad (2.12)$$

$$\Gamma^2 \frac{\partial \Theta}{\partial t} + \mathbf{u} \cdot \nabla (\Theta + \bar{T}) - \nabla^2 \Theta = W \left((r-1) \frac{\partial \Theta}{\partial r} + \left(\frac{\partial \bar{T}}{\partial r} \right)^+ \Theta \right) \quad (2.13)$$

$$\Gamma^2 \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla (c + \bar{C}) = W(r-1) \frac{\partial c}{\partial r}. \quad (2.14)$$

189 \mathbf{u} is the velocity field, p the dynamic pressure, Θ the temperature perturbation
 190 with respect to the reference profile \bar{T} and c the composition perturbation with
 191 respect to the reference profile \bar{C} . $\langle x \rangle$ denotes the lateral average of the quantity
 192 x . Ra is the thermal Rayleigh number, Rc is the compositional Rayleigh number.
 193 The terms on the right hand side of eqs. (2.13) and (2.14) are due to the time
 194 dependence of the scales L and ΔT , which brings new advection terms associated
 195 with the change of frame, with $W = L\dot{R}^+/\kappa$ the dimensionless velocity of the
 196 freezing front. See table 1 for the definition and values of the various symbols.

197 Note that these equations are written under the assumption that $\dot{R}^- = 0$.
198 Other terms would appear on the right hand side of eqs. (2.13) and (2.14) in the
199 general case involving the crystallization of a basal magma ocean. For Earth's
200 case, we assume the basal ocean crystallizes much slower than the surface ocean,
201 and as such we neglect \dot{R}^- (Labrosse et al., 2007). We assume the diffusion of the
202 compositional field is negligible since the diffusion coefficient of composition is
203 much smaller than that of heat. Moreover, diffusion of \bar{T} is neglected while that
204 of Θ is retained in order to ease the linear stability analysis. This is justified a
205 posteriori by the fact that the diffusion timescale is much longer than the other
206 timescales considered in this study.

207 2.4. Phase change boundary condition

208 In the classical Rayleigh-Bénard setup, convecting matter arriving near an
209 horizontal boundary forms a topography whose height is limited by the weight
210 viscous forces can sustain. This topography is often neglected and a non-
211 penetrative boundary condition is assumed at the interface ($u_r(R^+) = 0$). How-
212 ever, in the system studied here, the boundary between the magma ocean and
213 the cumulate is a phase change interface. A topography of the solid with respect
214 to the equilibrium position of the solid/liquid interface can then be eroded by
215 melting or freezing. Provided that the melting/freezing time is short compared
216 to the time needed to build the topography by viscous forces, it is thus possible
217 to have a non-zero normal velocity across the interface. This is taken into ac-
218 count with the help of the boundary condition introduced for the inner core by
219 Deguen et al. (2013). This boundary condition, which translates the continuity
220 of normal stress across the interface, is written in dimensional form as:

$$\Delta\rho_m g \tau_\phi u_r + 2\eta \frac{\partial u_r}{\partial r} - p = 0. \quad (2.15)$$

221 where $\Delta\rho_m$ is the density difference between the solid and liquid phases and τ_ϕ
222 is the phase change timescale. Note that our definition of the dynamic pressure
223 (defined here as $p = P - \langle P \rangle$) differs from that of \hat{p} used by Deguen et al. (2013).
224 The laterally constant term $\Delta\rho_m g \tau_\phi \dot{R}$ is thus included in p instead of explicitly
225 appearing in the boundary condition. The dimensionless form of the boundary
226 condition is

$$\pm\Phi^\pm u_r + 2 \frac{\partial u_r}{\partial r} - p = 0 \quad (2.16)$$

227 where Φ is the phase change number defined as:

$$\Phi^\pm = \frac{|\Delta\rho_m|^\pm g L \tau_\phi}{\eta} \quad (2.17)$$

228 (the superscript $+$ denotes the interface between the SMO and the solid at R^+
229 while the superscript $-$ denotes the interface between the BMO and the solid at
230 R^-). Moreover, the continuity of tangential stress is simply written as a classic
231 free-slip boundary condition.

232 The phase change timescale τ_ϕ is related to the time needed to transport
 233 latent heat in the magma ocean from the areas that freeze to the areas that
 234 melt (Deguen et al., 2013):

$$\tau_\phi = \frac{\rho L_h}{(\rho - \Delta\rho_m)^2 C_p (\partial_P \mathcal{T}_m - \partial_P \mathcal{T}_{\text{isen}}) g u'} \quad (2.18)$$

235 where u' is the velocity scale in the magma ocean. A reasonable value for the
 236 latter is $u' \sim 1 \text{ m s}^{-1}$ (Lebrun et al., 2013). Using nominal values for the other
 237 parameters, we find that $\tau_\phi \sim 10^4 \text{ s}$. Plugging this in the expression of the phase
 238 change parameter eq. (2.17) yields $\Phi \sim 10^{-5}$.

239 The phase change number Φ compares the phase change timescale τ_ϕ (i.e.
 240 the time needed to erode topography via melting and freezing) to the viscous
 241 timescale (i.e. the time needed to build topography with viscous forces). The
 242 value of Φ allows to tune continuously the boundary condition between a non-
 243 penetrative classical condition ($\Phi \rightarrow \infty$) and a fully permeable condition ($\Phi \rightarrow$
 244 0). Although this number should depend on time since L depends on time and
 245 τ_ϕ depends also on time but in a non trivial way, it is kept constant in this
 246 study. Two extreme values are tested for the SMO/solid interface: $\Phi = \infty$
 247 which leads to the classical non-penetrative boundary condition and $\Phi = 10^{-2}$
 248 which leads to a flow-through boundary (we use this value rather than 10^{-5}
 249 because the resolution of radial modes is more computationally demanding as Φ
 250 decreases, while the overturn timescale is not affected as shown in the results).
 251 For the Earth, these two values are also considered at the bottom of the solid,
 252 accounting for the possible presence of a basal magma ocean (BMO, Labrosse
 253 et al., 2007). For Mars and the Moon, we do not consider the possibility of a
 254 BMO and the bottom interface is hence non-penetrative, $u_r(R^-) = 0$. Rather
 255 than being realistic, these extreme constant values are used to study how the
 256 possibility of melting and freezing at the interface affects the stability of the
 257 solid, both in terms of onset time of overturn and preferred mode of motion.
 258 The estimated nominal value being $\Phi \sim 10^{-5}$, we expect the real system should
 259 be closer to the flow-through case than to the classical non-penetrative case.

