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Asynchronous Switched Observers Design for Switched Takagi-Sugeno Systems Subject to Output Disturbances

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Abstract: This paper deals with the design of asynchronous switched observers for a class of switched continuous-time Takagi-Sugeno (TS) systems subject to output disturbances. The considered switched TS observer is proposed for the continuous state estimation when the active switched modes of the system are unknown or not precisely known. The design conditions, obtained via switched Lyapunov-like function candidates, are expressed as Linear Matrix Inequality (LMI) and satisfy H_∞ disturbance attenuation performances. The effectiveness of the proposed approach is illustrated through the simulation of an academic example.

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Keywords: Switched Takagi-Sugeno systems, Asynchronous switched observer design, LMIs.

1. INTRODUCTION

The use of new technological devices, for instance real-time embedded control plants, makes switched systems suitable and realistic to model several physical systems involving both continuous and discrete dynamic behaviors. In this context, fruitful theoretical results have been proposed for the state estimation of switched systems, see e.g. (Alessandri and Coletta, 2001; Hamdi et al., 2009; Jabri et al., 2012; Belkhiat et al., 2011). These approaches can be classified into two categories: In one hand, some authors assume that the active switched mode is known at any time and, consequently, the observer switches synchronously with the switched systems (Alessandri and Coletta, 2001; Belkhiat et al., 2015; Ethabet et al., 2018). In the other hand, when the systems' active mode is unavailable from measurements, which is often the case in practice, the observer's switching laws cannot precisely match the system's ones, leading to asynchronous switches between the system and the observer (Wang and Tong, 2017; Xiang et al., 2012). The observer design conditions for switched systems are often obtained via the direct Lyapunov methodology. In this framework, pioneer works considered common quadratic Lyapunov functions, see e.g. (Alessandri and Coletta, 2001). Although the convergence of the estimation error is guaranteed regardless of the systems and observers' switches, these common quadratic approaches remain very conservative. Therefore, some results have considered switched quadratic Lyapunov functions to relax the design conditions and expand them to a larger class of switched systems (Belkhiat et al., 2015; Xiang et al., 2012).

Furthermore, Takagi-Sugeno (TS) fuzzy systems are nowadays extensively used for the modeling and control design

of switched systems exhibiting nonlinear dynamics, see e.g. (Jabri et al., 2011, 2012, 2018b,a; Cherifi et al., 2019). However, although several progress have been made in some fields of switched TS systems as the stability and stabilization issues, there is still space for improvements on state estimation and switched TS observers design. To this ends, Lendek et al. (2012) have proposed a non-quadratic Lyapunov function-based approach to design switched TS observers for a class of periodic discrete-time TS models. Then relaxed conditions, considering the α -sample variation of the discrete-time Lyapunov function, have been proposed (Lendek et al., 2014). Recently, to estimate the state vector of discrete-time TS fuzzy switched system under synchronous switching, Liu et al. (2018) have proposed a synthesis approach to perform a design of switched Luenberger-type observers. Several sufficient conditions were established by constructing multiple Lyapunov functions and common Lyapunov functions. In the continuous time framework, a design method for observer-based output feedback control for a class of switched fuzzy systems with time-varying delay is proposed in (Zhang et al., 2018). Moreover, the estimation of the lateral velocity and yaw rate using switched Takagi-Sugeno fuzzy interval observers have been developed in (Ifqir et al., 2017). In the latter mentioned study, the active mode is assumed to be available at any time, which allows to automatically select the appropriate candidate observer. The main results have been expressed in terms of sufficient LMI-based conditions through the use of a switched fuzzy Lyapunov function. Note that most of the available studies in the switched TS framework consider that the observer and the system to be estimated share the same switched sets and that both switch at the same instants. This is obviously a drawback for two reasons: 1) the observer state doesn't

match the system's state (especially during transients) leading to different switching instants even if the same switched sets are considered, 2) in practical applications, measuring both the switching instant and modes from the plant remains a tricky challenge to implement such observers. To circumvent these drawback, the goal of this paper is to design asynchronous observers for continuous-time switched TS systems subject to output disturbances (i.e. additive sensor fault) with H_∞ performance specifications and under arbitrary switching sets and sequences. Indeed, to the best of the authors' knowledge, this hasn't been explored so far, which motivates this study.

