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Collision risk prediction for constellation design ☆

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Abstract

INDEMN is an object-oriented program dedicated to the modeling of the evolution of the densities of space objects. Following the work achieved by D. Kessler (1978) and by other authors more recently (G. L. Somma, IAC 2016, A6-IP3 ; A. Rossi, DPPS 2004, 197), the dynamical model is based on a source and sink approach for various altitudes. The source terms represent the future launches, the explosion of intact spacecrafts, and the collision between objects. Different collision cross sections are used for the various types of objects and the number of debris generated is based on the NASA break-up model. The sink terms are the drag and the end-of-life de-orbitation for the satellites launched after 2009, with a controllable success rate. The code was validated against former simulations performed with statistical and semi-deterministic models. In addition to the classical object types featured in several statistical codes, which are intact objects, explosion debris, and collision debris, a new type representing the satellites of a specific constellation is included. These satellites orbit with altitudes close to 1 200 km and they can perform collision avoidance maneuvers as long as they are fully operational. It is shown that, under realistic assumptions, if only one primary collision occurs at an altitude of 800 km, the probability of a collision involving a constellation satellite becomes larger than 2% by 2035, which highly jeopardizes the satellite constellation as a whole.

Keywords: LEO environment modeling, Source-sink debris model, Constellation management

1. Introduction

The Low Earth Orbit (LEO) economy is driven by the growing market of global and real-time connectivity, and new space activities represent the major share of the growth of the overall space sector today [1]. The emerging needs are addressed by the deployment of large constellations of small satellites, as announced by several telecommunication companies. This involves new platform designs that could also be used for Earth surveillance and business intelligence applications. On the other hand, Cubesat technologies, that allow in-situ data collection, for instance for science and climate change monitoring, and Earth observation services, contribute significantly to the increase of the number of LEO objects. Nevertheless, they are not considered a major threat for the long-term stability of the space environment due to their short orbital lifetime.

Multiple numerical studies have investigated the risk caused by constellations both on the stability of the environment, and the long-term profitability of large constellations. In particular, some of them addressed the relevance of the 25-year rule [2, 3]. The tools developed by national and international agencies were primarily made to assess how much a given satellite was in danger on a given orbital path, and only recent improvements accurately account for small satellite launches [4, 5, 6]. Due to the large number of satellite constellations that have

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been unveiled in the last few years, knowing the effect of the other satellites of the same constellation or the consequences of the deployment of another constellation by a competitor on the orbital environment becomes crucial. Each satellite constellation of interest is treated as a specific population in the code. Constellation-level parameters such as the reliability of the de-orbitation system, altitude spread, or de-orbitation time can be accounted for in the model, and they can be varied to perform sensitivity analysis. Finally, the number of LEO objects is several orders of magnitude higher than above 2 000 km, with many different intersecting trajectories, which makes a statistical description more relevant.

The INDEMN code was developed internally at Share My Space in Paris and uses a statistical model that derives the evolution of various populations at multiple altitudes. The model was initially inspired by the work of G.-L. Somma [7, 8, 9], improved by an object-oriented code architecture (the reader can refer to Appendix B for some details about the code architecture), that allows for an arbitrary number of different object populations and a more accurate description of the size distribution function of each population. The code is essentially one-dimensional (1D) and computes the time evolution of the object densities as a function of the altitude, for each population.

Section 2 provides a general overview of the model. Section 3 shows how this approach can be used to manage risk for a satellite constellation. Finally, some practical analysis are presented in Section 4.

2. A statistical model for the LEO environment

2.1. Overview

INDEMN is an object-oriented Python 3.X code developed by Share My Space. It features a statistical model deriving the evolution of various populations in various altitude shells and it can be used both for scientific and industrial purposes. The model is based on source and sink terms that include future launches, explosions, collisions, drag, and de-orbitation policies and compliance. Many populations can be implemented and their interactions are computed at each time step. The collision frequency between two given populations depends on the local density of each population, at a given time. After being initialized using object databases for the larger objects (> 10 cm) and statistical models for smaller ones (< 10 cm) according to the NASA breakup model [10], the density of objects belonging to each population is computed dynamically with its radial dependence. The overall operating principle of INDEMN is illustrated in Fig. 1

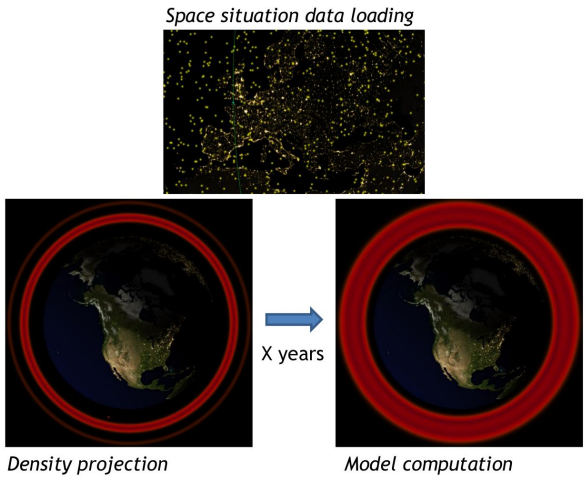

Figure 1: Illustration of the main steps of the INDEMN code

The main part of the code consists in the computation of the evolution of the density of various object populations depending on their altitude only. Choice was made to limit the boundaries of the LEO environment from 200 to 2 000 km. Below 200 km, the lifetime of an object is of a few days only and orbit propagation models are more relevant. Above 2 000 km, the drag compensation model does not apply anymore and the very low object density limits the relevance of any statistical approach. The whole model relies on the somewhat strong assumption that the orbital plane angles follow a chaotic law on a long time scale. Indeed, some objects are organized into specific orbit planes such as sun-synchronous satellites or constellations. However, once these objects are not under control anymore (due to collision or other mission termination cause), the RAAN starts drifting at a speed that depends both on altitude, inclination and eccentricity, such that all orbits of passive objects can intersect. This is justified by the fact that the drifts of the orbital planes depend on external perturbation terms that are very difficult to predict accurately enough for long-term orbit propagation, and on object parameters such as the the attitude or

the magnetic moment, which are impossible to predict or control in most cases for passive objects. For active satellites that feature a propulsion system, it is assumed that all collisions are avoided through successful maneuvers.

