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# Illustration of the Inclusion of Sound–Flow Interactions in Lighthill’s Equation

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## Introduction

**L**IGHTHILL’S equation<sup>1</sup> is an exact reformulation of the flow equations

$$\left( \frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2 \right) \rho' = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}(\mathbf{x}, t) \quad (1)$$

where  $\rho' = \rho - \rho_0$  is the density fluctuation,  $\rho_0$  and  $c_0$  the ambient density and sound speed, and  $T_{ij} = \rho u_i u_j + (p - c_0^2 \rho) \delta_{ij} - \tau_{ij}$  the Lighthill stress tensor, with  $u_i$  the velocity components,  $p$  the pressure, and  $\tau_{ij}$  the viscous stresses. The classical interpretation of Eq. (1) consists of regarding the aerodynamic noise as solution of a wave equation in a fictitious medium at rest. The sound generation is assigned to the right-hand side, through the tensor  $T_{ij}$ , which is reduced to  $T_{ij} = \rho u_i u_j$  in unheated flows at high Reynolds numbers. Thus, as long as  $T_{ij}$  is known, evaluated from the unsteady Reynolds averaged Navier–Stokes equations,<sup>2</sup> from large eddy simulation,<sup>3</sup> or from direct numerical simulation,<sup>4,5</sup> Eq. (1) can be solved for noise predictions.

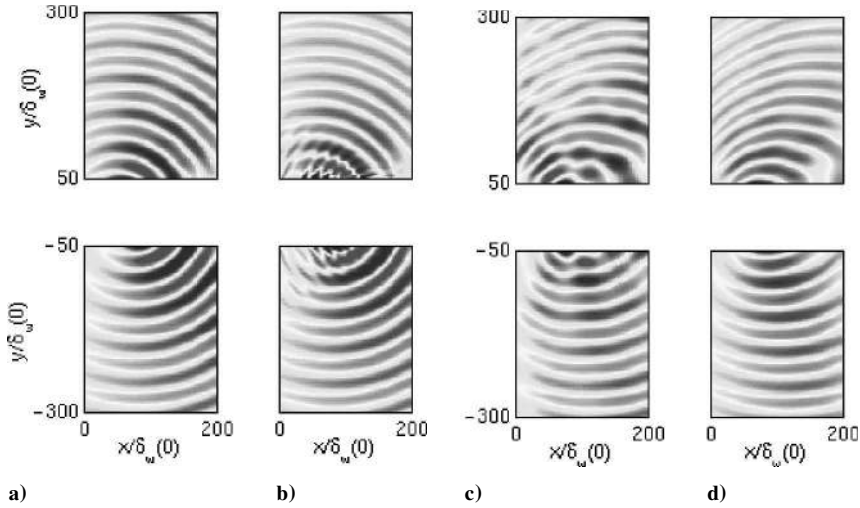
Because Lighthill’s equation is based on a wave equation in a medium at rest, the right-hand side contains both sound generation and flow effects on propagation. Two parts can actually be identified

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**Fig. 1** Snapshots of the dilatation fields obtained at the same time: from Lighthill's equation using a)  $T_{ij}^f$  and c)  $T_{ij}^r$ ; b) from LEE without mean flow; and d) from Navier–Stokes equations. The dilatation scale is defined for levels from  $-1.5$  to  $1.5 \text{ s}^{-1}$ .

in the Lighthill tensor: the terms quadratic in velocity fluctuations responsible for the noise generation by turbulence and the terms linear in acoustic fluctuations including flow–acoustic interactions. Then, for computing noise when flow effects on propagation are significant, the simulation providing  $T_{ij}$  must be compressible, and Eq. (1) must be integrated on the region encompassing all noise sources and sound–flow interactions. At this point, it should be noted that wave operators including some mean flow effects on propagation have been proposed, such as the third-order Lilley's wave operator<sup>6</sup> for a unidirectional sheared mean flow or the linearized Euler equations (LEE) for general mean flows.<sup>7</sup>