In what follows, new LMI-based conditions for the design of asynchronous switched TS observers for a class of switched TS fuzzy systems subject to external bounded output disturbances, under arbitrary switching sets and sequences, is presented. In this study, we assume that the exact switching sequence is unknown (or not precisely known) but only the set of all the admissible switches is known. The proposed approach ensures that, without external disturbances, the estimation error asymptotically converges to zero regardless the switches of the system and the observer. In addition, it ensures the robustness of the observer against output disturbances.

The paper is organized as follows. Section II presents the considered class of switched TS fuzzy systems as well as the problem statement. Section III details the proposed LMI-based conditions for the design of the proposed robust switched observer. Finally, an academic example is considered in section IV to illustrate the effectiveness of the proposed approach.

2. PRELIMINARIES AND PROBLEM STATEMENT

Let us consider the class of nonlinear switched systems represented by switched TS models given as follows:

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^m \sum_{i_j=1}^{r_j} \sigma_j(t) h_{i_j}(z_j(t)) (A_{i_j} x(t) + B_{i_j} u(t)) \\ y(t) = Cx(t) + Ww(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^v$, $w(t) \in \mathbb{R}^v$ represent respectively the state, the measurement (output), the input vectors and a L_2 -norm bounded output disturbance vector. m is the number of switching modes. r_j is the number of fuzzy rules associated to the j^{th} mode. For $j = 1, \dots, m$, $i_j = 1, \dots, r_j$, $A_{i_j} \in \mathbb{R}^{n \times n}$, $B_{i_j} \in \mathbb{R}^{n \times v}$, $C \in \mathbb{R}^{p \times n}$ and $W \in \mathbb{R}^{p \times v}$ are constant matrices describing the local dynamics of each polytopes. $z_j(t)$ are vectors of premise variables and $h_{i_j}(z_j(t)) \geq 0$ are membership functions satisfying the convex sum proprieties $\sum_{i_j=1}^{r_j} h_{i_j}(z_j(t)) = 1$. The piecewise continuous functions $\sigma_j(t) \in \{0, 1\}$ represent the system's switching laws defined such that the active system in the l^{th} mode leads to:

$$\begin{cases} \sigma_j(t) = 1 & \text{if } j = l \\ \sigma_j(t) = 0 & \text{if } j \neq l \end{cases} \quad (2)$$

Note that, in this paper the switching functions $\sigma_j(t)$ of switched TS models (1) are supposed to be unavailable from measurements but occur at defined switching sets S_{j,j^+} where j and j^+ denote respectively the active modes

before and after switching. Without loss of generality, these switching sets are described in this paper by linear hyper-planes such that:

$$S_{j,j^+} = \{x \in \mathbb{R}^n | s_{j,j^+} x = 0\}, \quad (j, j^+) \in I_s \quad (3)$$

where s_{j,j^+} are constant scalars (possibly unknown), I_s denotes the set of admissible switches, which are assumed known in this study, i.e. $(j, j^+) \in I_s$ if a switch from mode j to j^+ is possible.

Remark 1. This work is not concerned with unmeasured premise variables, see e.g. (Ichalal et al., 2018) and references therein for studies dealing with that topic (without considering switched systems). Thus, in this paper, the premise variables $z_j(t)$ are assumed to be known and depend only on the measured output, i.e. $z_j(t) \equiv y(t)$.

Our goal is to design the following robust switched fuzzy observer, dedicated to estimate the continuous state $x(t)$ of (1).

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{\hat{j}=1}^m \sum_{i_{\hat{j}}=1}^{r_{\hat{j}}} \hat{\sigma}_{i_{\hat{j}}}(t) h_{i_{\hat{j}}}(y(t)) \left(A_{i_{\hat{j}}} \hat{x}(t) + B_{i_{\hat{j}}} u(t) \right. \\ \left. + K_{i_{\hat{j}}}(y(t) - \hat{y}(t)) \right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \quad (4)$$

where $\hat{x}(t) \in \mathbb{R}^n$, $\hat{y}(t) \in \mathbb{R}^p$ represent respectively the estimated state and output vectors. $\hat{j} = 1, \dots, m$ are the observer's switched modes having each $r_{\hat{j}} \equiv r_j$ fuzzy rules, i.e. $i_{\hat{j}} = 1, \dots, r_{\hat{j}}$. $K_{i_{\hat{j}}}$ are the observer gain matrix to be synthesized. The piecewise continuous function $\hat{\sigma}_{i_{\hat{j}}}(t) \in \{0, 1\}$ is the observer's switching law, whose switches can occur according to the observer's own switching sets $\bar{S}_{\hat{j},\hat{j}^+}$, i.e. possibly mismatching the switching sets of the system (3), and given as follows:

$$\bar{S}_{\hat{j},\hat{j}^+} = \{\hat{x} \in \mathbb{R}^n | \bar{s}_{\hat{j},\hat{j}^+} \hat{x} = 0\}, \quad (\hat{j}, \hat{j}^+) \in I_s \quad (5)$$

where $\bar{s}_{\hat{j},\hat{j}^+}$ are constant scalars.