INDEMN cannot be used for just-in-time collision avoidance, but rather aims at providing a mission risk profile. It is particularly relevant when the user aims at assessing the correlated risk related to the launch of multiple satellites (at various dates) on orbital trajectories that may intersect. It can be used to predict how many collision avoidance maneuvers will be required every year for a given spacecraft (SC), which has a significant impact on the operations [11]. Finally, a simplified interface was made available online since February 2018 at `https://indemn.herokuapp.com`. The access to the *INDEMN online calculator* is free and can be provided on demand by the authors.

2.2. Inputs of the model

2.2.1. Space objects catalogs

The main input of the code consists of the data of the space objects orbiting around Earth in the LEO environment. The main catalog is maintained by the US STRAT COM and made available on the space-track.org platform [12]. This data is mainly stored in the Two Line Element (TLE) format and can be retrieved automatically by an Application Program Interface (API) request. The information for each object is stored in a Python data structure whose primary characteristics are the orbital elements directly initialized from the TLE data, and that includes various methods for secondary orbital properties, and for orbit propagation. However, orbit propagation of the objects is not used in the core of the code. A flag is added to the object definition that specifies the type of object (collision/explosion debris, rocket body, intact object, mission related object etc.). All these objects are listed together and are used at initialization of the LEO environment.

The collision and explosion object populations initialized with the object databases represent exclusively objects larger than 10 cm, due to the performance of the RADARs currently used to maintain the public catalog [13]. However, these populations can be statistically extrapolated to smaller objects, using the NASA breakup model [10].

2.2.2. Atmosphere description

The residual atmospheric density is extracted from the NRLMSISE-00 Atmosphere model [14] that provides angle and altitude resolved density data since 1960. The data was averaged over the orbital angles and fitted to a sinusoidal function. This fit tends to minimize the amplitude of the temporal variations of density compared to other models such as the ones implemented in ESAs MASTER or NASAs ORDEM.

The effect of the drag varies considerably depending on the altitude. The residual atmosphere mass density is fitted to :

$$\rho(t,h) = \rho_m(h) + \rho_A(h)\cos\left[\frac{2\pi}{T_s}(t-t_0)\right] \qquad (1)$$

where T_s is the period of solar activity (11.2 years), and t_0 is a date of maximum solar activity (e.g. January 1st, 2002). The parameters $\rho_m(h)$ and $\rho_A(h)$ were fitted using the NRLMSISE-00 atmosphere model. ρ_m is simply the time-averaged density and ρ_A is the standard deviation divided by $\sqrt{2}$. A comparison between the historical atmosphere density data and the fitted approximation used here is presented in Fig. 2. The mean square error of this fit is 20.0%, which is enough to capture the influence of the residual atmosphere on the population of space debris. In general, it is quite challenging to perform accurate predictions of the residual atmosphere density, provided the highly non-predictible nature of solar activity that plays a great role. Another approach would be to reproduce in the future a sequence of measured historical data (with no real physical ground however).

2.2.3. General parameters

The input parameters are sorted in Table 1 according to the level of expertise of the user. The mission time, the ballistic coefficient, and the explosion rate can be defined for each population independently. The date of the catalog t_{cat} corresponds to the date when the data was retrieved from the NORAD database. The relative velocity between two colliding objects v_{ref} is assumed constant, which is a quite strong assumption that was however historically accepted [15]. The relative velocity primarily depends on the orbital plane angles. However, since the model depends only on the altitude, this influence is averaged out. The fraction of catastrophic collisions is fixed to 0.3187,

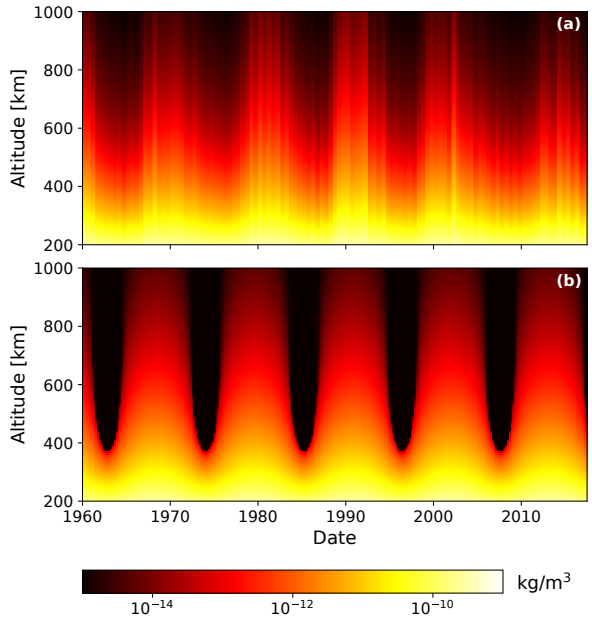

Figure 2: Comparison between the historical data retrieved from the NRLMSISE-00 Atmosphere model [14] averaged over orbital angles (a), and the fit proposed in Eq. 1) of this paper (b).

according to former studies of past collision events [18]. The relative velocity should also depend on the altitude but the difference is not critical when only the LEO environment is treated. The reader can refer to a paper by Dolado-Perez et al. [16] for further details on this matter. All the *expert* parameters except the fraction of catastrophic collisions come from the NASA break-up model. These parameters are experimental fits and should not be changed, except if new experiments are carried out, or an uncertainty quantification study is needed.

The parameters listed in Table 1 deal exclusively with the background environment. More parameters describing the behavior of the constellation satellites will be introduced in Subsection 3.3.