260 2.5. Determination of overturn timescale

261 We start from a completely molten primitive mantle ($R^+ = R^-$ and $\mathcal{T}^+ =$
 262 \mathcal{T}^-). We numerically integrate eq. (2.10) to obtain R^+ as a function of time (the
 263 potential surface temperature \mathcal{T}_p and the surface temperature \mathcal{T}_s are computed
 264 using eq. (2.8) and eq. (2.9)).

265 At each timestep of this integration, we compute the reference tempera-
 266 ture and composition profiles in the solid as shown in section 2.1 as well as
 267 the dimensionless numbers $\text{Ra}(t)$, $\text{Rc}(t)$, $W(t)$ and $\Gamma(t)$. Using a Chebyshev-
 268 collocation approach (e.g. Guo et al., 2012; Canuto et al., 1985), the set of lin-
 269 earized equations around the reference state is written as an eigenvalue problem
 270 (see appendix B). Solving numerically this problem yields the growth rate and
 271 shape of the most unstable mode of overturn. The inverse of that growth rate
 272 is the timescale for convection to set in in the solid shell. We compute this

273 timescale at each timestep of the evolution of the SMO. By comparing this
274 timescale with the corresponding time in the evolution of the SMO, we can as-
275 sess whether convection is able to take place before the entire magma ocean is
276 crystallized. Three different models are considered for the bulk of the solid:

- 277 1. full model: compositional, thermal, and moving frame terms are taken
278 into account;
- 279 2. thermal model: compositional terms are left out, modeling the ideal case
280 where no fractional crystallization occurs and the sources of instability
281 are purely thermal (eq. (2.14) and the corresponding buoyancy term in
282 eq. (2.12) are ignored);
- 283 3. frozen-time model: moving frame terms (right-hand-side of eqs. (2.13)
284 and (2.14)) are left out, resulting in a frozen-time approach where all
285 long term evolution terms are ignored when studying the stability of the
286 system at a given instant.

287 We also compare the timescale obtained by linear stability analysis with the
288 Stokes time $\tau_{\text{Stokes}} = \eta L^2 / (\Delta \rho g L_M^3)$ computed at each time to check whether
289 this time is a relevant proxy of the stability of the solid mantle.

290 3. Results

291 The destabilization timescales for the Earth, Mars, and the Moon with var-
292 ious boundary conditions along with the time needed to crystallize the remain-
293 ing SMO are shown on Figure 2. Comparison of the destabilization timescales
294 obtained for various bulk setups and boundary conditions yields information re-
295 garding their contribution to the destabilization of the solid.

296 The simplest cases are the one neglecting the compositional effects on den-
297 sity. For such cases, the destabilization timescale tends to infinity for a given
298 non-zero thickness of crystallized mantle. This thickness corresponds to the one
299 needed for instabilities to overcome diffusion of perturbations of the reference
300 state. In other words, it corresponds to the thickness above which the Rayleigh
301 number in the solid part is above the critical Rayleigh number. For the Moon,
302 this thickness is never reached and the Moon’s mantle stays stable with respect
303 to purely thermal convection. For the Earth and Mars, this thickness is reached
304 rather early, after ~ 500 km of solid mantle is formed. As crystallization pro-
305 gresses, the thickness and the temperature contrast between the top and the
306 bottom of the solid mantle increase. The available buoyancy in the system there-
307 fore increases. This leads to a strong decrease of the destabilization timescale,
308 which becomes much shorter than the time needed to crystallize the remaining
309 surface magma ocean (up to 6 orders of magnitude, depending on which bound-
310 ary conditions are considered). This suggests that even in the purely thermal
311 case, solid-state convection sets in before the mantle is completely crystallized
312 for planets larger than Mars.

313 The cases taking compositional effects on density into account are always
314 unstable. This contrasts with the purely thermal cases and is due to the fact

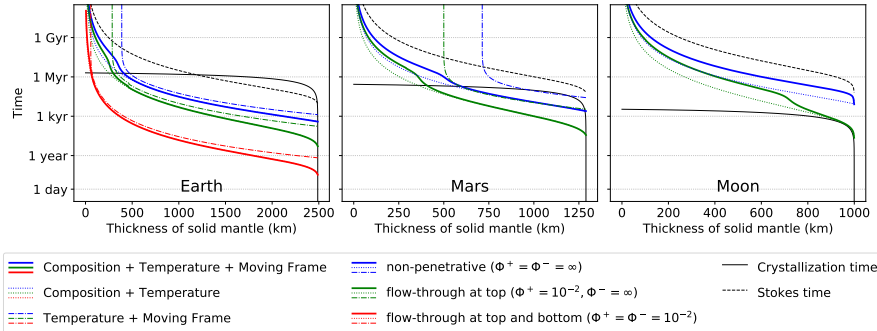


Figure 2: Growth time of the most unstable mode as a function of the crystallized mantle thickness for the Earth, Mars, and the Moon. The solid black line is the time necessary to crystallize the remaining surface magma ocean. Colors represent different boundary conditions: both horizontal boundaries non-penetrative (blue); flow-through boundary condition between the solid and the surface magma ocean to model the possibility of melting and freezing (see section 2.4 for details) (green); and flow-through boundary conditions for both horizontal boundaries assuming the presence of a basal magma ocean (red). Linestyles represent different approximations regarding compositional effects (fractional crystallization and effect on density) and moving frame contributions: both are taken into account (solid lines), compositional effects are neglected (dash-dotted lines), or moving frame terms are neglected (dotted lines). The black dashed line is the Stokes time for each thickness, given for comparison.