Notations. In the sequel, to lighten the mathematical expression, the premise entries z_j will be omitted when there is no ambiguities and the following notations will be employed for fuzzy matrices:

$$M_{h_\sigma} = \sum_{j=1}^m \sum_{i=1}^{r_j} \sigma_j(t) h_{i_j}(y(t)) M_{i_j} \quad (6)$$

and:

$$N_{h_\sigma} = \sum_{\hat{j}=1}^m \sum_{i_{\hat{j}}=1}^{r_{\hat{j}}} \hat{\sigma}_{i_{\hat{j}}}(t) h_{i_{\hat{j}}}(y(t)) N_{i_{\hat{j}}} \quad (7)$$

Moreover, the subscript \dagger denotes the Moore-Penrose pseudo-inverse and $\mathcal{H}_e(M) = M + M^T$.

Note that the switching sets $\bar{S}_{\hat{j},\hat{j}^+}$ depend on the estimated state \hat{x} from the observer, whereas the switching sets S_{j,j^+} depend on the system's state x . Moreover, as mentioned in the introduction, the synchronous case ($\sigma(t) = \hat{\sigma}(t)$) is not realistic, especially during transient since $\hat{x}(t) \neq x(t)$. Therefore, in the sequel, we will not consider the synchronous case but the more general case where the switched system (1) and the switched observer (4) operate asynchronously in different modes ($\sigma(t) \neq \hat{\sigma}(t)$) and with

possibly mismatching switching sets ($\bar{S}_{\hat{j}, \hat{j}^+} \neq S_{j, j^+}$). In this asynchronous case, the dynamics of the estimation error $e = x(t) - \hat{x}(t)$ can be expressed as:

$$\dot{e}(t) = (A_{h_\sigma} - K_{h_\sigma} C) e(t) + (A_{h_\sigma} - A_{h_{\hat{\sigma}}}) x(t) + (B_{h_\sigma} - B_{h_{\hat{\sigma}}}) u(t) - K_{h_{\hat{\sigma}}} W w(t) \quad (8)$$

Moreover, let us notice that the dynamics of the estimation error (8) depends on the state vector $x(t)$, the input vector $u(t)$ and also on the disturbance vector $w(t)$. Thus, the following statements are defined as the observer design requirements:

- With zero disturbance input condition ($w(t) = 0$), the estimation error $e(t)$ asymptotically converges to zero.
- For all non zero $w(t) \in L_2[0, \infty)$, the following H_∞ criterion is considered:

$$\int_0^\infty e^T(t) e(t) dt \leq \gamma^2 \int_0^\infty w^T(t) w(t) dt \quad (9)$$

where $\gamma > 0$ is an attenuation level from $w(t)$ to $e(t)$, to be minimized.

To conclude these preliminaries, let us recall the following useful lemma for the proof of the conditions proposed in the next section.

Lemma 2. (S-procedure(Derinkuyu and Pinar, 2015)). Let $L \subset L^2$ be time invariant subspace and $\psi_k : L \rightarrow R$, ($k = 0, \dots, M$), be continuous time invariant quadratic forms. Suppose that there exist $f_* \in L$ such that $\psi_1(f_*) > 0, \dots, \psi_M(f_*) > 0$, then the following statement are equivalent:

- $\psi_0(f) \leq 0$ for all $f \in L$ such that $\psi_1(f) > 0, \dots, \psi_M(f) > 0$.
- There exist $\tau_1 \geq 0, \dots, \tau_M \geq 0$ such that $\psi_0(f) + \tau_1 \psi_1(f) + \dots + \tau_M \psi_M(f) \leq 0$ for all $f \in L$.

3. MAIN RESULT

The goal of this section is to provide sufficient LMI-based conditions to design the fuzzy gain matrices $K_{h_{\hat{\sigma}}}$ of the observer (4) such that the robust H_∞ requirements defined above are satisfied. The main result is summarized by the following theorem.