2.3. Object populations

INDEMN was designed to run with an arbitrarily large number of populations. All the populations are stored in dictionary data structures indexed by their names. All the objects of a same population have the same ballistic coefficient C_x. All the populations are assumed to have the same mass density ρ_{mat} in the current version. The populations are sorted in four categories:

1. **Intact objects** populations are typically old satellites left in orbit at the end of their orbital lifetime or rocket bodies. Intact objects of the same population all have the same given size. The cross section used to compute the drag and the collision probability is calculated from the size of the object assuming a spherical geometry. They are subject to explosion with a given annual probability, that is greater than zero in general. During their de-orbitation phase, constellation satellites are treated as intact objects.
2. **Constellation objects** are object populations that are under the control of an operator. They are provided with a mission scenario that includes de-orbitation with a given success rate, and physical properties (see Table 2).
3. **Explosion debris** have a distribution function in size l that follows the NASA breakup model.

$$f_e \propto l^{-2.6} \tag{2}$$

Each population is truncated with minimum and maximum characteristic sizes (l_{min}, l_{max}).

Basic		
Parameter	*Default*	
Mission time	8 years	
De-orbitation time	25 years	[17]
Compliance to the space law, C	90%	[16]
Explosion rate, p_e	0/year	[16]
Drag coefficient, C_x	2.2	[8]
Minimum altitude, h_{min}	400 km	
Maximum altitude, h_{max}	2 000 km	
Simulation start, t_0	20/02/2018	
Physical duration of the run, Δt	80 years	

Advanced		
Parameter	*Default*	
Date of the catalog, t_{cat}	20/02/2018	
Typical size of intact objects, l_i	2.1 m	[7]
Min. size of explosion debris, $l_{e,min}$	0.1 m	
Max. size of explosion debris, $l_{e,max}$	0.66 m	[7]
Min. size of collision debris, $l_{c,min}$	0.1 m	
Max. size of collision debris, $l_{c,max}$	0.66 m	[7]
Object mass density, ρ_{mat}	444 kg/m^3	[7]
Mean number of objects created per launch, n_L	4.0	[13]
Reference velocity between colliding objects, v_{ref}	10 km/s	[8]
Number of meshes, N_{shell}	160	

Expert		
Parameter	*Default*	
Size exponent of the distribution function for collision debris	1.71	[10]
Mass exponent of the distribution function for collision debris	0.75	[10]
Pre-factor of the collision debris distribution function	0.03	[10]
Size exponent of the distribution function for explosion debris	1.6	[10]
Pre-factor of the explosion debris distribution function	6.0	[10]
Fraction of catastrophic collisions	0.3187	[18]

Table 1: Table of the input parameters of INDEMN

4. **Collision debris** follow the same rule but with the exponent corresponding to the collision debris predicted also by the NASA breakup model

$$f_c \propto l^{-2.71} \tag{3}$$

The power exponent for collision and explosion fragments populations were assumed constant, which is a limitation of the model. In reality, the atmospheric drag affects smaller debris more due to a higher surface to mass ratio, which means that the absolute value of the exponents should be slightly higher at lower altitudes.

The density of objects of a population at altitude h, described with a continuous size distribution function f, is

$$n(h) = \int_{l_{min}}^{l_{max}} f(h, l)\, dl \tag{4}$$

For a circular orbit, each object at altitude r affects the local density according to a linear weighting algorithm:

$$\delta n_i = \left[i + 1 - \frac{r - h_{min}}{h_{max} - h_{min}} \cdot N_{shell} \right] \frac{1}{V_i} \tag{5}$$

$$\delta n_{i+1} = \left[\frac{r - h_{min}}{h_{max} - h_{min}} \cdot N_{shell} - i \right] \frac{1}{V_i} \tag{6}$$

where δn_i is the density increment at the i-th grid point caused by the object at altitude r, and

$$V_i = \frac{4}{3}\pi(r_{i+1}^3 - r_i^3) \tag{7}$$

is the volume of the i-th spherical shell, the i-th grid point altitude being

$$r_i = \frac{1}{N_{shell}} \left[i h_{max} + (N_{shell} - i)\, h_{min} \right] \tag{8}$$

and $i = 0, ..., N_{shell}$. For an object that has an elliptical orbit, the density increment δn_i needs to be weighted according to the residence time of the object in each spherical shell. The simplest way to account for this effect is to use an elliptic function defined by [19]

$$g(\theta, e) = \int_0^{\theta} \frac{dx}{(1 + e\cos x)^2} \tag{9}$$

$$= 2\arctan\left[\left(\frac{1-e}{1+e} \right)^{\frac{1}{2}} \tan\frac{\theta}{2} \right] - \frac{e(1-e^2)^{\frac{1}{2}} \sin\theta}{1 + e\cos\theta}$$

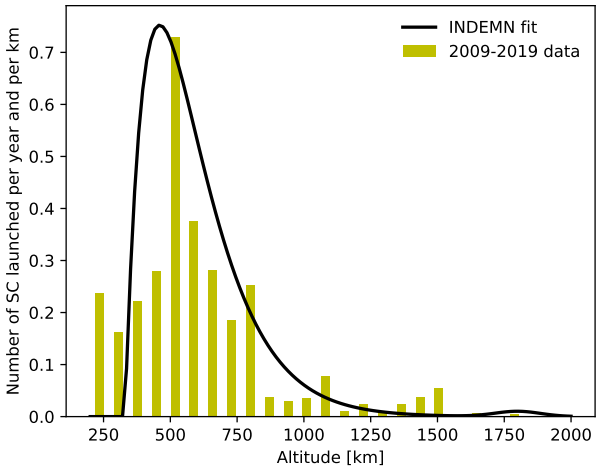

Figure 3: Spacecrafts launched per year and per altitude unit as a function of the altitude. The INDEMN fit is compared to historical data from spacetrack.

where e is the orbit eccentricity. The density increment writes

$$\delta n_i = [g(\theta_{i+1}, e) - g(\theta_i, e)]/\pi \tag{10}$$

where θ_i is the phase angle corresponding to where the orbit crosses the edge of the i-th shell, and defined by the relation

$$\frac{p}{1 + e\cos\theta_i} = r_i, \tag{11}$$

p being the ellipse parameter.