315 that diffusion of the composition field is neglected. There is no mechanism
316 to damp perturbations around the reference state, the latter is hence always
317 unstable. Similarly to what is observed for the thermal cases, the destabilization
318 timescale drops dramatically as the solid mantle thickens. For the Earth and
319 Mars, the destabilization timescale ends up being shorter than the crystallization
320 time of the remaining SMO by several orders of magnitude. The case where
321 moving frame terms are neglected exhibits a shorter destabilization time scale
322 at small thickness. The moving frame terms play a stabilising role only at the
323 beginning of mantle crystallization for the Earth and Mars but are significant
324 through the entire Moon’s mantle crystallization. The stabilising effect of the
325 moving terms can be understood from the energy conservation eq. (2.13). Taking
326 a temperature perturbation $\theta > 0$ and the associated velocity perturbation
327 $u_r > 0$, one can notice there is a competition between the advection term
328 $u_r \partial_r \bar{T} < 0$ and the moving frame term $W(r-1) \partial_r \theta$ whose average is negative.
329 The same reasoning can be made with a negative perturbation and on the iron
330 conservation eq. (2.14).

331 For the Moon, the destabilization timescale is always greater than the time
332 needed to crystallize the SMO. However, it should be noted that in this study
333 the time to crystallize the SMO is computed assuming a well-mixed SMO with a
334 surface behaving like a black body. The formation of a light solid crust enriched
335 in plagioclase when around 80% of the SMO is crystallized is expected to slow
336 down the solidification of the SMO by a few million years (e.g. Elkins-Tanton
337 et al., 2011). This would leave enough time for convection to set in in the solid

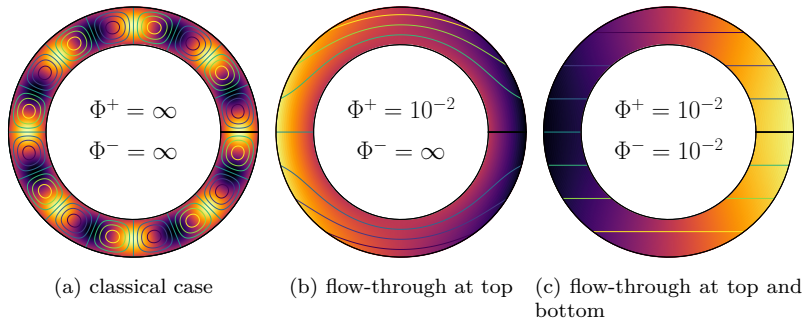


Figure 3: Most unstable convection modes for the Earth when a 1700km thick mantle has crystallized, for different boundary conditions represented by the values of the Φ parameters at the top and the bottom, as indicated. The dark zones represent negative temperature anomalies while the bright zones represent positive temperature anomalies. The streamlines are superimposed. Note that the linear stability analysis offers no constraint on the orientation and amplitude of these modes, only their harmonic degree and radial shape. (a): both boundaries non-penetrative, the convection rolls have an aspect ratio approximatively equal to 1; (b): flow-through top boundary, the flow pattern is of spherical harmonic degree one, the streamlines go through the top boundary but go around the central part; (c): flow-through conditions at both boundaries, the flow pattern is of spherical harmonic degree one, the streamlines go through both boundaries, resulting in a translation mode of convection. Similar behavior is obtained for the other bodies.

338 since the destabilization timescale we find is much shorter than that.
 339 The three boundary conditions exhibits different destabilization timescales.
 340 The case where both boundaries are non-penetrative (which is the case classically considered) needs more time to destabilize than the case where the boundary between the surface magma ocean and the solid allows melting and freezing.
 341
 342 Convective patterns obtained with a flow-through boundary are substantially different than the classical ones (Figure 3). Aspect-ratio-1 rolls are obtained with classical boundary conditions. However, when the top boundary allows phase change, a spherical-harmonic-degree-1 near-translation mode develops.
 343
 344 Matter freezes on one side of the spherical shell, goes around the core or basal magma ocean, and melts on the other side. In the case with a basal magma ocean and its boundary with the solid of flow-through type, matter also crosses the inner boundary of the spherical shell, resulting in a true translation mode.
 345
 346 These two translation modes involve very little or no deformation of the solid compared to the classical case, and therefore less viscous forces acting against convection. This explains the smaller destabilization timescale associated with these modes as well as the lower critical thickness in the purely thermal case.
 347
 348 Figure 4 shows the transition between the non-penetrative and the flow-through regime occurs over a rather short range of values of the phase change number. $\Phi^+ \lesssim 1$ leads to near-translation while $\Phi^+ \gtrsim 100$ leads to classical aspect-ratio-one rolls.
 349
 350 A notable feature on Figure 2 is the steep decrease of the destabilization
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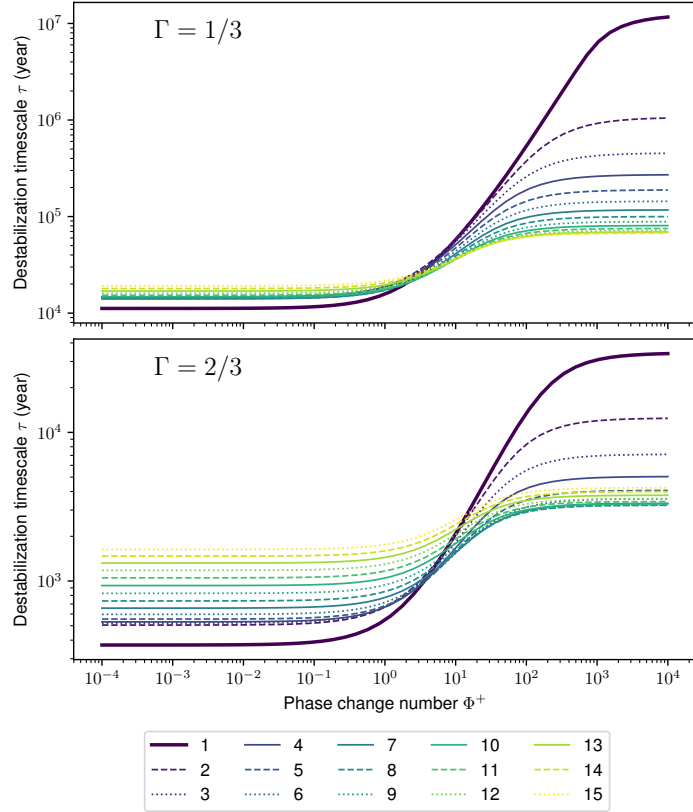