Theorem 3. Let us consider the switched TS system (1) and the asynchronous switched TS observer (4). For given scalar $\varepsilon > 0$, if, for all combinations of $i = 1, \dots, r_j$, $(\hat{j}, \hat{j}^+) \in \{1, \dots, m\}^2$ and $(\hat{j}, \hat{j}^+) \in I_s$, there exist a scalar $\lambda > 0$ and the matrices $P_{\hat{j}} = P_{\hat{j}}^T > 0$, $G_{\hat{j}}$, $Y_{i_{\hat{j}}}$ such that the scalar $\gamma > 0$ is minimized and satisfying the following conditions:

- The following equality and set of LMIs are satisfied:

$$P_{\hat{j}^+} = P_{\hat{j}} + G_{\hat{j}}^T C + C^T G_{\hat{j}} \quad (10)$$

$$\begin{bmatrix} \Gamma_{i_{\hat{j}} i_{\hat{j}^+}} & (*) & (*) & (*) \\ A_{i_{\hat{j}}}^T P_{\hat{j}} - A_{i_{\hat{j}^+}}^T P_{\hat{j}} & -\lambda \varepsilon & 0 & 0 \\ B_{i_{\hat{j}}}^T P_{\hat{j}} - B_{i_{\hat{j}^+}}^T P_{\hat{j}} & 0 & -\lambda \varepsilon & (*) \\ -Y_{i_{\hat{j}}} W & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (11)$$

with $\Gamma_{i_{\hat{j}} i_{\hat{j}^+}} = \mathcal{H}_e(P_{\hat{j}} A_{i_{\hat{j}}} - Y_{i_{\hat{j}}} C) + (1 + \lambda) I$,

- $\forall \hat{x} \in \bar{S}_{\hat{j}, \hat{j}^+}$, the state of the switched observer is updated according to:

$$\hat{x}^+ = \left(I - R_{\hat{j}}^{-1} (C R_{\hat{j}}^{-1})^\dagger C \right) \hat{x} + R_{\hat{j}}^{-1} (C R_{\hat{j}}^{-1})^\dagger y \quad (12)$$

with:

$$R_{\hat{j}} = V_{\hat{j}} \sqrt{\Lambda_{\hat{j}}} V_{\hat{j}}^T \quad (13)$$

and where $V_{\hat{j}} \in \mathbb{R}^{\eta \times \eta}$ is composed of the orthonormal eigenvectors of $P_{\hat{j}}$ and $\Lambda_{\hat{j}} \in \mathbb{R}^{\eta \times \eta}$ is the spectral matrix for $P_{\hat{j}}$, i.e. a diagonal matrix composed with the eigenvalues of $P_{\hat{j}}$ and $\sqrt{\Lambda_{\hat{j}}}$ denotes a diagonal matrix containing the square root of these eigenvalues,

then the observer gains are given by $K_{i_{\hat{j}}} = P_{\hat{j}}^{-1} Y_{i_{\hat{j}}}$ and the observer's state $\hat{x}(t)$ converge asymptotically to the system's state $x(t)$ (without external disturbances), i.e. the estimation error $e(t) \rightarrow 0$ when $t \rightarrow +\infty$, and the H_∞ criterion (9) is satisfied (for external disturbances attenuation).

Proof. Let us consider the following multiple Lyapunov function candidate:

$$v(t) = e^T(t) P_{\hat{\sigma}} e(t) \quad (14)$$

where $P_{\hat{\sigma}} = \sum_{j=1}^m \sigma_j(t) P_j$ with $P_j = P_j^T > 0$.

The time derivative of (14) can be written as:

$$\dot{v}(t) = 2(e^T P_{\hat{\sigma}} (A_{h_\sigma} - K_{h_{\hat{\sigma}}} C) e + e^T P_{\hat{\sigma}} (A_{h_\sigma} - A_{h_{\hat{\sigma}}}) x + e^T P_{\hat{\sigma}} (B_{h_\sigma} - B_{h_{\hat{\sigma}}}) u - e^T P_{\hat{\sigma}} K_{h_{\hat{\sigma}}} W w(t)) \quad (15)$$