2.4. Source terms

Constellation objects and intact objects populations are increased according to predictions of **future launches**. The default background launch profile is calculated from a fit of the number of launches performed in year 2016 as a function of the altitude. Even though the fit was performed with the data of year 2016, Fig. 3 shows that it matches well with the launch data for the period 2009-2019, which indicates that the background population of satellites in LEO is replenished in a quite steady manner. Specific constellations can have customized launch scenarios implemented, as a function of time and altitude.

Explosion objects are generated from populations of intact objects and constellation satellites. The number of objects generated by one explosion for a population of explosion objects bounded by (l_{min}, l_{max}) is:

$$N_e = 6\left(l_{min}^{-1.6} - l_{max}^{-1.6}\right) \tag{12}$$

according to the NASA breakup model again [10]. At each altitude and each time step the explosion source term is defined by

$$S_e = N_e \sum p_e n_e \tag{13}$$

where p_e is the annual probability of explosion, and n_e is the source object density. The summation is performed over all the populations of intact objects, including constellations.

The source term corresponding to the **collisions** is the most difficult to calculate because it requires to integrate the collision cross section over the two distribution functions of the colliding populations. The cross section is defined by

$$\sigma(l_i, l_j) = \frac{\pi}{4}\left(l_i + l_j\right)^2. \tag{14}$$

The number of debris generated by a collision between two populations i and j that have single masses m_i and m_j is [10]

$$N_{c,ij} = 0.03\,(m_i + m_j)^{0.75}(l_{min}^{-1.71} - l_{max}^{-1.71}) \tag{15}$$

But the primary object mass depends on the size according to – again for a spherical geometry

$$m = \rho_{mat}\pi\, l^3/6. \tag{16}$$

The production rate due to collisions between populations i and j is hence:

$$S_{c,ij} = v_{ref}\,\frac{\tilde{n}_{ij}}{n_j}\,(1 - s_{cam,ij}) \times \int_{l_{i,min}}^{l_{i,max}}\int_{l_{j,min}}^{l_{j,max}} f_i(l_i)\, f_j(l_j)\,\sigma(l_i, l_j)\, N_{c,ij}(l_i, l_j)\, dl_i\, dl_j \tag{17}$$

where $\tilde{n}_{ij} = \frac{n_j - \delta_{ij}/V_s}{1+\delta_{ij}}$, V_s being the volume of the shell at the altitude considered and δ_{ij} is a Kronecker operator ($\delta_{ij} = 1$ if $i = j$, 0 otherwise), introduced here to avoid

to count twice the same collision between objects of the same population. v_{ref} is the typical impact velocity at the altitude considered, which is close 10 km/s in most cases. In Eq. 17, $s_{cam,ij}$ is the probability of success of a collision avoidance maneuver when a collision between objects of populations i and j occurs. Of course, $s_{cam,ij} = 0$ when both populations are passive. The production generated by collisions between all populations is then the sum over all the population pairs:

$$S_c = \sum_{i,j} S_{c,ij} \tag{18}$$

This population based approach allows for refining the model as much as needed for the simulation purposes. Finer description of fragmentation events is available in recent studies [20].

2.5. Loss terms

The loss terms modeling **collisions** and **explosions** are computed the same way as the corresponding source terms. The only difference is that only one object is discarded when a collision or an explosion occurs and not N_c or N_e as is the case for the source terms.

$$L_e = p_e\, n_e \tag{19}$$

$$L_{c,i} = \sum_j v_{ref}\, \frac{\tilde{n}_{ij}}{n_j}\, (1 - s_{cam,ij}) \times \int_{l_{i,min}}^{l_{i,max}} \int_{l_{j,min}}^{l_{j,max}} f_i(l_i)\, f_j(l_j)\, \sigma(l_i, l_j)\, dl_i\, dl_j \tag{20}$$

The model also includes **end-of-life maneuvers** for populations of active satellites. If $S_l(t, h)$ is the source term due to new launches, the loss term corresponding to end-of-life maneuvers is

$$L_{eol}(t, h) = S_l(t - T_{mission} - T_{eol}, h)\, C_{eol} \tag{21}$$

where $T_{eol} = 25$ years is the de-orbitation time after the end of operations recommended by the IADC, $T_{mission}$ is the typical mission time of non-failing satellites, and C_{eol} is the level of compliance (between 0 and 1) to the 25-year rule. According to this model, that was already used by Somma et al. [8], all the objects remain at their initial altitude during T_{eol} before being eliminated. This model neglects the interaction of end-of-life satellites with objects at other altitudes. We have improved this model for constellation satellite end-of-life maneuvers.

For the constellations, another model was included to account for the residence time of de-orbited objects between their initial altitude and the altitude for which the effect of the drag becomes significant on a short time scale. In this description, after the de-orbitation has started, the constellation objects start descending with a uniform velocity

$$v_{deorbit} = (h_0 - h_{min})/T_{eol} \tag{22}$$

h_0 being the mean altitude of the constellation. This model represents a low thrust de-orbitation maneuver. This descending velocity is not the real radial velocity of the object since the de-orbitation maneuver is most likely performed by lowering only the perigee first. It rather represents the speed at which the mean altitude of the object evolves with time. Let ν be the probability of failure per unit of time. If what happens at each time-step is statistically uncorrelated, if N_0 is the number of objects starting their de-orbitation, the number of objects whose de-orbitation system is still active after T_{eol} is $N_{deorbit} = N_0 \exp(-\nu T_{eol})$. Moreover, the compliance C is defined by the ratio $N_{deorbit}/N_0$. Therefore,