Figure 4: Destabilization timescale of several harmonics degree ($l = 1$ to 15) as a function of the phase change number value for the Earth. The bottom boundary is non-penetrative. Top: 833 km are crystallized (mid-radius $\bar{r} \sim 4288$ km), bottom: 1667 km are crystallized (mid-radius $\bar{r} \sim 4704$ km). The most unstable mode is the one with the shortest destabilization timescale. One can notice that in the non-penetrative case ($\Phi^+ \rightarrow \infty$), the most unstable mode corresponds to aspect-ratio-1 rolls. The typical roll size of the most unstable mode ($\bar{r}\pi/l$) is roughly 900 km for the top case ($l = 15$) and 1850 km for the bottom case ($l = 8$). However, with a flow-through boundary ($\Phi^+ \rightarrow 0$), the most unstable mode is the near-translation mode for both cases.

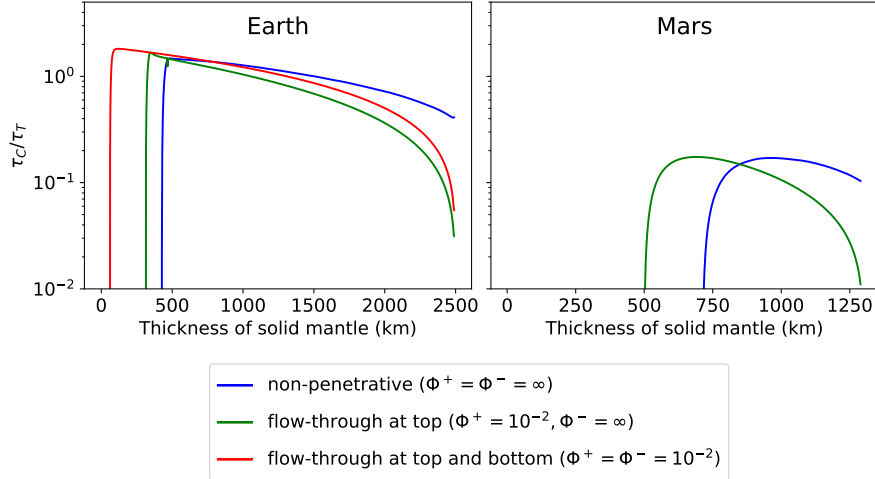


Figure 5: Ratio between the destabilization timescale obtained for the purely compositional case τ_C (thermal terms are left out) and the timescale obtained for the purely thermal case τ_T (compositional terms are left out). When this ratio is above one, it means the thermal reference profile is more unstable than the compositional reference profile. The Moon is not shown here since the purely thermal case is never unstable ($\tau_T \rightarrow \infty$). The colors are the same as in Figure 2, blue: non-penetrative condition for both horizontal boundaries ($\Phi^\pm = \infty$); green: flow-through condition at the boundary between the solid and the surface magma ocean; and red: flow-through condition at both horizontal boundaries.

360 timescales at the end of the crystallization when compositional terms are taken
 361 into account. That decrease is due to the strong (i.e. very unstable) compositional
 362 gradient appearing at the end of the crystallization. It does not affect
 363 the destabilization timescale obtained with non-penetrative boundary condi-
 364 tions; this can be explained by the fact that the strong compositional gradient
 365 is in a very thin layer at the top of the domain where vertical velocities vanish,
 366 and therefore does not contribute to the driving of the down- and up-welling
 367 currents.

368 A comparison between the purely thermal and purely compositional cases for
 369 the Earth and Mars is shown on Figure 5. The ratio between the destabilization
 370 timescales for these two cases is 0 before the critical thickness for the purely
 371 thermal case is reached. For Mars, the compositional profile is always more
 372 unstable than the thermal profile and controls the destabilization timescale of
 373 the system. For the Earth, however, the ratio between the two cases is fairly
 374 close to 1 for a large part of the crystallization history: neither the thermal nor
 375 the compositional profile dominates the destabilization timescale of the system.

376 Figure 6 shows that the destabilization timescale τ_{LSA} is proportional to the
 377 Stokes time $\tau_{Stokes} = \eta L^2 / (\Delta \rho g L_M^3)$. Two effects alter this relation: moving
 378 frame terms whose effects are not included in the Stokes time, and the strong
 379 compositional gradient at the end of the crystallization whose effects depend

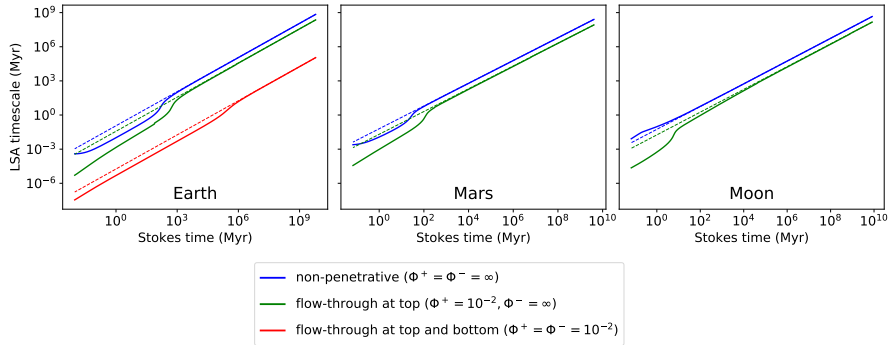


Figure 6: Growth time of the most unstable mode versus the Stokes time for the Earth, Mars, and the Moon. The solid line is the destabilization timescale obtained with the linear stability analysis τ_{LSA} (case with all terms accounted for). The dashed lines correspond to $\tau_{\text{LSA}} \propto \tau_{\text{Stokes}}$. Composition, temperature and moving frame terms are all taken into account. The colors are the same as in Figure 2, blue: non-penetrative condition for both horizontal boundaries ($\Phi^{\pm} = \infty$); green: flow-through condition at the boundary between the solid and the surface magma ocean; and red: flow-through condition at both horizontal boundaries.