Moreover, the H_∞ criterion (9) is satisfied if:

$$\dot{v}(t) + e^T(t) e(t) - \gamma^2 w^T(t) w(t) < 0 \quad (16)$$

which can be rewritten as:

$$\psi(t)^T \Omega(t) \psi(t) < 0 \quad (17)$$

with $\psi(t) = [e^T \ x^T \ u^T \ w^T]^T$ and:

$$\Omega(t) = \begin{bmatrix} \mathcal{H}_e(P_{\hat{\sigma}} A_{h_\sigma} - P_{\hat{\sigma}} K_{h_{\hat{\sigma}}} C) + I & (*) & (*) & (*) \\ A_{h_\sigma}^T P_{\hat{\sigma}} - A_{h_{\hat{\sigma}}}^T P_{\hat{\sigma}} & 0 & 0 & 0 \\ B_{h_\sigma}^T P_{\hat{\sigma}} - B_{h_{\hat{\sigma}}}^T P_{\hat{\sigma}} & 0 & 0 & 0 \\ -P_{\hat{\sigma}} K_{h_{\hat{\sigma}}} W & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

To deal with the inequality (17), with a scalar $\varepsilon > 0$, let us consider the following constraint:

$$\|e\|_2^2 - \varepsilon (\|x\|_2^2 + \|u\|_2^2) > 0 \quad (18)$$

From (18) and applying lemma 1, the inequality (17) is satisfied if there exists a scalar $\lambda > 0$ such that:

$$\begin{bmatrix} \Gamma(t) & (*) & (*) & (*) \\ A_{h_\sigma}^T P_{\hat{\sigma}} - A_{h_{\hat{\sigma}}}^T P_{\hat{\sigma}} & -\lambda \varepsilon & 0 & 0 \\ B_{h_\sigma}^T P_{\hat{\sigma}} - B_{h_{\hat{\sigma}}}^T P_{\hat{\sigma}} & 0 & -\lambda \varepsilon & (*) \\ -P_{\hat{\sigma}} K_{h_{\hat{\sigma}}} W & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \quad (19)$$

where $\Gamma(t) = \mathcal{H}_e(P_{\hat{\sigma}} A_{h_\sigma} - Y_{h_{\hat{\sigma}}} C) + (1 + \lambda) I$.

With the bijective change of variable $Y_{h_{\hat{\sigma}}} = P_{\hat{\sigma}} K_{h_{\hat{\sigma}}}$, the inequality (19) is satisfied if the LMI conditions (11) hold. Now, we have to prove that the Lyapunov Function candidate (14) is decreasing at the observer's switching instants, i.e.:

$$(x - \hat{x}^+)^T P_{\hat{j}^+} (x - \hat{x}^+) \leq (x - \hat{x})^T P_{\hat{j}} (x - \hat{x}) \quad (20)$$

where, at switching instants, \hat{j} and \hat{j}^+ denotes respectively the observer's previous and forthcoming switched modes

and \hat{x}^+ the estimated state to be updated, whose value computation will be further detailed.

Following the pioneer work of Petterson (2005), let us first assume that \hat{x}^+ satisfies $y = C\hat{x}^+$. In that case we have $C(x - \hat{x}^+) = y - y = 0$ and, for any matrix M of appropriate dimension, the following equality holds:

$$(x - \hat{x}^+)^T (M^T C + C^T M) (x - \hat{x}^+) = 0 \quad (21)$$

Then, assume that there exist the matrices $G_{\hat{j}}$ such that the equality (10) is satisfied, according to (21) the inequality (20) can be rewritten as:

$$(x - \hat{x}^+)^T P_{\hat{j}} (x - \hat{x}^+) \leq (x - \hat{x})^T P_{\hat{j}} (x - \hat{x}) \quad (22)$$

Now, we need to determine the updated value of the observer's state $\hat{x}^+(t) \in \bar{S}_{\hat{j}, \hat{j}^+}$ such that the inequality (22) is verified. To this end, let us consider $R_{\hat{j}} \in \mathbb{R}^{n \times n}$ such that $P_{\hat{j}} = R_{\hat{j}}^T R_{\hat{j}}$. Note that, according to the spectral theorem (Derinkuyu and Pinar, 2015), the matrices $R_{\hat{j}}$ can be obtained as given in (13). Thus, the inequality (22) is equivalently satisfied if:

$$\|R_{\hat{j}}(x - \hat{x}^+)\| \leq \|R_{\hat{j}}(x - \hat{x})\| \quad (23)$$

To find the updated value \hat{x}^+ , lying on the hyper plane $y = C\hat{x}^+$, that minimizes the distance $\|R_{\hat{j}}(x - \hat{x}^+)\|$, let us consider the following optimization problem:

$$\begin{aligned} \min_{\hat{x}^+} & \|R_{\hat{j}}(\hat{x}^+ - \hat{x})\| \\ \text{subject to : } & C\hat{x}^+ = y \end{aligned} \quad (24)$$