$$\nu = -\ln(C)/T_{eol} \tag{23}$$

Finally, a downward de-orbitation velocity due to the drag is estimated by conservation of energy: during a revolution of period T at a distance r from the center of the Earth, an object undergoes an energy dissipation

$$\mathcal{W}_{friction} = \frac{C_x}{2} \rho v^2 S \times 2\pi r \tag{24}$$

where C_x, S, and v are the ballistic coefficient, the cross section and the orbital velocity. This must be equated to the loss in the total mechanical energy, which can be written as an effective potential energy for circular orbits:

$$E_m = -\frac{\mu m}{2r} \tag{25}$$

where $\mu = GM_T = 3.98 \times 10^{14}\ \mathrm{m^3 s^{-2}}$ is the Earth gravitational constant. To the first order in Δr the change of

mechanical energy is

$$\Delta E_m = \mu m \frac{\Delta r}{2r^2}. \tag{26}$$

Defining the de-orbitation velocity

$$v_d \approx \Delta r / T \tag{27}$$

where Δr is here the altitude loss during one revolution,

$$v_d = C_x \rho S (\mu r)^{1/2} / m. \tag{28}$$

The number of objects lost due to the drag is the divergence of the flux in spherical coordinates

$$L_{drag} = -\frac{1}{r^2} \frac{\partial (r^2 n v_d)}{\partial r}. \tag{29}$$

The velocity defined by Eq. 28 is analogous to a fluid velocity of a 1D fluid model. Therefore, the time and space discretization must satisfy the Courant-Friedrich-Lewy (CFL) condition for the numerical scheme to be stable. Thus, it is necessary that at the minimum altitude of the domain – where the downward velocity is the largest:

$$v_d \Delta t < \Delta x \tag{30}$$

Eq. 30 can be quite restrictive if low altitudes need to be included. Fig. 4 is an example of how this radial drag velocity changes with altitude. At 200 km, the typical velocity is 300 km/y, which means that for a spatial mesh of 10 km, the time step should be lower than 11 days.

2.6. Numerical model

The time derivative of the densities are computed at each time step and each altitude for all populations. The differential system is solved using an explicit second order modified Euler method (midpoint method). The densities and their derivatives are totally vectorized in the altitude dimension, which yields faster memory access, and therefore a faster code execution.

3. Satellite constellation risk modeling

3.1. Motivation

Due to the steady rise of the total number of LEO objects, keeping track of each individual object is becoming more and more challenging. Simulation codes based on orbit propagation [21, 22] have limited prediction capabilities due to the uncertainty on the initial orbital data, on the modeling of the forces acting on the spacecraft, and the chaotic nature of orbital mechanics equations. It is therefore very challenging to perform collision risk prediction with orbit propagation and Monte-Carlo collisions beyond a few days in the future. In most populated regions, concepts such as *debris densities* and *debris fluxes*, inspired from fluid mechanics and used in the INDEMN model, become relevant. Statistical and semi-deterministic models were already used in the past to investigate for example the criticality of the environment on the long run, i.e. several centuries [23, 24, 25]. Large constellations of satellites such as OneWeb or Starlink will run with over one thousand satellites for several decades. At a lower orbit, the Iridium NEXT constellation will be made of 75 satellites in the most populated region at 780 km altitude. Other nanosat constellation operators such as Spire, Planet or BlackSky (690 km) tend to increase the altitude of their satellites and therefore increase their long-term impact on the orbital environment. In total, more than 15 000 satellite deployment have been announced for the decade to come, although some of these projects are still uncertain. The present study focuses on a

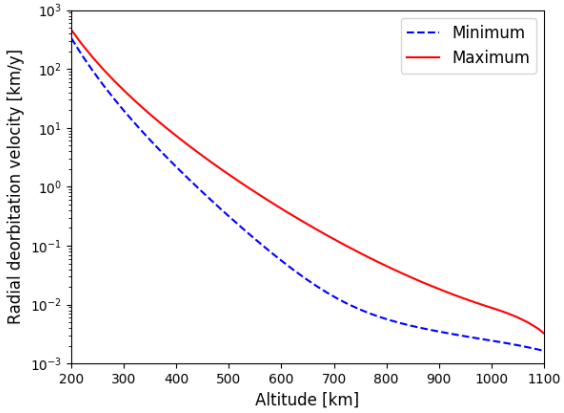

Figure 4: Radial drag velocity (pointing toward the Earth) as a function of the altitude for a spherical object of 2 m of diameter, 1t/m^3 of mass density, and a ballistic coefficient C_x equal to 3, for the minimum and maximum densities corresponding to the solar activity.

scenario with only one large constellation. To that regard, the results presented below are quite optimistic.

Due to the limited lifetime of each invividual satellite, the total number of satellites launched for one full operation of these constellations will outreach 10 000 satellites. This represents a major threat for the sustainability of the LEO environment, as emphasized in recent papers [5, 26]. The way these satellites are manufactured on semi-automated production lines is totally new to the industry, which makes the reliability of the whole system unpredictable. Several studies have shown that the stability of the environment relies on the level of compliance of the satellites to the 25-year de-orbitation rule, recommended by the IADC. About ten constellations with 100 to 4 000 satellites were announced since year 2015, many of which are quite hypothetical, but may well be implemented at full capacity by the mid 2020s. The financial scheme is still to be found, especially because of the high industrial risk uncertainty – including the risk of collision. The INDEMN software uses system-level input parameters together with the latest updates of space-track.org. It provides an estimate of the collision risk and the debris density, and helps to identify what are the critical parameters for the profitability of an industrial project or the sustainability of the environment as a whole, with respect to orbital pollution.

Before launching a constellation, the model evolution can be tested in various configurations, varying parameters such as the system reliability, the altitude spread of the various satellites, and the end-of-life scenario. It can also be used to assess the necessity of implementing new end-of-life constraining measures, and active debris removal (ADR) actions [35].