380 on the boundary condition. It should be noted that the ratio $\tau_{\text{LSA}}/\tau_{\text{Stokes}}$ de-
 381 pends on the body and the boundary conditions considered. Notably, permeable
 382 boundary conditions lead to a decrease of τ_{LSA} .

383 4. Discussion

384 We showed for the Earth and Mars that the growth timescale of convective
 385 instabilities in a crystallizing mantle from the bottom up is several orders of
 386 magnitude smaller than the time needed to fully crystallize that mantle. This
 387 holds even without taking into account fractional crystallization and the un-
 388 stable density gradient it induces. This contrasts with the assumptions made
 389 in several studies (Hess and Parmentier, 1995; Elkins-Tanton et al., 2003; Tosi
 390 et al., 2013) where the overturn is assumed to take place because of the compo-
 391 sitionally induced density gradient after the entire mantle is crystallized. The
 392 numerical simulations performed by Ballmer et al. (2017) for Earth-like objects
 393 lead to a destabilization of the solid after a few Myr, and those performed by
 394 Maurice et al. (2017) for Mars-like objects lead to a destabilization after roughly
 395 1 Myr. These times are not easily comparable to the timescales we compute via
 396 linear stability analysis since the physical problems are different in non-trivial
 397 ways: the simulations of Ballmer et al. (2017) are in a 2D aspect-ratio-1 carte-
 398 sian box, those of Maurice et al. (2017) are in cylindrical geometry with a vari-
 399 able viscosity, a melt extraction mechanism and a solidus temperature that de-
 400 pends only on pressure. However, despite these differences, the destabilization
 401 time uncovered by these simulations are rather similar to the one we predict for
 402 the non-penetrative cases: of the order of 1 Myr for the Earth and 0.5 Myr for

403 Mars. This confirms the linear growth rate of instabilities is a relevant proxy
404 for the timescale at which convection sets in.

405 Moreover, allowing transfer of matter via melting and freezing at the inter-
406 face between the solid and the surface magma ocean reduces dramatically the
407 timescale with which solid-state convection can set in. It also changes the shape
408 and harmonic degree of the most unstable mode: a degree-one translation mode
409 is preferred. Therefore, the possibility of melting and freezing at the interface
410 should be accounted for when studying the overturn of the primitive mantle of
411 planetary bodies. For example, the case of the Moon is an interesting potential
412 application. This body has a strong dichotomy: the near-side presents more
413 mare basalts, more KREEP material, and a thinner crust than the far-side.
414 Wasson and Warren (1980) already proposed that such features could be due to
415 a slower cooling of the lunar magma ocean on the near side than on the far-side.
416 A permeable boundary would allow the solid mantle to overturn with a domi-
417 nant degree-one before the entire crystallization of the mantle (keeping in mind
418 that the end of the crystallization is much slower than what we predict with
419 our simple model, see Elkins-Tanton et al. (2011)). The mechanisms proposed
420 to build a degree 1 at the scale of the Moon involve the dynamics of an entirely
421 crystallized lunar mantle (e.g. Parmentier et al., 2002; Zhong et al., 2000). The
422 possibility to form a degree one while the crystallization of the magma ocean
423 is still ongoing is therefore worth exploring with more complete models to test
424 whether this dominant degree-one can be conserved after crystallization of the
425 magma ocean and/or helps the development of degree-one instabilities such as
426 the ones predicted in the aforementioned studies. It is also tempting to asso-
427 ciate the degree-one feature of Mars (the Marsian dichotomy) to the same pro-
428 cess but, as explained above, the first degree-one overturn of the solid mantle
429 is expected to happen long before its complete crystallization. Secondary over-
430 turning instabilities are possible after the first one that we cannot investigate
431 with the tools presented above. A more complete study investigating the finite
432 amplitude dynamics is necessary to understand the implications of this work to
433 planets larger than the Moon.

434 It should be noted that several parameters involved in the problem are badly
435 constrained. The viscosity of the solid mantle and even its rheology is such a pa-
436 rameter. It is highly dependent on how close the temperature in the solid is from
437 the solidus and could easily vary by a few orders of magnitude (e.g. Solomatov,
438 2015). Since the destabilization timescale scales as the Stokes time (Figure 6),
439 it is directly proportional to the viscosity and could therefore vary by a few or-
440 ders of magnitude. The strong relation between the viscosity of the cumulate
441 and the overturn scaling has been investigated by Ballmer et al. (2017): their
442 numerical experiments confirm the overturn onset scales as the Stokes time. It
443 should be noted that our flow-through boundary conditions does not affect this
444 result, it only reduces the proportionality factor between the Stokes time and
445 the growth time of instabilities (Figure 6). This validates the general approach
446 proposed by Boukaré et al. (2018) to assess whether solid-state convection sets
447 in before the magma ocean is entirely crystallized: they compare the Stokes
448 time with the time needed to crystallize the magma ocean and their numeri-