By denoting $\alpha_{\hat{j}} = R_{\hat{j}}(\hat{x}^+ - \hat{x})$, i.e. $R_{\hat{j}}\hat{x}^+ = \alpha_{\hat{j}} + R_{\hat{j}}\hat{x}$, the optimization problem (24) can be rewritten as:

$$\begin{aligned} \min_{\hat{x}^+} & \|\alpha_{\hat{j}}\| \\ \text{subject to : } & CR_{\hat{j}}^{-1}\alpha_{\hat{j}} = y - C\hat{x} \end{aligned} \quad (25)$$

which solution, being the minimum length least square solution to $y - C\hat{x}$, is:

$$\alpha_{\hat{j}} = (CR_{\hat{j}}^{-1})^\dagger (y - C\hat{x}) \quad (26)$$

Hence:

$$R_{\hat{j}}\hat{x}^+ = R_{\hat{j}}\hat{x} + (CR_{\hat{j}}^{-1})^\dagger (y - C\hat{x}) \quad (27)$$

Left multiplying (27) by $R_{\hat{j}}^{-1}$ yields to (12). This ends the proof. \square

4. NUMERICAL EXAMPLE

The present section is dedicated to illustrate the effectiveness of the proposed switched TS observer design approach. To this end, let us consider the following second order switched TS system with two switched modes and two rules in each of them:

$$\begin{cases} \dot{x}(t) = \sum_{j=1}^2 \sum_{i=1}^2 \sigma_j(t) h_{ij}(z_j(t)) (A_{ij}x(t) + B_{ij}u(t)) \\ y(t) = Cx(t) + W\omega(t) \end{cases} \quad (28)$$

with the state vector $x^T = [x_1 \ x_2]$ and the matrices

$$\begin{aligned} A_{11} &= \begin{bmatrix} -0.2 & 10 \\ -2 & -1 \end{bmatrix}, A_{21} = \begin{bmatrix} 1.5 & 2 \\ -6 & -0.5 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ B_{21} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ in the first mode and } A_{12} = \begin{bmatrix} -2 & 12 \\ -2 & -1 \end{bmatrix}, \end{aligned}$$

$A_{22} = \begin{bmatrix} 1.5 & 2 \\ -2.5 & 1.9 \end{bmatrix}$, $B_{12} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $B_{22} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ in the second one, $C = [-1 \ -1.5]$, $W = [0.1 \ 0.1]$ and the membership functions $h_{11}(y(t)) = 1 - \sin^2(y(t))$, $h_{12}(y(t)) = 1 - h_{11}(y(t))$, $h_{21}(y(t)) = 1 - (\sin(3y(t)))^2$ and $h_{22}(y(t)) = 1 - h_{21}(y(t))$.

Solving the LMI-based conditions given in the theorem 3, we obtain the gains $K_{11} = \begin{bmatrix} 53.5 \\ -62.8 \end{bmatrix}$, $K_{12} = \begin{bmatrix} 31.2 \\ -40.3 \end{bmatrix}$, $K_{21} = \begin{bmatrix} 20.3 \\ -21.8 \end{bmatrix}$, $K_{22} = \begin{bmatrix} 17.4 \\ -20.5 \end{bmatrix}$ for the switched observer (4) and the Lyapunov matrices $P_1 = \begin{bmatrix} 17.9 & 15.5 \\ 15.5 & 14.4 \end{bmatrix}$, $P_2 = \begin{bmatrix} 10.8 & 10.1 \\ 10.1 & 14.3 \end{bmatrix}$, with $\gamma = 5.5$, $\varepsilon = 20$, $G_1 = [3.6 \ 0.04]$ and $\lambda = 8.6$.

In the sequel, we assume that the system (28) switches within the frontier defined by the hyper planes $S_{1,2} = x_1 + 0.25x_2$ and $S_{2,1} = x_1 + x_2$. Moreover, the simulation of the switched system (28) is performed with the input signal:

$$\begin{cases} u(t) = 1 & 0 \leq t \leq 2s \\ u(t) = 10 \sin(\pi t + 2) & 2 < t \leq 5s \end{cases} \quad (29)$$

and the output disturbance:

$$\begin{cases} w(t) = [0 \ 0]^T & 0 \leq t \leq 2.5s \\ w(t) = [1 \sin(0.8\pi t + 10) \ 0]^T & 2.5 < t \leq 4s \\ w(t) = [0 \ 0]^T & 4 < t \leq 5s \end{cases} \quad (30)$$

Then, two cases are considered in simulation for the switching hyper planes of the designed asynchronous switching TS observer.