3.2. Collision risk assessment

The collision frequency of one single object with population i is computed in a similar way to the term of losses by collisions

$$\nu_{ci} = \sum_j \nu_{ref} \frac{\tilde{n}_{ij}}{n_j n_i} (1 - s_{cam,ij}) \times \int_{l_{i,min}}^{l_{i,max}} \int_{l_{j,min}}^{l_{j,max}} f_i(l_i)\, f_j(l_j)\, \sigma(l_i, l_j)\, dl_i\, dl_j \qquad (31)$$

The individual risk of collision between two dates t_1 and t_2 is computed using Poisson's law

$$r_i(t_1, t_2) = 1 - \exp\left(- \int_{t_1}^{t_2} \sum_i \nu_{ci}\, t \right). \qquad (32)$$

3.3. Simplified model of a constellation

We propose here a simplified model of the orbital environment with a typical constellation depicted in Table 2. Although the model could theoretically describe popuations of Lethal Non-Trackable (LNT) objects below 10 cm, it was chosen here to focus on the objects bigger than 10 cm, for which more information is available in the literature. Indeed, one should keep in mind that a statistical model should preferably be calibrated with observation data and other simulation tools. The probability of having a mission terminated due to a LNT object is typically 25 times greater than that with a tracked object.

In the INDEMN model, the collision risk for a population of N satellites is not just the sum of the collision risk of each individual satellite in a fixed background, but also accounts for the interactions between the satellites of a same population.

Satellites of a same constellation are spread over various orbital planes, with altitudes that are close to the reference value of the constellation altitude h_{const}. It is assumed that the satellites are spread uniformly in a spherical shell between $h_{const} - \Delta h/2$ and $h_{const} + \Delta h/2$. The parameter Δh is called the *altitude spread*.

$$h = h_{const} \pm \Delta h/2 \qquad (33)$$

The parameters chosen for the following case study correspond approximately to the OneWeb constellation, taken from publicly available information [26].

The cross section is a tumbling-averaged cross section, such that the satellite attitude does not affect the model. Since the 1990s, satellite operators have enforced passivation measures that have made in-orbit explosions very rare. Consequently, the explosion rate is set to 0 in all the simulations carried out in the next sections. The default value for the success rate of the post-mission disposal maneuver is 90%, which is slightly lower than what the operator claims, but much higher than typical figures observed for former satellites. During their de-orbitation maneuver,

Parameter	*Default*
Altitude	1 200 km
Launch date	2019/01/01 [a]
Constellation build-up	2019-2021
Full operation	2022-2072
Satellite lifetime	5 years
Satellite cross section	2.7 m^2
Altitude spread	40 km
Post-mission disposal success rate	90 %
De-orbitation time	2 years
De-orbitation scenario	Low thrust
Yearly explosion rate	0 /year
Collision avoidance success rate	100 %
Launches per year	20 (deployment) 12 (replenishment)
Satellites per launch	18

Table 2: General parameters for the reference constellation used in this paper. The data is taken from Radtke et al. [26] (2017) and Bastida et al. [27] (2016) and are close to the scenario of the OneWeb constellation.

[a] As of August 2018, the launch date of the first satellites of OneWeb is expected to be between December 2018 and February 2019 (spacenews.com)

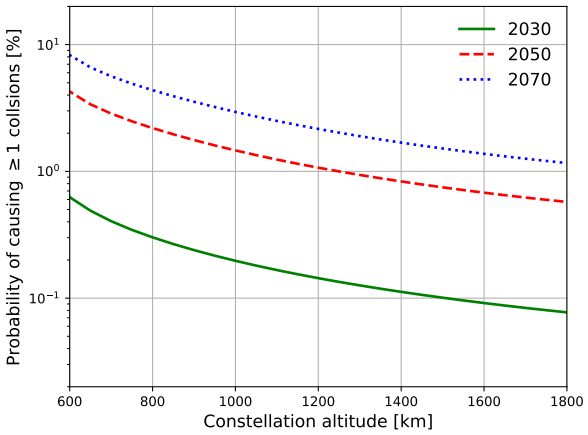

Figure 5: Risk for a de-orbiting satellite of the constellation to cause a collision as a function of the constellation altitude. The de-orbitation time is set to 2 years.

end-of-life satellites form a new population, that is not able to perform collision avoidance maneuvers anymore (0% success rate). This assumption has already been discussed in the past by other authors [28].

On-orbit collisions are critical events that may engage the third-party liability of the operator if it is shown that he is at fault. Moreover, it makes the operations of the other constellation satellites much more difficult to manage because the orbit becomes polluted. Thanks to constellation-level redundancies, the main service is likely to be still delivered, but at higher operational costs due to more frequent collision avoidance maneuvers, such that the profitability of the constellation as a whole would be at stake. Therefore, the probability of causing one collision during the de-orbitation phase is a key parameter, that will be thoroughly investigated in Section 4.

4. Practical cases

4.1. Influence of the constellation altitude

The altitude of the constellation characterized by Table 2 was varied with a fixed de-orbitation time of two years. The consequences on the probability of causing one or more collisions during the de-orbitation phase is presented in Fig. 5 in semi-logarithmic scale. The collision risk is larger for lower altitudes because the de-orbitation speed increases with altitude. Consequently, the density of de-orbiting satellites is larger for lower altitudes, hence the increase in collision risk. For an altitude of 1 000 km, the probability of causing at least one collision by 2050 is approximately 1.8%. The investigated scenario features one single large constellation, and with no fragmentation due to explosions. The post-mission disposal maneuver is performed within two years with a success rate of 90%. Overall, quite optimistic values were chosen for the constellation, and the model does not account for for LNTs. Therefore, we believe that the results presented are quite conservative.