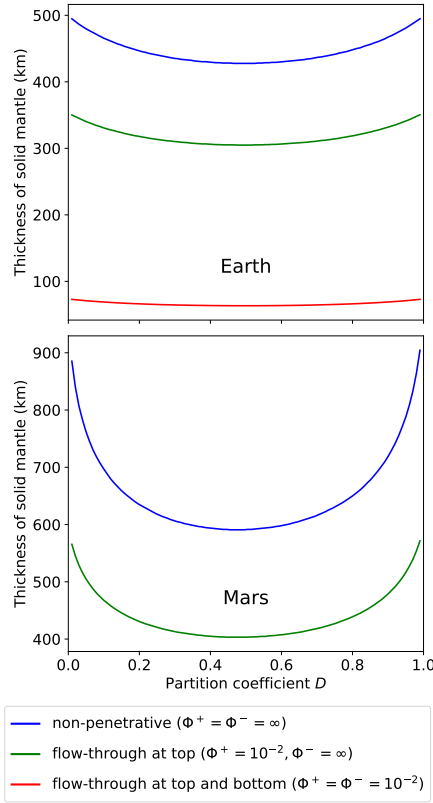


Figure 7: Thickness of the solid cumulate at which the destabilization timescale equals the time needed to crystallize the rest of the SMO for several values of the partition coefficient, $D \in [0.01, 0.99]$. The Moon is not shown here since the destabilization timescale is greater than the time needed to crystallize the SMO. The colors are the same as in Figure 2, blue: non-penetrative condition for both horizontal boundaries ($\Phi^\pm = \infty$); green: flow-through condition at the boundary between the solid and the surface magma ocean; and red: flow-through condition at both horizontal boundaries.

449 cal experiments yields that syn-crystallization convection is possible when the
 450 ratio between these two times exceeds $\sim 5 \times 10^4$. This value however was de-
 451 termined with non-penetrative boundary conditions, the actual threshold should
 452 be a few orders of magnitude higher (meaning syn-crystallization convection
 453 is allowed for shorter solidification timescales) since the flow-through bound-
 454 ary condition leads to a faster destabilization of the cumulate for the same
 455 Stokes time. Another aspect that deserves care is that for Earth-sized bodies,
 456 the Stokes time should incorporate both the thermal and compositional density
 457 contrasts. Boukaré et al. (2018) compare the “compositional” Stokes time with
 458 the solidification timescale; while this is perfectly valid for the Moon and Mars
 459 for which the thermal density contrast is much smaller than the compositional
 460 one, this does not hold for the Earth where both terms have similar magnitudes
 461 (Figure 5). The tremendous importance of the viscosity is why a viscosity of
 462 10^{18} Pas is assumed in this study since it is a higher bound for the near-solidus
 463 viscosity (see Solomatov, 2015, and references therein) and gives the most con-
 464 servative estimate for the destabilizing time. The viscosity could be significantly
 465 lower if the melt fraction is important in the cumulate, Solomatov (2015) sug-
 466 gests 10^{14} Pas as a lower bound at 40% melt fraction (roughly the rheological
 467 transition). Another potential effect of viscosity that is neglected in the study is
 468 dynamical: since solid state convection occurs during the crystallization of the
 469 magma ocean, the temperature in the solid departs from the solidus tempera-
 470 ture profile and as a result the viscosity increases. Moreover, the compositional
 471 profile becomes gravitationally stable with iron-enriched heavy material being
 472 transported from the top to the bottom of the solid. These two effects com-
 473 bined may lead to the stopping of the solid state convection (Solomatov, 2015).
 474 Depending on the size of the magma ocean considered, it could then be possible
 475 either that the magma ocean crystallizes completely before convection may start
 476 again in the solid, or that convection sets in again in the solid before it is en-
 477 tirely crystallized. Studying this scenario requires a more complex method than
 478 a simple linear stability analysis since it involves a non-linear feedback between
 479 the dynamics of the solid part and its viscosity, temperature, and compositional
 480 fields.

481 Another unconstrained parameter is the partition coefficient of iron between
 482 the solid and liquid. An exploration of this parameter shows that the effect
 483 of the partition coefficient is rather limited for the Earth, and slightly more
 484 important for Mars (Figure 7). This is in agreement with Figures 2 and 5
 485 showing the difference between the purely thermal case (corresponding to the
 486 extreme value $D = 1$) and the purely compositional case is rather small for the
 487 Earth but more important for Mars.

488 Finally, our choice of a constant emissivity results in a roughly constant
 489 solidification rate, whereas more sophisticated cooling models including an at-
 490 mosphere predict most of the mantle crystallizes quickly, and the solidification
 491 slows down when only a shallow magma ocean remains. Although such effects
 492 are important to build realistic solidification models, they should not affect dra-
 493 matically our results. Indeed, a faster crystallization at the beginning would lead
 494 to a destabilization of the solid mantle at a larger thickness, but we expect this

495 difference to be small since the destabilization timescale is rapidly much lower
496 than the solidification timescale.

497 **5. Conclusions**

498 Upward crystallization of the silicate mantle of planets within a magma
499 ocean is expected to produce a unstably stratified situation, because of both
500 temperature and composition. In this study, we have addressed the question
501 of whether the overturning instability develops faster than the time it takes to
502 crystallize the magma ocean. To that end, we have developed a linear stability
503 analysis tool to compute the growth rate of the fastest overturning mode and
504 studied systematically the effect of the most important parameters: the planet's
505 size (Moon to Earth size), the partition coefficient and the type of boundary
506 condition between the solid and the liquid. In particular, we have introduced a
507 boundary condition that accounts for the possibility of melting and freezing at
508 the interface between the solid mantle and the magma ocean.

509 This study shows convection is likely to start in the solid mantle of the
510 Earth, Mars and the Moon before the entire crystallization of the surface magma
511 ocean. Evolution models of the primitive mantle of planetary bodies should
512 therefore account for convection and the associated mixing in the solid part of
513 the crystallizing mantle.

514 This result holds for the Earth and Mars even without fractional crystalliza-
515 tion and the unstable compositional gradient it creates in the cumulate. The
516 value of the partition coefficient is found to have little impact on the timing of
517 mantle overturn.

518 The timescale at which convection sets in scales as the Stokes time. Specifi-
519 cally, it is proportional to the viscosity of the solid. However, it should be kept
520 in mind that these results are obtained assuming a newtonian rheology and a
521 constant viscosity in the solid mantle. Given the central role of viscosity in this
522 problem, better knowledge of the viscosity and rheology of the primitive solid
523 mantle is of primary importance to study its dynamics.