Case 1: The designed switched observer (4) and the switched system (28) share the same switching sets, i.e. $\bar{S}_{\hat{j}, \hat{j}^+} = S_{j, j^+}$. For simulation purpose, the system and observer initial state vectors have been respectively set as $x^T(0) = [3 \ 16.5]$, $\hat{x}^T(0) = [-2 \ -10]$. That is to say, the observer and the system have been respectively initialized in their first and second modes.

The simulation results for this case are shown in Fig.1 to Fig.4. Fig. 1 exhibits the phase planes of the observer and the system. Fig. 2 shows the trajectories of both state and estimated vectors. Moreover, Fig. 3 shows the estimation error evolution. In addition, Fig. 4 depicts the evolution of both system's and observer's modes. This simulation illustrates the ability of the designed observer to accurately estimate the system's state when they both share the same switching sets. Moreover, let us highlight that, even if the system and the observer share the same switching sets, they remain asynchronous (i.e. $\hat{\sigma}(t) \neq \sigma(t)$) since they don't cross the hyper-planes' frontiers exactly at the same instants. This is pointed-out in the zooms shown in Fig. 4. However, when the transient is passed, i.e. when the observer's state get close to the system state, this case also allows the estimation of the system's switching instants from the observer ones.

Case 2: This second case is dedicated to show what happens when only poor knowledge regarding to the system's switching law are available. For instance, this is

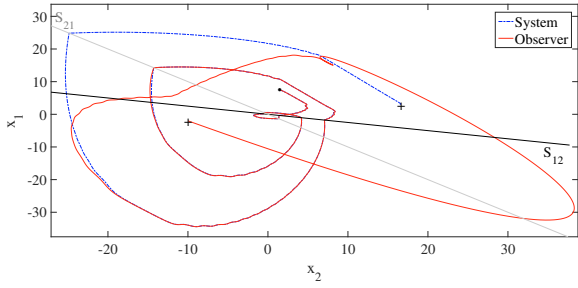


Fig. 1. System’s and observer’s phase planes, + denotes the initial conditions and • the final states (Case 1).

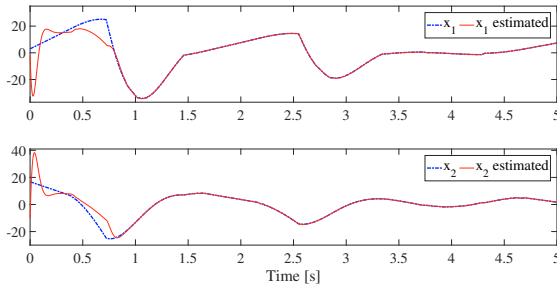


Fig. 2. States and estimated states evolution (Case 1).

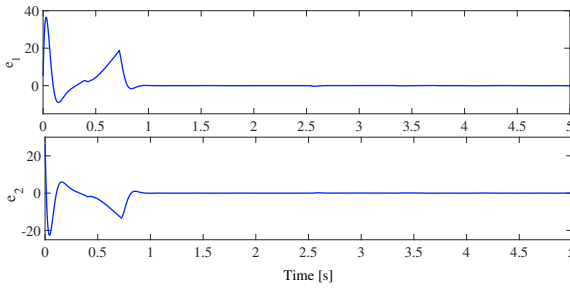


Fig. 3. Evolution of the estimation errors (Case 1).

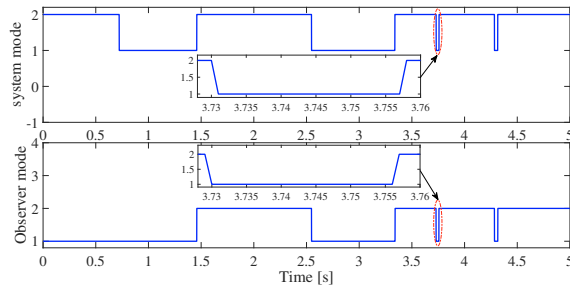


Fig. 4. Evolution of system and observer modes (Case 1).

often the case in practical application when parametric identification of the system’s switching law are biased. The only difference with regard to the previous case (Case 1) is that the observer’s switching sets defer from the ones of the switched system ($\hat{S}_{ij} \neq S_{ij}$). Thus, in this case, to perform the simulation we define the observer’s switching sets as $\hat{S}_{1,2} = 0.97\hat{x}_1 + 0.22\hat{x}_2$ and $\hat{S}_{2,1} = 0.97\hat{x}_1 + 0.90\hat{x}_2$.