4.2. Time of the de-orbitation maneuver

As discussed in former studies [27], the de-orbitation time has a strong influence on the long term impact of a constellation on the LEO environment. These studies revealed that the IADC 25-year rule might not be sufficient to sustain the stability of the environment. Indeed, Fig. 6 shows that the asymptotic slope of the long-term evolution of the collision risk is strongly affected by the time needed for constellation satellites to de-orbit after

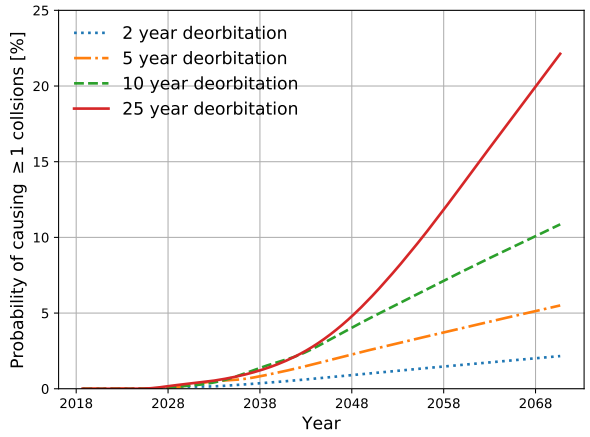

Figure 6: Probability of causing at least one collision for various de-orbitation scenarios: de-orbitation performed in 2 years, 5 years, 10 years, or 25 years. Constellation parameters are those specified in Table 2 .

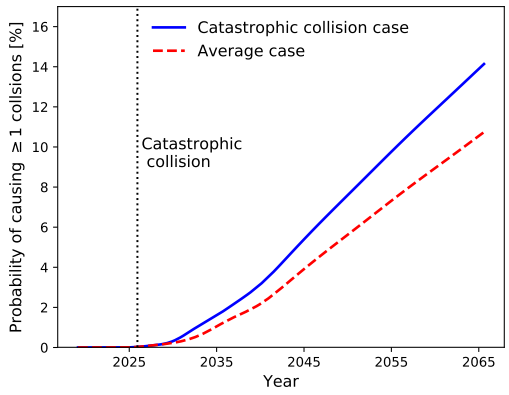

Figure 7: Probability for a de-orbiting satellite to be involved in a collision with a space debris, given that a catastrophic collision occurred in 2025, generating 3 000 objects larger than 10 cm.

their mission ended. In the 25 year de-orbitation case, the satellites start crossing the most population LEO region around 800 km later than in the other cases. In the other cases, this time shift is not really visible on the graph (but still there). The asymptotic slope is mainly driven by the time it takes to the de-orbiting satellites to cross the LEO region. By 2065, the risk of causing more than one collision is 2% in the 2-year de-orbitation scenario chosen by the OneWeb operator and 20% in the 25-year de-orbitation scenario recommended by the IADC, which may not be accepted neither by the operator, nor insurers, nor financers. Interestingly, the private sector has anticipated this risk and has implemented mission requirements that are more constraining than the international recommendations. It seems that the 25-year rule, that was decided at a time when announcements of large constellations were not so numerous, is not very well suited for the current paradigm.

4.3. Conditional collision probability

We call here conditional collision probability the probability that a collision occurs due to a satellite of the constellation provided that another independent collision has occurred at a given date.

By default, INDEMN provides mean values of space debris density, from which parameters such as the collision frequency can be derived. In order to test the robustness of the outputs with respect to some rare events, external conditions can be imposed in the evolution scenario, and compared with the reference case. In Fig. 7, the red dashed line represents the average temporal risk profile for the constellation depicted by Table 2. In the scenario represented by the solid blue line, it was assumed that a fragmentation has occurred at 800 km in year 2025 that generated 3 000 debris of more than 10 cm, independently of the constellation operation. This event is comparable with the collision between Iridium-33 and Kosmos-2251 that took place in 2009. Computation was performed for a 10-year de-orbitation scenario. Even though the collision considered here took place 400 km kilometers away from the constellation operational altitude, the influence on the collision risk for de-orbiting satellites is significant. In 2065, 40 years after the hypothetical fragmentation, the collision risk is as high as 15%, while it is only 11% in the reference case , which corresponds to a relative increase of 36%. By 2035, the collision risk reaches 2% in case of a catastrophic collision, as opposed to only 1.2% in the averaged reference scenario.

5. Conclusions

Some of the first results obtained with the INDEMN software were presented in this paper. Among all the parameters of the constellation, the de-orbitation time was

identified as a key driver of the total collision risk for end-of-life satellites with trackable objects. The cumulated collision probability for constellation satellites was estimated over five decades, based on size distribution functions for fragmentation debris. For industrial purposes, it would be useful to implement best and worst case evolution scenarios, and to perform an uncertainty quantification analysis. The worst case scenario would include all the constellations that have been announced since 2015, which mounts up to 15 000 more satellites in LEO. The figures presented in this paper are the results of a 1D model where all the objects have the same relative speed when they collide. It does not account for the discrete orbital planes of the constellation satellites. However, the long-term correction corresponding to this effect is supposed to be relatively weak [23]. The model could be improved to 2D or 3D, so as to account for the orbit angles more precisely, as it was performed in some former studies [29, 30]. Solar and Earth radiation fluxes were also neglected in the present study [31]. The deorbitation scenario of the background satellites is very simplified and could cause some discrepancy with respect to an ideal model. Nevertheless, as shown in Appendix B, the model was successfully benchmarked with reference cases of the literature.

In summary, INDEMN can provide useful estimates of the collision probability for mission analysis, constellation planning, and policy impact assessment. The risk related to LNT objects was not investigated in this paper. Statistical models of collision cascades require to account correctly for the behavior of the size distribution function of objects. The present work is a first step towards a kinetic theory of space debris inspired from the kinetic theory of gases, but with the object size as an additional phase parameter.