The motivation of the present study is to illustrate the inclusion of sound–flow interactions in Lighthill's equation and to show that they can be taken into account provided that they are properly enclosed in the  $T_{ij}$ . It is a continuation of earlier works dealing with the accuracy of three-dimensional integral solutions of Lighthill's equation<sup>3</sup> and with the hybrid method based on LEE developed by the authors.<sup>7</sup> The flow involved in these two works is still considered in the present study because its sound field calculated directly from the Navier–Stokes equations is used as a reference solution. It is a mixing layer between two streams of velocity  $U_1 = 0.12c_0$  and  $U_2 = 0.48c_0$  in the lower and the upper parts, respectively. The flow, computed by large-scale simulation,<sup>3</sup> is forced at discrete frequencies so that only the sound produced by the first vortex pairings is observed with a wavelength  $\lambda_p = 51.5\delta_\omega(0)$  corresponding to the pairing period  $T_p$ ,  $\delta_\omega(0)$  being the initial vorticity thickness of the shear layer. Flow effects on sound propagation are important, and the Lighthill source region will enclose the shear flow for refraction<sup>8</sup> and the acoustic domain for the convection by the two streams. Results obtained by solving Eq. (1) with the two-dimensional Green function will be compared to the reference solution and to a solution given by the LEE without mean flow for the propagation. The solution given by the LEE with the mean flow will not be used in this study because it is similar to the reference solution.<sup>7</sup>

## Methodology

Two source terms are considered to study the different contributions of the Lighthill tensor: the first one,  $T_{ij}^f = \rho u'_i u'_j$  with  $u'_i = u_i - \bar{u}_i$  the velocity fluctuations, can be associated to the sound generation by the turbulence itself, and the second one,  $T_{ij}^r = \rho u_i u_j$ , is the full Lighthill tensor. They are recorded from the flow simulation every  $T_p/11$  during  $8 \times T_p$  on the whole computational domain defined by  $0 \leq x_1 \leq 200\delta_\omega(0)$  and  $-300\delta_\omega(0) \leq x_2 \leq 300\delta_\omega(0)$ .

The time-domain solution of Eq. (1) in two dimensions is an infinite integral. Thus, in the present work, this equation is solved in the frequency domain.<sup>9</sup> By applying the Fourier transform  $\mathcal{F}[\phi(x, t)] = \hat{\phi}(x, \omega) = \int \phi(x, t) \exp(-i\omega t) dt$ , an inhomogeneous

Helmholtz equation is derived from Eq. (1) for the angular frequency  $\omega$ :

$$(\omega^2 + c_0^2 \nabla^2) \hat{\rho}'(x, \omega) = - \frac{\partial^2 \hat{T}_{ij}(x, \omega)}{\partial x_i \partial x_j} \quad (2)$$

The two-dimensional Green function associated to this equation is  $\hat{G}(x|y, \omega) = (i/4c_0^2) H_0^{(2)}(\omega|x-y|/c_0)$ , where  $H_0^{(2)}$  is the Hankel function of the second kind and order zero. By convolving Eq. (2) with the Green function and applying the differential operator on  $\hat{G}$ , it yields

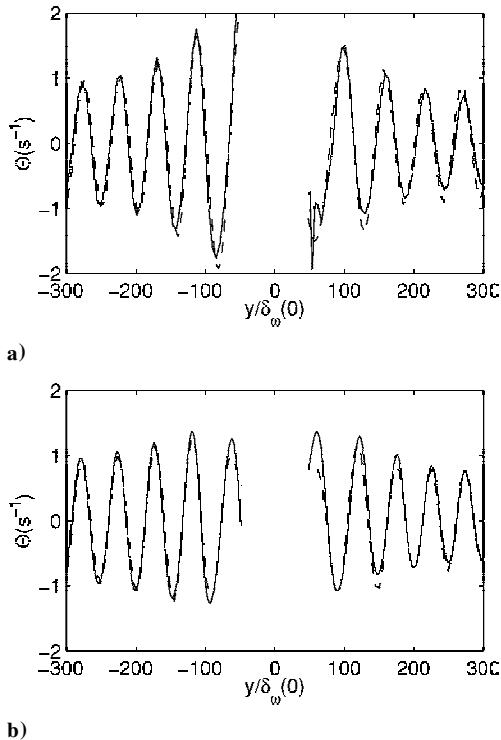
$$\hat{\rho}'(x, \omega) = - \int_{S(y)} \hat{T}_{ij}(y, \omega) \frac{\partial^2 \hat{G}}{\partial y_i \partial y_j}(x|y, \omega) dy$$