524 Finally, the possibility of exchange of matter between the solid mantle and
525 the magma ocean(s) should be accounted for in dynamical models of the primi-
526 tive mantle since it greatly alters the pattern of convection as well as the desta-
527 bilization timescale. It could even be a way of producing degree-one structures
528 such as the ones observed on the Moon and Mars.

529 **Acknowledgements**

530 We thank the editor Bruce Buffet and an anonymous reviewer for their useful
531 remarks that helped to improve the clarity of our paper. This study is funded
532 by the French Agence Nationale de la Recherche (grant number ANR-15-CE31-
533 0018-01, MaCoMaOc).

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628 **A. Composition profile resulting from the fractional crystallization**
629 **of the surface magma ocean**

630 Conservation of the heavy component implies that

$$\frac{d}{dt} \left(\int_{R^-}^{R^+} C(r)r^2 dr + \frac{1}{3} (R_T^3 - R^{+3}) C_l \right) = 0, \quad (\text{A.1})$$

631 where no compressibility effect on density is considered, allowing the bulk den-
632 sity to drop out of the equation. Using eq. (2.1) and $\dot{C}_l = \dot{R}^+ \frac{dC_l}{dR^+}$, assuming
633 $\dot{R}^+ > 0$ at all time and R^- constant:

$$\frac{1}{C_l} \frac{dC_l}{dR^+} = 3(1 - D) \frac{R^{+2}}{R_T^3 - R^{+3}}. \quad (\text{A.2})$$

634 Using eq. (2.1), eq. (A.2) can be written for C :

$$\frac{1}{C} \frac{dC}{dR^+} - \frac{1}{D} \frac{dD}{dR^+} = 3(1 - D) \frac{R^{+2}}{R_T^3 - R^{+3}}. \quad (\text{A.3})$$

635 Since $C(r)$ does not depend on time, this equation holds for any $r \leq R^+(t)$ (i.e.
636 everywhere in the solid) and can be written as:

$$\frac{1}{C} \frac{dC}{dr} - \frac{1}{D} \frac{dD}{dr} = 3(1 - D) \frac{r^2}{R_T^3 - r^3}. \quad (\text{A.4})$$

637 Equation (A.4) is general and allows to take into account variations of D .
 638 However, it is useful to consider the limiting case of a constant partition coeffi-
 639 cient D . In that case, a solution to this equation is

$$C = C_0 \left(\frac{R_T^3 - R^{-3}}{R_T^3 - r^3} \right)^{1-D}, \quad (\text{A.5})$$

640 with $C_0 = DC_{l0}$ the mass fraction of FeO in the first solid formed.

641 Note that eq. (A.5) diverges when $r \rightarrow R_T$ but is in fact only valid as long
 642 as $C < 1$ and $C_l < 1$. When C_l reaches 1, the solid formed has the same
 643 composition as the liquid. The complete solution therefore is

$$C = \begin{cases} C_0 \left(\frac{R_T^3 - R^{-3}}{R_T^3 - r^3} \right)^{1-D} & \text{if } r < R_s \\ 1 & \text{if } r > R_s, \end{cases} \quad (\text{A.6})$$

644 with

$$R_s = \left((R^-)^3 C_0^{\frac{1}{1-D}} + R_T^3 \left(1 - C_0^{\frac{1}{1-D}} \right) \right)^{1/3} \quad (\text{A.7})$$

645 the value of $R^+(t)$ such that $C_l(t) = 1$.

646 B. Linear Stability

Since the solid is considered isoviscous and no source of toroidal flow is imposed at the boundaries, the velocity field can be expressed in terms of the scalar poloidal potential \mathcal{P} : $\mathbf{u} = \nabla \times \nabla \times (\mathcal{P} \mathbf{r})$ (e.g. Ricard and Vigny, 1989; Ribe, 2007). Linearizing eqs. (2.11) to (2.14) around the reference state ($\mathbf{u} = \mathbf{0}; \bar{T}; \bar{C}$) gives:

$$\mathcal{Q} = \nabla^2 \mathcal{P} \quad (\text{B.1})$$

$$\nabla^2 \mathcal{Q} = \text{Ra} \frac{\Theta - \langle \Theta \rangle}{r + \lambda} + \text{Rc} \frac{c - \langle c \rangle}{r + \lambda} \quad (\text{B.2})$$

$$\Gamma^2 \frac{\partial \Theta}{\partial t} + \frac{\partial \bar{T}}{\partial r} \frac{\mathcal{L}^2 \mathcal{P}}{r + \lambda} - \nabla^2 \Theta = W \left((r-1) \frac{\partial \Theta}{\partial r} + \left(\frac{\partial \bar{T}}{\partial r} \right)^+ \Theta \right) \quad (\text{B.3})$$

$$\Gamma^2 \frac{\partial c}{\partial t} + \frac{\partial \bar{C}}{\partial r} \frac{\mathcal{L}^2 \mathcal{P}}{r + \lambda} = W(r-1) \frac{\partial c}{\partial r}. \quad (\text{B.4})$$

The boundary conditions on the temperature and composition perturbations are trivial:

$$\Theta^\pm = 0, \quad (\text{B.5})$$

$$c^\pm = 0. \quad (\text{B.6})$$

The boundary condition eq. (2.16) and the free-slip boundary condition are written in term of the poloidal potential as:

$$\pm\Phi^\pm \frac{1}{r+\lambda} \mathcal{L}^2 \mathcal{P} + \frac{\partial}{\partial r} \left(\frac{2}{r+\lambda} \mathcal{L}^2 \mathcal{P} - (r+\lambda) \mathcal{Q} \right) = 0 \quad (\text{B.7})$$

$$\frac{\partial^2 \mathcal{P}}{\partial r^2} + (\mathcal{L}^2 - 2) \frac{\mathcal{P}}{(r+\lambda)^2} = 0. \quad (\text{B.8})$$

647 $\lambda = R^-/L - 1$ is a curvature term due to the definition of the dimensionless
648 radius. \mathcal{L}^2 is the horizontal laplacian: $\mathcal{L}^2 \bullet = \partial_r((r+\lambda)^2 \partial_r \bullet) - (r+\lambda)^2 \nabla^2 \bullet$. The
649 quantity \mathcal{Q} is introduced to ease the formulation of this system as an eigenvalue
650 problem involving square matrices.