6. Acknowledgment

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Appendix A. Acronyms

API	Application Program Interface
ADR	Active Debris Removal
CFL	Courant-Friedrich-Lewy
IADC	Inter-Agency Space Debris Coordination Committee
INDEMN	INtelligent DEbris MaNagement
LEO	Low Earth Orbit
LNT	Lethal Non-Trackable
RAAN	Right Ascension of the Ascending Node
SC	Spacecraft
SSA	Space Situational Awareness
SSR	Space Sustainability Rating
TLE	Two Line Elements

Appendix B. Benchmarking INDEMN

Appendix B.1. IADC 2012 benchmark for the stability of the Future LEO environment

The IADC 12-08 report *Stability of the Future LEO environment* [17] is an initiative of the Inter-Agency Space Debris Coordination Committee aiming at comparing the simulation results of the simulation codes of six national or international space agencies against each other, starting with the same model assumptions and the same input data. The estimated 2012 initial population was provided by the Master 2009 for objects of size greater than 10 cm. It was assumed that there were no on-orbit explosion occurring (strong assumption) and that the compliance to the 25-year rule would be 90%. Instead of integrating the solar flux as it is done by five of the codes, INDEMN directly embeds a model for atmospheric density. The simulation is run for the LEO environement between 200 and 2 000 km.

Only two populations were considered in this case, intact objects and collision debris. Following G.-L. Somma [7], all the intact objects are assumed to have a typical size of 2.1 m and the collision debris of 0.3 m (the distribution functions are Dirac for both populations), and the cross sections and masses are computed accordingly. The orbital collision velocity is fixed to 10 km/s for all altitudes. The ballistic coefficients are $C_x = 2$ also for both populations.

INDEMN shows an agreement better than 18% with all the other code at all time and better than 5% agreement

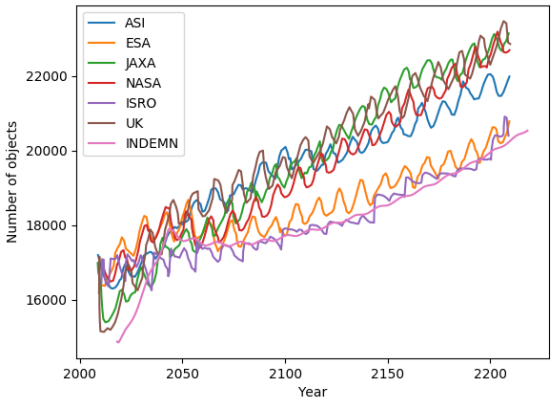

Figure B.8: Comparison of the simulation results by various codes developed by national and international agencies with the results provided by INDEMN for the IADC 12-08 benchmark.

with the ISRO sand ESA simulation tools. Considering the large discrepancy between the simulation codes, this is quite satisfactory.

Appendix B.2. Risk to space sustainability from large constellations of satellites

We propose a second simulaton case to improve the validation of INDEMN based on a paper by Bastida et al. [27]. The assumptions for the background populations are the same as for the IADC-2012 benchmark and it is still assumed that there will be no spacecraft (SC) explosion. A population corresponding to a satellite constellation is added whose characteristics are summarized in Table B.3. The constellation satellites, like all other objects, are assumed to be passivated with a 100 % success rate, but are de-orbited at the end of their operation time with a success rate varying between 50 and 100 %.

These simulations emphasize the critical role of the reliability of the de-orbitation phase. Paradoxically, it is when the satellite is the least likely to be fully operational – at the end of its lifetime – that the most critical and challenging maneuver is required. The success of the de-orbitation maneuver is so critical for the sustainability of the orbit of operation that it should be totally independent and isolated from the rest of the system. The propulsion system used for de-orbitation, including its power supply

Parameter	*Value*
Mass	200 kg
Cross section	1 m^2
Altitude	1 100 km
Number of satellites	1 080
Starting year	2018
Number of satellites launched per year	360 (2018-2020) 216 (2021-2071)
De-orbitation rate after 25 years	50-100 %
Mission life-time	5 years

Table B.3: Characteristic of the constellation simulated in Bastida et al.

if relevant, should be exclusively dedicated to this maneuver.

Appendix C. Code architecture

INDEMN is divided in three main classes:

- The TLE objects
- The population class
- The shell environment class

Appendix C.1. The TLE class

The TLE class contains the raw TLE data as attributes. A documentation of the TLE data is available at space-track.org. The launch date is converted to a Python datetime format using the `dtlaunch` method, which makes it easier to extract directly the year of the launch. Orbital parameters such as the instantaneous right ascension, the semi-major axis, the angular momentum, the instantaneous longitude, the apogee and perigee are also calculated with methods that have been implemented into the `TLE` class.

Appendix C.2. The population class

The `population` class mainly contains a vector of densities at the various altitude shells. The type of population is also an attribute of the `population` class, and so are the ballistic coefficient, and the level of compliance to the 25-year rule *C*. The minimum and maximum sizes of the population objects, and the exponent coefficients giving

the size distribution functions (1.6 for explosion debris, 1.71 for collision debris) are also attributes. Each source and loss terms (in # $km^{-3}s^{-1}$) are computed using dedicated methods. Some integrals used in the computation of the collision terms are calculated and stored in dedicated dictionary data structures at code initialization.

Appendix C.3. The environment class

The environment class (`shell_space`) encapsulates most of the information used during a run. Its attributes are summarized below:

- A 1D dictionary of populations ;
- A 2D dictionary of collision avoidance success rates between each pair of populations ;
- The integrals used for the computation of the terms of source by collision (2D dictionary) and loss by collision (1D dictionary) ;
- The altitude shells stored in a simple vector ;
- The parameters for atmosphere modeling ;
- The relative date.

The TLE data are read directly from the `shell_environment` class through the `add_TLE` and `add_TLElist` methods. The evolution of the densities of the various populations is also computed within the `shell_environment` class, using a time vector (in years) as input, and embedding the modified Euler solver.

Appendix C.4. Output management

The successive environment data objects during a run are stored together in a list of states in memory. This allows to access all the information easily at the end of the run, for data storage and post-processing.

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