For this simulation, the same initial states as in Case 1 for both the system and the observer have been considered for comparison purpose. Therefore, with the considered

switching sets, the observer and the system are still respectively initialized in their first and second modes.

The simulation results are shown in Fig.5 to Fig.8. In contrast to the first case, Fig.8 shows that the system and the observer switch within different time frames because they do not share the same switch sets. Indeed, in some larger time-intervals, the switched system (1) and the switched observer (4) operate asynchronously in different modes ($\sigma(t) \neq \hat{\sigma}(t)$). This is clearly shown in Fig.8 for $t \in [1.5, 2.6]$ where the system belongs to mode 2 while the observer belongs to mode 1, leading to poorer state estimation (see Fig 7 and 5). Note also that the external output disturbance acts during this interval, which doesn’t help the observer to quickly converge. However, after a while (for $t > 2.7$), the observer better catches the system’s state as their respective switching frontiers gets closer and, in this case, we can observe that their respective switching laws tend to synchronize (see the zooms in Fig. 8). This illustrates that the state estimation is produced with good accuracy after a finite time interval, asserting the validity of the proposed asynchronous switched TS observer design methodology.

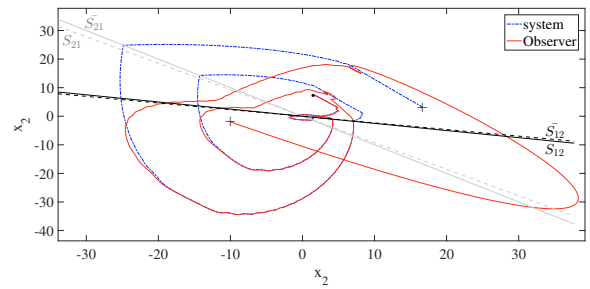


Fig. 5. System’s and observer’s phase planes, + denotes the initial conditions and • the final states (Case 2).

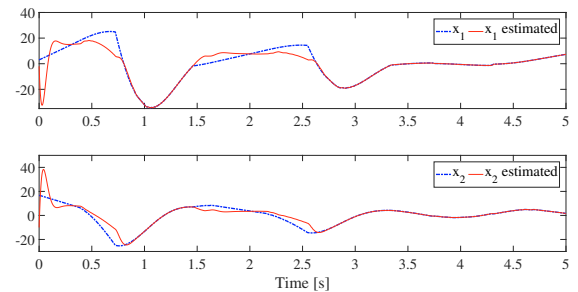


Fig. 6. States and estimated states evolution (Case 2).

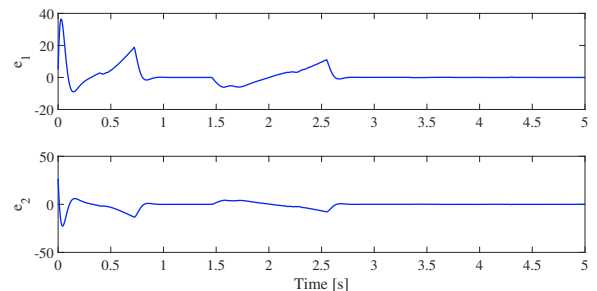


Fig. 7. Evolution of estimation error (Case 2).

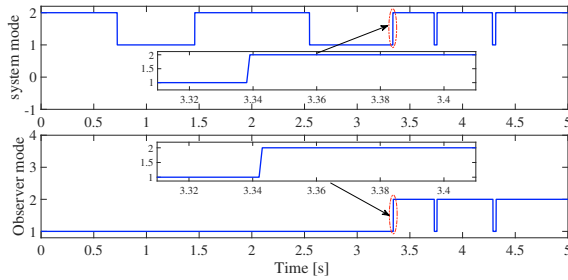


Fig. 8. Evolution of both system's and observer's mode (Case 2).

5. CONCLUSION

This work has dealt with the design of a robust observer for a class of switched TS systems subject to external output disturbances. The proposed observer delivers the continuous state estimation under arbitrary switching sequence. Thanks to a candidate multiple switched Lyapunov functional, LMI-based conditions, satisfying H_∞ rejection disturbance performances, for the robust observer design has been obtained. Finally, illustrative examples, showing clearly the efficiency of the designed observer, have been presented. Note that, in this study, it assumed that the premise variables are measurable. Thus, the extension of the proposed design method to the case of unmeasurable premise variables will be the focus of our future work.

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