651 The perturbations \mathcal{P} , \mathcal{Q} , Θ and c are developed using spherical harmonics,
652 e.g.

$$\mathcal{P} = \sum_{l=1}^{\infty} \sum_{m=-l}^l \mathcal{P}_l(r) Y_l^m(\theta, \phi) e^{\sigma_l t} \quad (\text{B.9})$$

653 where l and m are the spherical harmonics degree and order and σ_l is the growth
654 rate associated to the harmonic degree l . The system is laterally degenerated
655 and m does not affect the growth rate of the perturbation nor the shape of the
656 radial modes $\mathcal{P}_l(r)$, $\mathcal{Q}_l(r)$, $\Theta_l(r)$ and $c_l(r)$. These radial modes are discretized
657 using a Chebyshev collocation approach (e.g. Guo et al., 2012; Canuto et al.,
658 1985). Each radial mode is expressed as a vector whose components are the
659 values at the $N + 1$ Chebyshev nodal points (respectively denoted \mathbf{P} , \mathbf{Q} , \mathbf{T}
660 and \mathbf{C}). Radial derivatives evaluated at the nodal points $r_i = \frac{1}{2} (3 + \cos \frac{i\pi}{N})$
661 can then be expressed with a differentiation matrix \mathbf{d} , e.g. $\partial_r \mathcal{P}(r_i) = (\mathbf{dP})_i$. We
662 formulate the system of linearized equations along with the associated boundary
663 conditions as

$$\mathbf{LX} = \sigma_l \mathbf{RX} \quad (\text{B.10})$$

with

$$\mathbf{X} = \begin{pmatrix} \mathbf{P} & 0 : N \\ \mathbf{Q} & 0 : N \\ \mathbf{T} & 1 : N - 1 \\ \mathbf{C} & 1 : N - 1 \end{pmatrix} \quad (\text{B.11})$$

$$\mathbf{L} = \begin{pmatrix} 0 : N & 0 : N & 1 : N - 1 & 1 : N - 1 \\ \mathbf{d}^2 + (l_2 - 2)\underline{\mathbf{r}}_\lambda^{-2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{D}^2 & -\mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{d}^2 + (l_2 - 2)\underline{\mathbf{r}}_\lambda^{-2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ l_2(\Phi^+ \underline{\mathbf{r}}_\lambda^{-1} - 2\underline{\mathbf{r}}_\lambda^{-2} + 2\underline{\mathbf{r}}_\lambda^{-1} \mathbf{d}) & -(\mathbf{1} + \underline{\mathbf{r}}_\lambda \mathbf{d}) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}^2 & -\text{Ra} \underline{\mathbf{r}}_\lambda^{-1} & -\text{Rc} \underline{\mathbf{r}}_\lambda^{-1} \\ l_2(-\Phi^- \underline{\mathbf{r}}_\lambda^{-1} - 2\underline{\mathbf{r}}_\lambda^{-2} + 2\underline{\mathbf{r}}_\lambda^{-1} \mathbf{d}) & -(\mathbf{1} + \underline{\mathbf{r}}_\lambda \mathbf{d}) & \mathbf{0} & \mathbf{0} \\ -(\partial_r \bar{T}) l_2 \underline{\mathbf{r}}_\lambda^{-1} & \mathbf{0} & \mathbf{D}^2 + W^+ ((\underline{\mathbf{r}} - \mathbf{1}) \mathbf{d} + (\partial_r \bar{T})^+ \mathbf{1}) & \mathbf{0} \\ -(\partial_r \bar{C}) l_2 \underline{\mathbf{r}}_\lambda^{-1} & \mathbf{0} & \mathbf{0} & W^+ (\underline{\mathbf{r}} - \mathbf{1}) \mathbf{d} \end{pmatrix} \begin{pmatrix} 0 \\ 1 : N - 1 \\ N \\ 0 \\ 1 : N - 1 \\ N \\ 1 : N - 1 \\ 1 : N - 1 \end{pmatrix} \quad (\text{B.12})$$

$$\mathbf{R} = \begin{pmatrix} 0 : N & 0 : N & 1 : N - 1 & 1 : N - 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Gamma^2 \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \Gamma^2 \mathbf{1} \end{pmatrix} \begin{pmatrix} 0 : N \\ 0 : N \\ 1 : N - 1 \\ 1 : N - 1 \end{pmatrix} \quad (\text{B.13})$$

664 where $\mathbf{1}$ is the identity matrix, $\underline{\mathbf{r}}_{ij} = r_i \mathbf{1}_{ij}$, $\underline{\mathbf{r}}_\lambda = \underline{\mathbf{r}} + \lambda \mathbf{1}$, $l_2 = l(l + 1)$ and
 665 $\mathbf{D}^2 = \mathbf{d}^2 + 2\underline{\mathbf{r}}_\lambda^{-1} \mathbf{d} - l_2 \underline{\mathbf{r}}_\lambda^{-2}$. The extra row and column on top and right of the
 666 matrices are respectively the column and row indices of each of the submatrices.
 667 For example, the top left submatrix of the matrix \mathbf{L} is only the first row (hence
 668 the 0 on the extra column) of the matrix $\mathbf{d}^2 + (l_2 - 2)\underline{\mathbf{r}}_\lambda^{-2}$.

669 At a given instant during the crystallization, all the dimensionless numbers
 670 W , λ , Γ , Ra and Rc appearing in the matrices \mathbf{L} and \mathbf{R} are known. For any
 671 harmonic degree l of the perturbation, finding its growth rate σ_l and associated
 672 vertical mode \mathbf{X} is an eigenvalue problem. The largest eigenvalue is the growth
 673 rate, and the associated eigenvector represent the vertical modes. At a given
 674 instant, we look for the harmonic degree l with the highest growth rate σ_l , which
 675 is then used to compute the dimensional destabilization time scale $L_M^2 / (\kappa \sigma)$.