The acoustic field is calculated at the last recording time step. Snapshots of the fluctuating pressure  $p' = c_0^2 \rho'$  are provided in a related AIAA Paper.<sup>10</sup> However, in this short Note, dilatation  $\Theta = \nabla \cdot \mathbf{u}$  is used to represent the sound field for the sake of the comparison with the reference solution. Dilatation is connected to the acoustic field by the relation  $\Theta = -(1/\rho_0) \partial \rho' / \partial t$  in a medium at rest and by  $\Theta = -(1/\rho_0) (\partial \rho' / \partial t + U_i \partial \rho' / \partial x_i)$  in a uniform flow of velocity  $U_i$  ( $i = 1$  for the lower stream and  $i = 2$  for the upper stream). The first relation is then applied to evaluate the dilatation when using  $T_{ij}^f$  in Lighthill's equation and the second one when using  $T_{ij}^r$ .

## Results

The dilatation fields computed using  $T_{ij}^f$  and  $T_{ij}^r$  are displayed in Figs. 1a and 1c. They are strongly different in terms of wave front pattern and directivity. In Fig. 1a, waves fronts are circular and originate from the region of pairings near  $x_1 = 70\delta_\omega(0)$ . The directivity is well marked in the downstream direction. In Fig. 1c, waves fronts are significantly ovalized by the two uniform streams, and the directivity is pronounced for large angles.

The dilatation field of Fig. 1a, obtained from Lighthill's equation with  $T_{ij}^f$ , is compared to the dilatation field of Fig. 1b calculated by the hybrid method based on LEE without mean flow for the sound propagation.<sup>7</sup> Wave fronts are in phase and have the same amplitude, and both directivities are well marked in the downstream direction. This good agreement is shown more quantitatively by the corresponding instantaneous dilatation profiles at  $x = 130\delta_\omega(0)$  in Fig. 2a. This shows the equivalence of the two acoustic methods, namely Lighthill's equation with the reduced tensor  $T_{ij}^f$  and the LEE solved without mean flow. Both combine a wave operator in a medium at rest with nonlinear source terms corresponding only to a



**Fig. 2** Instantaneous dilatation profiles for  $x=130\delta_{\omega}(0)$ , obtained at the same time as in Fig. 1: a) —, from LEE without mean flow and ---, from Lighthill's equation using  $T_{ij}^f$ ; and b) —, from Navier-Stokes equations and ---, from Lighthill's equation using  $T_{ij}^f$ .

sound generation problem, and therefore no sound-flow interaction is calculated.

The dilatation field of Fig. 1c, obtained from Lighthill's equation with  $T_{ij}^f$ , is now compared to the dilatation field of Fig. 1d computed directly by the flow simulation. Wave fronts are modified in the same way by the flow. Directivities are also affected similarly with preferred radiations for large angles from the downstream direction. The accordance is supported by Fig. 2b plotting the instantaneous dilatation profiles at  $x = 130\delta_{\omega}(0)$ . These profiles are located  $60\delta_{\omega}(0)$  downstream the sound sources, in a region where wave fronts are significantly deformed by the flow. They superimpose fairly well with an agreement in phase and in amplitude. This demonstrates that flow effects on sound propagation are properly taken into account through the full Lighthill tensor, providing in this case both noise generation and sound-flow interactions.

## Conclusions

The present application illustrates clearly that it is possible using Lighthill's equation to compute the noise radiated by a turbulent flow accounting for flow effects on sound propagation, provided that these sound-flow interactions are accurately included in the Lighthill tensor. This implies that the Lighthill tensor should be compressible and that it should be known on a region including all sound sources and all sound-flow interactions. Practically, it is difficult with the conventional flow simulation codes. Therefore, it is generally convenient to solve Lighthill's equation when flow effects on sound propagation are small but to use hybrid methods with wave operators including mean flow effects, such as Lilley's equation or the LEE, when these effects are significant.

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